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Hydrometric uncertainty guidance (HUG)

Lignes directrices relatives à l'incertitude en hydrométrie

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Foreword

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Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TS 25377 was prepared by the European Committee for Standardization (CEN) Technical Committee CEN/TC 318, *Hydrometry*, in collaboration with Technical Committee ISO/TC 113, *Hydrometry*, in accordance with the Agreement on technical cooperation between ISO and CEN (Vienna Agreement).

Introduction

The management of a natural environment requires knowledge, by measurement, of what is happening. Only then can effective action be taken and the effectiveness of the action assessed. Much depends on the quality of the knowledge itself.

The quality of measurable knowledge is stated in terms of measurement uncertainty. The internationally agreed method for assessing measurement quality is the *Guide to the estimation of uncertainty in measurement* (GUM). Without this uniformity of measurement standards, equitable sharing of the environment is not possible and international obligations to care for the environment would be weakened.

The essential purpose of the GUM is that a statement of the quality of a measurement result will be presented with all measurements described in technical standards. Without this, no two measurements can be compared or standards set. Whereas the GUM is a reference document serving the universal requirements of metrology, the *Hydrometric uncertainty guidance (HUG)* document is specific to hydrometry, i.e. to the measurement of the components of the hydrological cycle. It borrows from the GUM the methods that are the most applicable to hydrometry and applies them to techniques and equipment used in hydrometry.

In the past, error analysis has provided an indication of measurement quality, but such statements cannot properly convey the quality of the result because it presupposes a knowledge of a true, error-free, value against which the measured result can be compared. The true value can never be known. Uncertainty therefore remains. For this reason, the GUM uses the concept of uncertainty and uses it for all stages and components of the measurement process. This ensures consistency.

The GUM defines standard uncertainty of a result as being equivalent to a standard deviation. This can be the standard deviation of a set of measured values or of probable values. This is broadly similar to the approach used in error analysis that preceded the uncertainty technique. However, the GUM provides additional methods of estimating uncertainty based on probability models. The two approaches are equivalent but uncertainty requires only a knowledge or estimate of the dispersion of measurement about its mean value, and not the existence of a true value. It is assumed that a careful evaluation of the components of measurement uncertainty brings the mean value close to a probable true value, at least well within its margin of uncertainty.

In more general terms, uncertainty is a parameter that characterizes the dispersion of measurable values that can be attributed to their mean value.

By treating standard deviations and probability models as if they approximated to Gaussian (or normal) distributions, the GUM provides a formal methodology for combining components of uncertainty in measurement systems where several input variables combine to determine the result.

Within this formal framework, the GUM can be consistently applied to a range of applications and, thereby, be used to make meaningful comparisons of results.

The HUG seeks to promote an understanding of the nature of measurement uncertainty and its significance in estimating the ‘quality’ of a measurement or a determination in hydrometry.

Hydrometry is principally concerned with the determination of flow in rivers and man-made channels. This includes

— environmental hydrometry, i.e. the determination of the flow of natural waters (largely concerned with hydrometric networks, water supply and flood protection),

— industrial hydrometry, i.e. the determination of flows within industrial plants and discharges into the natural environment (largely concerned with environment protection and also irrigation).

Both are the subject of international treaties and undertakings. For this reason, measured data needs to conform to the GUM to assure that results can be compared.

Hydrometry is also concerned with the determination of rainfall, the movement/diffusion of groundwater and the transport by water flow of sediments and solids. This version of the HUG is concerned with flow determination only.

The results from hydrometry are used by other disciplines to regulate and manage the environment. If knowledge is required of biomass, sedimentary material, toxins, etc., the concentration of these components is determined and their uncertainty estimated. The uncertainty of mass-load can then be determined from the uncertainty of flow determination. The components of this calculation are made compatible through compliance with the GUM.

For practitioners of hydrometry and for engineers, the GUM is not a simple document to refer to. The document has been drafted to provide a legal framework for professional metrologists with a working knowledge of statistical methods and their mathematical representation. A helpful document, NIST Technical Note 1297 [12], is an abbreviated version of the GUM written to be more accessible to engineers and to specialists in fields other than metrology.

The HUG, although simplifying the concepts, in no way conflicts with the principles and methods of the GUM. Accordingly, the HUG interprets the GUM to apply its requirements to hydrometry in a practical way, and, hopefully, in a way accessible to engineers and those responsible for managing the environment.

In addition, the HUG introduces and develops Monte Carlo Simulation, a complementary technique, which has benefits for hydrometry, insomuch as complex measurement systems can be represented realistically.

The HUG summarizes basic hydrometric methods defined in various technical standards. The HUG develops uncertainty estimation formulae from the GUM for these basic methods. The basic hydrometric methods described in the HUG may not be identical to those recited in the published technical standards. In such cases, the methods described in these standards are to be taken as authoritative. However, clauses in technical standards that concern uncertainty should be adapted to be in accordance with the HUG.

NOTE There is no unified definition of space coordinates within the hydrometric standards. The textbook conventional axes are adopted in this document when describing open channel flow: the x axis being horizontal and positive in the mean flow direction, the y axis being orthogonal to the x axis in the horizontal plane and the z axis being vertical positive.

Hydrometric uncertainty guidance (HUG)

1 Scope

This Technical Specification provides an understanding of the nature of measurement uncertainty and its significance in estimating the 'quality' of a measurement or a determination in hydrometry.

It is applicable to flow measurements in natural and man-made channels. Rainfall measurements are not covered.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 772, *Hydrometric determinations — Vocabulary and symbols*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 772 and the following apply.

NOTE For a complete appreciation of the scope of definitions used in measurement uncertainty, the reader is referred to the GUM [10] or to NIST Technical Note 1297 [12].

3.1
standard uncertainty
uncertainty of the result of a measurement expressed as a standard deviation

3.2
type A evaluation of uncertainty
method of evaluation uncertainty by the statistical analysis of a series of observations

3.3
type B evaluation of uncertainty
method of evaluation uncertainty by means other than the statistical analysis of a series of observations

3.4
combined standard uncertainty
standard uncertainty of the result of a measurement when that result is obtained from the values of a number of other quantities, equal to the positive square root of a sum of terms, the terms being the variances or covariances of these other quantities weighted according to how the measurement result varies with changes in these quantities

3.5
expanded uncertainty
quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the values that could be attributed to the measurand

3.6
coverage factor
numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty

4 Symbols and abbreviations

α	coefficient representing the effects of non-uniform energy (velocity) in a channel
$\gamma_{xx}, \gamma_{xy}, \gamma_{xz}$	angles between boat axes and the x axis.
σ	standard deviation
$\Delta'x, \Delta'y$	dispersion of measurement from the mean value of the set of x,y measurements for a symmetric distribution: $\Delta'x = 0{,}5(x_{max} - x_{min})$, etc.
$\Delta'x^{+}, \Delta'x^{-}$	± dispersion about the mean value, $\bar{x}$, for an asymmetric distribution of measurements where $\Delta'x^{+} = (x_{max} - \bar{x})$ and $\Delta'x^{-} = (\bar{x} - x_{min})$.
Δ	small difference in a measured quantity $\Delta Q, \Delta h, \Delta T$, etc.
$\Delta y, \Delta z$	notional small distances in the y and z directions at a cross-section in the channel
Dc_2	in the dilution method, the downstream mixed change ($c_m - c_b$) of concentration of the tracer
A, $A(z)$, $A(h)$	cross-section area (in the y,z plane) of the flow
B	channel width
b	contracted channel width or flume throat width
c_b	dilution method, the background concentration of tracer
c_T	dilution method, the feed concentration of tracer
c_m	dilution method, the downstream mixed concentration of the tracer
C	discharge coefficient
C_v	velocity coefficient
d_i	deviation of an measurement (the ith measurement of a series) from the mean value of that series
E	datum elevation of a range measuring device
$f(h)$	relationship between head, h, and cross-section area, A
F_x, F_y	multiplying factors to be applied to the summation of velocity-area elements to account for the approximation of a summation process to a true integration of continuously varying parameters.
g	gravitation acceleration
h	head of water relative to a defined datum level in the channel
H	total head relative to a defined datum level in the channel
i,j	indices of a count i = 1 to n, or j = 1 to m of a series
J	false measurement detection factor
K	constant of a flow determination equation for a weir or flume
k_1, k_2	constants for the determination of flow by the dilution method
M	dilution method, the mass of tracer introduced into the stream

n	exponent of a flow determination equation for a weir or flume
n, m	number of measurement in a series
$p(x)$	probability function
Q	flow
Q_p	estimated flow passing close to boundaries or any region where measurement cannot be determined by the primary means
Q_T	dilution method, the flow of tracer into the stream
S	standard deviation of a set of measurements
t_e	factor to be applied to small numbers of samples to enable the standard deviation to be representative of large numbers of samples (see Annex A)
t_1, t_2	in the dilution method, the interval during which a change in concentration is detectable
T	absolute temperature, in Kelvin
T_n	Grubbs' test parameter
$U(x)$, $u(y)$	uncertainty of measured variables x, y, etc.
$u_\mathrm{c}(p)$, $u_\mathrm{c}(q)$	the combined uncertainty of determined results p, q, etc.
$u^*(x)$	the percentage uncertainty of a measurement of any quantity x
U_{95}	measurement uncertainty expanded to the 95 % level of confidence
$V_{\bar{x}}$	mean velocity through a yx plane intersecting a channel cross-section of the channel
$V_x(y,z)$	velocity in the x direction at point y, z in the channel
$\vec{V}$	water velocity vector relative to channel
$\vec{V}_\mathrm{b}$	boat velocity vector relative to the channel
$\vec{V}$	water velocity vector relative to boat
$V_{x'}{}'$, $V_y{}'$, $V_z{}'$	water velocity components relative to boat along boat coordinate axes
$V_{\mathrm{b}x'}{}'$, $V_{\mathrm{b}y}{}'$, $V_{\mathrm{b}z}{}'$	components of boat velocity relative the boat axes
γ_{xx}, γ_{xy}, γ_{xz}	angles between boat axes and the channel x axis.
x,y,z	channel coordinates
x',y',z'	boat coordinates
x,y	measurable variables

In this document, the term "uncertainty" refers to measurement uncertainty and the following forms of equation are used to signify

— a sum of n values of x $$x_1 + x_2 + x_3 + \ldots\ldots x_i + \ldots\ldots x_n = \sum_{i=1}^{n} x_i,$$

— a difference, $\Delta f(x)$, in the function, $f(x)$, due to a small change, Δx, in the value x $$\Delta f(x) = \frac{\mathrm{d}f}{\mathrm{d}x}\Delta x,$$

— a value of an integral, F, of a function, $f(x)$, between, $x = x_1$, and $x = x_n$ $$F = \int_{x_1}^{x_n} f(x)\,\mathrm{d}x.$$

5 ISO/IEC Guide 98 (GUM) — Basic definitions and rules

5.1 General

This section summarizes the methods described in the GUM for the expression of uncertainty in measurement. For a general introduction to measurement uncertainty, refer to Annex A.

5.2 The uncertainty of sets of measurements

The GUM describes measurement uncertainty as a value that characterizes the dispersion of measurements that could reasonably be attributed to the result. The GUM goes on to define standard uncertainty as uncertainty expressed as a standard deviation, s.

So, for a set of n measurements, uncertainty is related to the difference between each measured value, x_i, from the average value, $\bar{x}$, of the set. The standard deviation, and hence the uncertainty, $u(x)$, is:

$$u(x) = s = \sqrt{\frac{1}{n-1}\left[(\bar{x}-x_1)^2 + (\bar{x}-x_2)^2 + (\bar{x}-x_3)^2 + ...(\bar{x}-x_n)^2\right]}$$

where components $(\bar{x}-x_i)^2$ are the deviation of the ith measurement, x_i, from the mean value, $\bar{x}$.

