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Cylindrical involute gears and gear pairs —

Part 1: Concepts and geometry

Roues et engrenages cylindriques en développante de cercle —

Partie 1: Concepts et géométrie

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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This document was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 1, *Nomenclature and wormgearing*.

This first edition of ISO 21771-1 cancels and replaces ISO 21771:2007, which has been technically revised.

The main changes are as follows:

— the sign convention for internal gears used in the ISO 6336 series[8] has been adopted. The negative value for the number of teeth of an internal gear is applied to the diameters and centre distance, so these dimensions of internal gears have negative values;

— flank direction has been renamed as hand of helix and sign (+/-) of helix angle is used;

— a definition of normal surface has been added and this is used rather than normal plane;

— the annex on tooth thickness was removed because it is now addressed in ISO 21771-2.

Additional material has been added to cover:

— calculation of form diameters for tooth tip corner radius and tooth root fillet radius in the transverse plane for an involute cylindrical gear (Clauses 10, 11 and Annex B);

— calculation of the tooth tip corner radius for a specified form diameter and tip diameter of an involute cylindrical gear;

— calculation of a radius tangent to the involutes of adjacent teeth at root or tip diameter (Annex A);

— generated tooth root fillet shape for individual involute cylindrical gears (Annex B);

— concepts and parameters for involute cylindrical gear pairs with crossed axes (Clause 6 and Annex C);

— geometry of surfaces in contact (Annex D);

— projection of a transverse plane profile of a tooth onto another plane (Annex E);

— interface to ISO 10828 for involute worm gear geometry (Annex F).

A list of all parts in the ISO 21771 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Cylindrical involute gears and gear pairs —

Part 1:
Concepts and geometry

1 Scope

This document specifies the geometric concepts and parameters for cylindrical gears with involute helicoid tooth flanks. Flank modifications are included. The formulae in this document apply to all pressure angles.

It also covers the concepts and parameters for involute cylindrical gear pairs with parallel or crossed axes, and a constant gear ratio. Gear and mating gear in these gear pairs have the same basic rack tooth profile.

2 Normative references

There are no normative references in this document.

3 Terms, definitions, symbols, subscripts and units

3.1 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1.1
basic rack tooth profile for involute gear teeth
tooth profile of a normal section through the teeth of a basic rack which corresponds to an external spur gear with number of teeth $z = \infty$ and diameter $d = \infty$

Note 1 to entry: The tooth of the basic rack tooth profile is bounded by the tip line at the top and by the parallel root line at the bottom. The fillet between the straight part of the profile and the root line is a circular arc with a radius equal to ρ_{fP}.

3.1.2
counterpart rack tooth profile
rack tooth profile symmetrical to the basic rack tooth profile about the datum line P-P and displaced by half a pitch relative to it

3.1.3
nominal involute flank
pure involute flank prior to any modifications

Note 1 to entry: See 4.4 for more information on a gear tooth involute flank.

3.1.4
tip alteration coefficient
k
change to the addendum relating to the standard addendum of one normal module, it is made non-dimensional by dividing by the normal module

Note 1 to entry: Tip alteration is also known as addendum modification, tip shortening or truncation.

3.1.5
generating gear pair
generating tool (rack, hob, pinion-type cutter, or grinding wheel) and the gear being machined during gear tooth machining process

3.2 Symbols

Table 1 provides all the symbols used in this document.

Table 1 — Symbols

Symbol	Description	Subclause
A	pinion lower end point of meshing (near pinion root)	5.5.6.1
a_w	working centre distance of a cylindrical gear pair	5.3.3, 5.3.5, 6.3.12
a_{w0}	centre distance in the generating process with pinion-type cutter	11.2
B	pinion lower point of single tooth contact (LPSTC)	5.5.6.1
b	facewidth	4.3.9
b_{Ea}	length of relief near tip	8.5.2
b_{Ef}	length of relief near root	8.5.2
b_F	usable facewidth	4.3.9
b_w	active facewidth (the facewidth used)	5.5.9.2
C	working pitch point with subscript: amount of relief for modifications	5.3.5 8
C_{ay}	modification of the profile by function	8.6
C_{Ea}	amount of triangular end relief modification at tip	8.5.2
C_{Ef}	amount of triangular end relief modification at root	8.5.2
$C_{H\alpha}$	amount of transverse profile slope modification	8.3.3
$C_{H\beta}$	amount of flank line slope modification	8.4.2
$C_{i,j}$	amount of modification at point (i,j)	8.5.1
C_{α}	amount of profile crowning (barrelling)	8.3.4
$C_{\alpha a}$	amount of tip relief	8.3.2
$C_{\alpha f}$	amount of root relief	8.3.2
C_{β}	amount of flank line crowning	8.4.3
$C_{\beta I}$, $C_{\beta II}$	amount of end relief	8.4.1
$C_{\beta y}$	modification of the flank line by function	8.6
$C_{\Sigma y}$	modification of the flank surface by function	8.6
c	tip clearance	5.3.8, 6.3.15
c_F	form over dimension	5.5.5
D	pinion highest point of single tooth contact (HPSTC)	5.5.6.1
d	reference diameter	4.3.4
d_a	tip diameter	4.6.5
d_{a0}	tip diameter of pinion-type cutter	11.1

Table 1 *(continued)*

Symbol	Description	Subclause
d_{b}	base diameter	4.4.2
d_{b0}	base diameter of the pinion-type cutter	11.1
d_{bK}	chamfering base diameter	8.2.2
$d_{C\alpha a}$	tip relief start diameter	8.3.2
$d_{C\alpha f}$	root relief start diameter	8.3.2
d_{Cf}	Profile control diameter	8.3.2
d_{Ea}	diameter for start of triangular end relief at tooth tip	8.5.2
d_{Ef}	diameter for start of triangular end relief at tooth root	8.5.2
d_{Fa}	tip form diameter	9.7
d_{Ff}	root form diameter	5.5.2, 9.7
d_{f}	root diameter (nominal dimension)	4.6.7
d_{fE}	root diameter produced	9.6
d_{Na}	active tip diameter	5.5.2, 6.4.3
d_{Nf}	start of active profile diameter (SAP diameter, active root diameter)	5.5.2, 6.4.3
d_{v}	V-circle diameter	4.6.2
d_{w}	working pitch diameter	5.3.5
d_{y}	Y-circle diameter	4.6.3
d_{0}	reference diameter of pinion-type cutter	11.1.1
E	pinion upper end point of meshing (near pinion tip)	5.5.6.1
E_{sni}	lower tooth thickness limit deviation for normal tooth thickness at reference cylinder	9.4
E_{sns}	upper tooth thickness limit deviation for normal tooth thickness at reference cylinder	9.4
e_{n}	normal space width on the reference cylinder	4.8.7
e_{P}	space width of the basic rack tooth profile	4.3.3
e_{t}	transverse space width on the reference cylinder	4.8.4
e_{yn}	normal space width on the y-cylinder	4.8.7
e_{yt}	transverse space width on the y-cylinder	4.8.4
F	force	D.3
g_{a}	length of addendum path of contact	5.5.6.2, 6.4.5.2
g_{f}	length of dedendum path of contact	5.5.6.2, 6.4.5.2
$g_{yft}(\theta)$	angle from line of centres to point on the gear fillet generated with pinion-type cutter	B.4.2
g_{α}	length of path of contact	5.5.6.2
$g_{\alpha y}$	distance of a point Y from working pitch point C	5.7.5
g_{β}	overlap roll length (arc of contact)	5.5.9.4
h	tooth depth (between tip line and root line)	4.7.1
h_{a}	addendum from reference pitch circle	4.7.2
h_{aP}	addendum of the basic rack tooth profile	4.3.2
h_{aP0}	addendum of the counterpart of the basic rack tooth profile	9.2.2
h_{aw}	addendum from working pitch circle	4.7.2
h_{a0}	addendum of pinion-type cutter	11.1.1
h_{FaP0}	straight part of tip flank of tool-generating profile	9.2.2
h_{FfP}	portion of the dedendum to root form line, of the basic rack tooth profile	4.3.2
h_{FfP0}	portion of the dedendum to root form line of the counterpart rack tooth profile	9.2.1
h_{fP0}	dedendum of the counterpart rack tooth profile and rack tool	9.2.2
h_{f}	dedendum from reference circle	4.7.2

Table 1 *(continued)*

Symbol	Description	Subclause
h_{fP}	dedendum of the basic rack tooth profile	4.3.3
h_{fw}	dedendum from working pitch circle	4.7.2
h_K	height of the tip corner chamfering or tip corner rounding	8.2.2
h_P	tooth depth of basic rack tooth profile	4.3.2
h_w	working depth of teeth in a gear pair	5.3.7, 6.3.14
i	transmission ratio of a gear pair	5.3.2
inv	involute function (not a variable)	4.4.5
j_{bn}	normal base backlash	5.6.4, 6.5.2
j_{bt}	transverse backlash	5.6.2
j_r	radial backlash	5.6.5
j_t	circumferential backlash at the reference circle	5.6.3, 6.5.3
j_{wn}	working normal backlash	6.5.2
j_{wt}	circumferential backlash at the working pitch circle	5.6.3, 6.5.3
K_g	sliding factor	5.7.6
K_{ga}	sliding factor at tooth tip	5.7.6
K_{gf}	sliding factor at tooth root	5.7.6
k	tip alteration coefficient	3.1.3, 4.6.4, 5.3.9, 6.3.16
L_{Ca}	tip relief roll length	8.3.2
L_{Cf}	root relief roll length	8.3.2
L_{CI}, L_{CII}	length of end relief	8.4.1
L_{Ea}	tip roll length of triangular end relief modification	8.5.2
L_{Ef}	root roll length of triangular end relief modification	8.5.2
L_y	length of roll to y-cylinder	4.4.10
L_{yt}	length of involute profile to y-cylinder	4.4.11
l_{max}	maximum length of a contact line	5.5.10
M_y	a point on a tooth flank where radius of curvature is calculated	7.1
m	module	4.3.7
m_n	normal module	4.3.7
m_t	transverse module	4.3.7
m_x	axial module	4.3.7
N	number of tooth or pitch	4.2.6
n_a	rotational speed of driving gear (rpm)	5.3.2
n_b	rotational speed of driven gear (rpm)	5.3.2
O	centre of a circle	6.3.4
P_d	diametral pitch	4.3.8
P_{nd}	normal diametral pitch	4.3.8
p_{bn}	normal base pitch	4.5.5.1
p_{bt}	transverse base pitch	4.5.5.1
p_{en}	normal base pitch on the path of contact	4.5.5.3
p_{et}	transverse base pitch on the path of contact	4.5.5.2
p_n	normal pitch on the reference cylinder	4.5.2.2
p_t	transverse pitch on the reference cylinder	4.5.2.1
p_{wn}	normal pitch at the working diameter	6.3.6, 6.5.2

Table 1 *(continued)*

Symbol	Description	Subclause
p_x	axial pitch	4.5.4
p_{yn}	normal pitch on the y-cylinder	4.5.3
p_{yt}	transverse pitch on the y-cylinder	4.5.3
p_z	lead	4.4.6
pr	protuberance of the tool (as seen in ISO 6336-3)	9.2.1
q	machining allowance on tooth flank	9.3
q_{Fs}	magnitude (amount) of undercut in transverse plane	8.2.1
R'	first principal radius of curvature of surface	7.4
R''	second principal radius of curvature of surface	7.4
R_c	radius of curvature of trochoid at point M	10.4
R_{fp}	radius of curvature of the basic rack profile at point Q	10.4
$R_{tro\text{-}y}$	radius of the fillet at point Q	10.4
r_{a0}	tip radius of the pinion-type cutter	11.1.1
r_{b0}	base radius of the pinion-type cutter	11.2
r_{ea}	x axis of ellipse	10.4
r_{eb}	y axis of ellipse	10.4
r_{Fa0}	tip form radius of the pinion-type cutter	11.1.1
r_{Ff}	root form radius	10.3
r_{inv}	radius to point on involute	10.1, 11.4.4.1
r_{M0}	radius for the centre of the tool tooth tip rounding of the pinion-type cutter	11.1.1
r_{tro}	radius to point on trochoid	10.1
r_w	manufacturing pitch circle radius of the gear	11.4.3
r_{w0}	manufacturing pitch circle radius of the pinion-type cutter	11.4.3
$r_{ya0}(\theta)$	radial polar coordinate of a point on tip radius of pinion-type cutter	Figure B.1
$r_{yft}(\theta)$	radial polar coordinate of a point on the gear fillet generated with pinion-type cutter	B.4.2
S_α	twist of the transverse profile	8.5.3
S_β	twist of the flank line	8.5.3
s_{aK}	tip transverse tooth thickness when tip chamfering or tip rounding	8.2.2
s_n	normal tooth thickness at the reference diameter	4.8.6
s_{ni}	minimum normal tooth thickness at the reference diameter	9.4
s_{ns}	maximum normal tooth thickness at the reference diameter	9.4
s_P	tooth thickness of the basic rack tooth profile	4.3.3
s_{pr}	residual fillet undercut (on normal surface)	10.1
s_{prt}	residual fillet undercut (transverse plane)	11.4.1
s_t	transverse tooth thickness at the reference diameter	4.8.2
s_{wn}	normal tooth thickness at working diameter	6.3.6, 6.5.2
s_{wt}	transverse tooth thickness at working diameter	6.3.8
s_{yn}	normal tooth thickness at the Y circle diameter	4.8.6
s_{yt}	transverse tooth thickness at the Y circle diameter	4.8.2
T	tangent point on base circle of line normal to involute	4.4.9
T_1	point of contact between the line of action and the base circle of pinion	5.5.6.1
T_2	point of contact between the line of action and the base circle of gear wheel	5.5.6.1
T_1	unit vector of reference helix	4.10
T_2	unit vector of generator	4.10

Table 1 *(continued)*

Symbol	Description	Subclause
T_{M1}	tangency point between the base cylinder of the pinion and a line normal to a contact line through point M_y	7.1
T_{M2}	tangency point on the base cylinder of the gear wheel and a line normal to a contact line through point M_y	7.1
U	involute point of origin	4.4.9
u	gear ratio	5.3.1
v	circumferential velocity at reference diameter	5.7.2
v_b	circumferential velocity at base diameter	5.7.2
v_g	sliding velocity	5.7.5
v_{ga}	sliding velocity at the active addendum	5.7.5
v_{gf}	sliding velocity at the active dedendum	5.7.5
v_{Mg}	sliding velocity at point M	6.6.4
v_n	normal velocity	5.7.3
v_{ry}	rolling velocity at diameter d_y	5.7.4
v_w	circumferential velocity of the working pitch circles	5.7.2, 5.7.6
v_y	circumferential velocity at diameter d_y	5.7.2
x	profile shift coefficient	4.3.10
x_E	generating profile shift coefficient	9.2.1
x_{Ei}	lower limit generating profile shift coefficient	9.5
x_{Es}	upper limit generating profile shift coefficient	9.5
x_{EsV}	profile shift coefficient for rough machining, upper limit	9.5
x_{EiV}	profile shift coefficient for rough machining, lower limit	9.5
x_{Eu}	generating profile shift coefficient at undercut limit	9.8
x_0	profile shift coefficient of the pinion-type cutter	11.1.1
X_1	point on x axis of coordinate system of surface 1	D.1
X_2	point on x axis of coordinate system of surface 2	D.1
Y	any point on a tooth flank or involute	4.4.3
Y_1	point on y axis of coordinate system of surface 1	D.1
Y_2	point on y axis of coordinate system of surface 2	D.1
z	number of teeth	4.2.5
z_a	number of teeth of driving gear	5.3.2
z_b	number of teeth of driven gear	5.3.2
z_0	number of teeth of pinion-type cutter	11.1.1
Z_1	point on z axis of coordinate system of surface 1	D.1
Z_2	point on z axis of coordinate system of surface 2	D.1
α_a	pressure angle at tip circle	5.5.8.3
α_{Fa}	pressure angle at the tip form diameter d_{Fa}	A.2.1
α_{Fa0}	pressure angle on the radius r_{Fa0} of pinion-type cutter	11.1.1
α_{Ff}	pressure angle at root form circle	9.7
α_{KP0}	normal chamfering pressure angle of the counterpart rack tooth profile	8.2.2
α_{KP0t}	transverse chamfering pressure angle of the counterpart rack tooth profile	8.2.2
α_{Mt0}	transverse pressure angle for the radius at the point M of the pinion-type cutter	11.1.1
α_{NP}	pressure angle at start of active profile	5.5.2.2
α_n	normal pressure angle	4.4.4
α_P	pressure angle of the basic rack tooth profile	4.4.4

