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Geometrical product specifications (GPS) — Filtration —

Part 22: Linear profile filters: Spline filters

Spécification géométrique des produits (GPS) — Filtrage —

Partie 22: Filtres de profil linéaires: Filtres splines

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Foreword

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The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In other circumstances, particularly when there is an urgent market requirement for such documents, a technical committee may decide to publish other types of normative document:

— an ISO Publicly Available Specification (ISO/PAS) represents an agreement between technical experts in an ISO working group and is accepted for publication if it is approved by more than 50 % of the members of the parent committee casting a vote;

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An ISO/PAS or ISO/TS is reviewed after three years in order to decide whether it will be confirmed for a further three years, revised to become an International Standard, or withdrawn. If the ISO/PAS or ISO/TS is confirmed, it is reviewed again after a further three years, at which time it must either be transformed into an International Standard or be withdrawn.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TS 16610-22 was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

ISO/TS 16610 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Filtration*:

— *Part 1: Overview and basic concepts*

— *Part 20: Linear profile filters: Basic concepts*

— *Part 22: Linear profile filters: Spline filters*

— *Part 29: Linear profile filters: Spline wavelets*

— *Part 31: Robust profile filters: Gaussian regression filters*

— *Part 32: Robust profile filters: Spline filters*

— *Part 40: Morphological profile filters: Basic concepts*

- *Part 41: Morphological profile filters: Disk and horizontal line-segment filters*
- *Part 49: Morphological profile filters: Scale space techniques*

The following parts are under preparation:

- *Part 21: Linear profile filters: Gaussian filters*
- *Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets*
- *Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets*
- *Part 30: Robust profile filters: Basic concepts*
- *Part 42: Morphological profile filters: Motif filters*
- *Part 60: Linear areal filters: Basic concepts*
- *Part 61: Linear areal filters: Gaussian filters*
- *Part 62: Linear areal filters: Spline filters*
- *Part 69: Linear areal filters: Spline wavelets*
- *Part 70: Robust areal filters: Basic concepts*
- *Part 71: Robust areal filters: Gaussian regression filters*
- *Part 72: Robust areal filters: Spline filters*
- *Part 80: Morphological areal filters: Basic concepts*
- *Part 81: Morphological areal filters: Sphere and horizontal planar segment filters*
- *Part 82: Morphological areal filters: Motif filters*
- *Part 89: Morphological areal filters: Scale space techniques*

Introduction

This part of ISO/TS 16610 is a geometrical product specification (GPS) Technical Specification and is to be regarded as a global GPS Technical Report (see ISO/TR 14638). It influences the chain links 3 and 5 of all chains of standards.

For more detailed information about the relation of this part of ISO/TS 16610 to the GPS matrix model, see Annex F.

This part of ISO/TS 16610 develops the terminology and concepts of spline filters.

The spline filter has an advantage over a conventional phase correct filter in that for open profiles, the ends of the measured profile are still usable.

Geometrical product specifications (GPS) — Filtration —

Part 22:
Linear profile filters: Spline filters

1 Scope

This part of ISO/TS 16610 specifies spline filters for the filtration of profiles. It specifies in particular how to separate the long and short wave component of a profile.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO/TS 16610-1:2006, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic terminology*

ISO/TS 16610-20:2006, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*

International vocabulary of basic and general terms in metrology (VIM). BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 2nd ed., 1993

3 Terms and definitions

For the purposes of this document, the terms and definitions given in VIM, ISO/TS 16610-1 and ISO/TS 16610-20, and the following apply.

3.1
spline
linear combination of piecewise polynomials, with a smooth fit between the pieces

NOTE The degree of the spline is equal to the degree of the polynomial of the highest degree used, e.g. a cubic spline is made of cubic polynomials.

3.2
cardinal spline
basic function of the space of **splines** (3.1) with infinite support

3.3
natural spline
spline (3.1) which is a straight line beyond the end points

3.4
spline profile filter
linear profile filter based on **splines** (3.1)

NOTE For the purposes of this part of ISO/TS 16610, the result of the low-pass filtering is a spline.

4 Spline profile filters

4.1 General

For a spline filter to conform with this part of ISO/TS 16610, it shall satisfy the filter equations given in 4.3.2, for open profiles, and 4.3.3, for closed profiles.