Or, more concisely:

$$u(x) = s = \sqrt{\frac{1}{n-1}\sum_1^n d_i^2} \quad (1)$$

where $d_i = \bar{x} - x_i$ is the deviation of the ith measurement from the mean value, $\bar{x}$,

and

$$\bar{x} = \frac{1}{n}\sum_{i=1}^n x_i \quad (2)$$

The larger the number, n, of measurements used to calculate the mean value, $\bar{x}$, the greater is the expectation that the mean value approaches the 'true' value. Therefore, the uncertainty of the mean value, $u(\bar{x})$, decreases as the number of measurements, n, increases. The GUM relationship for this is

$$u(\bar{x}) = \frac{1}{\sqrt{n}} u(x) \quad (3)$$

5.3 Random and systematic effects

Equation (3) applies only to the random variations of the measured quantity. This random effect is determined from the measured data and, as such, is evaluated after a set of measurements have been taken. Random effects can be determined from analysis of the historic data or by the instrumentation itself if it is designed to analyse the data in real time. Random effects diminish the average value of a set of n measurements by the factor $\frac{1}{\sqrt{n}}$. Random conditions often exist as natural turbulence. However, random variation can sometimes occur through human interpretation of a reading of an indicator, such as a staff gauge.

Uncertainties that are inherent to the measurement equipment or to the method are systematic. Systematic effects cannot be diminished by the use of Equation (3). During each measurement session, systematic effects can usually be taken as constant for the measurement device. Systematic components are

a) assessed as part of an installation or commissioning procedure, and/or

b) specified beforehand for the equipment by the manufacturer. Refer to Clause 9.

Refer to A.6 in Annex A for more information on random and systematic effects.

For the evaluation of the uncertainty of a continuous process, include unsteady effects as a random component. The quantity being measured may be varying slowly during the measurement process. This will widen the dispersion of measured values and hence add to the assessment of the random component. Such variation shall be part of the randomness of the measurement. If during the measurement process the rate of change is such that it significantly exceeds the natural dispersion of measurements, then the result shall be discarded.

5.4 Uncertainty models — Probability distributions

In hydrometry, measurements are often made using automated instruments. They have a margin of measurement within which measured values can vary randomly in steady conditions. If this uncertainty is inherent to the measurement process, it is a systematic component. It is commonly expressed as a probability distribution. Probability distributions have standard deviations about the mean value which are equivalent to the standard deviation of discrete measurements as defined above. The probability distributions equivalent to Equations (1) and (2) are

$$u(x) = \sqrt{\int_{-\Delta x}^{\Delta x} d(x)^2 \cdot p(x)\, \mathrm{d}x} \tag{4}$$

and

$$\bar{x}' = \int_{-\Delta x}^{\Delta x} x \cdot p(x) \mathrm{d}x \tag{5}$$

where $p(x)$ is a probability function and d is the dispersion. Refer to Annex A for details.

5.5 Combining uncertainties — The law of propagation

The GUM also defines a rule for combining uncertainties from several sources. It is called ‘the law of the propagation of uncertainties’. For a relationship, f, between a result, y, and variables, $x_1, x_2 \ldots\ldots x_n$, defined as $y = f(x_1, x_2 \ldots\ldots x_n)$, the combined uncertainty, $u_c(y)$, of y is

$$u_c(y)^2 = \sum_{i=1}^{i=n} \left(\frac{\partial f}{\partial xi} u(x_i) \right)^2$$

or

$$u_c(y)^2 = \left(\frac{\partial f}{\partial x_1} u(x_1) \right)^2 + \left(\frac{\partial f}{\partial x_2} u(x_2) \right)^2 + \ldots\ldots + \left(\frac{\partial f}{\partial x_n} u(x_n) \right)^2 \tag{6.A}$$

where $x_1, x_2 \ldots\ldots x_n$ are independent variables.

Equation (6.A) applies only where the variables $x_1, x_2 \ldots\ldots x_n$ are uncorrelated, i.e. if variable x_i changes value, no other x variable is affected by that change. If two or more variables x do influence each other (i.e. they are correlated), then an additional component of uncertainty exists. Equation (6.A) then becomes

$$u_c(y)^2 = \sum_{i=1}^{i=n} \left(\frac{\partial f}{\partial xi} u(x_i) \right)^2 + 2 \sum_{i=1}^{i=n-1} \sum_{j=i+1}^{j=n} \frac{\partial f}{\partial xi} \frac{\partial f}{\partial xj} u(x_i x_j) \tag{6.B}$$

Almost all hydrometric uncertainty estimations require the use of the simpler form, i.e. Equation (6.A).

The components can be random or systematic. The partial derivatives $\frac{\partial f}{\partial x_n}$ are referred to as 'sensitivity coefficients'.

5.6 Expressing results

Equation (6.A) expresses the final result in terms of standard uncertainty. For the Gaussian probability, used as a model distribution for general analysis, one standard deviation covers only 68 % of the range of possible results. This means that for a result expressed as

Flow rate = 10,8 l/s ± 0,6 l/s

or

$$Q = \bar{Q} \pm u(Q)$$

Only 68 % of the measurement will lie between 10,2 l/s and 11,4 l/s. Almost one third of the measurement can be expected to lie outside this band. Such a statement is of little value in hydrometry. A more meaningful statement is required that will cover a larger portion of possible results.

Subclause A.9 defines expanded uncertainty. By expanding the margin of uncertainty, a greater portion of the expected range of measurements is covered. For the Gaussian probability distribution, it can be shown that by doubling the uncertainty margin, 95 % of expected measurements are covered.

The same result expressed in the form

Flow rate = 10,8 l/s ± 1,2 l/s at the 95 % confidence level

or

$$Q = \bar{Q} \pm U_{95}(Q)$$

means that 95 % of the measurements are expected to lie between 9,6 l/s and 12,0 l/s. This is a more practical expression of the result.

In hydrometry, all measurements shall be expressed at the 95 % confidence level with a statement of the form:

Quantity = Value ± uncertainty at the 95 % confidence level

or

Quantity = Value ± percentage uncertainty at the 95 % confidence level

Refer to A.9 for more detail.

6 Open channel flow — Velocity area methods

6.1 General

Figure 1 shows the coordinate system used in this document with orthogonal axes x, y, z. The mean velocity is calculated in the x direction. The xy-plane is horizontal. The z axis is vertical. Note that a velocity $\vec{V}$ vector representing the mean velocity does not have to align with the x axis. The flow in the channel can be determined from velocities passing obliquely through an intersecting yz plane.

The origin of the coordinate system may be located at any point relative to the channel but is typically located at the hydraulic datum for weirs and flumes or, for velocity-area methods, on a gauge datum alongside the stream.

For example, vertical measurement can be $h(z)$, expressed from a hydraulic datum relative to the z_0 coordinate system origin.

The determination of flow in open channels requires the following:

a) the determination of the mean velocity $\bar{V}_x$ across the channel section; and

b) the measurement of the cross-section area $A(h)$, in the yz plane, through which the flow passes; h is the water depth.

The product of these two quantities is the discharge, Q.

$$Q = \bar{V}_x A(h)$$

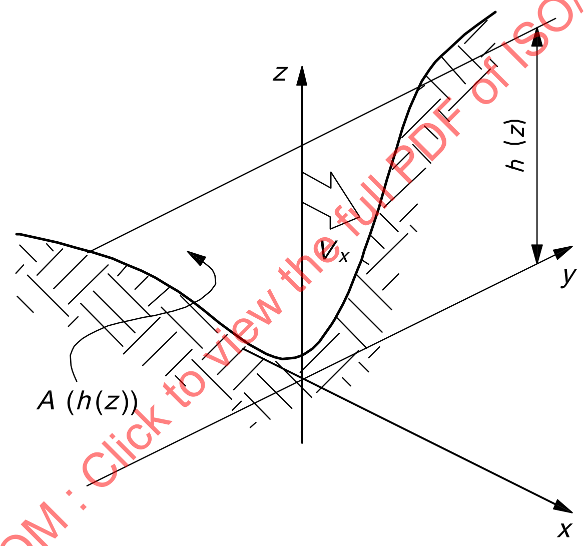


Figure 1 — Co-ordinate relationship at a channel cross-section

6.2 Mean velocity, $\bar{V}_x$

The evaluation of mean velocity shall deal with the V_x variability with respect to position, y,z, across the $V_x(y, z, t)$ channel and with respect to time, t. At the walls, friction slows the mainstream velocity to zero which causes steep velocity gradients to occur, illustrated in Figure 2. Velocity gradients and shear stress within the body of the flow induce vortices which causes turbulent conditions. Turbulence exists in a moving body of water even when the water surface appears tranquil.

The evaluation shall therefore scan the cross-section while integrating and averaging the velocity component in the x direction. The flow can be steady and hence $\bar{V}_x$ can be constant, but turbulence causes the local value of $V_x(y, z, t)$ to be unsteady.

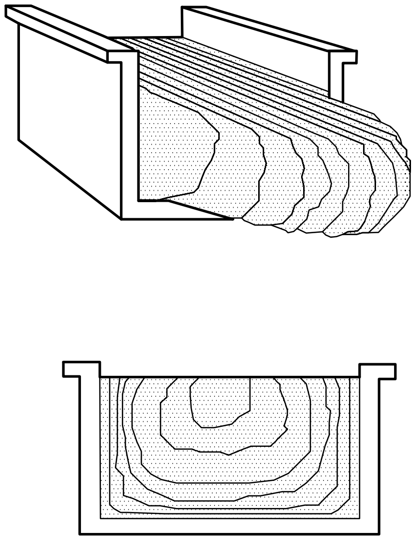


Figure 2 — Typical current profiles and contours

6.3 Velocity-area determination

The quantity, $\bar{V}_x$, is determined across the channel from instantaneous point velocities, $V_x(y, z, t)$. In this subclause, it is assumed that steady flow conditions prevail. If the flow does not vary with time, t, during the integration process, then

$$Q = \int_A \bar{V}_x (y,z) \mathrm{d}A \tag{7}$$

The ‘arithmetic’ method of integration is summation of velocity through notional stream tubes of defined area.

This is typically done by dividing the cross-section into a number of horizontal segments or vertical segments, then measuring velocity at frequent intervals along the centreline of each segment to determine the segment mean velocity. Flow through the segment is the mean velocity through the segment multiplied by the segment area. The flows through each segment are summated to give the total flow in the channel. Therefore, Equation (7) becomes

$$Q = F_y F_z \sum_1^m \sum_1^n V_x \left(y_i z_j \right) \Delta z_j \Delta y_i + Q_\mathrm{p} \tag{8}$$

where

F_y is a factor, often assumed to be unity, relating the discrete summation in the y direction to an ideal integration of a true continuous velocity profile;

F_z is a factor, often assumed to be unity, relating the discrete summation in the z direction to an ideal integration of true continuous velocity profile; and

Q_p represents perimeter flow passing between the region of segments and the channel boundary.

The summation method divides the area into $m \times n$ rectangular stream tubes of height, Δz_j, and width, Δy_i. A set of Δy_i stream tubes makes up each horizontal segment, and a set of Δz_j stream tubes makes up each vertical segment. For small values of m or n, special consideration shall be given to the F_y and F_z functions. The uncertainty of these factors is systematic to the summation process.

The term Q_p is the flow passing through a perimeter region that exists close to the channel floor and walls and the water surface where the velocity, $V_x(y_i, z_j)$, cannot be reliably determined. This can be due to the coarse y-z resolution of the measuring device, the presence of steep velocity gradients through a boundary region or interference from the walls on the measurement process (sonar reflections for example). In the perimeter region, the flow is estimated by extrapolating velocity profiles determined in the body of the flow.

6.4 Stationary determination of velocity

6.4.1 General

There are two methods of scanning the velocity profiles:

— stationary scans where the scanning device is static relative to the x,y,z coordinates when measurement are made; and

— moving scans in which the scanning device moves across the channel at a known velocity.

6.4.2 Vertical segments

A range of meter types may be used to determine point velocities within vertical segments. Various techniques are used to assess the mean velocity within each segment: single point, three points, five points, continuous lower/raise traverse.

The Doppler sonar provides a method of 'snapshot' integration along vertical segments by rapidly recording and processing velocities at a sufficient number of points to minimize the integration uncertainties.

Equation (8) then becomes

$$Q = F_y F_z \sum_1^n h_i \bar{V}_x (y_i) \Delta y_i + Q_p \qquad (9)$$

where $\bar{V}_x(y_i)$ is the processed mean value by automated summation (integration) of the velocities in the jth vertical segment. The Doppler sonar is also used in small channels where only a single vertical profile is integrated. In such cases, special attention shall be given to the evaluation of F_y and the area term, $h_1 \Delta y_1$.

