AEROSPACE INFORMATION REPORT

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PARTICLE COUNT DATA CONVERSION AND EXTRAPOLATION

1. SCOPE

This report describes a mathematical model which can be used to analyze particle count data. Particle counts which fit the model can be graphically displayed, converted from one counting size-frequency range to another, and extrapolated to estimate counts beyond the measured range. Derivation, applications and calculations are described.

2. METHOD OUTLINE

Cumulative particle counts larger than stated size-range are fitted against a modified log-normal distribution function by plotting on special log-log^2 graph paper. Many real particle distributions approximate a straight line, showing good fit for this log-normal model. The resultant plot provides a basis for further analysis, including size-range conversion and extrapolation.

3. MODIFIED LOG-NORMAL MODEL

Many real particle counts have been shown to approximate a Gaussian distribution with a logarithmic variate. This cumulative distribution function is represented by an equation having the form:

$$3.1 \qquad n_c = \frac{\Sigma n}{\sqrt{2\pi}\,\ln\sigma}\int_{\infty}^{\ln x} \exp - \left[\frac{\ln x - \ln M}{\sqrt{2}\,\ln\sigma}\right]^2 d\ln x \qquad (1)$$

where: n_c is cumulative particle count, Σn is total number of particles, $\ln \sigma$ is the natural logarithm of the geometric standard deviation, M is the geometric mean particle size, and x is the particle size variate.

Assuming that $|\ln M|$ is small compared to ln x, which means that about half of the particles are smaller than one micron in size, the rather formidable expression (2) may be reduced to a simpler equivalent expression using lumped constants:

$$3.2 \qquad \log n_c = \log A - 2.303\, B \log^2 x \qquad (2)$$

where: n_c is still cumulative particle count expressed as a base-10 logarithm, x is the particle size variable, and A and B are characteristic intercept-slope constants which describe quantity and distribution respectively.

If a particle count fits the mathematical model (2), data plots on special log-log^2 graph paper are linear and provide a basis for further analysis. The necessary graph paper is made by laying out a logarithmic n-axis and an x-axis proportional to the square of the same logarithmic modulus. (A sheet of log-log^2 graph paper is attached to this report.)

4. DATA PLOTTING

Contamination particle counts are plotted on log-log^2 graph paper as cumulative counts larger than each lower size-range limit. This method provides a discrete size to be plotted, rather than some "average" value of the size range, and tends to correct classification errors for the smaller size ranges.

EXAMPLE: Contamination analysis according to ARP 598 was performed on 100 ml of hydraulic oil. The following microscopic particle counts were obtained.

Size Range	>100	50-100	25-50	15-25	5-15 microns
No Particles	4	19	150	450	3780

These frequency counts are converted to corresponding cumulative counts by successive addition from left to right.*

Size Limit	>100	>50	>25	>15	>5 microns
No. Particles	4	23	173	623	4403

The greater number of significant digits in the cumulative counts (4403 particles larger than five microns) are not meaningful and may be rounded off (4400) for convenience. These cumulative counts are plotted as shown in Fig. 1.

Many such contamination plots are essentially linear on log-log^2 graph paper showing good fit to the modified log-normal function. The example count has been faired to a straight line because all data points except the largest (4 particles >100 microns) fit quite closely. The largest and smallest particle count size-ranges usually exhibit the greatest counting error and should be less heavily weighted in drawing the mean curve.

Deviations from linearity are also significant when based on accurate count data. Downward concave curves may imply a closed distribution within the counting range or a

*Total particles greater than 100 microns is 4.
Total particles greater than 50 microns is 4 + 19 = 23.
Total particles greater than 25 microns is 23 + 150 = 173, etc.

mechanical breakdown which generates large particles. Upward-concave curves may imply the action of a wear process which preferentially generates small particles. Inflected or multiple curves may imply the presence of more than one type of particle population showing contamination from different sources.

5. CALCULATIONS

The cumulative particle count plot demonstrated in Fig. 1 may be used for further analysis or modification of the original data. Data conversion to other particle count size-ranges or extrapolation of data to uncounted size-ranges may be accomplished analytically or graphically. The latter method is easier and is sufficiently accurate for most purposes.

EXAMPLE: The particle count given in Section 4 and plotted on Fig. 1 must be converted to equivalent frequency counts in the different size-ranges of >80, 40-80, 20-40, 10-20, and 5-10 microns. Cumulative particle counts corresponding to these lower size-range limits of >80, >40, >20, >10, and >5 microns are read directly from the plot of Fig. 1.

Size Limit	>80	>40	>20	>10	>5 microns
No Particles	5	45	320	1450	4400

Counts are read from the straight line plot, not the dashed curve (4.5 particles >80, not 6.5), and are rounded off. Frequency counts are obtained by the reverse process given in Section 4, by successive subtraction from left to right.