NOTE A comparison with Gaussian filters is given in Annex B, and illustrative examples are given in Annex C. A concept diagram for the concepts of spline filters is given in Annex D, and the relationship to the filtration matrix model is given in Annex E.

4.2 Weighting function

The weighting function of a spline profile filter cannot be given by a simple closed formula. Filter equations are therefore used instead of weighting functions to describe the spline profile filters. However, where necessary, a numerical calculation of the weighting function for a spline profile filter is always possible. If the sampling interval Δx is small enough, and the spline profile filter is based on cardinal cubic splines, the weighting function $s(x)$, with a default value of $\beta = 0$, can be approximated by the continuous function

$$s(x) = \frac{\pi}{\lambda_c} \sin\left(\sqrt{2}\frac{\pi}{\lambda_c}|x| + \frac{\pi}{4}\right) \exp\left(-\sqrt{2}\frac{\pi}{\lambda_c}|x|\right) \quad (1)$$

NOTE This is the weighting function of the ideal spline profile filter, based on cardinal cubic splines. A graph of this function is shown in Figure 1.

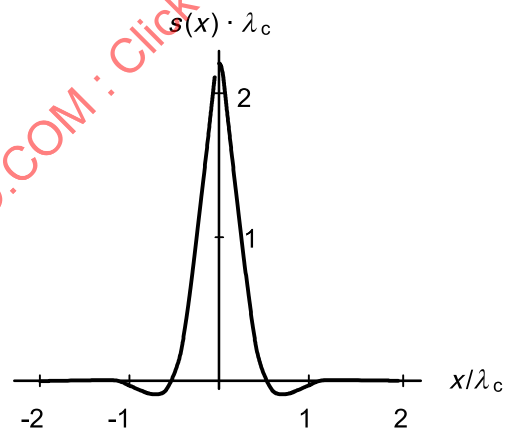


Figure 1 — Weighting function of the ideal spline profile filter, based on cardinal cubic splines

4.3 Filter equations

4.3.1 General

The following filter equations for spline profile filters are based on cardinal cubic splines (for more details, see [1] and [2]).

4.3.2 Filter equation for the non-periodic spline profile filter

Non-periodic spline profile filters should be used in cases of filtering open profiles. The filter equation for the non-periodic spline profile filter is given by

$$\left[1+\beta\alpha^2 P+(1-\beta)\alpha^4 Q\right] w = z \qquad (2)$$

with the matrix

$$P=\begin{pmatrix} 1 & -1 & & & & & \\ -1 & 2 & -1 & & & & \\ & -1 & 2 & -1 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & -1 & 2 & -1 & & \\ & & & -1 & 2 & -1 & \\ & & & & -1 & 1 & \end{pmatrix} \quad Q=\begin{pmatrix} 1 & -2 & 1 & & & & \\ -2 & 5 & -4 & 1 & & & \\ 1 & -4 & 6 & -4 & 1 & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & 1 & -4 & 6 & -4 & 1 \\ & & & 1 & -4 & 5 & -2 \\ & & & & 1 & -2 & 1 \end{pmatrix} \qquad (3)$$

with n rows, n columns and the parameters

$$\alpha=\frac{1}{2\sin\frac{\pi\Delta x}{\lambda_c}} \text{ and } 0 \leqslant \beta \leqslant 1 \qquad (4)$$

where

n is the number of extracted data values of the profile;

z is the vector of dimension n of the profile values before filtering;

w is the vector of dimension n of the profile values in the filtered profile;

λ_c is the cut-off wavelength of the profile filter;

Δx is the sampling interval.

NOTE 1 The vector w gives the profile values of the long wave component. The short wave component r can be obtained by the difference vector $r = z - w$, i.e. by subtracting the long wave component values obtained by the filtering process from the extracted data values of the profile.

NOTE 2 β is called the tension parameter, and controls how tightly the spline curve fits through the data points.

NOTE 3 Algorithms to solve matrix equations can be found in [6].