Table 1 *(continued)*

Symbol	Description	Subclause
α_{P0}	pressure angle of the counterpart rack tooth profile	9.2.1
α_{pr0}	pressure angle of the protuberance section of the counterpart rack tooth profile	9.2.1
α_t	transverse pressure angle	4.4.3
$\alpha_{w\,min}$	working normal pressure angle at minimum (zero-backlash) centre distance	6.3.12
α_{wn}	working normal pressure angle of gear pair	6.3.13
α_{wt}	working transverse pressure angle of gear pair	5.3.4, 6.3.7
α_{wt0}	working transverse pressure angle in the generating gear pair	11.2
α_{yn}	normal pressure angle at the y-cylinder	4.4.4
α_{yt}	transverse pressure angle at the y-cylinder	4.4.3
β	helix angle	4.4.7
β_b	base helix angle	4.4.7
β_R	Helix angle of right flanks	4.4.7
β_{M0}	helix angle on the circle of radius r_{M0}	11.1.1
β_L	Helix angle of left flanks	4.4.7
β_v	helix angle at diameter d_v	4.3.9
β_w	helix angle at working pitch diameter	5.3.6
β_y	helix angle at y-cylinder	4.4.7
β_0	helix angle of pinion-type cutter	11.1.1
γ	lead angle at reference cylinder	4.4.7
γ_y	lead angle at y-cylinder	4.4.7
Δ	sum of inverse radius of curvature of surfaces	7.4
δ	tilt angle of the contact line at the reference cylinder	4.10
δ_{pr0}	residual fillet undercut angle in transverse plane	11.4.1
δ_w	angle between the principal direction of curvature and working pitch plane	7.2
δ_y	tilt angle of the contact line at y-cylinder	4.10
ε_α	transverse contact ratio	5.5.9.1, 6.4.7.1
$\varepsilon_{\alpha n}$	contact ratio in the common normal working plane of crossed axis gear pair	6.4.7.2
ε_β	overlap ratio (axial contact ratio)	5.5.9.3
ε_γ	total contact ratio	5.5.9.5
ζ	specific sliding	5.7.7
ζ_f	specific sliding at end points of path of contact	5.7.7
η	transverse space width half angle at reference circle	4.8.5
η_b	base transverse space width half angle	4.8.5, 6.3.11
η_{inv}	angle between the start of involute profile on base circle and the point r_{inv} on involute	10.1
η_M	pressure angle at tip form radius minus the angle from tip form radius point to centre of tip rounding circle on pinion-type cutter.	11.1.1
η_{tro}	angle between the start of involute profile on base circle and the point r_{tro} on trochoid	10.1
η_{wt}	transverse space width half angle at working pitch diameter	6.3.8
η_y	transverse space width half angle at Y-circle	4.8.5
θ_{a1}	polar angle on the pinion tooth flank from pitch point to tip circle	5.5.8.3
θ_{a2}	polar angle on the gear tooth flank from pitch point to tip circle	5.5.8.3
θ_{aa1}	angle from the pitch point to the intersection of the tip circles on the pinion’	5.5.8.3
θ_{aa2}	angle from the pitch point to the intersection of the tip circles of an internal gear	5.5.8.3
θ_M	angle used to define point M on ellipse	11.4.5

Table 1 *(continued)*

Symbol	Description	Subclause
$\kappa_{\text{yft}}(\theta)$	angular polar coordinate of a point on the gear fillet generated with pinion-type cutter	B.4.2
λ	angle between the pitch line of the basic rack and the line joining the common normal at the generated point Q crossing the pitch point	10.1
λ_{M0}	polar angle on a pinion-type cutter from centre of tip rounding circle to point where the tip rounding circle transitions to the involute (at tip form diameter)	11.1.1
ξ_{Fa0}	roll angle at tip form circle of pinion-type cutter	9.7
ξ_{Ff}	roll angle at root form circle	9.7
ξ_{Na}	roll angle at active tip circle	5.5.2
ξ_{Nf}	roll angle at active root circle	5.5.2
ξ_{y}	roll angle of the involute at point Y	4.4.9
ρ_{A}	radius of curvature in transverse plane at point A, start of engagement	5.5.7
ρ_{a}	tooth tip corner radius of gear	Annex A
ρ_{aP0}	tooth tip corner radius of the hob	9.2.2
ρ_{a0n}	tooth tip corner radius of pinion-type cutter (on normal surface)	11.1.1
ρ_{B}	radius of curvature in transverse plane at inner point of single pair contact on the pinion	5.5.7, 7.1
ρ_{C}	radius of curvature in transverse plane at point C, working pitch point	5.5.7, 7.1
ρ_{D}	radius of curvature in transverse plane at outer point of single pair contact on the pinion	5.5.7, 7.1
ρ_{E}	radius of curvature in transverse plane at end point of engagement	5.5.7, 7.1
ρ_{f}	tooth root fillet radius of gear	Annex A
ρ_{fP}	tooth root fillet radius on the basic rack tooth profile	10.4
ρ_{fP0}	tooth root fillet radius of the counterpart rack tooth profile	8.2.2
ρ_{M}	radius of curvature in transverse plane at point M	7.1
ρ_{y}	radius of curvature of the involute at diameter d_{y}	4.4.10
ρ_{yn}	radius of curvature on normal surface at diameter d_{y}	4.9
ρ_{yt}	radius of curvature in transverse plane at diameter d_{y}	4.9
Σ	skew angle of crossed axis gear pair	6.3.5, 6.3.12
Σ_{l}	sum of individual lines of contact	5.5.10
Σ_{R}	sum of inverse of main radii of curvature of equivalent ellipsoid	7.4
Σx	sum of profile shift coefficients	5.4
Σx_{E}	sum of profile shift coefficient, non-zero backlash	5.4
τ	angular pitch	4.5.1
ϕ_{yt}	angle of vector radius OM according Y axis	E.1
φ	trochoid/tip parameter	10.1
φ_{w}	angle between the principal radii of curvature in a crossed axis gear pair	7.3
φ_{j}	angular backlash	5.6.6
φ_{α}	transverse angle of transmission	5.5.9.1, 6.4.7.1
φ_{β}	overlap angle	5.5.9.3
φ_{γ}	total angle of transmission	5.5.9.5
$\chi_{\text{ya0t}}(\theta)$	angular polar coordinate of a point on tip radius of the pinion type cutter	Figure B.1
ψ	transverse tooth thickness half angle at reference circle	4.8.3
ψ_{b}	base transverse tooth thickness half angle	4.8.3
ψ_{y}	transverse tooth thickness half angle at Y-circle	4.8.3
ψ_{wt}	transverse tooth thickness half angle at working pitch diameter	6.3.8

Table 1 *(continued)*

Symbol	Description	Subclause
ω	angular velocity	5.5.8.3, 5.7.1, 6.6.2
ω_a	angular velocity of driving gear	5.3.2
ω_b	angular velocity of driven gear	5.3.2
ω_0	angle between the centre line of gear pair and centre point of tooth tip rounding of pinion-type cutter	11.4.2

The variables in Table 1 can have an appended subscript 1 to indicate the variable is for a quantity associated with the pinion or a subscript 2 to indicate the variable is for a quantity associated with the gear wheel.

3.3 Subscripts

Table 2 provides the subscripts used in this document.

Table 2 — Subscripts

Subscript	Description
—	no subscript designates quantities associated with the reference cylinder
a	for quantities associated with the tip of a tooth or for the driving gear
b	for quantities associated with the base cylinder or for the driven gear
E	relating to “generating” (e.g. quantities generated on the cylindrical gear) or to triangular end relief modification
e	for quantities associated with the plane of action
F	for quantities determining form circles and maximum usable flank area
f	for quantities associated with the root
g	for “sliding”
i	for the lower limit in the case of deviations or an index value
K	for quantities resulting from tip corner chamfering
k	for a number of teeth, tooth spaces, pitches or spans
L	for designating left flanks
l	for “left-hand”
M	for designating a measured value
M0	relating to centre of tip rounding circle or ellipse on tool tooth tip
m	for a mean value
max	for a maximum value
min	for a minimum value
N	for active circles
n	for quantities in a normal surface
P	for quantities of the basic rack tooth profile
P0	for quantities of the tool basic rack tooth profile and for counterpart rack tooth profile
R	for designating right flanks
r	for “right-hand”
s	relating to “tooth thickness” or for the upper limit in the case of deviations
t	for quantities in a transverse section
V	for working pitch element, for rough gear cutting
v	for quantities associated with the V-cylinder
W	for measuring base tangent length

Table 2 *(continued)*

Subscript	Description
w	for quantities associated with the working pitch cylinder and working values of a gear pair
x	for quantities in an axial section
y	for values at a point y (on the y-cylinder)
Z	for quantities associated with cylinder dimensions
α	for quantities associated with contact
β	for quantities associated with a tooth trace
γ	for total contact ratio
Σ	for "sum"
0	for quantities associated with the generating tool or the generating gear pair
1	for quantities associated with the pinion (smaller gear) of a gear pair
2	for quantities associated with the gear wheel (larger gear) or internal gear
I	for locating face
II	for the face opposite the locating face

3.4 Units and conventions

3.4.1 Units

The quantities dealt with in this document are to be stated in the following units:

— modules, lengths and linear dimensions in millimetres (mm);

— linear velocities (circumferential, normal, rolling, sliding) in metres per second (m/s);

— angles which are to be used in formulae in radians (rad);

— angles which can be used for entries or to display results in degrees (°);

— angular velocity in radians per second (rad/s).

3.4.2 Conventions

For internal gears, the number of teeth is given a negative value. So, in this document, as in the ISO 6336 series, the centre distance and diameters of internal gears have negative values. The notation $|z|$, denotes the absolute value, which is always positive, e.g. $|-50| = +50$. The expression $\frac{z}{|z|}$ is used to extract the sign of the tooth number and is convenient for programming. In particular, it is often used to determine the appropriate sign for an element of an expression; the result is 1 for external gears and −1 for internal gears.

In this document, the driving gear is not used to define point A on the path of contact. Instead, point A is where the line of action intersects the active tip diameter, d_{Na2} , of the gear wheel. The last point, E, is where the line of action intersects the active tip diameter, d_{Na1} , of the pinion. See 5.5.6.1.

4 Individual cylindrical gears

4.1 General

In this clause, the geometry of gear teeth is described using a generation process based on zero-backlash engagement with a basic rack. The relationships are valid for any basic rack, but the standard basic rack tooth profile (see ISO 53) is used for illustration purposes. The standard basic rack tooth profile of the tooth system has straight flanks. Its datum line is the straight line on which the nominal dimensions of tooth thickness and space width are defined as equal to half the pitch. The standard basic rack tooth profile has the same pressure angles for the left and right flanks and the dedendum equal to the addendum plus tip clearance for the mate. All the teeth have the same nominal helix angle. Formulae for calculation of start of involute diameter are given in Clause 10 for hob and rack type cutters and in Clause 11 for pinion-type cutters.

Information on projection of the transverse profile of a gear tooth to other planes is given in Annex E.

Information helpful to interface this document to the geometry of worms defined with ISO–10828 is given in Annex F.

4.2 Concepts for an individual gear

4.2.1 Gear, cylindrical gear, external gear and internal gear

A gear is a rotationally symmetrical object (gear blank) with a tooth system located at the rim. A cylindrical gear is a gear with a cylindrical reference surface. A distinction is made between external and internal gears according to the radial arrangement of the teeth in each case. The tips of the teeth point outwards in an external gear and inwards in an internal gear.

4.2.2 Tooth system, external teeth and internal teeth

The tooth system refers to all the teeth and tooth spaces around the rim of a gear. As in 4.2.1, a distinction is made between internal and external gear teeth.

4.2.3 Tooth and tooth space

A tooth is a geometrical element on the gear that enables the transmission of force and motion. The form and dimensions of the teeth and the distance between consecutive teeth are defined by the tooth system parameters. The tooth space is the gap between two consecutive teeth, it is bounded by the tooth flanks, fillets, root, and tip cylinder.

4.2.4 Tooth system parameters

The nominal dimensions of involute cylindrical gear teeth are uniquely determined by the diameter of the reference cylinder, the associated basic rack and its position in relation to the reference cylinder. The nominal dimensions are defined by the following parameters, which are independent of each other:

— internal or external gear;

— number of teeth, z ;

— basic rack tooth profile;

— normal module, m_n ;

— helix angle, β , with sign or helix angle along with hand of helix;

— profile shift coefficient, x ;

— tip diameter, d_a ;

— facewidth, b ;

— root diameter, d_f .

4.2.5 Number of teeth and sign of number of teeth

The number of teeth around the rim of the gear is denoted by z .

The number of teeth, z , of an external cylindrical gear shall be taken as a positive value in the formulae in this document, while the number of teeth, z , in an internal cylindrical gear is to be taken as a negative value. In keeping with the ISO 6336 series, for internal gears, in addition to the number of teeth, the diameters and centre distances are taken as negative values.

NOTE 1 The negative sign for number of teeth z corresponds to the idea that in the transition from an external gear to an internal gear, the gear diameter d and number of teeth z is increased until d=+∞ and z=+∞ is reached. In the course of the transition, d and z change to -∞ and then assume finite negative values.

With this rule and the definition of the sign of the profile shift coefficient it is possible to use the same formulae without any change for external and internal gears.

NOTE 2 For internal gears, although the diameters used in calculations are negative, on drawings the diameters are usually shown as positive numbers.

In the case of segments, the number of teeth, z , used in calculations is the number that there would be on the whole circumference.

4.2.6 Tooth number

When numbering teeth, the designations tooth 1, tooth 2, etc. are to be defined on a transverse surface (datum face) viewed in an agreed direction so that the teeth are numbered in ascending order (moving in a clockwise direction). If the letter N is used to denote a reference tooth, the next tooth in the direction of counting is denoted by N + 1 and the previous tooth going in the opposite direction by N − 1. Tooth z is followed by tooth 1 in the direction of counting, see Figure 1.

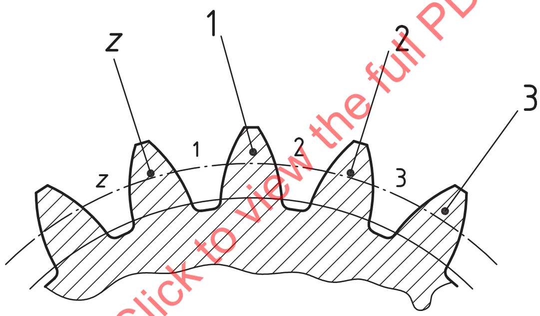


Figure 1 — Numbering of teeth and tooth spaces on datum face

4.2.7 Top land and bottom land

4.2.7.1 Top land

The top land of a tooth is the outermost (innermost in the case of internal gears) periphery of the tooth, it is concentric to the reference cylinder, see Figure 2.

4.2.7.2 Bottom land

The bottom land is the innermost (outermost in the case of internal gears) periphery of the tooth space, it is concentric to the reference cylinder, see Figure 2.

4.2.8 Tooth flanks and flank sections

4.2.8.1 Tooth flank

Tooth flanks are those parts of the surface of a tooth that are located between the top land and the bottom land, see Figure 2.

4.2.8.2 Right flank and left flank

The right flank (or left flank) is the tooth flank that an observer sees on the right-hand (or left-hand) side when viewing the datum face of a tooth when it is pointing upwards. This definition applies to both external and internal gears, see Figure 2.

Right flank parameters are indicated by the subscript R and left flank parameters by the subscript L .

a) Internal tooth b) External tooth

Key

1 top land
2 addendum flank
3 reference cylinder
4 dedendum flank
5 bottom land
6 datum face

Figure 2 — Top land, bottom land and tooth flank (from datum face)

4.2.8.3 Addendum flank

The addendum flank is that part of a tooth flank that is located between the reference cylinder and the top land, see Figure 2.

4.2.8.4 Dedendum flank

The dedendum flank is that part of a tooth flank that is located between the reference cylinder and the bottom land, see Figure 2.