6.4.3 Horizontal segments

An alternative method of 'snapshot' integration is to divide the cross-section into horizontal segments. The mean velocity is determined in each horizontal segment and hence the flow through each segment. This method is often used with transit-time sonar (see ISO 6416).

Equation (8) then becomes

$$Q = F_y F_z \sum_1^m b_j \bar{V}_x (z_j) \Delta z_j + Q_p \qquad (10)$$

where

$\bar{V}_x(z_j)$ is the processed mean value derived from scanning the velocities in the jth horizontal segment;

b_j is the length of the jth segment across the channel.

Compared with vertical segment methods, relatively few horizontal segments are used: F_z is not unity. Its value and its uncertainty shall be determined from 'typical' velocity profiles at the site.

6.5 Moving determination of velocity

NOTE The complex nature of the measurement process described here is either rationalized as a tabular procedure in the relevant standard or incorporated into equipment.

Acoustic Doppler current profilers (ADCPs) scan from a moving boat to determine the velocity of the boat and the velocities of the flow relative to a boat coordinate system.

Two additional components are required to be determined. They are

1) the boat velocity vector, $\vec{V}_\mathrm{b}$, relative to the channel coordinates, x,y,z, and

2) the alignment of the boat coordinates, x',y',z', relative to the fixed, x,y,z axes.

The boat velocity vector, $\vec{V}_\mathrm{b}$, can be determined using GPS or by bottom tracking when the floor of the stream is static. Note that the boat velocity need not be parallel to the y' axis of the boat.

If the velocity of the water relative to channel is $\vec{V}$, then the velocity of the water relative to the $x'y'z'$ axes of the boat is $\vec{V}' = \vec{V} + \vec{V}_\mathrm{b}$ where $\vec{V}'$ is a velocity vector of the water relative to the boat. Note that ' denotes "relative to the boat".

In the two-dimensional space of Figure 3, this can be expressed in components relative to the boat coordinate system as

$$\left(V_{x'}, V_{y'}\right) = \left(V'_{x'} - V_{\mathrm{b}x'}, V'_{y'} - V_{\mathrm{b}y'}\right)$$

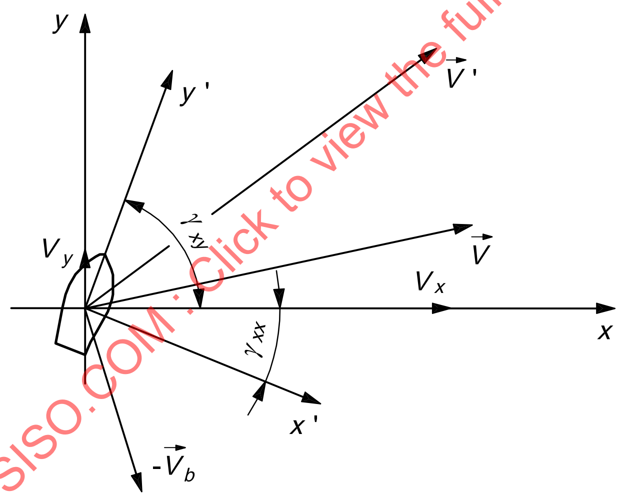


Figure 3 — 2D velocity vector and coordinates

Alignment of the boat axes, x',y', relative to the channel coordinates, x,y, can be expressed as direction angles γ_{xx}, γ_{xy} where $\cos\left(\gamma_{xx}\right)^2 + \cos\left(\gamma_{xy}\right)^2 = 1$

In the three-dimensional space of Figure 4, these relationships become

$$\left(V_{x'}, V_{y'}, V_{z'}\right) = \left(V'_{x'} - V_{\mathrm{b}x'}, V'_{y'} - V_{\mathrm{b}y'}, V'_{z'} - V_{\mathrm{b}z'}\right)$$

$\gamma_{xx}, \gamma_{xy}, \gamma_{xz}$ are the direction angles relative to the boat axes, x',y',z', of the channel x axis where $\cos\left(\gamma_{xx}\right)^2 + \cos\left(\gamma_{xy}\right)^2 + \cos\left(\gamma_{xz}\right)^2 = 1$. The velocity component of the flow along the channel x axis is

$$V_x = \left(V'_{x'} - V_{\mathrm{b}x'}\right)\cos\gamma_x + \left(V'_{y'} - V_{\mathrm{b}y'}\right)\cos\gamma_y + \left(V'_{z'} - V_{\mathrm{b}z'}\right)\cos\gamma_z \qquad (11)$$

or

$$V_x\left(y_i z_j\right)=\left(V'_{x'}\left(y_i z_j\right)-V_{\mathrm{b}x'}\left(y_i z_j\right)\right)\cos\left(\gamma_{xx}\left(y_i z_j\right)\right)+\left(V'_{y'}\left(y_i z_j\right)-V_{\mathrm{b}y'}\left(y_i z_j\right)\right)\cos\left(\gamma_{xy}\left(y_i z_j\right)\right)$$

$$+\left[V'_{z'}\left(y_i z_j\right)-V_{\mathrm{b}z'}\left(y_i z_j\right)\right]\cos\left[\gamma_{xz}\left(y_i z_j\right)\right] \tag{12}$$

Equation (12) is a general statement for determining the velocity components for Equation (8) with the relative velocity resolved along the x axis from $y'z'$ scans.

The relationship is simpler when the boat has no yaw or pitch component, i.e. $\gamma_{xz}=90$. In this situation, $\cos\gamma_{xz}=0$ and $\cos\left[\gamma_{xy}\left(y_i z_j\right)\right]=\sin\left[\gamma_{xx}\left(y_i z_j\right)\right]$. So, Equation (12) becomes

$$V_x\left(y_i z_j\right)=\left[V'_{x'}\left(y_i z_j\right)-V_{\mathrm{b}x'}\left(y_i z_j\right)\right]\cos\left[\gamma_{xx}\left(y_i z_j\right)\right]+\left[V'_{y'}\left(y_i z_j\right)-V_{\mathrm{b}y'}\left(y_i z_j\right)\right]\sin\left[\gamma_{xx}\left(y_i z_j\right)\right] \tag{13}$$

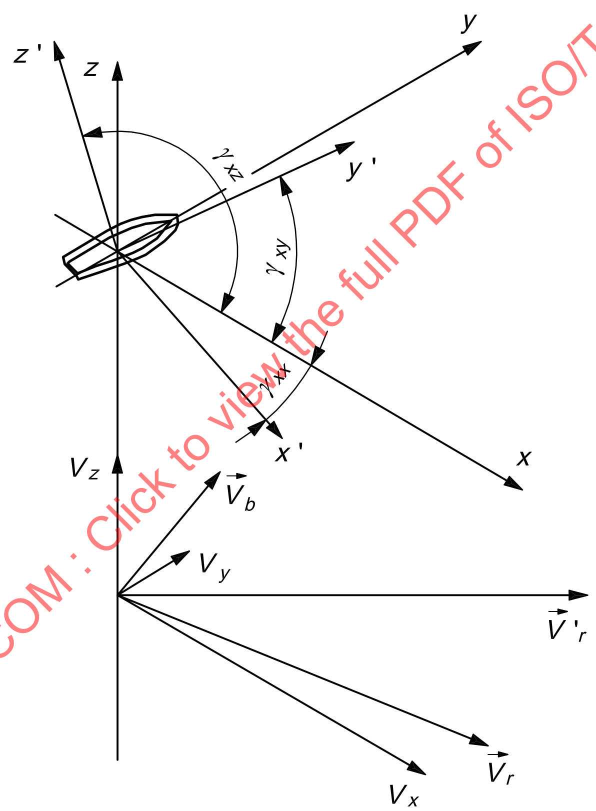


Figure 4 — 3D velocity vector and boat coordinates

6.6 Velocity-area uncertainties

6.6.1 Stationary scans

6.6.1.1 General

Equation (8) can be applied to propeller or manual EM methods, or automated methods such as ultrasonic Doppler, transit time or echo correlation. Each segment $\Delta y_i \cdot \Delta z_j$ has a portion of area uncertainty together with the uncertainty of each velocity determination, $V_x\left(y,z\right)$.

There is also uncertainty associated with the F_y and F_z factors that relate the discrete summations in the y and z directions compared to the theoretical integration of true continuous velocity profiles. The terms F_y and F_z can also include any effects on the integration process of unsteadiness of $V_x(y,z,t)$.

The general uncertainty relationship is derived from Equation (6). Refer to Annex A for examples of combining uncertainties.

In terms of percentage uncertainties, $u^*(x)$, Equation (8) becomes

$$u^*(Q)^2 = \sum_i^m \sum_j^n \left(V_x(y_i z_j) \Delta y_i \Delta z_j \right) \left[u^*(F_y)^2 + u^*(F_z)^2 \right] +$$

$$\sum_i^m \sum_j^n \left(V_x(y_i z_j) \Delta y_i \Delta z_j \right) \left[u^* \left(V_x(y_i z_j) \right)^2 + u^*(\Delta y_i)^2 + u^*\left(\Delta z_j\right)^2 \right) \right] + u(Q_\mathrm{p})^2 \tag{14}$$

where

$u^*(F_y)$ is the percentage uncertainty inherent in the integration of a continuous velocity profile by a summation of discrete determinations along a line in the y direction;

$u^*(F_z)$ is the percentage uncertainty inherent in the integration of a continuous velocity profile by a summation of discrete determinations along a line in the z direction;

$V_x(y_i z_j)$ is the total uncertainty of the velocity determination.

These include

$$u^*\left[V_x(y_i z_j) \right]^2 = u_k^*\left[V_x(y_i z_j) \right]^2 + u_r^*\left[V_x(y_i z_j) \right]^2 \tag{15}$$

where

u_k^* is the uncertainty of the velocity measurement device specified by a calibration certificate. Note that the value of u_k^* is a systematic effect which can vary with V_x. u_r^* is a component of uncertainty relating to the unsteady nature of flow in channels: see below;

$u^*(\Delta y_i)$ is the total width measurement percentage uncertainty of the means and method used to define channel width. The same percentage can be applied to individual segments. Thus

$$u^*(\Delta y_i) = u^*(B) \tag{16}$$

or can be estimated for each segment individually.

$u^*(\Delta Q_\mathrm{p})$ is the estimated uncertainty of flow passing outside the area of the segments;

$u^*(\Delta z_j)$ is the depth measurement uncertainty of the equipment and method used to define segment depth.

These include

$$u^*(\Delta z_j)^2 = u_k^*(\Delta z_j)^2 + u_\mathrm{d}^*(\Delta z_j)^2 + u_r^*(\Delta z_j)^2 \tag{17}$$

where

$u_k^*(\Delta z_j)$ is the systematic effect of the depth measurement equipment specified by a calibration certificate; note that the value of $u_k^*(\Delta z_j)$ can vary with $h(z)$;

$u_{\mathrm{d}}^{*}\left(\Delta z_j\right)$ is the uncertainty of the datum setting of the depth measurement apparatus;

$u_{\mathrm{r}}^{*}\left(\Delta z_j\right)$ is a random component of uncertainty (see 6.6.1.2).

6.6.1.2 Random and systematic effects

Random effects are determined by collecting sets of data from which a standard deviation can be determined using Equation (1) or Equation (3). This applies to velocity component $u^{*}\left[V_x\left(y_i z_j\right)\right]$ and the depth of water $u^{*}\left(z_j\right)$ across the channel.

Systematic effects are $u^{*}\left(F_y\right)$, $u^{*}\left(F_z\right)$, $u_{\mathrm{c}}^{*}\left[V_x\left(y_i z_j\right)\right]$, $u_{\mathrm{c}}^{*}\left(\Delta z_j\right)$, $u_{\mathrm{d}}^{*}\left(\Delta z_j\right)$ and $u^{*}\left(\Delta y_i\right)$ for which a probability distribution and realistic measurement dispersion values are required. Refer to Clause 9.

Refer to Annex A for more information on random and systematic effects.