4.3.3 Filter equation for the periodic spline profile filter

Periodic spline profile filters should be used in cases of filtering closed profiles. The filter equation for the periodic spline profile filter is given by

$$\left[1+\beta\alpha^2\tilde{P}+(1-\beta)\alpha^4\tilde{Q}\right]\tilde{w}=\tilde{z} \tag{5}$$

with the matrix

$$\tilde{P}=\begin{pmatrix} 2 & -1 & & & & -1 \\ -1 & 2 & -1 & & & \\ & -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots \\ & & -1 & 2 & -1 & \\ & & & -1 & 2 & -1 \\ -1 & & & & -1 & 2 \end{pmatrix} \quad \tilde{Q}=\begin{pmatrix} 6 & -4 & 1 & & & & 1 & -4 \\ -4 & 6 & -4 & 1 & & & & 1 \\ 1 & -4 & 6 & -4 & 1 & & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & & \\ & & 1 & -4 & 6 & -4 & 1 & \\ 1 & & & 1 & -4 & 6 & -4 & \\ -4 & 1 & & & 1 & -4 & 6 & \end{pmatrix} \tag{6}$$

with n rows, n columns and the parameters

$$\alpha=\frac{1}{2\sin\dfrac{\pi\Delta x}{\lambda_c}} \text{ and } 0\leqslant\beta\leqslant 1 \tag{7}$$

where

n is the number of extracted data values of the profile;

$\tilde{z}$ is the vector of dimension n of the profile values before filtering;

$\tilde{w}$ is the vector of dimension n of the profile values in the filtered profile;

λ_c is the cut-off wavelength of the profile filter;

Δx is the sampling interval.

NOTE 1 The vector $\tilde{w}$ gives the profile values of the long wave component. The short wave component $\tilde{r}$ can be obtained by the difference vector $\tilde{r}=\tilde{z}-\tilde{w}$, i.e. by subtracting the long wave component values obtained by the filtering process from the extracted data values of the profile.

NOTE 2 β is called the tension parameter, and controls how tightly the spline curve fits through the data points.

NOTE 3 Algorithms to solve matrix equations can be found in [6].

4.4 Transmission characteristics

4.4.1 General

The following transmission characteristics for spline profile filters are based on cardinal cubic splines. For more details, see [3]. For the influence of the sampling interval, see Annex A.

4.4.2 Transmission characteristic of the long wave profile component

The filter characteristic (see Figure 2) is determined from the filter equations of the spline profile filter by means of the Fourier transformation. The filter characteristic for the long wave component is approximated (for very small Δx) by the following equation:

$$\frac{a_1}{a_0} = \left[1 + \beta\alpha^2 \sin^2 \frac{\pi\Delta x}{\lambda} + 16(1-\beta)\alpha^4 \sin^4 \frac{\pi\Delta x}{\lambda}\right]^{-1} \quad (8)$$

where

a_0 is the amplitude of the sinusoidal profile before filtering;

a_1 is the amplitude of the sinusoidal profile in the long wave component;

λ is the wavelength of the sine profile $(\lambda \geqslant 2\Delta x)$;

Δx is the sampling interval.

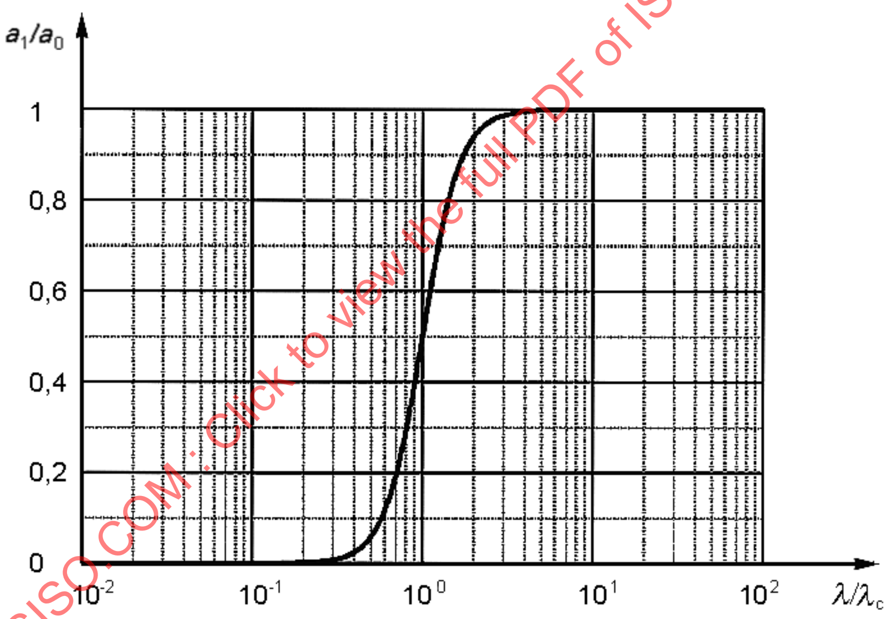


Figure 2 — Transmission characteristic for the long wave profile component $(\beta = 0\,;\, \Delta x = \lambda_c / 200)$