Size Range	>80	40-80	20-40	10-20	5-10 microns
No. Particles	5	40	275	1130	2950

These frequency counts are equivalent to the original ARP 598 frequency counts.

Data extrapolation may also be estimated from the plot of Fig. 1. For example, suppose that a pump-wear study or an electro-hydraulic servo-valve investigation required knowledge of the number of particles in the 1-5 micron size-range. ARP 598 data is limited to the size-range above five microns. Extrapolation of this data according to Fig. 1 shows a cumulative count of 14,000 particles greater than one micron. Subtraction of 4400 particles greater than five microns (14,000-4400=9600) gives 9600 particles in the 1-5 micron size-range.

6. ANALYSIS

Particle count data may be further manipulated after it has been fitted to the modified log-normal model represented by log-log^2 graph paper. The original particle count of Section 4 is repeated below.

Size Range	>100	50-100	25-50	15-25	5-15 microns
No. Particles	4	19	150	450	3780

Within the errors of the ARP 598 method (± 20% or more), this tabulation represents the hydraulic oil con-

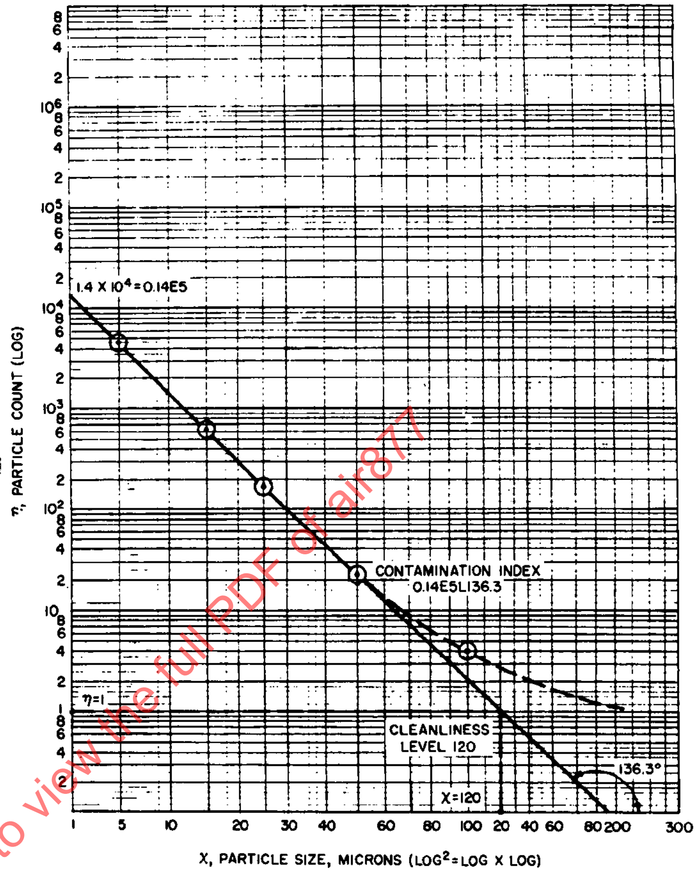

Figure 1
MODIFIED LOG-NORMAL
CONTAMINATION PLOT

tamination. The same information may be expressed by an equivalent "contamination index" derived from the plot of Fig. 1. This straight-line plot is uniquely determined by its ordinate intercept at x = 1, n = 14,000 = $1.4 \times 10^4 = 0.14 \times 10^5$ and its angle of inclination 136.3 deg measured with an ordinary protractor. The contamination index .14E5 $\angle$136.3, using Fortran format for 0.14×10^5, expresses the complete particle count tabulation in concise notation for reference, comparison, communication, or further mathematical analysis. The vector or "phasor" form of the contamination index, .14E5 $\angle$36.3, encourages mathematical description and analysis of otherwise complex processes such as contamination generation or filtration.

Other geometrical characteristics of the log-log^2 linear plot have been used for specification convenience. The x-axis intercept at n = 1 also uniquely describes contamination quantity if the distribution slope is held constant. Particle size distribution tends to approach a typical average value for similar systems. Determination and specification of the applicable distribution-slope allows contamination quantity to be expressed as a single number or "cleanliness level" instead of an ordered number-pair. The cleanliness level thus defined for Fig. 1 is shown to be 120. This hydraulic oil passes "cleanliness level 200" requirements but fails "cleanliness level 100" requirements.

The methods for particle count analysis and calculations described in this report have been verified in limited engineering applications. Further verification or determination of practical application limits is necessary for continuing progress. Further discussion and modification of the proposed log-normal model by other investigators is encouraged.

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