6.6.2 Additional factors — Moving scans

For moving scans with ADCP, the velocity component, $V_x\left(y_i z_j\right)$, in Equation (14) needs to include additional terms for the boat velocity, $\vec{V}_{\mathrm{b}}$, and for the alignment of the boat axes with the x,y,z axes. From Equation (11),

$$V_x = \left(V'_{x'} - V_{\mathrm{b}x'}\right)\cos\gamma_x + \left(V'_{y'} - V_{\mathrm{b}y'}\right)\cos\gamma_y + \left(V'_{z'} - V_{\mathrm{b}z'}\right)\cos\gamma_z$$

Using Equation (6.A),

$$\left[u(V_x)\right]^2 = \left[V_{\mathrm{b}x'}\cos\left(\gamma_x\right)u\left(V_{\mathrm{r}x'}\right)\right]^2 + \left[V'_{x'}\cos\left(\gamma_x\right)u\left(V_{\mathrm{b}x'}\right)\right]^2 + \left[\left(V'_{x'} - V_{\mathrm{b}x'}\right)\sin\left(\gamma_x\right)u\left(\gamma_x\right)\right]^2$$

$$+\left[V_{\mathrm{b}y'}\cos\left(\gamma_y\right)u\left(V_{\mathrm{r}y'}\right)\right]^2 + \left[V'_{y'}\cos\left(\gamma_y\right)u\left(V_{\mathrm{b}y'}\right)\right]^2 + \left[\left(V'_{y'} - V_{\mathrm{b}y'}\right)\sin\left(\gamma_y\right)u\left(\gamma_y\right)\right]^2$$

$$+\left[V_{\mathrm{b}z'}\cos\left(\gamma_z\right)u\left(V_{\mathrm{r}z'}\right)\right]^2 + \left[V'_{z'}\cos\left(\gamma_z\right)u\left(V_{\mathrm{b}z'}\right)\right]^2 + \left[\left(V'_{z'} - V_{\mathrm{b}z'}\right)\sin\left(\gamma_z\right)u\left(\gamma_z\right)\right]^2$$

$u\left[V_x\left(y_i z_j\right)\right]$ is evaluated for each i,j cell.

Even simplifying the relationship by assuming that the roll and pitch of the boat is very small so that $\cos\left(\gamma_z\right) = 0$ and $\cos\left(\gamma_y\right) = \sin\left(\gamma_x\right)$ does not significantly reduce the complexity of these relationships.

6.6.3 Random and systematic effects

6.6.3.1 Random effects

The additional random components are from $\vec{V}_{\mathrm{b}}$, and γ_x, γ_y, γ_z.

The movement of the boat shall be sufficiently slow and steady in comparison with the measurement rate that local values of $V_x\left(y_i z_j\right)$ are randomly dispersed about a mean value for each cell i,j. Such a set of random measurements shall be used to identify random effects from Equations (1) and (3). If each i,j determination is recorded, this can be done on completion of a transect.

6.6.3.2 Systematic effects

The means of determination of $\vec{V}_{\mathrm{b}}$ and of γ_x, γ_y, γ_z have additional uncertainties for which a probability distribution and realistic estimates of the dispersion of detectable values are required.

$u\left[V_x\left(y_i z_j\right)\right]$ is a systematic effect relating to measuring velocity in the *i,j*th cell. The systematic effects are determined using the above equation where $u\left(V_x\right)$, $u\left(V_y\right)$, $u\left(V_{bz}\right)$, etc. and the alignment angle uncertainties, γ_x, γ_y, γ_z, are defined in the product specification as probability distributions with dispersion limits.

Refer to Annex A for more information on random and systematic effects.

6.6.4 Determining uncertainty for moving scans

It can be seen that analysis of ADCP systems through partial derivatives of Equation (6.A) is complex. A simpler and more rigorous method can be used based on numerical methods rather than mathematical analysis. One such method, advocated in Supplement 1[11] to the GUM, is the Monte Carlo Simulation. Refer to Annex B.

Monte Carlo Simulation can be used to predict the uncertainty of complex systems, such as an ADCP system, from Equations (8) and (13) using model probability distributions and a realistic dispersion of values.

The following components are required to be modelled:

a) to determine V_x and $u\left(V_x\right)$ for *i,j* values

1) $\gamma_x, \gamma_y, \gamma_z$,

2) V_{bx}, V_{by}, V_{bz},

3) V_{rx}, V_{ry}, V_{rz},

b) to determine Q for *i,j* values

1) F_x, F_y,

2) $\Delta x, \Delta y$, and

c) around the perimeter

Q_p.

The simulation can relate the dispersions to a value representing a systematic component plus a value from the measured data which represents a random component. Unless there is evidence of a particular probability distribution, the simulation shall use rectangular distributions.

This method is a simulation of a measurement process and does not need to use actual measurements. To use this method to determine the uncertainty with Equation (12), the dispersions of measured values are assessed from recorded data and then used with a model probability distribution to determine a result for $u^*\left(Q\right)^2$.

6.7 Integration uncertainties $\left[u^*\left(F_y\right), u^*\left(F_z\right)\right]$

6.7.1 General

Annex E of ISO 748:—[1] provides tables of values for $u^*\left(F_y\right)$ and $u^*\left(F_z\right)$ with Table E.4 stating the values of integration uncertainty for a range of numbers of point measurement within a segment and with Table E.6 stating the values of integration uncertainty for a range of numbers of segments across the channel.

These tables are to be used in Equation (8) and those derived from it.

6.7.2 Vertical scanning uncertainties

Manufacturers of equipment for scanning and integrating velocity profiles in vertical sections through the flow shall provide:

a) $u^*\left(F_z\right)$: a calibration statement, estimations of measurement uncertainty derived by formal means based on verifiable dispersions of measurement and probability distributions in accordance with 9.3 in terms of two-dimensional fully developed velocity profiles using mean-section velocity as reference velocity;

   Ultrasonic Doppler and echo correlation methods shall include a statement of the particle characteristics to which the uncertainty statement applies.

b) $u^*\left(F_y\right)$: the one or several vertical scans have to be extrapolated horizontally to define a velocity profile for the channel. This may be done by modelling the channel profile, usually a parabolic relationship using a power coefficient, n. Factors determining the coefficient n include

   1) channel length, geometry and aspect ratio,

   2) channel wall roughness,

   3) the Reynolds number of the flow in the channel.

Suppliers of such equipment shall provide data equivalent to Table E.4 of ISO 748:— for their recommended method of interpolation/extrapolation, taking into account an appropriate range of factors listed above.

6.7.3 Horizontal scanning uncertainties

Manufacturers of equipment for scanning and integrating velocity profiles in horizontal sections through the flow shall provide

a) $u^*\left(F_z\right)$: a calibration statement, estimations of measurement uncertainty derived by formal means based on verifiable dispersions of measurement and probability distributions in accordance with 9.3 in terms of two-dimensional fully developed velocity profiles using mean-section velocity as reference velocity;

b) $u^*\left(F_y\right)$: the one or several horizontal scans have to be extrapolated vertically to define a velocity profile for the channel. This may be done by modelling the channel profile, usually a parabolic relationship using a power coefficient, n. Factors determining the coefficient n include

   1) channel length, geometry and aspect ratio,

   2) channel wall roughness,

   3) the Reynolds number of the flow in the channel.

1) To be published. Revision of ISO 748:1997.

Suppliers of such equipment shall provide data equivalent to Table E.4 of ISO 748:— for their recommended method of interpolation/extrapolation, taking into account an appropriate range of factors listed above.

6.8 Perimeter flow uncertainties, $u\left(Q_{\text{p}}\right)$

The velocity area method applies to the main part of the cross-section, the part in which the velocity is measurable within the margins of uncertainty for the equipment used. The measurements from such equipment may have to be discarded close to a boundary or where the shear rate is high.

In these situations, the velocity shall be estimated using other means. These are described in the appropriate standard for that method.

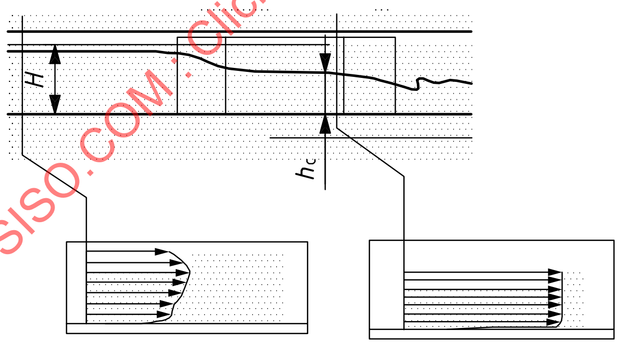
The estimation of uncertainty in these regions is problematic. Any statement shall include the critical thinking on which the estimate is based. It is unlikely that uncertainties below 20 % can be justified for the total perimeter flow. A norm of 40 % can be taken as a safe estimation unless there is evidence to the contrary.

Such estimates can overwhelm the estimated uncertainty of the main body of the flow. To minimize the effect of $u\left(Q_{\text{p}}\right)$ on $u(Q)$, equipment should be used that can minimize the perimeter region. This also has the effect of minimizing the mean velocity with the perimeter area, thus minimizing Q_{p}.

7 Open channel flow — Critical depth methods

7.1 General

When flow passes through a hydrometric structure, a weir flume or notch, a unique relationship exists between the upstream water level and the flow, and that relationship is largely independent of the velocity profile approaching the structure and conditions downstream of it. Analysis shows that by accelerating the flow at a hydrometric structure, the velocity distortions are greatly reduced and that, for practical purposes, the velocities adopt a geometrically consistent pattern for each class of weir or flume. This is shown in Figure 5 for a rectangular flume.



Key

H total head

h_{c} critical depth

Figure 5 — Rectangular flume

A unique relationship therefore exists between the upstream water level, the cross-section of the accelerated flow and the mean velocity in the accelerated section. This relationship is the basis of the critical depth method.

7.2 Head and geometry determination

From critical depth theory, the flow through a flume or over a weir can be expressed in the form

$$Q = KC_vCbh^n \tag{18}$$

where

K is a constant;

C is a predetermined calibration or discharge coefficient;

C_v is a velocity coefficient;

b is a geometric scale factor;

h is the head of water; and

n is a parameter with a normal value of n = 1,5 for rectangular cross-sections and n = 2,5 for v-shaped cross-sections.

The uncertainty expression for this Equation follows the example given in Annex A for an Equation of the type $r = Bxy^n$.

$$u^*(Q)^2 = u^*(C)^2 + u^*(C_v)^2 + u^*(b)^2 + \left[n \cdot u^*(h)\right]^2 \tag{19}$$

The component C_v is defined as

$$C_v = \left(\frac{H}{h}\right)^{1,5} \tag{20}$$

where H is the total head.

In current standards, the $u^*(C_v)$ value is assumed to be zero. This is also the case in this version of the HUG, but $u^*(C_v)$ should not be assumed to be negligibly small. Refer to 7.3 with respect to the variability of energy distribution in the channel, α.

Equation (20) cannot be directly applied because H is not a directly measurable quantity (unless the hydrometric structure is immediately downstream of a reservoir). Water surface level and hence h_1 is substantially constant after the flow has travelled in a straight path for a short distance. For this reason, it is normal practice to measure h_1 and then compute H from

$$H = h_1 + \alpha \frac{\bar{V}_{x1}^2}{2g} \tag{21}$$

where

$\bar{V}_1$ is the mean approach velocity;

h_1 is the head of water in the upstream channel;

α is a coefficient related to the velocity distribution in the approach channel and is defined by a mass-weighted relationship:

$$\alpha = \frac{1}{\bar{V}_{x1}^3} \frac{1}{A_1} \int_{A_1} \left[V_{x1}(y,z) \right]^3 \mathrm{d}A_1 \qquad (22)$$

α is related to a non-uniform distribution of velocity in the approach channel.

7.3 Iterative calculation

In Equation (21), $\bar{V}_{x1}$ is derived from the relation

$$\bar{V}_1 = \frac{Q}{A_1(h_1)} \qquad (23)$$

Because $\bar{V}_{x1}$ is a function of the flow rate, Q, these equations can only be solved by iteration. The process begins by setting $H = h_1$ (h_1 being the measured quantity), then calculating Q from Equation (18), then $\bar{V}_1$ from Equation (23), and then re-estimating H from Equation (21) with α before repeating the cycle. The process continues until converged values are achieved for H and for $\bar{V}_1$ for the given measured quantity h_1.

7.4 Evaluating uncertainty

Equation (19) defines the relationship for evaluation of $u^*(Q)$ for a weir, flume or notch.