4.4.3 Transmission characteristic of the short wave profile component

The transmission characteristic of the short wave profile component is complementary to the transmission characteristic of the long wave profile component. The short wave profile component is the difference between the surface profile and the long wave profile component. The filter characteristic for the short wave profile component (see Figure 3) has the following equation:

$$\frac{a_2}{a_0} = 1 - \frac{a_1}{a_0} = \left[4\beta\alpha^2 \sin^2 \frac{\pi\Delta x}{\lambda} + 16(1-\beta)\alpha^4 \sin^4 \frac{\pi\Delta x}{\lambda}\right]\left[1 + \beta\alpha^2 \sin^2 \frac{\pi\Delta x}{\lambda} + 16(1-\beta)\alpha^4 \sin^4 \frac{\pi\Delta x}{\lambda}\right]^{-1} \quad (9)$$

where

a_2 is the amplitude of the sinusoidal profile short wave component.

NOTE The sum of the transmission characteristics of the short wave profile component and the long wave profile component equals one.

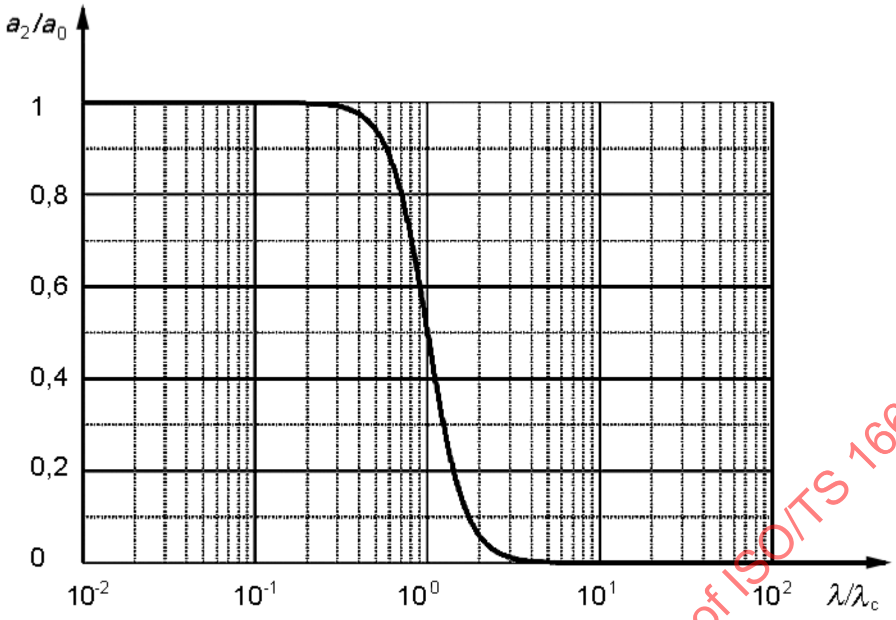


Figure 3 — Transmission characteristic for the short wave profile component $\left(\beta = 0; \Delta x = \lambda_c / 200\right)$

5 Recommendations

5.1 Nesting Index (cut-off values)

It is recommended that the nesting index (the cut-off value λ_c) be chosen from a logarithmic series (constant ratio) of values. Experience has shown that a constant ratio of approximately the square root of ten between successive scale values is optimum. The nesting index should be chosen from the following series of values:

... 2,5 µm; 8 µm; 25 µm; 80 µm; 250 µm; 0,8 mm; 2,5 mm; 8 mm; 25 mm;

5.2 Tension parameter (β)

If not otherwise specified, the tension parameter β takes the value zero.