4.2.8.5 Usable flank

The usable flank is that part of a tooth flank that can be used to engage with a mating flank. On a cylindrical gear, it is part of the involute helicoid including any flank modifications.

4.3 Reference surfaces, datum lines and reference quantities

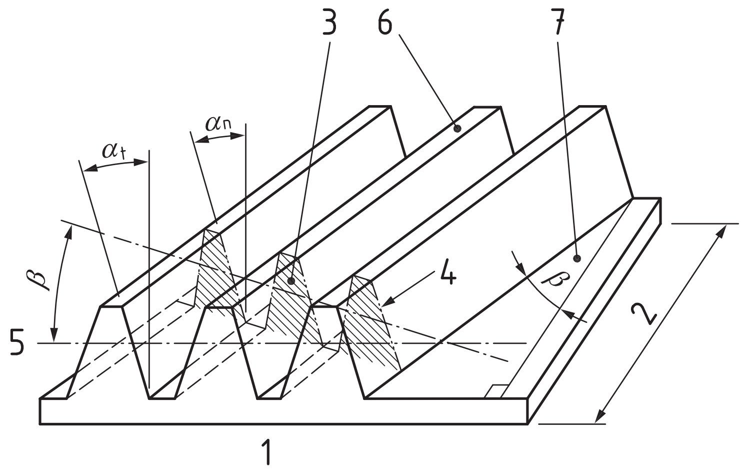
4.3.1 Reference surface and datum face

The reference surface of the teeth is an imaginary cylindrical surface to which the geometrical parameters relate. In the case of cylindrical gears, the reference surface is termed the reference cylinder, see Figure 2.

The agreed front of the gear (usually used for text or suitably marked) is defined as the datum face, see Figure 2. Parameters which relate to the datum face are denoted by the subscript I while parameters which relate to the opposite face are denoted by the subscript II.

4.3.2 Reference rack

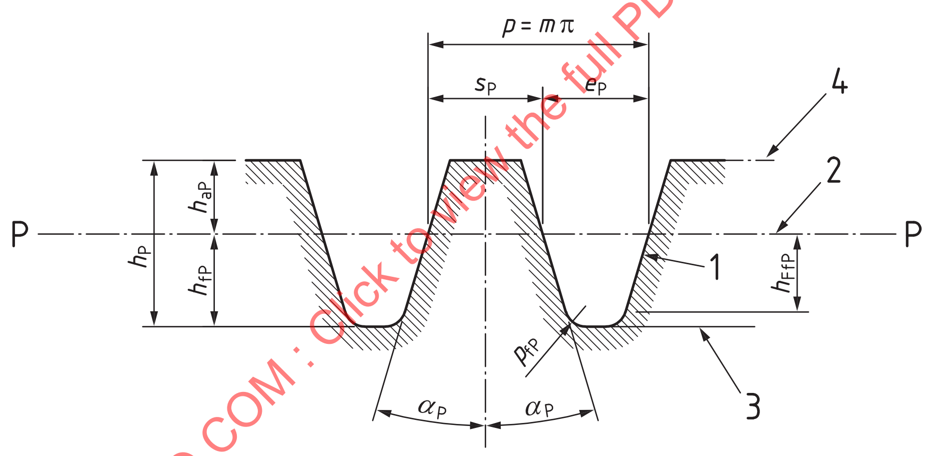
The reference rack is the rack that can be produced using the same gear-cutting tool, gear-cutting method and pitch point (pitch axis) as the actual cylindrical gear. It is characterized by its basic rack tooth profile, the direction of its teeth in relation to the pitch axis of the generating gear, tip plane, root plane and facewidth, see Figure 3.



Key

1	transverse section
2	facewidth
3	normal section
4	basic rack tooth profile
5	pitch axis
6	tip plane
7	root plane

Figure 3 — Concepts and parameters relating to reference rack



Key

1	basic rack profile
2	datum line
3	root line
4	tip line

Figure 4 — Terms and parameters relating to basic rack tooth profile in normal section

4.3.3 Basic rack tooth profile for involute gear teeth

The basic rack tooth profile is defined in a normal section of the reference rack. The flank lines of the basic rack tooth profile of involute teeth are straight lines. Tooth thickness, s_P , and space width, e_P , on the datum line of the basic rack (P-P) in the normal section are equal, see Figures 4 and 47.

Both flanks are symmetric with the same pressure angle and have a tooth root fillet radius ρ_{fP} with a maximum value corresponding to a full/complete radius. No peaks are allowed in the root.

The default basic rack tooth profile does not have any flank modifications, i.e. neither undercut near the root or tip chamfering are considered. Those aspects are considered in Clauses 7 and 8.

This document is valid for any basic rack tooth profile. Some standardized rack tooth profiles are given in ISO 53, these should just be considered as examples of profiles that can be used.

4.3.4 Reference cylinder, reference circle and reference diameter

The reference cylinder is the reference surface for the cylindrical gear teeth. Its axis coincides with the axis of the gear (datum axis). The reference circle is the intersection of the reference cylinder with a transverse plane section. The reference diameter, d , is determined by Formula (1):

$$d = z m_{\mathrm{t}} = \frac{z m_{\mathrm{n}}}{\cos\beta} \tag{1}$$

NOTE 1 For internal gears, since the number of teeth is negative, the reference diameter is also negative.

NOTE 2 The reference diameter is the location where a gear's tooth thickness, pressure angle and helix angle are defined.

NOTE 3 At the reference diameter, the normal pressure angle is equal to the basic rack pressure angle ($\alpha_{\mathrm{n}} = \alpha_{\mathrm{p}}$).

4.3.5 Gear axis

The axis of a gear (datum axis) is the axis defined by the geometrical axis of the datum surfaces.

4.3.6 Sections through a cylindrical gear

4.3.6.1 Transverse section and transverse profile

The sectioning of cylindrical gear teeth by a plane perpendicular to the gear axis yields a transverse section. For helical gears, quantities in the transverse section are denoted by the subscript t. The intersection of a tooth with a transverse plane is termed the transverse profile.

NOTE The terms "transverse plane" and "transverse section" can be used interchangeably.

4.3.6.2 Normal surface, normal plane and normal profile

A normal plane is normal (perpendicular) to a flank line (see 4.3.6.4) at a specified point. The sectioning of gear teeth by a normal plane yields a normal section. A normal section applies to racks and spur gears, it generally is not applicable to helical gears.

For a helical gear, for any helix (or flank line) on a given y-cylinder, a normal plane is normal to the helix at the considered point. This normal plane is not normal to other helices.

A complementary helix is normal to helices at the intersection of right and left flanks of a tooth with a y-cylinder. The complementary helix has a helix angle of 90°-|β_{y}| (equivalent to lead angle γ_{y}).

A normal surface is defined by the complementary helices that intersect a radial axis of symmetry of a tooth. It is curved three dimensionally.

A normal profile is the intersection of a normal surface with the tooth flanks. For a helical gear, the normal profile does not lie in a plane.

Subscript "n" is used for parameters relating to the normal profile.

For spur gears and racks, as flank lines are all parallel straight lines, the normal surface as well as the normal profile are in a plane.

For spur gears the normal surface lies in a transverse plane, so the transverse profile and normal profile of a spur gear are identical.

NOTE In the past, the normal surface has been referred to as the normal plane; this is not correct for helical gears.

4.3.6.3 Axial section and axial profile

The sectioning of cylindrical gear teeth by a plane containing the gear axis yields an axial section.

Quantities in the axial section are denoted by the subscript x. The intersection of a tooth with an axial plane is termed the axial profile.

4.3.6.4 Cylindrical section and flank lines

The flank lines are the intersections of the right and left flanks with a cylinder that has an axis which coincides with the gear axis. Hence, right and left flank lines are to be distinguished.

The flank lines are helices in the case of helical gear teeth and straight lines in the case of spur gear teeth.

The reference flank line (tooth trace) is the intersection of the flank with the reference cylinder. The base flank line is the intersection of the unmodified involute flank - possibly imagined as extended - with the base cylinder. The origin of the involute helicoid is a base flank line. The tip flank line is the line of intersection of the unmodified involute flank - possibly imagined as extended - with the tip cylinder.

4.3.7 Module

The module of a basic rack is found as the pitch of the rack divided by the number π (see Figure 4). The normal module, m_n , of the cylindrical gear is found as the module of the basic rack tooth profile.

For a helical gear, the transverse module, m_t , is found as:

$$m_t = \frac{d}{z} = \frac{m_n}{\cos\beta} \tag{2}$$

and the axial module, m_x , as

$$m_x = \frac{m_n}{\sin|\beta|} = \frac{m_n}{\cos\gamma} = \frac{m_t}{\tan|\beta|} \tag{3}$$

For a spur gear, $m_t = m_n$.

4.3.8 Diametral pitch

The transverse diametral pitch is the ratio of the number of teeth to the reference diameter in inches. The diametral pitch has units of inverse inches.

$$P_d = \frac{z}{d} = \frac{\pi}{p_t} \tag{4}$$

NOTE P_d is a deviation from the symbol convention where “t” is used to denote the transverse plane.

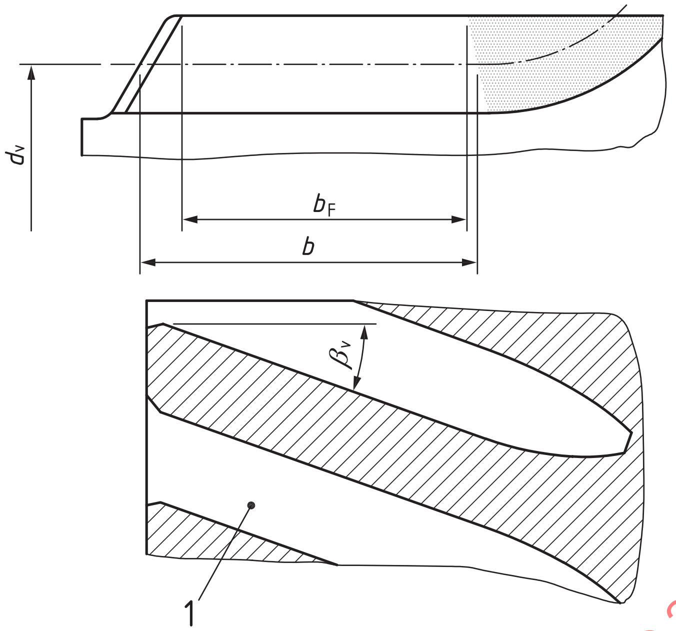
The normal diametral pitch is the value of diametral pitch on a normal surface of a helical gear or worm.

$$P_{nd} = \frac{25{,}4}{m_n} = \frac{P_d}{\cos\beta} \tag{5}$$

4.3.9 Facewidth

The facewidth, b , is the length of the toothed part of the cylindrical gear measured in the axial direction on the V-cylinder. See 4.6.2.

The usable facewidth, b_F , is the distance between two transverse sections that contain the fully developed height of the tooth flank. See Figure 5.



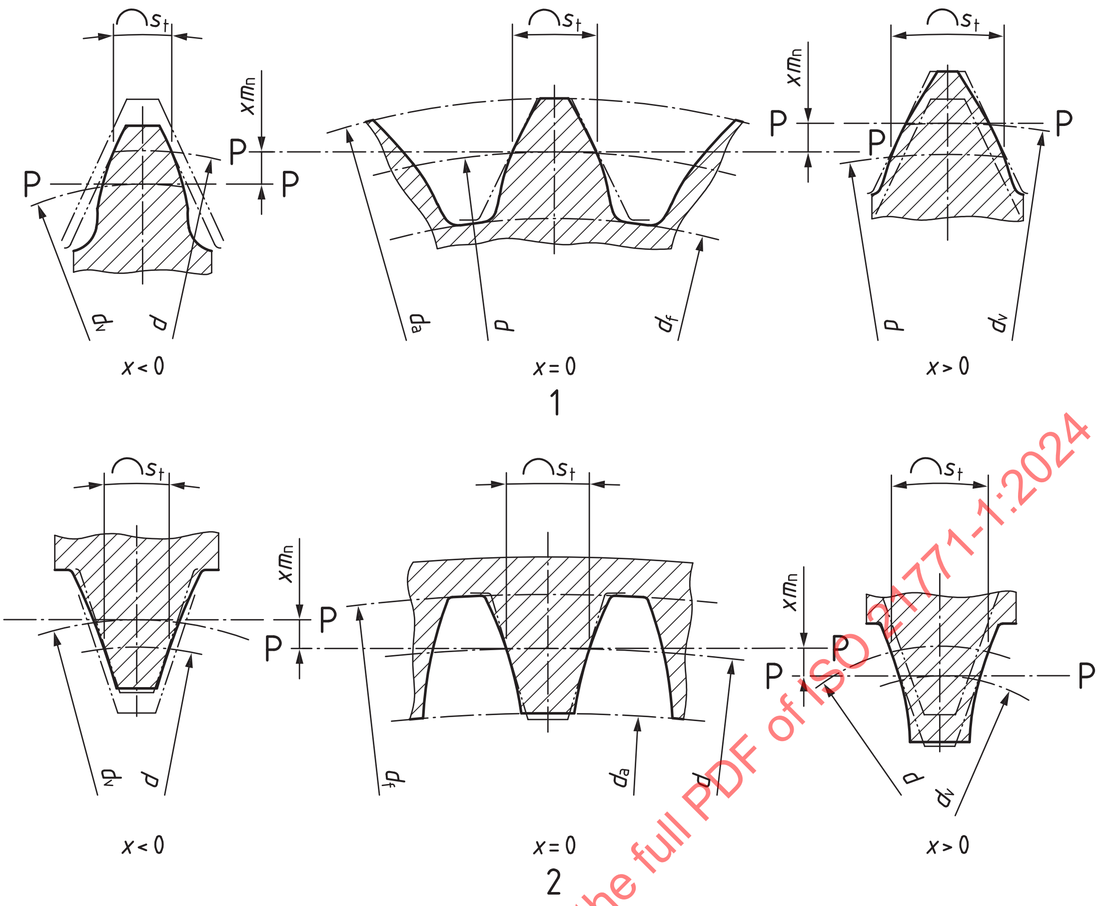
Key

1 developed view of V-cylinder

Figure 5 — Facewidth b, usable facewidth b_F

4.3.10 Profile shift, profile shift coefficient and sign of profile shift

The profile shift, xm_n, for involute gear teeth is the radial displacement of the basic rack datum line from the reference cylinder of the gear away from (or towards) the centre of the gear. The magnitude of the profile shift can be made non-dimensional by dividing by the normal module, and it is then expressed by the profile shift coefficient, x. Positive profile shift increases the tooth thickness on the reference cylinder on both external and internal gears. See Figure 6.



Key

⌒	measurement along an arc	1	external gear
P–P	datum line of basic rack	2	internal gear

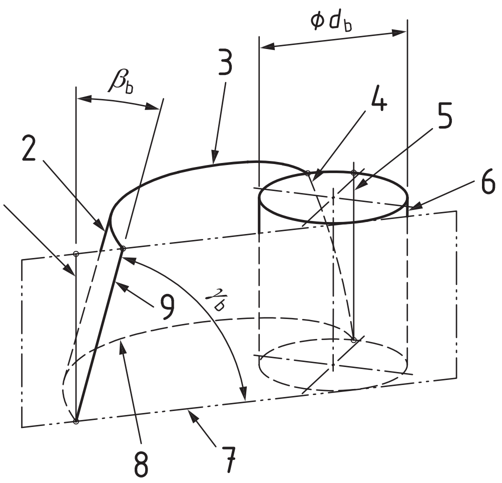
NOTE For internal gears, since the number of teeth is negative, the diameters used in calculations are negative. On drawings, the diameters are usually shown as positive numbers.

Figure 6 — Profile shift for external and internal gear teeth

4.4 Involute helicoids

4.4.1 Generator of involute helicoids

Figure 7 shows that the intersection of the theoretical flank (key 3) with the base cylinder (key 6) is a helix (key 4). If the base cylinder is unwrapped to a plane [base cylinder tangential plane (key 7)] the flank line becomes a straight line (key 9) inclined by the base helix angle, β_b , to an axial line (key 1), and this is the generator of the involute helicoid.