$u^*(C)$ The value $u^*(C)$ is defined by the characteristics of the particular hydrometric structure set out in International Standards for each particular hydrometric structure. It quantifies a systematic effect by the type B method, normally stated in hydrometric standards at the 95 % confidence level as a Gaussian distribution. The value shall be transformed to the 68 % level of confidence (i.e. half the value) before being used for the calculation of uncertainty. The value of $u^*(C)$ varies with head h, progressively increasing as h diminishes.

$u^*(C_v)$ In this version of the HUG, $u^*(C_v) = 0$ (this may not be justifiable for flows with a high approach velocity).

$u^*(b)$ This represents the uncertainty of the geometry of the hydrometric structure, throat width, crest length and horizontal alignment, V-angle, etc. The value is determined by measurement and should be stated as a dispersion and probability distribution.

$u^*(h)$ The value of head measurement uncertainty is related to the type of measurement system used (float, ultrasonic, etc.), refer to Clause 9. The value is taken from the manufacturer's specification, specified in terms of probability distribution and dispersion of h.

The value of $u^*(h)$ shall include a component of datum uncertainty.

The percentage head measurement uncertainty increases steeply as the head approaches zero.

n This is a constant value, i.e. the exponent of h from Equation (18).

8 Dilution methods

8.1 General

There are two dilution methods:

1) a method that maintains a continuous feed of tracer of known concentration, the flow being determined by the ratio of the known concentration with a measured fully mixed concentration in the stream;

2) a method that determines the flow from the dilution of a known mass of tracer added instantaneously to the stream, the flow being determined by integrating the mixed concentration of the tracer to determine its mean dilution.

Both methods rely on the stream to thoroughly mix the tracer between the point where the tracer is added and the point where its concentration is measured.

8.2 Continuous feed

The flow in the stream Q is determined by

$$Q = Q_{\mathrm{T}} k_1 k_2 \frac{c_{\mathrm{T}} - c_{\mathrm{m}}}{c_{\mathrm{m}} - c_{\mathrm{b}}} \tag{24}$$

where

Q_{T} is the flow of tracer into the stream;

c_{b} is the background concentration of tracer;

c_{T} is the feed concentration of tracer;

c_{m} is the downstream mixed concentration of the tracer;

k_1 is a mixing factor (approaching 100 %);

k_2 is a factor (approaching 100 %) to account for any degradation of the tracer while in contact with the stream.

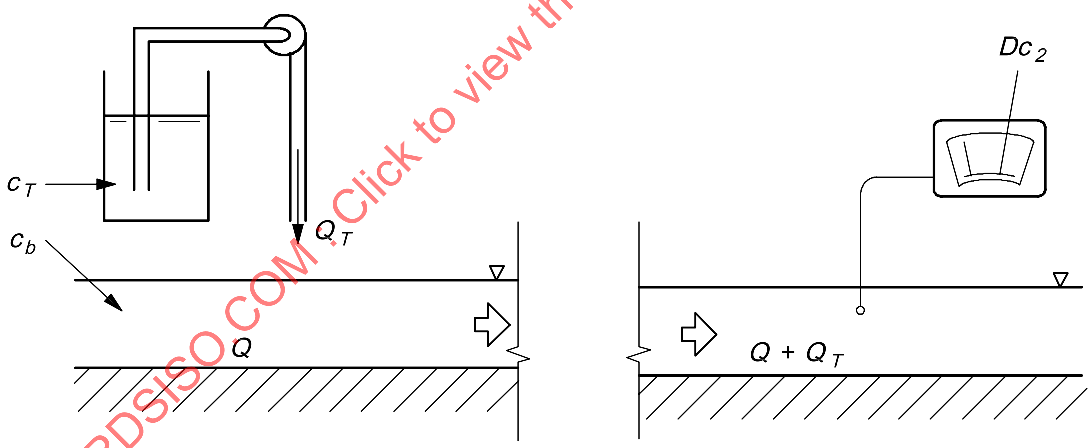


Figure 6 — Continuous feed dilution method

The datum uncertainty of the measurement concentration, Dc_2, can be eliminated if the difference between the mixed stream concentration, c_{m}, and its background level, c_{b}, is measured: $Dc_2 = c_{\mathrm{m}} - c_{\mathrm{b}}$. If the background concentration is very low compared to the feed concentration, i.e. $c_{\mathrm{b}} << c_{\mathrm{T}}$ then Equation (24) becomes

$$Q = Q_{\mathrm{T}} k_1 k_2 \frac{c_{\mathrm{T}} - Dc_2}{Dc_2} \tag{25}$$

For the purpose of estimating uncertainty, this Equation can be further simplified by assuming that $c_T >> c_m$ from which it follows that

$$u^*(Q)^2 = u^*(Q_{\mathrm{T}})^2 + u^*(k_1)^2 + u^*(k_2)^2 + u^*(c_{\mathrm{T}})^2 + u^*(Dc_2)^2 \tag{26}$$

The value of all measured concentrations shall be tabulated from a prior calibration using procedures described in Clause 10. The value of $u^*(c)$ can vary with the measured concentration, c.

8.3 Transient mass

The flow in the stream, Q, is determined by

$$Q = \frac{M}{k_1 k_2 \int_{t_1}^{t_2} (Dc_2)\,\mathrm{d}t} \tag{27}$$

where

- M is the mass of tracer introduced into the stream;
- Dc_2 is the downstream mixed change ($c_m - c_b$) of concentration of the tracer;
- k_1 is a mixing factor (approaching 100 %);
- k_2 is a factor (approaching 100 %) to account for any degradation of the tracer while in contact with the stream;
- t_1, t_2 is the interval during which a change in concentration is detectable.

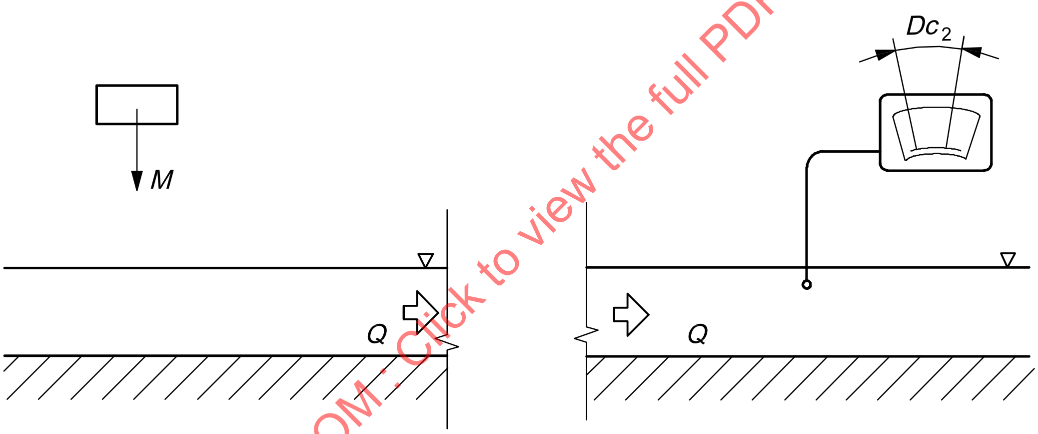


Figure 7 — Transient mass dilution method

From Equation (27),

$$Q = \frac{M}{k_1 k_2 S} \tag{28}$$

where

S is the sum of the product of concentration and time-increment

$$S = \sum_{i=1}^{N} Dc_{2i} \Delta t_i \tag{29}$$

and

N is the number of summation intervals;

Δt_i is the time increment. For equal increments, $\Delta t_i = \frac{(t_2 - t_1)}{N}$

From which it follows that

$$u^*(Q)^2 = u^*(M)^2 + u^*(k_1)^2 + u^*(k_2)^2 + u^*(S)^2 \tag{30}$$

and

$$u^*(S)^2 = u^*(\Delta t)^2 + \left(\frac{1}{\sum_{i=1}^{N} Dc_{2i}} \sum_{i=1}^{N} Dc_{2i} \right)^2 \tag{31}$$

9 Hydrometric instrumentation

9.1 Performance specifications

To have confidence in measurement equipment, it is necessary to verify the performance specified by the manufacturer. A plain statement of uncertainty is not as easily verified as the equivalent statement by which measurement dispersion, $\Delta' x$, is defined together with a probability distribution. Common probability distributions are listed in Table 1.

Table 1 — Probability distribution

Probability distribution	Uncertainty	Derived from Annex A
Triangular	$u(x) = \frac{1}{\sqrt{6}} \Delta' x$	Equation (A.8)
Rectangular	$u(x) = \frac{1}{\sqrt{3}} \Delta' x$	Equation (A.8)
Bimodal	$u(x) = \Delta' x$	Equation (A.8)
Tabular distribution	$u(x) = u^*(\text{Table}) \Delta' x$	Equation (A.5)

Refer to Annex A (A.2 and A.3) for a derivation of the relationships of Table 1. The forms of the triangular and rectangular distributions are shown in Figure A.2.

9.2 Validity of uncertainty statements

Hydrometric instrumentation is used both in the laboratory and in the field. Not all instruments are suited for field use because extremes of environment can influence the measurement performance. Equipment rated only for laboratory use should not be used in the field. For this reason, a product specification shall include, in addition to the statement of measurement uncertainty, a statement defining the operating environment in which the uncertainties are valid. This ensures that the claimed uncertainty is realistic for hydrometric purposes.

A validity statement shall list all known influences on measurement performance that can increase measurement uncertainty above 0,2 % at the 68 % confidence limit (or 0,4 % at the 95 % confidence limit). Factors common to all environmental measurements shall be stated as in the example in Table 2.

Table 2 — Environmental factors

Environmental factor	Effect	Uncertainty statement	Dispersion or (uncertainty)
Ice	yes	Measurement invalid	
Rainfall	yes	Rectangular probability distribution	0,4 %
Snowfall	yes	Measurement invalid	
Humidity	yes	Triangular probability distribution	0,2 %
Temperature	no		
Wind speed	yes	Rectangular probability distribution	0,3 %
Solar radiation	yes	Unclassified probability distribution	(0,1 % uncertainty)
Water surface waves	yes	Rectangular probability distribution	15 % wave height
Water turbidity	no		
Water-surface active agent	no		
Other factors (*product- and application-specific*)			

If a combination of factors together do affect measurement performance, while the same factors individually do not, those factors that apply in combination shall be stated.

If the influence has an unclassified probability distribution, the manufacturer shall state a method for combining the stated uncertainty with other components of measurement uncertainty.

If a product's measurement processes is susceptible to environmental and other factors, as indicated in the example above, such information shall be illustrated by a representative example of the measurement performance in the presence of these influences.

Hydrometric equipment shall have the means for, and/or description of, a performance verification procedure to ensure that the performance of a measurement system can be verified in the environment where it is deployed.

9.3 Manufacturer's performance specifications

The presentation of the measurement performance of equipment requires a unifying framework to allow one product to be compared with another or one measurement system to be compared with another.

Measurement performance specifications shall state clearly the confidence limit to which they apply. This may be the standard uncertainty, in which case the 68 % confidence limit shall be stated, or more commonly, the statement may be at the 95 % confidence limit.

The 95 % confidence limit is the norm for hydrometry performance specifications.

Note that before combining type A or type B estimations with uncertainty values at the 95 % confidence limit, these values shall first be converted by using a coverage factor k (typically $k = 2$) to values at the 68 % confidence limit. The subsequent result can then be expanded to the 95 % confidence limit using a coverage factor $k = 2$.

The determination of systematic effects is by repeated measurement under steady conditions. The standard deviation of a set of 'static' measurements defines the systematic component of the device's measurement uncertainty.

This specification requires the maximum and minimum rating of the equipment to be stated. The maximum rating is then used to define 25 %, 50 % and 75 % of maximum ratings. To each of these five ratings, a measurement uncertainty is specified for the product.

These are to follow the pattern set out in the Table 3.

Table 3 — Maximum and minimum ratings

		Probability distribution	**Corresponding dispersion of measurement**				
			min.	**25 % max.**	**50 % max.**	**75 % max.**	**max.**
Measured values from the instrument							
Measurement uncertainty (%)							
Environmental factor	1						
Environmental factor	2						
Environmental factor	3						
etc.							
Total measurement uncertainty (%)							

The manufacturer shall provide a table with more columns if the uncertainty characteristic cannot be adequately interpolated (by linear interpolation) by five points.