5.3 Implementation

It is strongly recommended that the matrix equations given in 4.3.2 and 4.3.3 be used to implement the spline filter.

6 Filter designation

Spline filters in accordance with this part of ISO/TS 16610 are designated

FPLS

See also ISO/TS 16610-1:2006, Clause 5.

Annex A
(informative)

Influence of the sampling interval

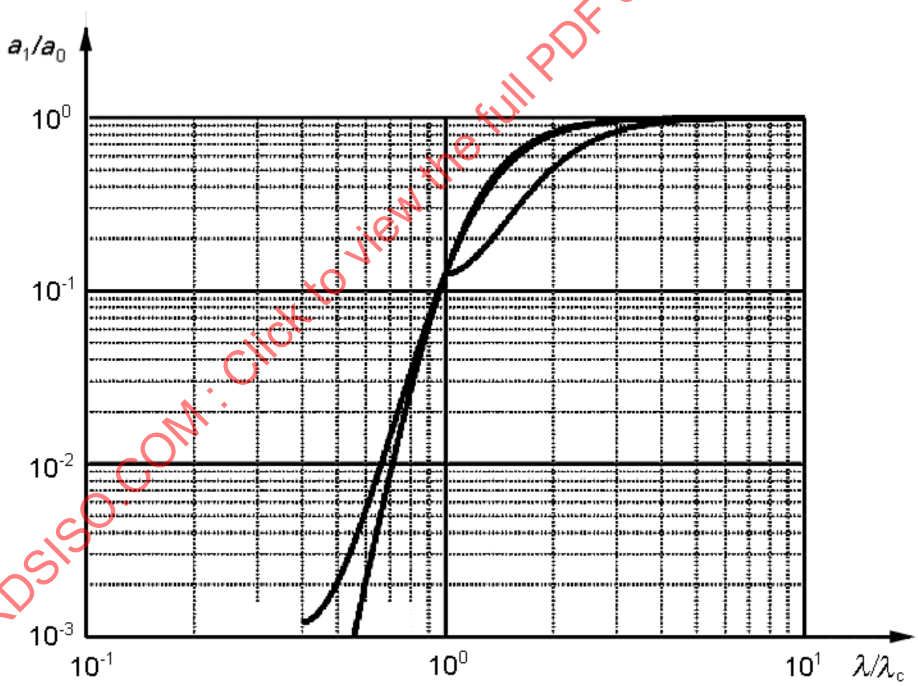
The transfer characteristic of the spline profile filter, as for every digital filter, depends on the sampling interval Δx (see Figure A.1). If Δx is small enough, the sine in all equations may be replaced by its argument.

EXAMPLE The error for $\Delta x = 0{,}01\lambda_c$ is $\varepsilon < 5{,}2 \times 10^{-6}$

Consequently, the equation for the parameter α in 4.2.3 simplifies to

$$\frac{a_1}{a_0} = \left[1 + \beta\left(\frac{\lambda_c}{\lambda}\right)^2 + (1-\beta)\left(\frac{\lambda_c}{\lambda}\right)^4 \right]^{-1} \tag{A.1}$$

i.e. it no longer depends on Δx. However, according to the sampling theorem (see ISO/TS 14406), this is only valid for $\lambda \geqslant 2\Delta x$.



NOTE According to the sampling theorem, $\lambda / \lambda_c \geqslant 2\Delta x / \lambda_c$ is valid.

Figure A.1 — Influence of the sampling interval Δx on the transfer function of the spline profile filter for the parameters $\beta = 0$; $\Delta x / \lambda_c$: 0,1; 0,2 and 0,5

Annex B
(informative)

Comparison of spline profile filter and Gaussian filter

In order to compare the transmission characteristics of the spline profiler filter and the Gaussian filter, a graphical representation of the deviation, in accordance with ISO 11562:1996, is given (see Figure B.1).