Key

1 developed axial line
2 involute helicoid surface
3 involute
4 base helix
5 base cylinder axial line
6 base cylinder
7 developed base cylinder envelope
8 involute
9 straight line generator

Figure 7 — Base cylinder with generator and involute helicoid

4.4.2 Base cylinder, base circle and base diameter

The base cylinder is that cylinder coaxial with the gear axis that is determinative for the generation of the involute helicoids, see Figures 7 and 8. Quantities associated with the base cylinder are denoted by the subscript b.

The base circle is the intersection of the base cylinder with a transverse section. The involutes from the base circle form the transverse profiles of the gearing. The base diameter, d_b, is given by Formula (6):

$$d_b = d\cos\alpha_t = z m_t \cos\alpha_t = z m_n \frac{\cos\alpha_t}{\cos\beta} = z m_n \frac{\cos\alpha_n}{\cos\beta_b} = \frac{z m_n}{\sqrt{\tan^2\alpha_n + \cos^2\beta}} \tag{6}$$

NOTE For internal gears, since the number of teeth is negative, the base diameter is also negative.

4.4.3 Transverse pressure angle at a point and transverse pressure angle

In a transverse section, the tangent to the involute through an arbitrary point Y is inclined to the radius to the involute at that point by the transverse pressure angle, α_{yt}, (see Figure 8):

$$\cos\alpha_{yt} = \frac{d_b}{d_y} = \frac{d}{d_y}\cos\alpha_t \tag{7}$$

The transverse pressure angle, α_t, is the acute angle between the tangent to the involute at the point of intersection with the reference circle and the radius through this point of intersection. It is expressed by Formula (8):

$$\cos\alpha_t = \frac{d_b}{d} \quad \text{or} \quad \tan\alpha_t = \frac{\tan\alpha_n}{\cos\beta} \tag{8}$$

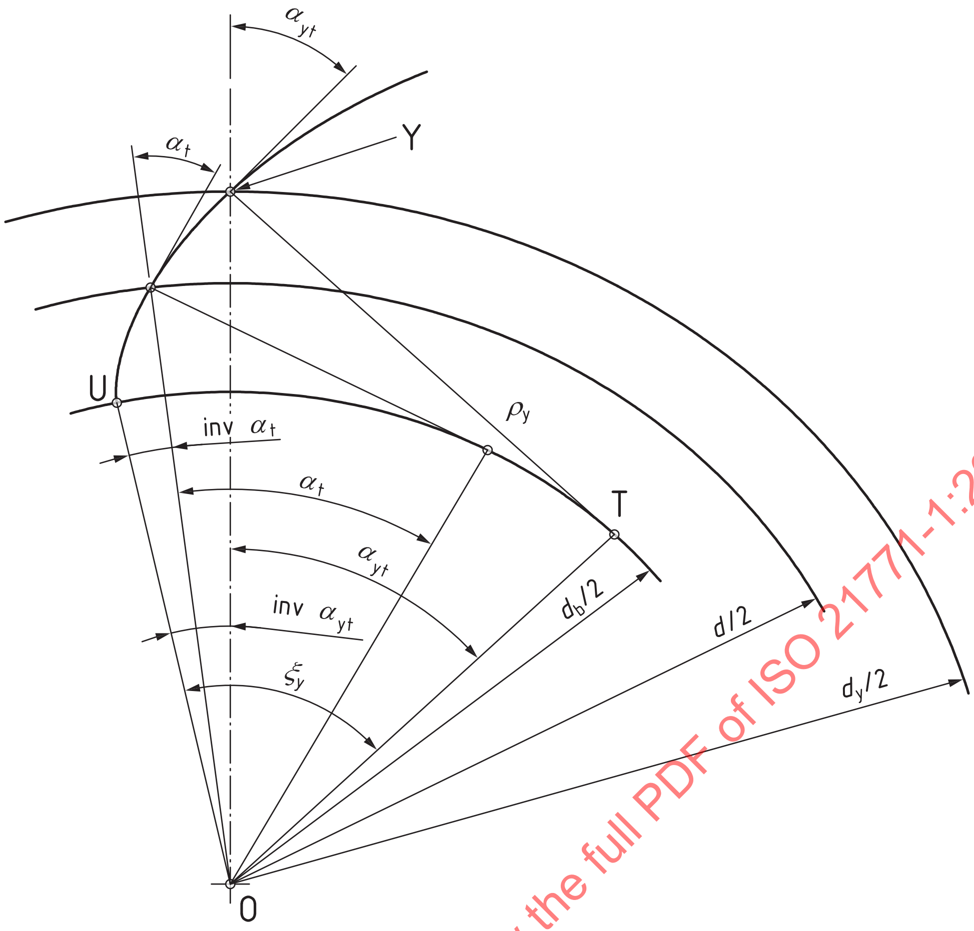


Figure 8 — Parameters relating to involute

4.4.4 Normal pressure angle at a point and normal pressure angle

At an arbitrary point Y on the flank, the tangent to the normal surface of the involute helicoid is inclined to the radius through Y by the normal pressure angle at that point, α_{yn}. The corresponding angle of inclination at the reference cylinder is the normal pressure angle, α_n, this is equal to the pressure angle, α_p, of the basic rack tooth profile.

$$\tan\alpha_n = \tan\alpha_t \cos\beta \quad (9)$$

$$\tan\alpha_{yn} = \tan\alpha_{yt} \cos\beta_y \quad (10)$$

For a spur gear, $\alpha_n = \alpha_t$ and $\alpha_{yn} = \alpha_{yt}$.

4.4.5 Involute function

The angular difference, $\xi_y - \alpha_{yt}$, is termed the involute function of angle α_{yt} and is denoted by $\mathrm{inv}\,\alpha_{yt}$ (to be read as “involute α_{yt}”):

$$\mathrm{inv}\,\alpha_{yt} = \xi_y - \alpha_{yt} = \tan\alpha_{yt} - \alpha_{yt} \quad (11)$$

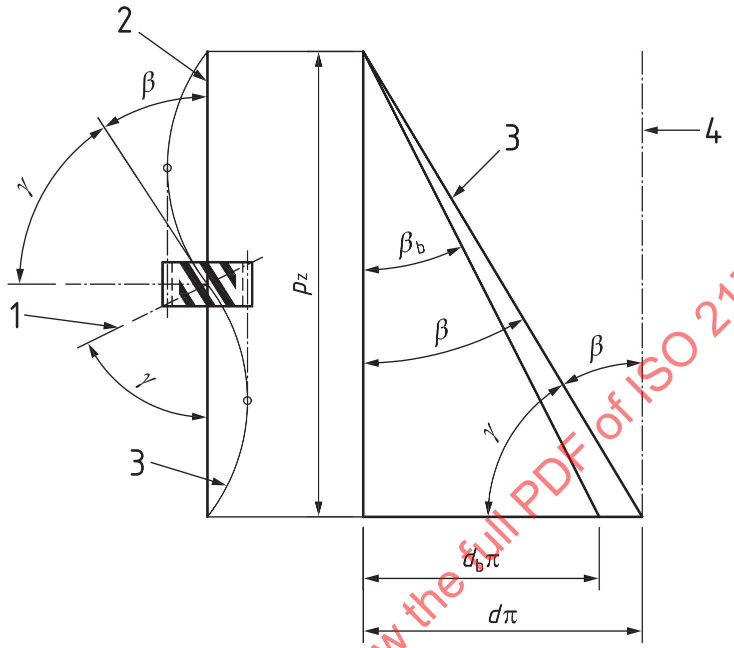
Radians shall be used in Formula (11).

4.4.6 Lead

The lead, p_z, is the distance between successive intersections of an axial line with the involute helicoid, see Figure 9. The lead is independent of the diameter of the cylinder.

$$p_z = \frac{z m_n \pi}{\sin|\beta|} = \frac{z m_t \pi}{\tan|\beta|} = z p_x \tag{12}$$

NOTE For internal gears, since the number of teeth is negative, the lead is also negative.



Key

1 normal surface

2 reference cylinder generator, gear axis

3 trace of reference helix

4 projection of the gear axis

Figure 9 — Lead triangle, lead, helix angle and lead angle

4.4.7 Helix angle and lead angle

The helix angle, β, is the angle between a tangent to a reference helix and an axial line on the reference cylinder through the tangent contact point. In special cases, the helix angle, β_R, of right flanks can differ from the helix angle, β_L, of left flanks; however, all formulae in this document are based on the left and right flanks having equal helix angles.

In some calculations, such as for crossed axis gears, the sign of the helix angles is required.

For an external gear with right-hand lead (right-hand helix) the helix angle is positive, for a left-hand lead (left-hand helix) the helix angle is negative.

For an internal gear the signs are opposite those of an external gear, i.e. an internal gear with a right-hand lead has a negative helix angle and an internal left-hand lead has a positive helix angle.

Therefore, the sum of the helix angles of a parallel axis gear pair is zero.

NOTE On gear drawings, the absolute values of the helix angles are usually listed together with the direction of the helix (r for right-hand, l for left-hand).

Relationships between β and the base helix angle, β_b, (helix angle on the base cylinder):

$$\tan\beta_b = \tan\beta \cos\alpha_t \tag{13}$$

$$\sin\beta_b = \sin\beta \cos\alpha_n \tag{14}$$

$$\cos\beta_b = \cos\beta \frac{\cos\alpha_n}{\cos\alpha_t} = \frac{\sin\alpha_n}{\sin\alpha_t} = \frac{\sin\alpha_{yn}}{\sin\alpha_{yt}} = \cos\alpha_n \sqrt{\tan^2\alpha_n + \cos^2\beta} \tag{15}$$

On a cylinder with arbitrary diameter, d_y, the helix angle, β_y, can be found:

$$\tan\beta_y = \frac{d_y}{d}\tan\beta = \tan\beta \frac{\cos\alpha_t}{\cos\alpha_{yt}} = \frac{d_y}{d_b}\tan\beta_b = \frac{\tan\beta_b}{\cos\alpha_{yt}} \tag{16}$$

$$\sin\beta_y = \sin\beta \frac{\cos\alpha_n}{\cos\alpha_{yn}} = \frac{\sin\beta_b}{\cos\alpha_{yn}} \tag{17}$$

$$\cos\beta_y = \frac{\tan\alpha_{yn}}{\tan\alpha_{yt}} = \frac{\cos\alpha_{yt}\cos\beta_b}{\cos\alpha_{yn}} \tag{18}$$

The lead angle, γ, is the angle between a tangent to a reference helix and a transverse plane through the tangent contact point, see Figure 9. Lead angle does not apply to internal gears. For external gears:

$$|\beta + \gamma| = 90^\circ \tag{19}$$

$$\gamma_y = (90^\circ - |\beta_y|)\frac{\beta_y}{|\beta_y|} \tag{20}$$

For spur gears, $\beta = 0°$ and $\gamma = 90°$.

4.4.8 Hand of helix

The flank lines can follow either a right-hand helix or a left-hand helix. See Figure 10.

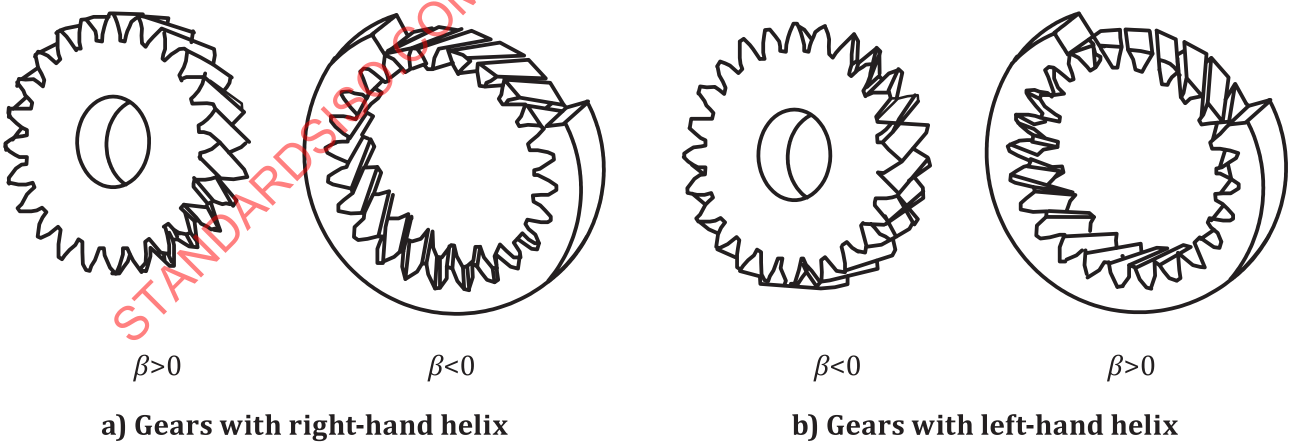


Figure 10 — Direction of helix

4.4.9 Roll angle of the involute

In the transverse plane, the angle at the centre over the base circle arc from the origin, U, of the involute to the contact point, T, of the tangent from point Y to the base circle is the roll angle, ξ_y, of the involute, see Figure 8. The length of the base circle arc, UT, is equal to the length of the tangent portion, YT, hence

$$\xi_y = \tan \alpha_{yt} \tag{21}$$

NOTE ξ_y is in radians.

4.4.10 Radius of curvature of the involute and length of roll

The tangent portion, YT, is the radius of curvature, ρ_y, of the involute at point Y and at the same time the length of roll, L_y, belonging to point Y, i.e. the length of the developed base circle arc from the involute origin, U. (See Figure 8.) In the triangle OTY it is the side opposite the transverse pressure angle, α_{yt}, at the centre of the circle O:

$$L_y = \rho_y = \frac{d_b}{2}\xi_y = \frac{d_b}{2}\tan \alpha_{yt} = \frac{z}{|z|}\frac{\sqrt{d_y^2 - d_b^2}}{2} \tag{22}$$

NOTE 1 For internal gears, since the number of teeth is negative, the length of roll is also negative.

NOTE 2 The length of roll is measured in a transverse plane.

4.4.11 Length of involute profile, L_{yt}

The length of an involute profile from its origin on the base circle to point Y with pressure angle α_{yt} (length along profile from U to Y on Figure 8) is:

$$L_{yt} = r_b \frac{\tan^2 \alpha_{yt}}{2} \tag{23}$$

4.5 Angular pitch and pitches

4.5.1 Angular pitch

The angular pitch, τ, is that angle laying in a transverse section that results from the dividing of the complete periphery of a circle into z equal parts.

$$\tau = \frac{2\pi}{|z|} = \frac{2p_{yt}}{|d_y|} \text{ in radians} \tag{24}$$

$$\tau = \frac{360}{|z|} = \frac{360 p_{yt}}{\pi |d_y|} \text{ in degrees} \tag{25}$$

4.5.2 Pitches on the reference cylinder

4.5.2.1 Transverse pitch

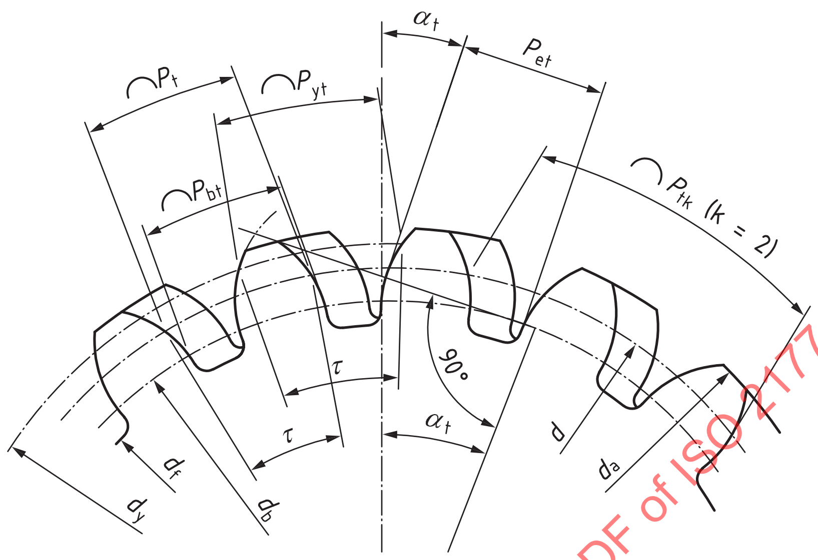
The transverse pitch, p_t, is the length of the reference circle arc between two successive right or left tooth flanks (see Figure 11):

$$p_t = \frac{\pi m_n}{\cos \beta} = \frac{|d|}{2}\tau = \frac{\pi d}{z} = \pi m_t \tag{26}$$

4.5.2.2 Normal pitch

The normal pitch, p_n , is the length of the helical arc on the reference cylinder that is between and normal to the flank lines of two successive right or left tooth flanks (see Figure 12):

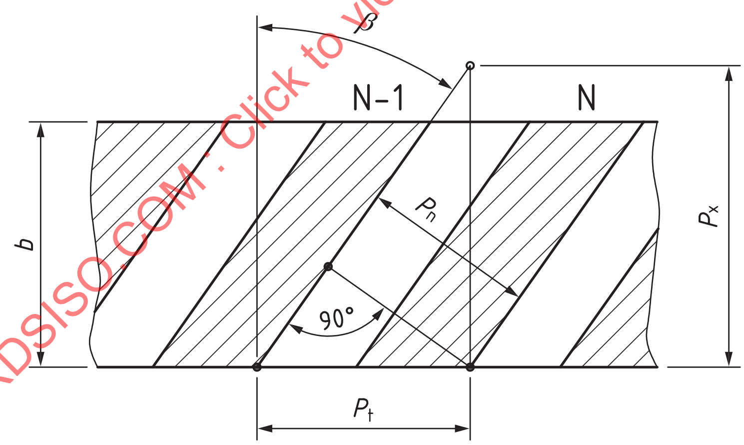
$$p_n = \pi m_n = p_t \cos\beta \qquad (27)$$



Key

⌒ measurement along an arc

Figure 11 — Diameter, angular pitch, and transverse pitches on helical cylindrical gear



NOTE This is a developed view of a reference cylinder.