9.4 Performance guide for hydrometric equipment for use in technical standard examples

Examples are given in Table C.1.

Many of the values presented in this table are provisional. They are intended to be norms of performance for the technology. Values are to be defined by consensus between users and should be representative of the broad range of equipment available. A formal testing programme can be required to establish the table entries.

When using Table C.1, the following shall be noted.

a) The percentage uncertainty for head measurement cannot be specified by the equipment manufacturer. It shall be derived from a relationship of the form

$$u^*(h) = \sqrt{\frac{u(E)^2 + u(h_1)^2}{h}}$$

where $u(E)$ is the uncertainty of the relative datum.

b) $$u^*(h) = \sqrt{\frac{u(E)^2 + u(R)^2}{h}}$$

where $u(R)$ is the uncertainty of range/extension.

c) The performance figures assume precise compensation for the effects of temperature on sonic velocity. The following formula is a practical approximation:

$$\text{sonic velocity} = 20{,}08\sqrt{\text{Absolute temperature of air}}$$

d) If unsteady conditions exist, a time-dependent component of uncertainty shall be defined. Instrumentation without this capability shall require a manufacturer's statement of uncertainty relating to unsteady conditions.

10 Guide for the drafting of uncertainty clauses in hydrometric standards

10.1 General

The main purpose of this specification is to simplify the drafting of hydrometric standards so that they meet the requirements of the GUM. This is a significant task: publication of the HUG is only one part.

In accordance with former practice in hydrometry, the statement of the result of an uncertainty estimation shall be at the 95 % confidence limit. However, care must be taken when combining uncertainties because Type A and Type B estimation methods derive uncertainty at the 68 % confidence limit and an instrument's performance is normally stated at the 95 % confidence limit.

This issue shall be emphasized, and illustrated by example in the drafted uncertainty clauses.

Measurement performance shall be stated in the terms set out in Table 3, not in any other way.

Standards which have classified measurement performance into categories such as Class 1, Class 2, etc. are not acceptable. Classifications can introduce confusions which can lead to poor choice of equipment. Intellectual commitment is required to select the appropriate equipment and method.

10.2 Equipment, methods and measurement systems

10.2.1 General

Flow is not a measurable quantity, it is determined from the measurement of other parameters such as velocity, depth, width and from the mass/concentration of a tracer.

10.2.2 Equipment

Equipment used for the measurement of parameters that determine discharge shall state measurement uncertainties in accordance with Table 3. Furthermore, “Hydrometric equipment shall have the means for, and/or description of, a performance verification procedure to ensure that the performance of a measurement system can be verified in the environment where it is deployed”.

Equipment standards may state the expected performance for the equipment type as described in that standard.

Where such verification is not available, an analysis of the component uncertainties may be used as a basis for a provisional statement of performance. Provisional statements should declare that they are an ‘Estimated performance, not based on formal ISO evaluation’. Note that a ‘means for performance verification' is required.

10.2.3 Methods

Methods have components of uncertainty. Some of these components require special attention, for example the evaluation of perimeter flow uncertainty, $u^*(Q_\text{p})$.

Additional data is required for any measurement system that does not determine flow across the total section of the channel. Where the method is incorporated within the product, as with ADCP equipment, sufficient information shall be disclosed to enable independent evaluation of Q_p and $u^*(Q_p)$.

10.2.4 Systems

Hydrometric structures and dilution equipment are examples of measurement devices that have a method of use which together define flow measurement systems.

Hydrometric structures are the primary component of a measurement system which requires a measurement of head to determine the flow.

Standards for hydrometric structures shall illustrate the measurement method by means of examples using one or more head measurement devices listed in Table C.1. The estimation of measurement uncertainty shall include systematic effects that define datum elevation of the head measurement device.

Standards for hydrometric systems shall include an uncertainty budget representative of typical measurement conditions over a tabulated range of flows including minimum flow, 25 %, 50 %, 75 % and maximum flow.

Annex A
(informative)

Introduction to hydrometric uncertainty

A.1 Basic definitions and rules

A measurement can never be truly exact: measured values always have a margin of uncertainty. So a measurement value should not be stated without an accompanying indication of its quality. The standard way of doing this is with a statement of its uncertainty.

The GUM (ISO/IEC Guide 98) describes measurement uncertainty as a value that characterizes the dispersion of measurements that could reasonably be attributed to the result. The GUM goes on to define standard uncertainty as uncertainty expressed as a standard deviation.

This definition has three features:

a) uncertainty can be quickly determined from a set of measurements;

b) uncertainty can be modelled for a measurement process; and

c) a 'law of propagation of uncertainties' can be used to combine uncertainties.

For a set of measurements, uncertainty is related to the difference between each measured value, x_i, from an average value, $\bar{x}$, of the set. The standard deviation, and hence the uncertainty, $u(x)$, is

$$u(x) = \sqrt{\frac{1}{n-1}\left[\left(\bar{x}-x_1\right)^2 + \left(\bar{x}-x_2\right)^2 + \left(\bar{x}-x_3\right)^2 + \ldots\ldots + \left(\bar{x}-x_n\right)^2\right]}$$

where components $\bar{x} - x_i$ are the deviation of the ith measurement, x_i, from the average value, $\bar{x}$.

Or, more concisely

$$u(x) = \sqrt{\frac{1}{n-1}\sum_1^n \left(d_i\right)^2} \quad \text{(A.1)}$$

where $d_i = \bar{x} - x_i$ is the difference of the ith measurement from the average value, $\bar{x}$

and

$$\bar{x} = \frac{1}{n}\sum_{i=1}^n x_i \quad \text{(A.2)}$$

The larger the number n of measurement used to calculate the mean value, $\bar{x}$, the closer it can approach a 'true' value. Therefore, the uncertainty of the mean value, $u(\bar{x})$, reduces as the number of measurements, n, increases. The GUM relationship for this is

$$u(\bar{x}) = \frac{1}{\sqrt{n}}u(x) \quad \text{(A.3)}$$

Under steady conditions, repeat sets of n measurements can be used to determine a mean value with a $\frac{1}{\sqrt{n}}$ reduction of measurement uncertainty. An instrument capable of working on a sequence of measurements representing the variability of the parameter being measured, can take advantage of this. However, this process only applies to random effects on the input quantity. Uncertainties that are inherent to the measurement process (systematic effects) are not reduced in this way.

The GUM defines a rule for combining uncertainties from several sources, the 'law of propagation of uncertainties'. For a relationship f between a result y and dependent variables $x_1, x_2 .... x_n$ in which $y = f(x_1, x_2 .... x_n)$, the uncertainty of y is

$$u_c^2(y) = \sum_{i=1}^{i=n} \left[\frac{\partial f}{\partial x_i} u(x_i) \right]^2$$

or

$$u_c^2(y) = \left(\frac{\partial f}{\partial x_1} u(x_1) \right)^2 + \left(\frac{\partial f}{\partial x_2} u(x_2) \right)^2 + ... + \left(\frac{\partial f}{\partial x_n} u(x_n) \right)^2 \qquad \text{(A.4)}$$

where the variables $x_1, x_2 ..... x_n$ are independent variables.

Equation (A.4) applies only where the variables $x_1, x_2 ..... x_n$ are uncorrelated, i.e. if variable x_i changes value, no other x variable is affected by that change. If two or more variables x do influence each other (i.e. they are correlated), then an additional component of uncertainty exists. Equation (A.4) then becomes

$$u_c^2(y) = \sum_{i=1}^{i=n} \left(\frac{\partial f}{\partial x_i} u(x_i) \right)^2 + 2 \sum_{i=1}^{i=n-1} \sum_{j=i+1}^{j=n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i x_j)$$

Almost all hydrometric uncertainty estimations require the use of the simpler form, i.e. Equation (A.4).

This law means that uncertainties are combined as root-sum-squared values. The partial derivatives $\frac{\partial f}{\partial x_n}$ are referred to as sensitivity coefficients. The result $u_c(y)$ can be more sensitive to some variables, x, than to others. For example, in percentage terms the flow through a V-notch is 2,5 times more sensitive to uncertainty of head measurement than it is to uncertainty of the angle of the V.

A.2 Introduction to the definitions

A.2.1 General

The definitions given in the previous clause can be interpreted using examples.

Table A.1 lists a set of measurements $(n = 20)$ to determine the width of a section of a rectangular channel. Equation (A.2) determines the mean width, $\bar{x}$, from values, x_i, measured at different positions, selected randomly from the channel.

Table A.1 — Sample width measurements of channel

i **index**	x_i **value**	$d_i = \bar{x} - x_i$	d_i^2
1	1 203	4,35	18,922 5
2	1 220	−12,65	160,022 5
3	1 218	−10,65	113,422 5
4	1 193	14,35	205,922 5
5	1 210	−2,65	7,022 5
6	1 194	13,35	178,222 5
7	1 206	1,35	1,822 5
8	1 185	22,35	499,522 5
9	1 227	−19,65	386,122 5
10	1 197	10,35	107,122 5
11	1 225	−17,65	311,522 5
12	1 219	−11,65	135,722 5
13	1 210	−2,65	7,022 5
14	1 194	13,35	178,222 5
15	1 221	−13,65	186,322 5
16	1 209	−1,65	2,722 5
17	1 205	2,35	5,522 5
18	1 204	3,35	11,222 5
19	1 197	10,35	107,122 5
20	1 210	−2,65	7,022 5
	$\bar{x} = 1\,207{,}3$		$\sqrt{\frac{1}{n-1}\sum_1^n (d_i)^2} = 11{,}7$

From Table A.1 and using Equation (A.2), the best estimate of channel width, its mean value, is $\bar{x} = 1\,207{,}3$ mm. The standard deviation is calculated using Equation (A.1) to be 11,7 mm. This is the uncertainty of the measured values. The uncertainty of the average channel width from this set of measurements is reduced by the factor $\frac{1}{n-1}$, where $n = 20$. So, the uncertainty of the mean value of channel width is 2,6 mm.

In Table A.2, the same 20 measurements have been sorted into size bins of width 5 mm.

Table A.2 — Width measurements grouped into 5 mm bins (n = 20)

1 185 to 1 190	1 190 to 1 195	1 195 to 1 200	1 200 to 1 205	1 205 to 1 210	1 210 to 1 215	1 215 to 1 220	1 220 to 1 225	1 225 to 1 230
					1 212			
	1 192			1 208	1 214			
	1 193	1 199	1 203	1 206	1 214	1 215	1 221	
1 189	1 192	1 198	1 203	1 206	1 211	1 219	1 220	1 225

A.2.2 Data histograms

The same data can be represented as a histogram where individual values are not explicitly given but are represented by counts in the bins. Each item in the bin is taken to have the mean bin value.

Table A.3 — Width measurements grouped as a histogram into 5 mm bins (n = 20)

Mean bin value 1 187,5	**Mean bin value** 1 192,5	**Mean bin value** 1 197,5	**Mean bin value** 1 202,5	**Mean bin value** 1 207,5	**Mean bin value** 1 212,5	**Mean bin value** 1 217,5	**Mean bin value** 1 222,5	**Mean bin value** 1 227,5

Larger sets of data can reveal a distinctive shape to the grouping; for example Table A.4 shows a histogram with a larger number of counts of measurements (up to 8 in some of the bins) for the same channel as that of Table A.1.

Table A.4 — Channel 1 width measurements grouped as a histogram into 5 mm bins (n = 78)

Mean bin value 1 182,5	**Mean bin value** 1 187,5	**Mean bin value** 1 192,5	**Mean bin value** 1 197,5	**Mean bin value** 1 202,5	**Mean bin value** 1 207,5	**Mean bin value** 1 212,5	**Mean bin value** 1 217,5	**Mean bin value** 1 222,5	**Mean bin value** 1 227,5	**Mean bin value** 1 232,5

Table A.5 is another rectangular channel of different size with different distribution of measured values.

Table A.5 — Channel 2 width measurements grouped as a histogram into 5 mm bins (n = 78)

Mean bin value	Mean bin value	Mean bin value	Mean bin value	Mean bin value	Mean bin value	Mean bin value	Mean bin value	Mean bin value	Mean bin value	Mean bin value
932,5	937,5	942,5	947,5	952,5	957,5	962,5	967,5	972,5	977,5	982,5

Table A.6 is a set of output signal recordings over a short period of time for a shaft encoder measuring the level of a float.