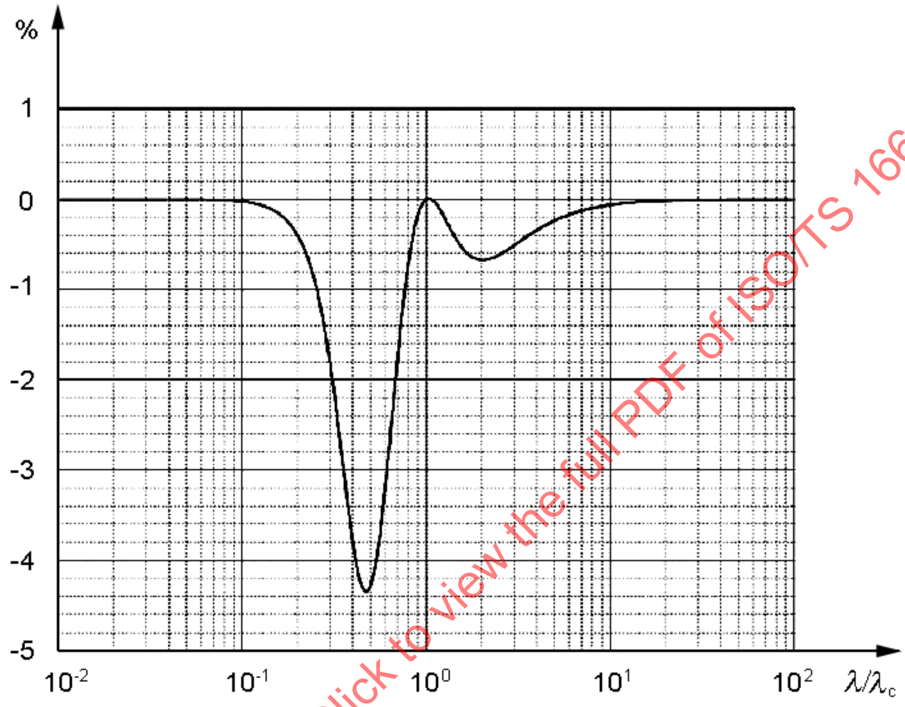


Figure B.1 — Deviation of the transmission characteristic of the spline profile filter from the Gaussian filter $(\beta = 0{,}625242; \Delta x = \lambda_c / 200)$

Annex C
(informative)

Illustrative examples

The following examples for the application of the spline profile filter and the Gaussian filter (currently standardized profile filter) are given for information purposes.

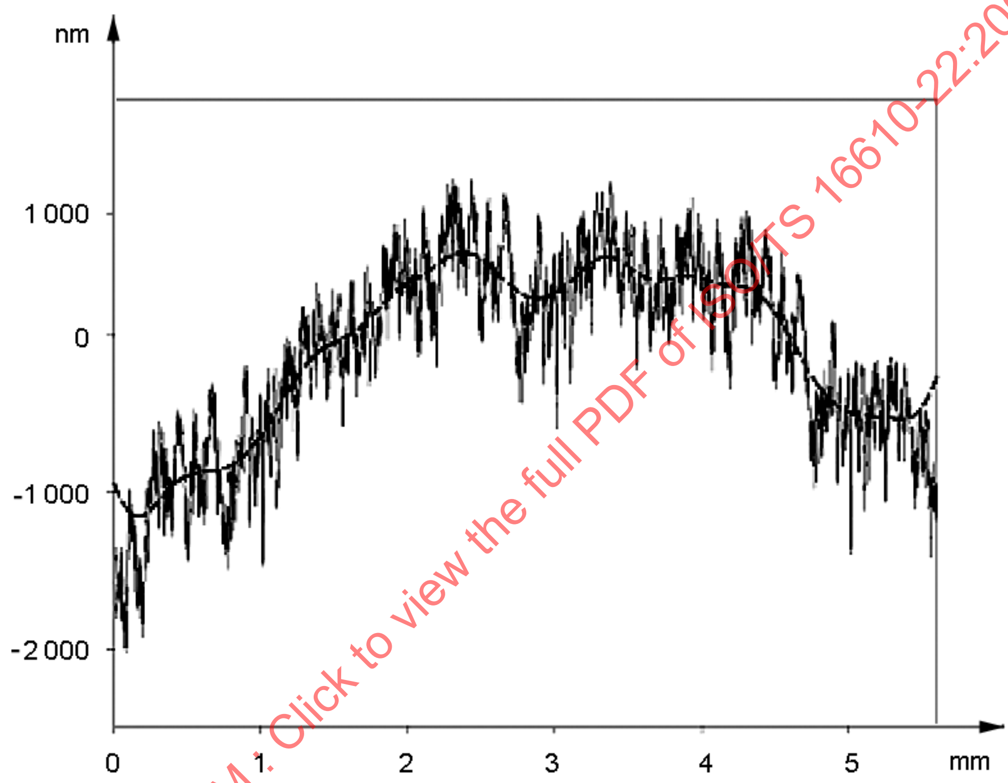


Figure C.1 — Gaussian filter with $\lambda_c = 0{,}8$ mm**, in accordance with ISO 11562:1996**