Figure 12 — Geometrical relations between transverse, normal and axial pitch

4.5.3 Pitches on any cylinder

It is necessary to distinguish between the transverse pitch, p_{yt} , and the normal pitch, p_{yn} , on a cylinder of any diameter, d_y , (y-cylinder):

$$p_{yt} = \frac{|d_y|}{2}\tau = \frac{\pi d_y}{z} = \frac{d_y}{d} p_t \qquad (28)$$

$$p_{yn} = p_{yt} \cos\beta_y \tag{29}$$

4.5.4 Axial pitch

The axial pitch, p_x, of a helical gear is the portion of a generation line of a cylinder concentric with the gear axis between two successive right or left tooth flanks, see Figure 12. The axial pitch is independent of the diameter of the cylinder. Axial pitch does not apply to spur gears. It is expressed by:

$$p_x = \frac{\pi m_n}{\sin|\beta|} = \pi m_x = \frac{p_z}{z} = \frac{\pi m_t}{\tan|\beta|} = \frac{p_{yt}}{\tan|\beta_y|} = \frac{p_{yn}}{\sin|\beta_y|} \tag{30}$$

4.5.5 Base pitch

4.5.5.1 General

The distance between successive right or left tooth flanks on the developed base cylinder tangential plane is the base pitch.

— Transverse base pitch:

$$p_{bt} = \frac{|d_b|}{2}\tau = p_t \cos\alpha_t = p_{yt} \cos\alpha_{yt} = \frac{\pi d_b}{z} = \frac{d_b}{d} p_t \tag{31}$$

— Normal base pitch:

$$p_{bn} = p_n \cos\alpha_n = p_{bt} \cos\beta_b \tag{32}$$

4.5.5.2 Transverse base pitch along a tangent to the base circle

The transverse base pitch, p_{et}, measured along a line tangent to the base circle in the transverse plane, is the distance between two successive right or left tooth flanks (see Figure 11):

$$p_{et} = p_{bt} \tag{33}$$

NOTE For a gear pair, the path of contact is on a line that is tangent to both base circles.

4.5.5.3 Normal base pitch on the plane of contact

In a gear pair, the plane of contact is a plane tangent to the two base cylinders. The normal base pitch, p_{en}, measured in the plane of contact is the distance between two parallel tangential planes which contact two successive right or left tooth flanks:

$$p_{en} = p_{bn} \tag{34}$$

4.6 Diameters of gear teeth

4.6.1 General

The position of the basic rack tooth profile relative to the reference cylinder gives rise to the following cylindrical surfaces and diameters with respect to the gear teeth.

4.6.2 V-cylinder and V-circle diameter

The V-cylinder is the cylinder concentric to the gear axis which is tangent to the datum line of the basic rack in its generating position (see Figure 6). Its nominal diameter, d_v, (V-circle diameter), is calculated using Formula (35).

$$d_v = d + 2xm_n \tag{35}$$

NOTE The V-cylinder is also the pitch surface at generation with a rack tool.

4.6.3 y-cylinder and y-circle diameter

The y-cylinder is a cylinder of arbitrary diameter, d_y, concentric to the gear axis that is used for calculation purposes.

4.6.4 Tip alteration coefficient

Tip alteration is a change to the addendum relating to the standard addendum of one normal module. The tip alteration is made non-dimensional by dividing by the normal module, and it is then expressed as the tip alteration coefficient, k.

The value to be used for k is signed.

A negative value yields a shorter addendum for either external or internal gears.

NOTE The value for k is specified by the designer.

4.6.5 Tip cylinder, tip circle and tip diameter

The tip cylinder is the cylinder that defines the tips of the gear tooth system. A transverse section yields the tip circle. The nominal dimension of the tip diameter, d_a, is calculated Formula (36).

$$d_a = d + 2(xm_n + h_{aP} + km_n) \tag{36}$$

NOTE 1 For a standard addendum of one normal module, $h_{ap} = m_n$.

NOTE 2 The value for the tip alteration coefficient, k, is determined by the gear designer. The value of k can be positive or negative. See 5.3.9.

4.6.6 Profile control diameter, d_{Cf}

The profile control diameter, also known as start of profile evaluation diameter, is a specified diameter beyond which the tooth profile is required to conform to the specified design. If not specified, the profile form diameter, d_{Ff}, is used as the profile control diameter. Note that $d_{Ff} \leq d_{Cf}$.

NOTE If the mating gear is known, the designer sometimes selects the minimum start of active profile diameter, d_{Nf}, as the profile control diameter.

4.6.7 Root cylinder, root circle and root diameter

The root cylinder is the cylindrical envelope surface that forms the bottom of the tooth space. A transverse section yields the root circle. The nominal dimension of the root diameter, d_f, is Formula (37).

$$d_f = d - 2(h_{fP} - xm_n) \tag{37}$$

For additional information and formulae related to detailed root geometry, see form factor in ISO 6336-3.

4.7 Gear tooth depth

4.7.1 Tooth depth

The tooth depth, h, of cylindrical gear (or rack) teeth is the difference between the tip and root radius:

$$h = \frac{d_a - d_f}{2} = h_{aP} + km_n + h_{fP} \tag{38}$$

NOTE The tooth depth is sometimes referred to as the tooth height.

4.7.2 Addendum, working addendum, dedendum and working dedendum

The addendum, h_a, and the dedendum, h_f, of a cylindrical gear are stated on the basis of the reference circle. Their values are obtained using Formulae (39) and (40):

$$h_a = \frac{d_a - d}{2} = h_{aP} + xm_n + km_n \tag{39}$$

$$h_f = \frac{d - d_f}{2} = h_{fP} - xm_n \tag{40}$$

The working addendum, h_{aw}, and working dedendum, h_{fw}, are stated on the basis of the working pitch circle. Their values are obtained using Formulae (40) and (41):

$$h_{aw} = \frac{d_a - d_w}{2} \tag{41}$$

$$h_{fw} = \frac{d_w - d_f}{2} \tag{42}$$

The direction of sliding on the working addendum is opposite of that on the working dedendum.

4.8 Tooth thickness and space width

4.8.1 General

The formulae in 4.8 yield the tooth thickness and space width and their half angles for any value of profile shift coefficient x. If x is the nominal profile shift coefficient, then nominal sizes for the tooth thickness and space width and their half angles result. If the generating profile shift coefficient, x_E, is used, then generated sizes for the tooth thickness and space width and their half angles result. See ISO 21771-2 for additional information on tooth thickness.

For cylindrical gears, unless chordal tooth thickness is explicitly mentioned, tooth thickness is defined on the surface of a specified cylinder. It is the length between the flanks of a tooth, measured either along a circular arc in a transverse plane or along a helix normal to the tooth flanks. The only exception to this is chordal tooth thickness, which is a straight-line measurement between points at a specified diameter on opposite flanks of a tooth.

4.8.2 Transverse tooth thickness

For spur and helical gears and involute worms, the transverse tooth thickness, s_{yt}, is the length between the two flanks of a gear tooth, measured at a specified diameter along a circular arc in the transverse plane. See Figure 13.

$$s_{yt} = |d_y| \psi_y = |d_y| \left[\psi + \frac{z}{|z|}(\text{inv}\,\alpha_t - \text{inv}\,\alpha_{yt}) \right] = d_y \left[\frac{\pi + 4x\tan\alpha_n}{2z} + \text{inv}\,\alpha_t - \text{inv}\,\alpha_{yt} \right] \tag{43}$$

NOTE The diameters of internal gears have negative values, so the absolute value function is used in some formulae.

The transverse tooth thickness, s_t, on the reference circle is produced from:

$$s_t = |d|\psi = d\left(\frac{\pi + 4x\tan\alpha_n}{2z}\right) = \frac{m_n}{\cos\beta}\left(\frac{\pi}{2} + 2x\tan\alpha_n\right) \tag{44}$$

4.8.3 Transverse tooth thickness half angle

Transverse tooth thickness angles are angles at the centre in a transverse section which are enclosed by the radii bounding a transverse tooth thickness, see Figure 13. The corresponding transverse tooth thickness half angles, ψ_y, for any transverse tooth thickness, s_{yt}, are expressed by:

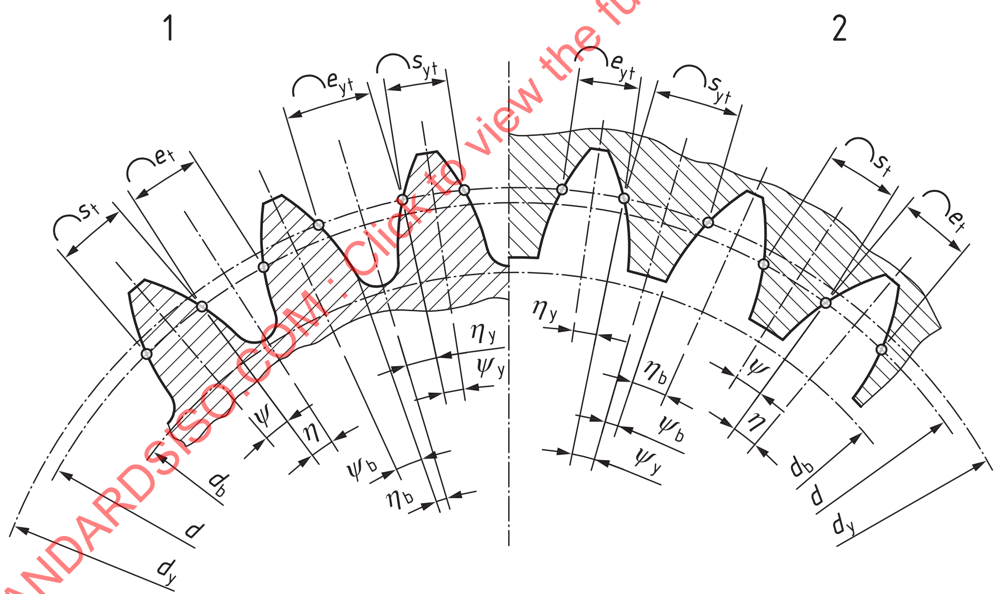
$$\psi_y = \frac{s_{yt}}{|d_y|} = \psi + \frac{z}{|z|}(\text{inv}\,\alpha_t - \text{inv}\,\alpha_{yt}) \tag{45}$$

The following transverse tooth thickness half angle applies to the reference circle:

$$\psi = \frac{\pi + 4x\tan\alpha_n}{2|z|} \tag{46}$$

In the case of the base circle, the base transverse tooth thickness half angle is produced by:

$$\psi_b = \psi + \frac{z}{|z|}\text{inv}\,\alpha_t \tag{47}$$



Key

1 external gear

2 internal gear

◠ measurement along an arc

NOTE For internal gears, since the number of teeth is negative, the diameters used in calculations are negative. On drawings, the diameters are usually shown as positive numbers.

Figure 13 — Transverse tooth thickness and space width (external and internal gear teeth)

4.8.4 Transverse space width

The transverse space width, e_{yt}, is the length in a transverse section of a circular arc of diameter, d_y, between the two involute helicoids of a tooth space (see Figure 13):

$$e_{yt} = |d_y| \eta_y = |d_y| \left[\eta - \frac{z}{|z|} (\text{inv}\,\alpha_t - \text{inv}\,\alpha_{yt}) \right] = d_y \left[\frac{\pi - 4x \tan\alpha_n}{2z} - (\text{inv}\,\alpha_t - \text{inv}\,\alpha_{yt}) \right] = p_{yt} - s_{yt} \tag{48}$$

The transverse space width, e_t, on the reference circle is produced by:

$$e_t = |d| \eta = d \left(\frac{\pi - 4x \tan\alpha_n}{2z} \right) = \frac{m_n}{\cos\beta} \left(\frac{\pi}{2} - 2x \tan\alpha_n \right) = p_t - s_t \tag{49}$$

4.8.5 Transverse space width half angle

Space width angles are angles at the centre in a transverse section which are enclosed by the radii bounding a tooth space, see Figure 13. The corresponding transverse space width half angles, η_y, for any transverse space width, e_{yt}, are produced by

$$\eta_y = \frac{e_{yt}}{|d_y|} = \eta - \frac{z}{|z|} (\text{inv}\,\alpha_t - \text{inv}\,\alpha_{yt}) \tag{50}$$

The following transverse space width half angle applies to the reference circle:

$$\eta = \frac{\pi - 4x \tan\alpha_n}{2|z|} \tag{51}$$

At the base circle, the base transverse space width half angle is produced by:

$$\eta_b = \eta - \frac{z}{|z|} \text{inv}\,\alpha_t \tag{52}$$

4.8.6 Normal tooth thickness

For helical gears and involute worms, the normal tooth thickness is the length between the two sides of a gear tooth that is measured at a specified diameter along a helical arc that is normal to the helix along the tooth flank. The normal tooth thickness, s_{yn}, on a cylinder with diameter d_y is calculated using Formula (53):

$$s_{yn} = s_{yt} \cos\beta_y \tag{53}$$

The following normal tooth thickness applies to the reference cylinder:

$$s_n = s_t \cos\beta = m_n \left(\frac{\pi}{2} + 2x \tan\alpha_n \right) \tag{54}$$

4.8.7 Normal space width

The normal space width is the length of a helical arc that is between and is normal to the two involute helicoids bounding a tooth space. The normal space width, e_{yn}, on a cylinder with diameter d_y is calculated using Formula (55):

$$e_{yn} = e_{yt} \cos\beta_y = p_{yn} - s_{yn} \tag{55}$$

The following normal space width applies to the reference cylinder:

$$e_n = e_t \cos\beta = m_n \left(\frac{\pi}{2} - 2x \tan\alpha_n \right) = p_n - s_n \tag{56}$$

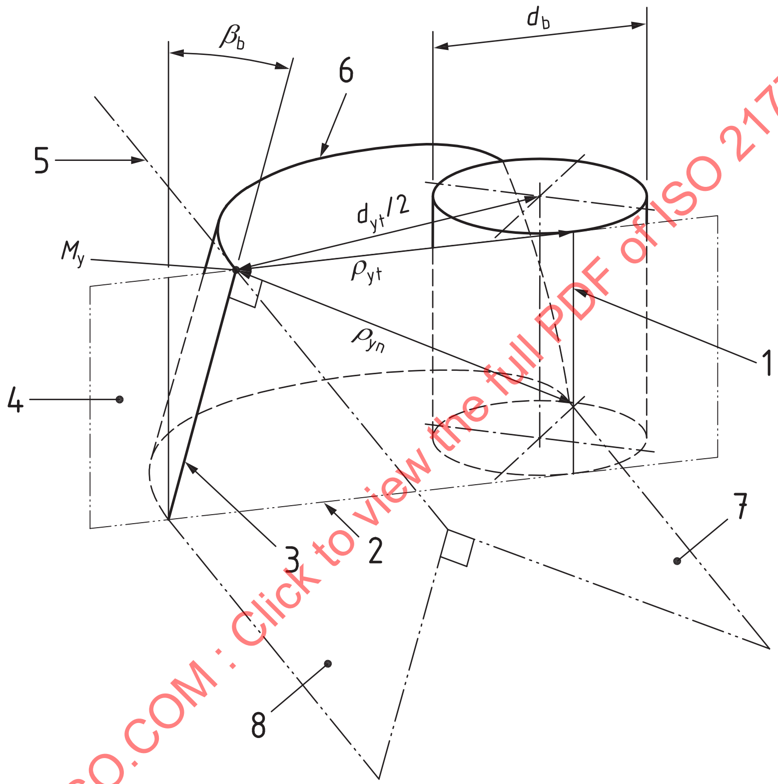
4.9 Radii of curvature of the tooth flanks in different planes

In general, at a specified point on the tooth flank of an involute cylindrical gear, the radius of curvature in any plane perpendicular to the surface can be determined. There are three important planes related to the radius of curvature at a specified point on an involute tooth flank.