Table A.6 — Shaft encoder — Float level measurements grouped as a histogram 0,001 m bins (n = 64)

Mean bin value	Mean bin value	Mean bin value	Mean bin value	Mean bin value	Mean bin value	Mean bin value	Mean bin value	Mean bin value
0,527 5	0,528 5	0,529 5	0,530 5	0,531 5	0,532 5	0,533 5	0,534 5	0,535 5

The definition of uncertainty from Equation (A.1) can be expressed in terms of measurements allocated to bins in a histogram. If the set of n measurements is grouped into k bin sizes, each of width $\frac{2\Delta d}{k}$, and in the jth bin there are m_j measurements, then $\sum_{j=1}^{k} m_j = n$.

So,

$$u(x) = \sqrt{\sum_{j=1}^{k} \left(d_j\right)^2 \frac{m_j}{n-1}} \tag{A.5}$$

Also,

$$\sum_{j=1}^{k} \frac{m_j}{n} = 1 \tag{A.6}$$

where d_j is the deviation of the jth bin from the mean value, $\bar{x}$.

Presenting measured data as sorted histograms shows the distribution of measured values between the smallest and largest.

Equation (A.5) can be applied to the histograms of Table A.4, A.5 and A.6. The results for the three histograms of Table A.4, Table A.5 and Table A.6 calculated in this way, are summarized in Table A.7.

Table A.7 — Results derived from Tables A.4, A.5 and A.6 using Equation (A.5)

Shape	Mean value $\bar{x}$	Dispersion $-\Delta' x, \Delta x$	Standard deviation
Table A.4 'Rectangular'	1 207,05	−24,55 to 25,45	15,57
Table A.5 'Triangular'	956,99	−24,49 to 25,51	12,24
Table A.6 'Bimodal'	0,531 3	−0,003 8 to 0,004 2	0,002 14

An inappropriate choice of the probability distribution can affect the determined uncertainty value. Where a choice is possible, the selection shall be justified. Where there is no clear choice, a rectangular probability distribution shall be used.

These histograms are close approximations to probability distributions. The probability distribution enables the mean value and the standard deviation to be determined from the range of measurements each side of the mean, i.e. from the dispersion.

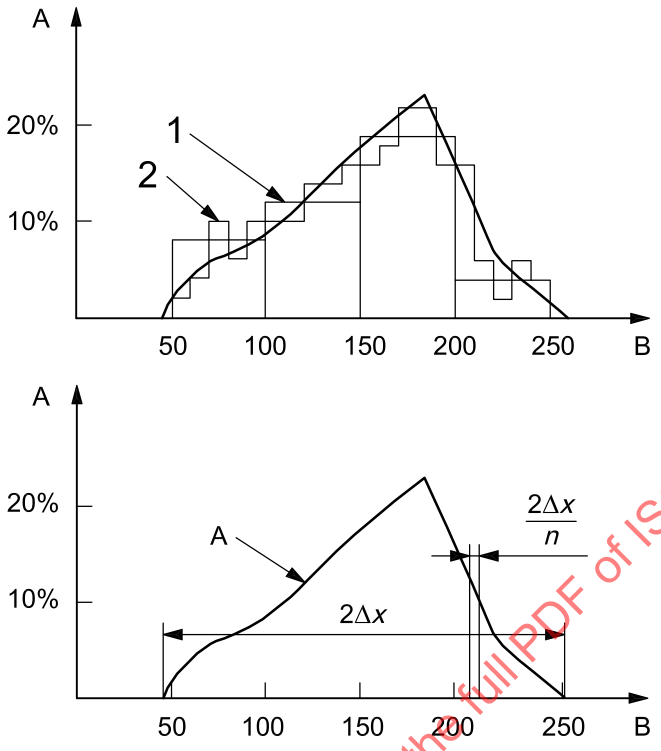
This is very significant for estimating the systematic component of the measurement uncertainty of sensors. Systematic effects are determined beforehand by testing the sensor under steady conditions in an environment similar to the one in which it is to be used. The results determine the dispersion of values in these conditions and define a probability distribution for the results. The two in combination define the systematic component of uncertainty of a sensor measurement in that environment.

A.3 Measurement histograms and probability distributions

The shapes outlined in Tables A.4, A.5 and A.6 approximate to rectangular, triangular and bimodal probability distributions

Figure A.1 shows distributions for two sets of measurements, one for a small set, $m = 4$, sorted into large bins, the other for a larger set, $m = 20$, sorted into small bins.

The ratio $\frac{m_j}{n}$ is the proportion of the total number of measurements, m, in the jth bin. This ratio is the probability that a measurement is in the jth bin.



Key

1 $m = 4$

2 $m = 20$

A probability, in percent

B measured value

Figure A.1 — Histograms for two sets of measurements and a probability distribution

The histograms are from a small set of data, $m = 4$, sorted into large bins and from a larger set of data, $m = 20$, sorted into smaller bins. The probability distribution is a continuous line. It is a model distribution equivalent to an infinite number of measurements distributed into infinitesimal bins.

For large numbers of measurements, $n-1 \rightarrow n$. The summation can be represented as an integral for large numbers of measurements in bins of size $\mathrm{d}x = \frac{2\Delta x}{n}$. So, the average, $\overline{x}'$, is

$$\overline{x}' = \int_{-\Delta' x}^{\Delta' x} x \cdot p(x)\,\mathrm{d}x \tag{A.7}$$

and

$$u(x) = \sqrt{\int_{-\Delta' x}^{\Delta' x} d(x)^2 \cdot p(x)\,\mathrm{d}x} \tag{A.8}$$

where

$d(x) = \bar{x}' - x$;

$p(x)$ is a probability function.

As the relationship $\sum_1^k \frac{m_k}{n} = 1$ applies to sets of measurements, so the relationship

$$\bar{x}' = \int_{-\Delta' x}^{\Delta' x} p(x)\,\mathrm{d}x = 1 \qquad \text{(A.9)}$$

applies to the probability function. The uncertainty expressed in Equation (A.9) is the standard deviation of the probability function for the measurement process it describes between the limits $x = \Delta' x$ and $x = -\Delta' x$.

Equation (A.7) gives the same results as those derived from the histogram Equation (A.5) for large data sets with small bin sizes. Probability distributions are exactly equivalent to data histograms for determining uncertainty.

A.4 Probability models

A.4.1 General

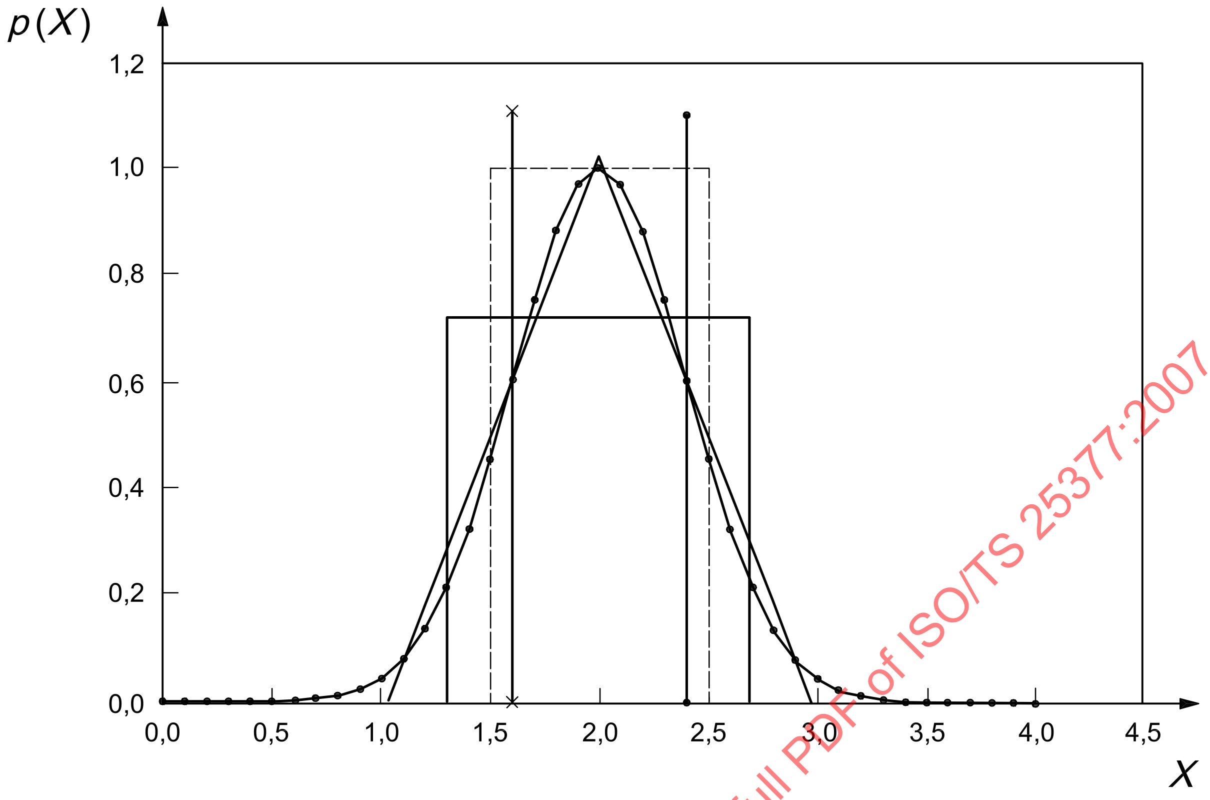
There are four probability distributions commonly used to calculate measurement uncertainty. They are

— rectangular probability distributions,

— triangular probability distributions,

— bimodal distributions, and

— Gauss distributions.



Key

——•——	A	normal (Gauss) distribution
————	B	triangular distribution
————	C	rectangular distribution
– – – –	D	unit area under distributions
——×——	E	one standard deviation
——•——	F	+ one standard deviation

Figure A.2 — Common probability distributions

The common probability distributions are shown on Figure A.2. All have the same area beneath their function line: the area is unity as required by Equation (A.9).

The Gauss distribution is of little practical significance in hydrometry because its dispersion is infinitely wide (the distribution approaches zero probability but never reaches it). It is referred to as the 'normal' distribution. The 'normal' distribution does not exist in practical science but is commonly used as a theoretical device. Note, it can be seen that the Gauss distribution can be approximated by the triangular distribution which has defined dispersion of values.

One value of the Gauss distribution is used in defining the concept of confidence limit. In the HUG, the term 'Gauss' distribution is used instead of 'normal' to avoid the suggestion that 'normal' distributions exist in hydrometry. The Gauss distribution is considered further in A.9.

The vertical lines indicate the standard deviation for the three distributions shown: all have the same standard deviation. The vertical lines also indicate the bimodal distribution. The bimodal has infinite probability at these lines and zero probability elsewhere.

In accordance with Equation (A.8), the area under each line (including a bimodal distribution) is one unit of area as depicted by the inner tall rectangle.

A.4.2 Probability models — General considerations

The distribution of measurements can be modelled if the shape of the distribution is always repeatable. In other words, when random measurements are repeated, they have similar distributions within their dispersion $\Delta' x$ about the mean value, $\bar{x}$.

The rectangular, triangular and bimodal distribution models are able to meet most of the requirements. Others can be derived and used if they represent the distribution more realistically.

Rectangular and triangular probability distributions are the natural forms for human estimation of readings from measurement devices such as rulers, staff gauges, verniers, etc. The choice between using rectangular or triangular is based on what seems probable to the user. If in doubt, a rectangular model shall be used.

Measurement instruments can be tested to determine their true dispersions under steady conditions. The effect of environmental factors can also be assessed at the same time. The scatter of measurements need not match that of a rectangular or triangular probability distribution, but it should be sufficiently well defined and repeatable that a standard deviation can be determined for any span of dispersions.

If an instrument shows small sensitivity to environmental changes, these changes shall be accommodated by defining a wider dispersion of measurement. The determined uncertainty then applies to a stated range of environmental conditions.

It is essential that all measurement processes have repeatable probability distributions. Any instrument that is susceptible to external factors which make its performance unpredictable will have indeterminate uncertainty. Such instruments shall not be used for hydrometry.