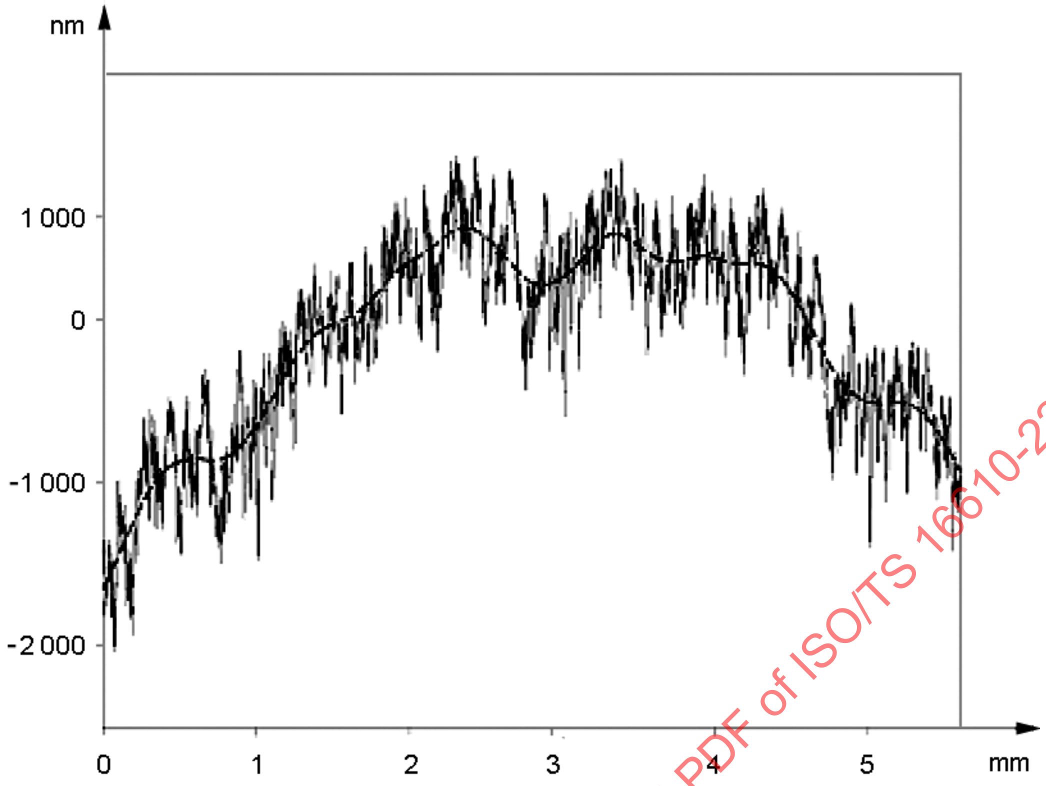


Figure C.2 — Spline profile filter with λ_c = 0,8 mm **and** β = 0

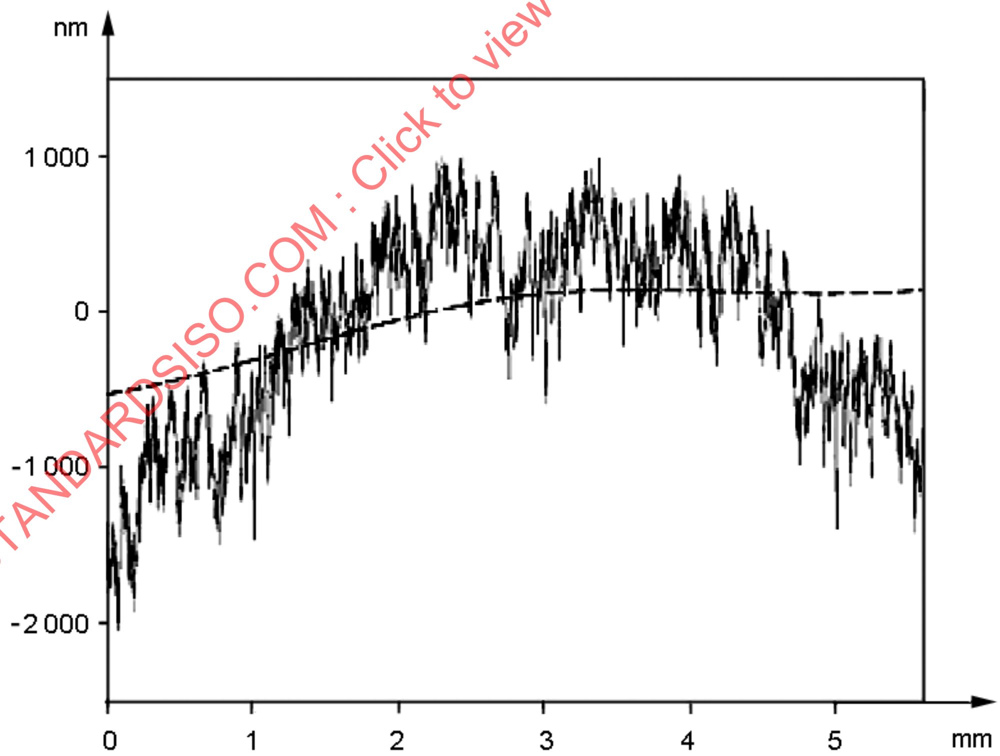