— The tangent plane to involute helicoid: a plane tangent to the tooth flank surface at the specified point. This plane (key 8 in Figure 14) is established by the tangent to the involute in the transverse plane (key 5 in Figure 14) and the straight-line generator (key 3 in Figure 14).

— The base plane: a plane that is tangent to the base cylinder and that contains both the specified point and the straight-line generator (key 3 in Figure 14).

— The normal plane: a plane containing the specified point that is perpendicular to both the tangent plane to involute helicoid and the base plane.



Key

1 axial line
2 developed base cylinder envelope
3 straight line generator
4 base plane
5 tangent to involute in transverse plane
6 involute
7 normal plane
8 tangent plane to involute helicoid

Figure 14 — Involute helicoid with generator and tangent to involute

The minimum and maximum radii of curvature are defined as the principal radii of curvature. These are always in two perpendicular planes that are also perpendicular to the tangent plane to involute helicoid. For a cylindrical involute gear, the first principal radius of curvature, ρ_{yn}, is in the normal plane and is the minimum radius of curvature. The second principal radius of curvature is in the base plane and is infinite since the generator of the flank surface is a straight line. See Clause 7.

NOTE 1 For an involute gear pair with parallel axis, the straight-line generator is equivalent to contact lines between tooth flanks during meshing.

Once the radius of curvature in the normal plane is known, the radius of curvature in any inclined plane can be determined according to the Meusnier theorem. The curvature radius in the transverse plane at an involute point is the length of roll at this point. The rotation of the transverse plane around the tangent of the involute by the base helix angle delivers the normal plane and the normal radius of curvature (first principal radius of curvature).

Radius of curvature in a transverse plane at diameter d_y :

$$\rho_{yt} = \frac{z}{|z|} 0{,}5 \sqrt{d_y^2 - d_b^2} \tag{57}$$

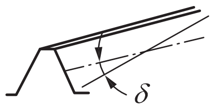
Normal radius of curvature at diameter d_y :

$$\rho_{yn} = \frac{z}{|z|} \frac{\sqrt{d_y^2 - d_b^2}}{2\cos\beta_b} = \frac{\rho_{yt}}{\cos\beta_b} \tag{58}$$

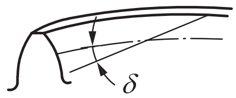
NOTE 2 The normal radius of curvature given in Formula (58) is also known as the first principal radius of curvature. The second principal radius of curvature is in a direction perpendicular to the first principal radius of curvature and is infinite except when there is face line crowning.

4.10 Tilt angle of contact line

For some considerations, the tilt angle between the tangent at the reference cylinder and the generator of the involute helicoid is of importance. It also describes the tilt angle between the instantaneous contact line between the basic rack and the helical gear tooth flank. See Figure 15.



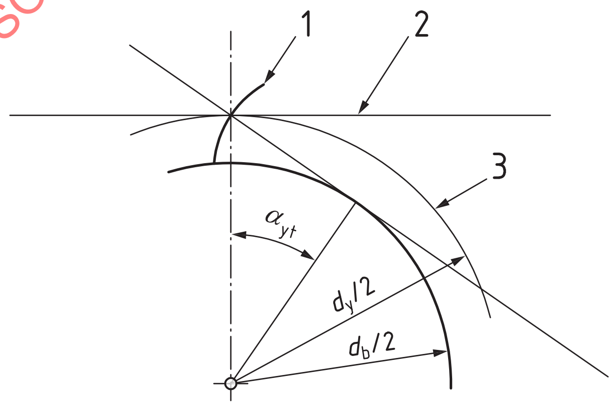
a) On generating rack



b) On helical gear tooth flank

Figure 15 — Tilt angle of generatrixes, δ

At a specified diameter, d_y , the tangent to the helix and the generator through that point are both members of the tangent plane at the point. See Figure 16. The dot product of the two vectors delivers the angle between the vectors.



Key

1 involute

2 tangent to circle of diameter d_y

3 circle with diameter d_y

Figure 16 — Tangent to the helix at diameter d_y and the generator

The unit vectors of helix, T_1 , and generator, T_2 , are:

$$T_1 = (\sin(\beta_y), 0, \cos(\beta_y)) \tag{59}$$

$$T_2 = (\cos(-\alpha_{yt}) \sin(\beta_b), \sin(-\alpha_{yt}) \sin(\beta_b), \cos(\beta_b)) \tag{60}$$

The scalar vector product delivers the tilt angle, δ_y , between the two vectors.

$$\cos\delta_y = \frac{\cos\beta_y}{\cos\beta_b} \text{ or } \tan\delta_y = \sin\alpha_{yn} \tan\beta_y \tag{61}$$

The tilt angle of the contact line varies with the diameter at the flank. For the counterpart rack, the tilt angle is constant and defined at reference diameter d .

$$\cos\delta = \frac{\cos\beta}{\cos\beta_b} \text{ or } \tan\delta = \sin\alpha_n \tan\beta \tag{62}$$

5 Parallel axis cylindrical gear pairs

5.1 General

The basic prerequisites for meshing of a parallel axis cylindrical gear pair (or rack and pinion) according to this document are:

— identical basic rack tooth profiles for gear and mating gear (rack), and

— the same base helix angle (or rack helix angle) with appropriate hands of the helices.

NOTE The concepts and formulae in Clause 5 apply to rack and pinion sets when both the rack and pinion have the same normal base pitch, even if the rack and pinion have different helix angles. With this combination, the axis of the pinion will not be perpendicular to the transverse plane of the rack. When the helix angles do not match, there will be an additional sliding component across the faces of the rack and pinion, this sliding is not covered in this document.

5.2 Concepts for a gear pair

5.2.1 Mating gear and mating flank

The mating gear in a gear pair is the gear which meshes with the other gear in question. The mating flanks for the tooth system in question are the contacting tooth flanks of the mating gears.

5.2.2 Working flank and non-working flank

The tooth flank which transmits torque during meshing is called the working flank. The other flank of this tooth is the non-working flank.

5.2.3 External gear pair

The mating of two external cylindrical gears gives an external gear pair. The mating of an external gear with a rack gives a rack and pinion.

In the case of an external gear pair, the subscript 1 is used in formulae for the smaller gear (pinion) and the subscript 2 for the larger gear (gear wheel). When the gears are of the same size, the subscripts can be allocated as desired. In the case of a parallel axis external gear pair with helical gear teeth, one gear has a left-handed and the other gear (mating gear) has a right-handed helix.

5.2.4 Internal gear pair

The mating of an external cylindrical gear with an internal cylindrical gear gives an internal gear pair.

In the case of an internal gear pair, the subscript 1 is used in formulae for the external gear and the subscript 2 for the internal gear. In the case of an internal gear pair with helical gear teeth, both gears have the same hand of helix. Both are either right-handed or left-handed.

5.3 Mating quantities

5.3.1 Gear ratio

The gear ratio, u , of a gear pair is the ratio of the number of teeth of the gear wheel (or internal gear), z_2 , to the number of teeth of the pinion, z_2 :

$$u=\frac{z_2}{z_1}=\frac{d_{b2}}{d_{b1}}=\frac{d_2}{d_1}=\frac{d_{w2}}{d_{w1}}, \qquad |u|\geq 1 \tag{63}$$

NOTE See 5.3.5 for d_{w1} and d_{w2} .

5.3.2 Driving gear, driven gear and transmission ratio

The driving gear introduces rotation to the gear pair and effects the rotation of the driven gear.

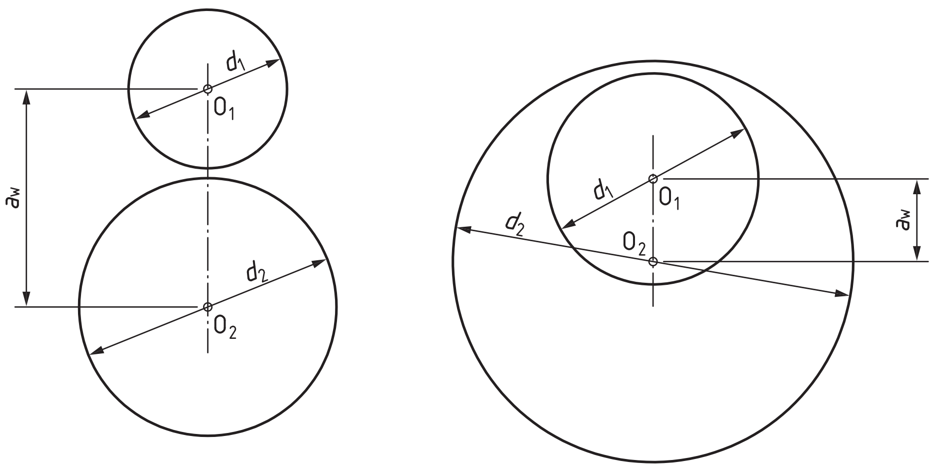
The transmission ratio, i , of a gear pair is the ratio of the angular velocity (rotational speed) of the driving gear (subscript a) to that of the driven gear (subscript b):

$$i=\frac{\omega_a}{\omega_b}=\frac{n_a}{n_b}=-\frac{z_b}{z_a} \tag{64}$$

In the case of an external gear pair, the two cylindrical gears rotate in opposite directions, i.e. their angular velocities or rotational speeds have opposite signs; the transmission ratio is negative. In the case of an internal gear pair, the two cylindrical gears have the same direction of rotation, i.e. their angular velocities or rotational speeds have the same sign; the transmission ratio is positive. If it is necessary to make a distinction, ratios such that $|i| > 1$ are said to be “speed reducing ratios” while ratios such that $|i| < 1$ are said to be “speed increasing ratios”.

5.3.3 Line of centres and centre distance

In a transverse section of two mating gears, the line which connects the two axes is called the line of centres. The centre distance, a_w , is the working distance between the gear axes of the two gears on the line of centres, see Figure 17.



NOTE For internal gears, since the number of teeth is negative, the diameters used in calculations are negative and the centre distance is negative. On drawings, the diameters are usually shown as positive numbers.

Figure 17 — Line of centres, centre distance

5.3.4 Working transverse pressure angle

The working transverse pressure angle, α_{wt}, is that pressure angle whose vertex lies on the working pitch circle. When the centre distance, a_w, is known, α_{wt} is calculated from:

$$\cos\alpha_{wt} = (z_1 + z_2)\left(\frac{m_n \cos\alpha_t}{2a_w \cos\beta}\right) \tag{65}$$

Alternatively, with nominal profile shift coefficients, α_{wt} results from:

$$\text{inv}\,\alpha_{wt} = \text{inv}\,\alpha_t + \frac{2\tan\alpha_n}{z_1 + z_2}(x_1 + x_2) \tag{66}$$

NOTE The inverse involute function can be used to solve for the working pressure angle, α_{wt}. One method for calculating the inverse involute is given in ISO 21771-2:—[1], Annex D.

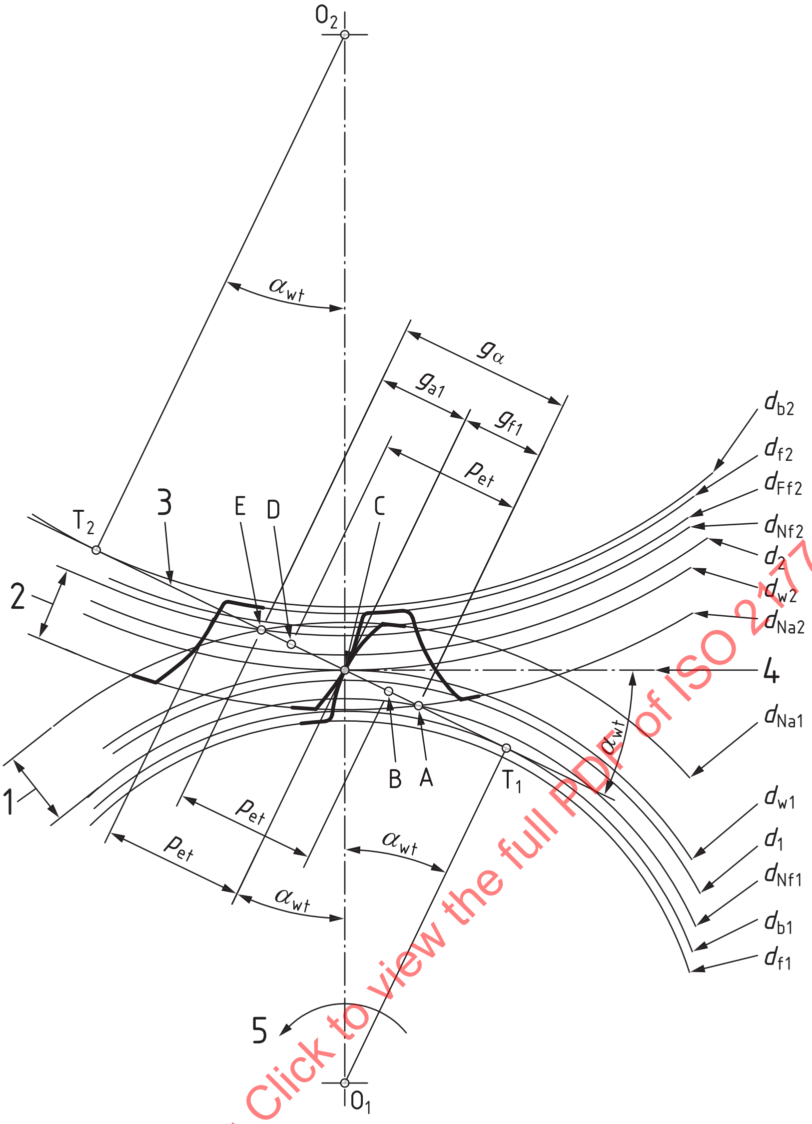
5.3.5 Working pitch point, pitch cylinders, pitch circles, pitch diameter and pitch axis

The working pitch point C divides the centre distance in the ratio of the tooth numbers. The working pitch cylinders (working pitch circles) are those cylinders (circles) which pass through the working pitch point. The working pitch diameter is the diameter of the working pitch circle. The working pitch axis is the axis through the working pitch point, parallel to the axis of the reference cylinders.

NOTE During operation, the circumferential velocities on the working pitch cylinders are the same.

The pitch circles established during the operation of a cylindrical gear pair (gear pair in a gear unit) are termed working pitch circles (d_w). (See Figures 18 and 20.) The pitch circles established by a generating cutter during the generating of a tooth system in a generating gear pair are termed generating pitch circles.

1) Under preparation. Stage at the time of publication: ISO/DIS 21771-2:2024.



Key

1 radial part of active flank gear 1

2 radial part of active flank gear 2

3 line of action

4 tangent to working pitch circles

5 direction of rotation of driving pinion

NOTE 1 See 5.5.6.1 for description of lettered points.

NOTE 2 Figure 18 is for a transverse section of an external gear pair.