A.5 Adjustment for small set of data — The t_e factor

The probability diagram results from a hypothetically large number of measurements and defines the dispersion Δx . If the set of measurements is very small, then the smallest and largest values may not be represented. Therefore, the uncertainty for small samples could be underestimated. This problem is dealt with by adding a factor t_e to Equation (1) to increase $u(x)$ for small sets of measurements.

$$u_{\mathrm{c}}(x) = t_e \sqrt{\frac{1}{n-1} \sum_1^n (d_i)^2} \qquad \text{(A.10)}$$

Values of t_e are shown in Table A.8 for numbers of measurements (minimum 2).

Table A.8 — Values of t_e

Number n	t_e
2	1,84
3	1,32
4	1,20
5	1,14
6	1,11
8	1,09
10	1,08
12	1,07
15	1,03
20	1,03

A.6 Random and systematic effects

The terms random and systematic are used in standards to distinguish between i) effects that represent an inherent dispersion of measured values under steady conditions, and ii) systematic effects that are part of the measurement procedure or apparatus.

For example, in hydrometry, the following systematic effects exist:

a) in setting the datum elevation of a head measurement device (float encoder, ultrasonic range sensor, staff gauge, etc). If its existence is known or suspected, then steps are taken to eliminate datum uncertainty to a level at which it exists only as a residual random component;

b) with a ruler used for measuring distance (here uncertainty is both in the ruler's calibration and in the use of the ruler);

c) in any calibration certificate (velocity probe, laser ranging device, etc). This requires traceability to international standard measurements, supported with the development of an uncertainty budget for the calibration certificate in accordance with the GUM.

d) where an algorithm is used to transform a measured parameter into another quantity.

Systematic effects remain constant for the measurement process whereas random effects can vary. Note that the GUM does not have different procedures to treat random and systematic effects. Both are assumed to exist as dispersions around the determined value.

Tables A.2 and A.3 are of channel width measurements. These are measurements taken randomly within the channel. Both tables show results with a resolution of 1 mm. The measurement device, a ruler or a laser ranging device, will have a margin of uncertainty of its own: a systematic component. Assume the dispersion of systematic effects about the recorded values is 0,5 mm. Without more information on the dispersions, it shall also be assumed that the probability diagram is rectangular. Therefore, the systematic uncertainty of the measurement process is

$$u(x) = \sqrt{\frac{1}{3}} \times 0{,}5 = 0{,}29 \text{ mm}$$

This does not significantly add to the channel width uncertainties of 15,8 mm and 12,3 mm for the channel data of Tables A.2 and A.3. However, situations can exist where the systematic effect is significant and the uncertainty shall be added according to the law of propagation, [see Equation (A.6)].

Another common source of uncertainty lies in setting the datum of a level measurement system. The float-encoder system of Table A.6 has been datumed relative to the invert of a channel. The fixed elevation of the encoder is estimated to be within 2,5 mm of the true value relative to a hydraulic datum of the channel. A dispersion of 2,5 mm is therefore assumed. In this case, care having been taken, a triangular probability distribution is considered to be appropriate. The systematic component of the head measurement, $u(x_s)$, attributable to its datum is therefore

$$u(x_s) = \sqrt{\frac{1}{6}} \times 2{,}5 = 1{,}02 \text{ mm}$$

This is significant compared with the random uncertainty, 2,2 mm, of the encoder signal.

These uncertainties are combined according to Equation (A.4).

$$u'_{\mathrm{c}}(x) = \sqrt{u(x)^2 + u(x_s)^2} = 2{,}42 \text{ mm}$$

A.7 Summary — Type A and Type B estimation methods

The fundamental definition of measurement uncertainty is standard deviation. This applies to sets of measurement results. Measurement uncertainty also exists within the measurement process that measures each value. This is a systematic effect. The systematic component has to be pre-estimated in another way using a probability distribution. Clause A.3 shows both methods are equivalent.

The GUM classifies these two methods of estimation:

a) Type A: a statistical method using a set of n discrete random measurements by

   1) evaluating the mean value $\bar{x} = \frac{1}{n}\sum_1^n x_i$,

   2) determining the set of deviations from the mean $d_i = \bar{x} - x_i$,

   3) determining the standard deviation $u(x) = t_e \sqrt{\frac{1}{n-1}\sum_1^n (d_i)^2}$;

b) Type B: a non-statistical method, based on probability by

   1) identifying the maximum and minimum values of x to determine $2\Delta' x$,

   2) selecting or declaring a probability distribution, $p(x)$, of the deviation,

   3) determining the standard deviation from $u(\bar{x}) = \sqrt{\int_{-\Delta' x}^{\Delta' x} d(x)^2 \, p(x) \, \mathrm{d}x}$.

For the sake of clarity and completeness, all Type B statements shall be a declaration of the expected dispersion of measurements and the associated probability distribution. This shall also apply to manual estimates of measurement uncertainty. It is easier to verify that measurements are within a specified dispersion of values. This is particularly true of complex systems requiring multiple inputs.

A.8 Gauss probability distribution

Figure A.2 shows the Gauss distribution with a probability function defined as

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp^{\frac{-(\bar{x}-x)^2}{2\sigma^2}} \qquad \text{(A.11)}$$

The function implies infinite dispersions so, for hydrometry, has little practical value. However, this function does meet the conditions required in Equation (A.9). By using the function in Equation (A.8) and integrating from $x = -\infty$ to $x = +\infty$, the standard deviation is found to be equal to σ .

$$u(x) = \sigma$$

This feature makes the Gauss function particularly useful. It provides a rationale for combining and manipulating uncertainties. For example, the following processes can be verified using the Gauss function for randomly varying values of x:

— the addition of uncertainties as root-sum squared values;

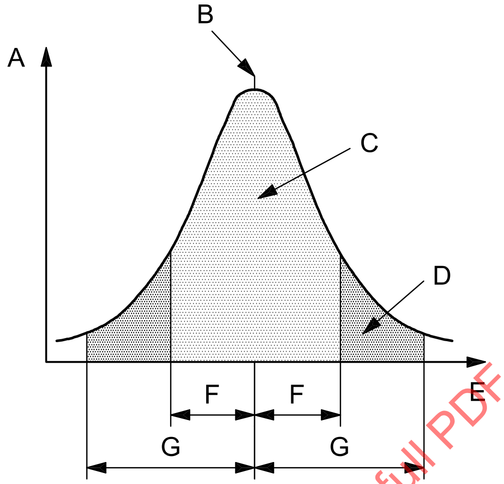
— defining the uncertainty of the mean value of a set of n measurement as $\bar{x} = \frac{\sigma}{n}$;

— defining confidence limits for an uncertainty statement.

However, it remains an artificial probability distribution. If used to represent large numbers of inputs, such as exist in acoustic Doppler systems, the combination of uncertainty using the law of propagation can produce erroneous estimates of uncertainty.

An alternative method, based on numeric methods, is a more rigorous approach. This method is known as the Monte Carlo Simulation.

A.9 Expanded uncertainty, $U(x)$, confidence limits and coverage factors



Key

A probability density

B best estimate (arithmetic mean)

C 68 % chance that true value is within this range

D 95 % chance that true value is within this range

E value

F one standard deviation

G two standard deviation

Figure A.3 — Confidence limits

Using the Gaussian distribution, it can be shown that uncertainty derived from Equation (A.11) covers only 68 % of the values represented by the function, 32 % are dispersed beyond the one standard deviation lines (see Figure A.2). This means that a significant portion of measurements will lie outside the stated band of uncertainty.

The GUM introduces the concept of 'expanded uncertainty' and 'level of confidence'. Again, this is based on the Gauss function. If the stated uncertainty is extended to double its value, from one standard deviation to two standard deviations, the area under the probability function with the expanded limits increases from 68,27 % of the total area to 95,45 % (68 % to 95 %).

Expanded uncertainties at the 95 % level of confidence are written in capitals (U_{95}) and are related to standard uncertainty simply as

$$U_{95} = 2u(x)$$

By doubling an estimate of standard deviation, confidence levels can be stated at the 95 % level of confidence instead of 68 %.

To express the uncertainty at the 95 % level of confidence, a factor, the coverage factor k, is applied to the computed value of standard uncertainty. As indicated, for a normal probability distribution, 95 % of the measurements are covered for a value of $k = 2$. The same approximation applies to all other types of distribution.

For Type A estimations, this applies to large sets of measurements only. Small sets require an adjustment factor, the t factor. An expanded version of Table A.8 is shown as Table A.9 in which t_e includes different confidence limits.

Table A.9 — t_e factors at 68 %, 95 % and 99 % confidence levels

Number n	t_e		
	68 % level	95 % level	99 % level
2	1,84	12,71	63,66
3	1,32	4,30	9,92
4	1,20	3,18	5,84
5	1,14	2,78	4,60
6	1,11	2,57	4,03
8	1,08	2,36	3,50
10	1,06	2,26	3,25
12	1,05	2,20	3,11
15	1,04	2,14	2,98
20	1,03	2,09	2,88

NOTE For small sets, the relationships between the factors is not equal to the coverage factor $k = 2$ or $k = 3$. For this reason, the GUM requires all estimations of uncertainty to be as standard deviations, $k = 1$. Only after the uncertainties have been combined [refer to Equation (A.10)] is a coverage factor applied to represent the final result at the appropriate level of confidence.

In hydrometry, results are presented and stated at the 95 % level of confidence as $U_{95}(x)$.

A.10 Examples of combined uncertainty calculation, U_c

For most measurement systems, the measurement results p, q, r, s are derived from several variables. Examples are

Example A $\quad p = Ax + By - Cz$

Example B $\quad q = \dfrac{Axy}{z}$

Example C $\quad r = Bxy^n$

Example D $\quad s = p + r$

where x, y and z are primary variables and A, B, C, n are constants.

Uncertainty of results is written with the suffix 'c' as follows: $u_c(p), u_c(q), u_c(r), u_c(s)$. Their values, due to uncertainty of primary measurements, $u(x), u(y), u(z)$, are determined by their sensitivity to the primary variables which are expressed as partial derivatives with respect to each primary variable. That is, restating Equation (3) here:

$$u_c(y)^2 = \left(\frac{\partial f}{\partial x_1} u(x_1)\right)^2 + \left(\frac{\partial f}{\partial x_2} u(x_2)\right)^2 + \ldots\ldots + \left(\frac{\partial f}{\partial x_n} u(x_n)\right)^2$$

From which

Example A

$$u_c(p)^2 = \left(\frac{\partial p}{\partial x} u(x)\right)^2 + \left(\frac{\partial p}{\partial y} u(y)\right)^2 + \left(\frac{\partial p}{\partial z} u(z)\right)^2 \text{ or}$$

$$\frac{u_c(p)}{p} = \left|\frac{1}{Ax + By - Cz}\right| \sqrt{\left(Au(x)^2 + Bu(y)^2 + Cu(z)^2\right)}$$

Example B

$$u_c(q)^2 = \left[\frac{\partial q}{\partial x} u(x)\right]^2 + \left[\frac{\partial q}{\partial y} u(y)\right]^2 + \left[\frac{\partial q}{\partial z} u(z)\right]^2$$

so,

$$\left(\frac{u_c(q)}{q}\right)^2 = \left(\frac{z}{Axy}\right)^2 \left[\left(\frac{Ay}{z}\right) \cdot u(x)\right]^2 + \left[\left(\frac{Ax}{z}\right) \cdot u(y)\right]^2 + \left[\frac{Axy}{z} \cdot \frac{-1}{z} \cdot u(z)\right]^2 \text{ or}$$

$$\frac{u_c(q)}{q} = \sqrt{\left(\frac{u(x)}{x}\right)^2 + \left(\frac{u(y)}{y}\right)^2 + \left(\frac{u(z)}{z}\right)^2}$$

Example C

$$u_c(r)^2 = \left(\frac{\partial r}{\partial x} u(x)\right)^2 + \left(\frac{\partial r}{\partial y} u(y)\right)^2 \text{ or}$$

$$\left(\frac{u_c(r)}{r}\right)^2 = \left|\frac{1}{Bxy^n}\right|^2 \left[By^n u(x)\right]^2 + \left[Bnxy^{n-1} u(y)\right]^2 \text{ or}$$

$$\frac{\Delta r}{r} = \sqrt{\left[\frac{u(x)}{x}\right]^2 + \left[n\frac{u_c(y)}{y}\right]^2}$$