Figure C.3 — Gaussian filter with λ_c = 8 mm**, in accordance with ISO 11562:1996**

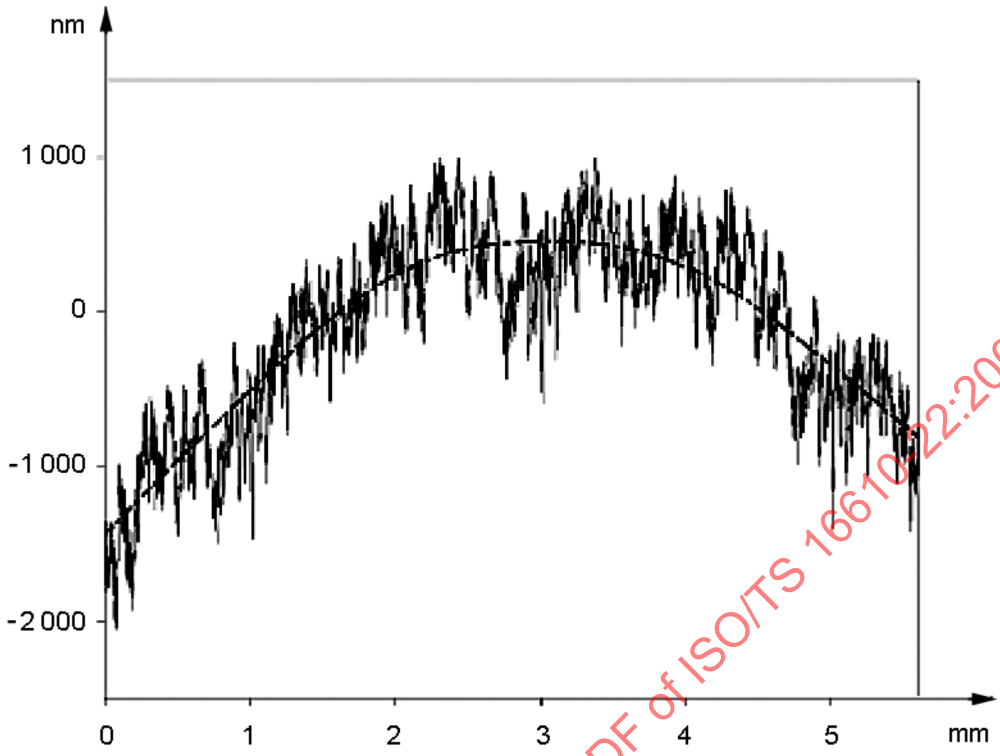


Figure C.4 — Spline profile filter with $\lambda_c = 8$ mm **and** $\beta = 0$