Figure 18 — Meshing conditions and active ranges on working flanks, external gear pair

The diameters of the working pitch circles are:

$$d_{\mathrm{w1}} = \frac{2z_1 a_{\mathrm{w}}}{z_1 + z_2} = d_1 \frac{\cos\alpha_{\mathrm{t}}}{\cos\alpha_{\mathrm{wt}}} = \frac{d_{\mathrm{b1}}}{\cos\alpha_{\mathrm{wt}}} \tag{67}$$

$$d_{\mathrm{w2}} = \frac{2z_2 a_{\mathrm{w}}}{z_1 + z_2} = d_2 \frac{\cos\alpha_{\mathrm{t}}}{\cos\alpha_{\mathrm{wt}}} = \frac{d_{\mathrm{b2}}}{\cos\alpha_{\mathrm{wt}}} \tag{68}$$

NOTE For an internal gear, d_{w2} is negative and for an internal gear pair, a_{w} is negative.

This gives a working centre distance:

$$a_w = \frac{1}{2}(d_{w2} + d_{w1}) \tag{69}$$

5.3.6 Working helix angles

For parallel axis gears, the helix angle at the working pitch diameter of the pinion has the same magnitude but opposite hand from the helix angle at the working pitch diameter of the gear wheel:

$$\beta_{w1} = -\beta_{w2} \tag{70}$$

The working helix angle can be determined from the working pitch diameter, base diameter, and base helix angle:

$$\tan\beta_{w1} = \frac{d_{w1}}{d_{b1}} \cdot \tan\beta_{b1} \tag{71}$$

$$\tan\beta_{w2} = \frac{d_{w2}}{d_{b2}} \cdot \tan\beta_{b2} \tag{72}$$

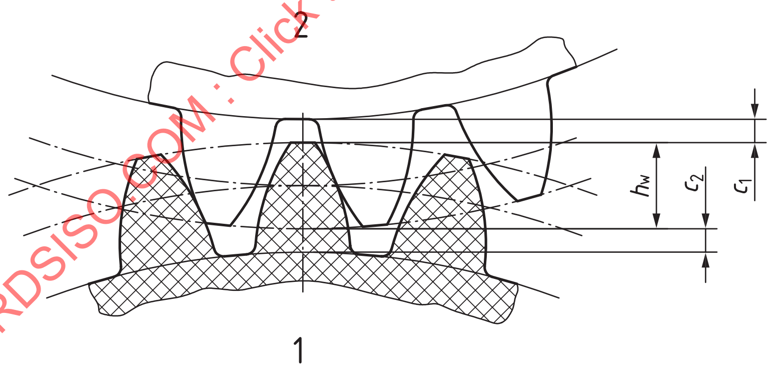
5.3.7 Working depth

The working depth, h_w, of a gear pair is the overlap of the tip circles of the two cylindrical gears on the line of centres, see Figure 19:

$$h_w = \frac{d_{a1} + d_{a2}}{2} - a_w \tag{73}$$

5.3.8 Tip clearance

The tip clearance, c, is the distance by which the tip circle of a gear is separated from the root circle of the mating gear, see Figure 19.



Key

1 pinion

2 gear wheel

Figure 19 — Working depth, h_w, and tip clearances c_1 and c_2 of a gear pair

The actual clearance follows from the centre distance, a_w, the manufactured tip diameter, d_a, and the generated root diameter, d_{fE}. For a pinion it is

$$c_1 = a_w - \frac{d_{a1}}{2} - \frac{d_{fE2}}{2} \tag{74}$$

and for a gear wheel

$$c_2 = a_w - \frac{d_{a2}}{2} - \frac{d_{fE1}}{2} \tag{75}$$

5.3.9 Calculation of tip alteration coefficient k (for parallel axis gear)

It is sometimes necessary for the addendum to be altered to suit the mating conditions and the specified minimum tip clearance of the mate. This is known as tip alteration, and is also sometimes called addendum modification, truncation, or tip shortening, and is usually given as a coefficient (i.e. normalized by the normal module so it is dimensionless). Tip alteration is usually used to shorten the teeth, but it can be used to enlarge the addendum.

If the tip clearance c_1 (or c_2) corresponding to the basic rack tooth profile is to be retained, then for many gear sets, the necessary tip alteration coefficient k (for both pinion and gear) can be estimated as:

$$\begin{aligned} k &= \frac{1}{m_n}\left(a_w - \frac{d_1 + d_2}{2}\right) - (x_1 + x_2) \\ &= \frac{a_w}{m_n} - \frac{z_1 + z_2}{2 \cdot \cos\beta} - (x_1 + x_2) \\ &= \frac{z_1 + z_2}{2 \cdot \cos\beta}\left(\frac{\cos\alpha_t}{\cos\alpha_{wt}} - 1\right) - (x_1 + x_2) \end{aligned} \tag{76}$$

The tip alteration coefficient, k, calculated in this way is obtained with correct sign, that is to say:

— negative values in the case of external gear pairs, so that the tip diameters become smaller;

— positive values in the case of internal gear pairs, so that the absolute value of the tip diameter of the internal gear becomes smaller while the diameter of the pinion tip circle becomes larger.

The calculated values are often so small that they are cancelled out by the deeper infeed of the cutting tool which is necessary for producing the backlash and by the negative root diameter deviations so that the remaining effective tip clearance of the mate is altered to only a slight extent (or within permissible limits). See 5.3.8.

In the case of internal gear pairs, the tip alteration coefficient, k, which is always positive in this case can usually not be realized because the special engagement and manufacturing conditions of internal gear pairs can limit the usable addendum of the internal gear and pinion if tip-to-tip interference is to be avoided. See 5.5.8.3.

5.4 Calculation of the sum of the profile shift coefficients

The sum of the profile shift coefficients which corresponds to the zero-backlash condition is related to the basic tooth parameters and centre distance by Formula (77), with α_{wt} from Formula (65):

$$\sum x = x_1 + x_2 = \frac{(z_1 + z_2)(\mathrm{inv}\,\alpha_{wt} - \mathrm{inv}\,\alpha_t)}{2\tan\alpha_n} \tag{77}$$

In the case of non-zero backlash, the normal backlash, j_{bn} (see 5.6.4), is included in the calculation:

$$\sum x_E = x_{E1} + x_{E2} = \frac{(z_1 + z_2)(\mathrm{inv}\,\alpha_{wt} - \mathrm{inv}\,\alpha_t)}{2\tan\alpha_n} - \frac{j_{bn}}{2m_n \sin(\alpha_n)} \tag{78}$$

The way in which $\sum x = x_1 + x_2$ is distributed between the two gears can be decided on the basis of aspects such as permissible stress, sliding velocities or other specified dimensions of the gear teeth such as root diameter.

5.5 Tooth engagement

5.5.1 General

Tooth engagement refers to the meshing of a gear (or a rack) with its mating gear. The tooth engagement is influenced by the geometry of the gear pair (or rack and pinion), the mutual contact of the tooth flanks and the sliding conditions, see Figures 18 and 20.

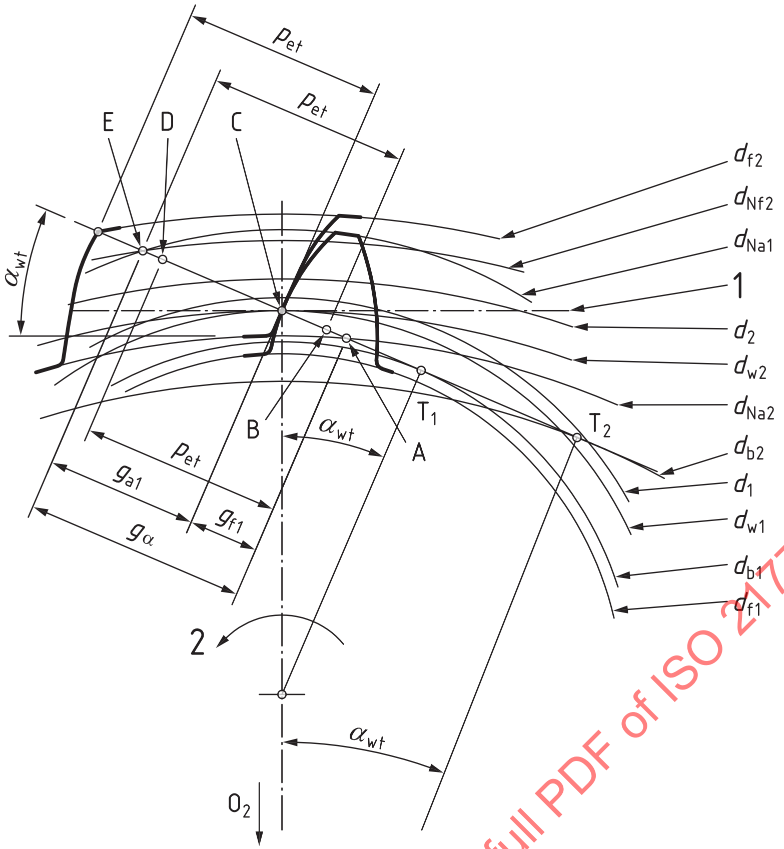
5.5.2 Active area of the tooth flanks, start of active profile and active tip diameters

5.5.2.1 General

The start of involute diameter (root form diameter), d_{Ff}, is the start of the involute portion of the profile. For an external gear it is the greater of the base diameter, d_b, or the diameter of the intersection of the flank with the tooth root fillet or trochoid (taking into account undercut, if necessary, see Clause 10).

In a defined gear pair, the active tip diameter, d_{Na}, of a gear can be governed either by its tip form diameter, d_{Fa}, or by the start of involute of its mate, d_{Ff}. The start of active profile (active root) diameter, d_{Nf}, can be governed either by the diameter of start of involute d_{Ff}, or by the tip form diameter of its mate, d_{Fa}. During rotation, the active tip diameter of a gear will contact the active root diameter of the mating gear. The active area of the flank extends from the active tip diameter, d_{Na}, to the active root diameter, d_{Nf}, and so is dependent on the characteristics of both gears and the centre distance.

Usually, contact in the dedendum on both gears is limited by the tip form diameter of the mating gear.



Key

1 tangent to working pitch circles

2 direction of rotation of driving pinion

NOTE 1 See 5.5.6.1 for a description of points A, B, C, D and E.

NOTE 2 For internal gears, since the number of teeth is negative, the diameters used in calculations are negative. On drawings the diameters are usually shown as positive numbers.

NOTE 3 Figure 20 is for a transverse section of an internal gear pair.

Figure 20 — Meshing conditions and active ranges on working flanks, internal gear pair

5.5.2.2 Pinion start of active profile and active tip diameters

When contact in the pinion dedendum is limited by the tip form diameter of the gear wheel:

$$d_{\mathrm{Nf1}} = \sqrt{\left(2\left|a_{\mathrm{w}}\right|\sin\alpha_{\mathrm{wt}} - \sqrt{d_{\mathrm{Fa2}}^2 - d_{\mathrm{b2}}^2}\right)^2 + d_{\mathrm{b1}}^2} \quad (79)$$

$$d_{\mathrm{Na2}} = d_{\mathrm{Fa2}}. \quad (80)$$

However, if d_{Ff1} is greater than the quantity calculated for d_{Nf1} , then:

$$d_{\mathrm{Nf1}} = d_{\mathrm{Ff1}} \quad (81)$$

$$d_{\mathrm{Na2}} = \frac{z_2}{\left|z_2\right|}\sqrt{\left(2a_{\mathrm{w}}\sin\alpha_{\mathrm{wt}} - \sqrt{d_{\mathrm{Ff1}}^2 - d_{\mathrm{b1}}^2}\right)^2 + d_{\mathrm{b2}}^2} \quad (82)$$

See 9.7, 10.3 or 11.3 for d_{Ff}.

It is important to verify if the gear teeth need to be truncated to avoid gear mesh interference with the tooth root fillet of the pinion.

With α_{Nf1} from:

$$\cos\alpha_{Nf1} = \frac{d_{b1}}{d_{Nf1}} \tag{83}$$

The corresponding roll angles (see 4.4.9) are:

$$\xi_{Nf1} = \frac{z_2}{z_1}(\xi_{wt} - \xi_{Na2}) + \xi_{wt} \tag{84}$$

$$\xi_{Na2} = \tan\left(\arccos\frac{d_{b2}}{d_{Na2}}\right) \tag{85}$$

5.5.2.3 Gear wheel start of active profile and active tip diameters

When contact in the gear wheel dedendum is limited by the tip form diameter of the pinion:

$$d_{Nf2} = \frac{z_2}{|z_2|}\sqrt{\left(2a_w \sin\alpha_{wt} - \sqrt{d_{Fa1}^2 - d_{b1}^2}\right)^2 + d_{b2}^2} \tag{86}$$

$$d_{Na1} = d_{Fa1} \tag{87}$$

However, if d_{Ff2} is greater than the quantity calculated for d_{Nf2}, then:

$$d_{Nf2} = d_{Ff2} \tag{88}$$

$$d_{Na1} = \sqrt{\left(2|a_w|\sin\alpha_{wt} - \sqrt{d_{Ff2}^2 - d_{b2}^2}\right)^2 + d_{b1}^2} \tag{89}$$

It is important to verify if the pinion teeth need to be truncated to avoid gear mesh interference with the tooth root fillet of the gear wheel.

With α_{Nf2} from:

$$\cos\alpha_{Nf2} = \frac{d_{b2}}{d_{Nf2}} \tag{90}$$

The corresponding roll angles (see 4.4.9) are:

$$\xi_{Nf2} = \frac{z_1}{z_2}(\xi_{wt} - \xi_{Na1}) + \xi_{wt} \tag{91}$$

$$\xi_{Na1} = \tan\left(\arccos\frac{d_{b1}}{d_{Na1}}\right) \tag{92}$$

5.5.3 Plane of action, zone of action and contact line

The plane of action of a cylindrical gear pair is tangent to the base cylinders of the gear and mating gear. In the case of an external gear pair, the plane of action passes between the base cylinders. The intersection of the two planes of action (one for each tooth flank) is parallel to the gear axis, and is the working pitch axis (see 5.3.5). The zones of action are the parts of the planes of action which are limited by the active tip diameters d_{Na1}, d_{Na2} of the gear and mating gear and by the facewidth. A zone of action is linked to the flank that is normal to it. Hence, one of the planes of action is linked to the right flanks and the other to the left flanks.

At any instant in time, the intersection of the zone of action with a pair of corresponding tooth flanks of a gear pair is known as the contact line. With the rotation of the gears around their axes, the contact lines move through the zone of action (see Figure 25). On tooth flanks, the contact lines are identical to the generators of flank and mating flank, see 4.4.1.

5.5.4 Line of action, path of contact and point of contact

Lines of action are where the planes of action intersect transverse sections. The left flank line of action is defined by the left flank of the driving gear. A line of action is inclined to the common tangent to the working pitch circles at the working pitch point (working pitch circle tangent) by the working transverse pressure angle, α_{wt} (see Figures 18 and 20), and it contacts the two base circles at the points T_1 and T_2.

A path of contact is that part of the line of action which is within the zone of action. The starting point, A, of the path of contact is at or near the root circle of the pinion. The finishing point, E, of the path of contact is at or near the tip circle of the pinion.

NOTE In Figures 18 and 20, only the line of action of the working flanks is shown in each case.

Both the left and right lines of action intersect the centre line at working pitch point C (see Figures 18 and 20).

A point of contact is a point where a path of contact intersects the corresponding tooth flanks in a specific working position of the two gears. It is a point on the contact line.

5.5.5 Form over dimension

The form over dimension, c_F, is the radial distance between the active root diameter and root form diameter.

$$c_F = \frac{1}{2}(d_{Nf} - d_{Ff}) \quad (93)$$

5.5.6 Designations and values relating to the line of action

5.5.6.1 Special points on the line of action

Special points on the line of action (see Figures 18 and 20) are as follows:

- T_1 is the point of contact between the line of action and the base circle of pinion (d_{b1});
- T_2 is the point of contact between the line of action and the base circle of gear wheel (d_{b2});
- C is the working pitch point, the intersection of the line of action with the line of centres.

In Figures 18 and 20, ε_α is less than 2. Special points on the path of contact are the following:

- A is the point at which the line of action intersects the active tip diameter, d_{Na2}, of the gear wheel (limit point of meshing near pinion root);
- B is the lowest point of single tooth contact (LPSTC) on the pinion, highest point of single tooth contact (HPSTC) on the gear wheel; where $\varepsilon_\alpha < 2$, it is the point within the path of contact which is one transverse base pitch away from point E;
- D is the highest point of single tooth contact (HPSTC) on the pinion, lowest point of single tooth contact (LPSTC) on the gear wheel; where $\varepsilon_\alpha < 2$, it is the point within the path of contact which is one transverse base pitch away from point A;
- E is the point at which the line of action intersects the active tip diameter, d_{Na1}, of the pinion (limit point of meshing near the pinion tip).

NOTE Points A through E go from pinion root to pinion tip, regardless of whether pinion or wheel drives.

5.5.6.2 Length of the path of contact

The length, g_α, of the path of contact (length between points A and E on the path of contact) of two mating cylindrical gears is:

$$g_\alpha = \overline{AE} = \frac{1}{2}\left[\sqrt{d_{Na1}^{2} - d_{b1}^2} + \frac{z_2}{|z_2|}\sqrt{d_{Na2}^2 - d_{b2}^2} - 2a_w \sin\alpha_{wt}\right] \quad (94)$$

NOTE The symbol $\overline{AE}$ indicates the distance between points A and E.

The length of the path of contact when a cylindrical gear (subscript 1) is mated with a rack is:

$$g_\alpha = \frac{1}{2}\left(\sqrt{d_{Na1}^2 - d_{b1}^2} - d_{b1}\tan\alpha_t\right) + \frac{h_{aP} - x_1 m_n}{\sin\alpha_t} \quad (95)$$

The path of contact is divided by working pitch point C into the addendum path of contact and the dedendum path of contact. The addendum path of contact is the portion of the path of contact between the active tip diameter, d_{Na}, and the working pitch point. The dedendum path of contact goes from the working pitch point towards the root to the start of active profile diameter, d_{Nf}, see Figures 18 and 20.

The addendum path of contact, g_a, of the pinion is equal to the dedendum path of contact, g_f, of the gear wheel:

$$g_{a1} = \overline{CE} = \frac{1}{2}\left(\sqrt{d_{Na1}^2 - d_{b1}^2} - d_{b1}\tan\alpha_{wt}\right) = g_{f2} \quad (96)$$

Similarly, the addendum path of contact of the gear wheel, g_{a2}, is equal to the dedendum path of contact of pinion, g_{f1}:

$$g_{a2} = \overline{AC} = \frac{1}{2}\left(\frac{z_2}{|z_2|}\sqrt{d_{Na2}^2 - d_{b2}^2} - d_{b2}\tan\alpha_{wt}\right) = g_{f1} \quad (97)$$

The two portions of the path of contact are also known as the approach path of contact and the recess path of contact. The approach path of contact corresponds to the addendum path of contact of the driven gear. So, when the pinion is driving and the gear wheel is the driven gear, the approach path of contact is the addendum path of contact of the gear wheel, and the recess path of contact is the dedendum path of contact of the gear wheel.

For the case where the gear wheel is the driver, the approach path of contact is the addendum path of contact of the pinion and the recess path of contact is the dedendum path of contact of the pinion.

5.5.7 Radii of curvature of the tooth flanks in a transverse plane

The following segments of the lines of action give rise to the radii of curvature of the tooth flanks in the transverse plane (see Figures 18 and 20):

$$\overline{T_1C} = \rho_{C1} = \frac{1}{2}\sqrt{d_{w1}^2 - d_{b1}^2} = \frac{1}{2}d_{b1}\tan\alpha_{wt} \quad (98)$$

$$\overline{T_2C} = \rho_{C2} = \frac{1}{2}\frac{z_2}{|z_2|}\sqrt{d_{w2}^2 - d_{b2}^2} = \frac{1}{2}d_{b2}\tan\alpha_{wt} \quad (99)$$

$$\overline{T_2A} = \rho_{A2} = \frac{1}{2}\frac{z_2}{|z_2|}\sqrt{d_{Na2}^2 - d_{b2}^2} \quad (100)$$

$$\overline{T_1E} = \rho_{E1} = \frac{1}{2}\sqrt{d_{Na1}^2 - d_{b1}^2} \quad (101)$$

$$\overline{T_1B} = \rho_{B1} = \rho_{E1} - p_{et} \quad (102)$$

$$\overline{T_2D} = \rho_{D2} = \rho_{A2} - p_{et} \quad (103)$$

$$\overline{T_1T_2} = \rho_{C1} + \rho_{C2} = a_w \sin\alpha_{wt} = \rho_{A1} + \rho_{A2} = \rho_{E1} + \rho_{E2} \quad (104)$$

NOTE The values obtained for the curvature radii of an internal gear and for the segment $\overline{T_1T_2}$ of an internal gear pair are negative values.

See 4.9 and Clause 7 for additional information on radius of curvature of the tooth flanks.

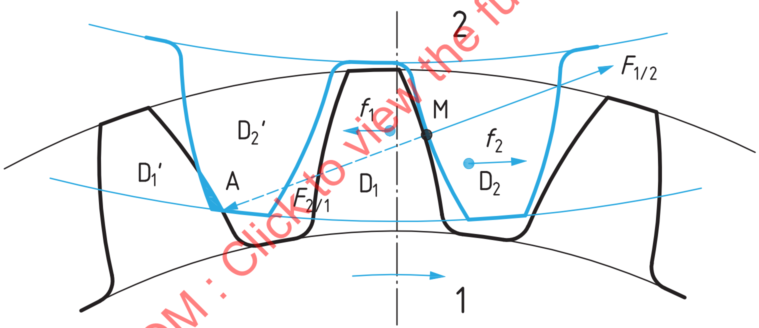
5.5.8 Tooth interference

5.5.8.1 General

Interference conditions occur if parts of the gear tooth flanks come into contact with any part of the mating gear other than the working flanks or if the top lands have any contact with the mating gear.

5.5.8.2 Tip to dedendum interference

For a parallel axis cylindrical gear pair, interference can occur either between the dedendum of active flank of the pinion and the tooth tip corner of the conjugate gear or between the dedendum of active flank of the gear and the tooth tip corner of the conjugate pinion, see Figure 21.



Key

1	driving pinion	f_1	pinion tooth bending deformation
2	driven gear	f_2	gear tooth bending deformation
D_1-D_2	pair of teeth in contact at point M	$F_{1/2}$	transmitted force
D_1'-D_2'	next pair of teeth	$F_{2/1}$	reaction force

Figure 21 — Interference between gear teeth

To avoid this interference the following conditions shall be satisfied:

— for external gear: $|\overline{CA}| < |\overline{CT_1}|$ and $|\overline{CE}| < |\overline{CT_2}|$;

— for internal gear: $|\overline{CA}| < |\overline{CT_1}|$ and $|d_{Nf2}| < |d_{Ff2}|$.

5.5.8.3 Tip-to-tip interference on internal gears

An internal gear pair can have a second type of interference, in which the tooth tip corner of the internal gear contacts the tooth tip corner of the pinion as they come into mesh.

In Figure 22, two mating gears are shown in contact at the pitch point, C. The clearance between the teeth can be seen three or four teeth away from the pitch point. However, as one moves further from the pitch point to the intersection of the two tip circles, interference between the tips of teeth on the two gears can be seen.

To avoid this secondary interference, the actual rotation of the gear tooth caused by the rotation of the pinion tooth shall be greater than the rotation of the gear tooth that would allow interference. The actual rotation of the pinion tooth is given by:

$$\omega_1 = \theta_{aa1} + \theta_{a1} \quad (105)$$

Where θ_{aa1} is the angle from the pitch point to the intersection of the tip circles on the pinion. It can be found from the law of cosines that:

$$\cos\theta_{aa1} = \frac{r_{a2}^2 - r_{a1}^2 - a_w^2}{2a_w r_{a1}} = \frac{d_{a2}^2 - d_{a1}^2 - 4a_w^2}{4a_w d_{a1}} \quad (106)$$

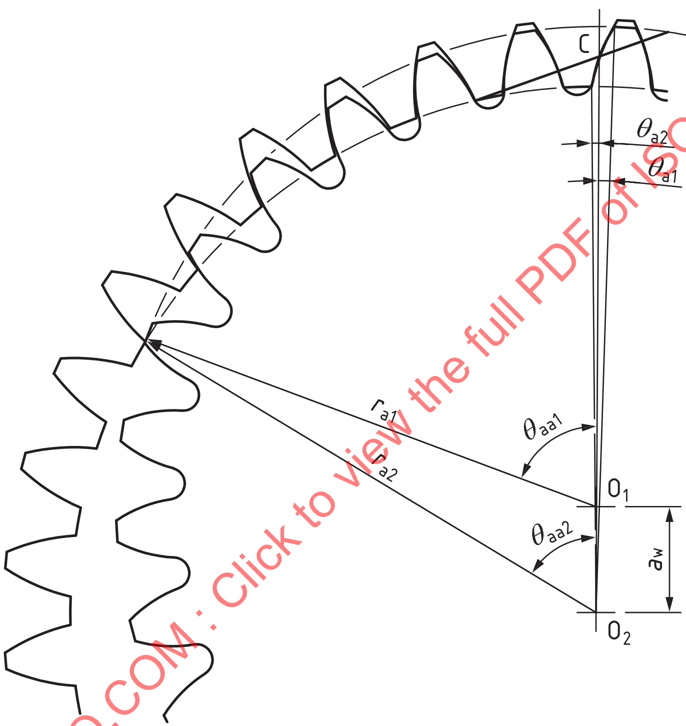


Figure 22 — Tip-to-tip interference with an internal gear

The angle θ_{a1} is the angle on the pinion tooth between the pitch circle and the tip circle. It is given by:

$$\theta_{a1} = \mathrm{inv}(\alpha_{at1}) - \mathrm{inv}(\alpha_{wt}) \quad (107)$$

where α_{wt} is the working transverse pressure angle and α_{at1} is the transverse pressure angle at the tip circle.

The rotation limit of the gear is given by:

$$\omega_2 = \theta_{aa2} - \theta_{a2} \tag{108}$$

where θ_{aa2} is the angle from the pitch point to the intersection of the tip circles on the gear, found from θ_{aa1} by the law of sines, such that:

$$\sin\theta_{aa2} = \frac{r_{a1}\sin\theta_{aa1}}{r_{a2}} \tag{109}$$

and where θ_{a2} is the angle on the gear tooth between the pitch circle and the tip circle, that is:

$$\theta_{a2} = \text{inv}(\alpha_{wt}) - \text{inv}(\alpha_{at2}) \tag{110}$$

Since the actual gear rotation shall be greater than this rotation limit for a rotation ω_1 of the pinion, $z_1\omega_1 > z_2\omega_2$ to avoid tip-to-tip interference.

5.5.9 Mesh geometry parameters

5.5.9.1 Transverse angle of transmission and transverse contact ratio

The transverse angle of transmission, φ_α, is the centre angle through which a gear of a gear pair rotates from start to finish of engagement of one active tooth flank transverse profile with its mating profile. The transverse angle of transmission of pinion and gear is given as follows:

$$\varphi_{\alpha 1} = \frac{2g_\alpha}{d_{b1}} = |u|\,\varphi_{\alpha 2} \tag{111}$$

$$\varphi_{\alpha 2} = \frac{2g_\alpha}{|d_{b2}|} = \frac{\varphi_{\alpha 1}}{|u|} \tag{112}$$

The transverse contact ratio, ε_α, is the ratio of the transverse angle of transmission, φ_α, to the angular pitch, τ, or the ratio of the path of contact to the transverse base pitch:

$$\varepsilon_\alpha = \frac{\varphi_{\alpha 1}}{\tau_1} = \frac{\varphi_{\alpha 2}}{\tau_2} = \frac{g_\alpha}{p_{et}} = \frac{g_f + g_a}{p_{et}} \tag{113}$$

where $p_{et} = p_{bt}$ [see Formula (33)].

5.5.9.2 Active facewidth

The active facewidth, b_w, is the minimum overlapping section of the usable facewidths of the gear pair, see Figure 23 and Figure 5.

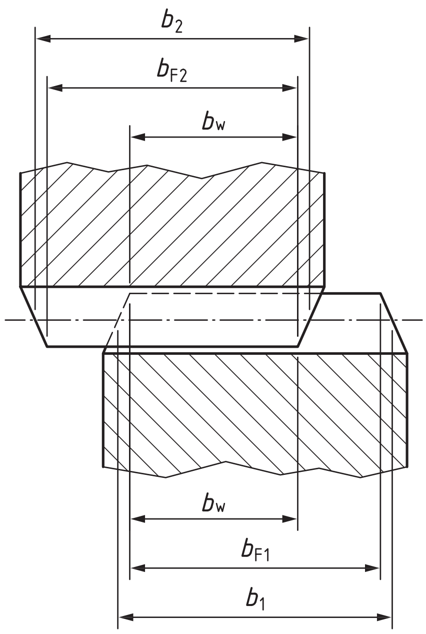


Figure 23 — Active facewidth, b_w

5.5.9.3 Overlap angle and overlap ratio (axial contact ratio)

The overlap angle, φ_β, is the angle between the two axial planes enclosing the end points of a tooth trace of the gear pair, see Figure 24:

$$\varphi_{\beta 1} = \frac{2\, b_w \tan|\beta|}{d_1} = \frac{2\, b_w \sin|\beta|}{m_n\, z_1} = |u|\, \varphi_{\beta 2} \tag{114}$$

$$\varphi_{\beta 2} = \frac{2\, b_w \sin|\beta|}{m_n\, |z_2|} = \frac{\varphi_{\beta 1}}{|u|} \tag{115}$$

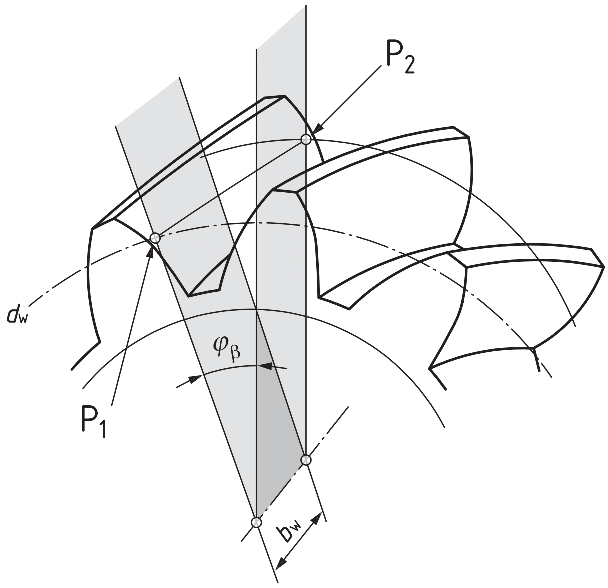
The overlap ratio, ε_β, is the ratio of the overlap angle, φ_β, to the angular pitch, τ, or the ratio of the facewidth, b_w, to the axial pitch, p_x. It is the contact ratio in the axial plane, and is sometimes referred to as the axial contact ratio.

$$\varepsilon_\beta = \frac{\varphi_{\beta 1}}{\tau_1} = \frac{\varphi_{\beta 2}}{\tau_2} = \frac{b_w}{p_x} = \frac{b_w \sin|\beta|}{m_n\, \pi} = \frac{b_w \tan|\beta|}{p_t} = \frac{b_w \tan|\beta_b|}{p_{et}} \tag{116}$$

5.5.9.4 Overlap roll length

The overlap roll length, g_β, of a helical gear is the length of the working pitch circle arc belonging to the overlap angle, φ_β:

$$g_\beta = \frac{|d_w|}{2} \varphi_\beta = b_w \tan|\beta_w| \tag{117}$$



Key

P1, P2 end points of an active tooth trace of a gear pair

Figure 24 — Overlap angle of gear pair, φ_β

5.5.9.5 Total angle of transmission and total contact ratio

The total angle of transmission, φ_γ, is the angle at the centre of a gear in a gear pair through which the gear rotates from start to finish of contact of one of its flanks with its mating flank. It is equal to the sum of the transverse angle of transmission and the overlap angle:

$$\varphi_{\gamma 1} = \varphi_{\alpha 1} + \varphi_{\beta 1} = |u|\, \varphi_{\gamma 2} \tag{118}$$

$$\varphi_{\gamma 2} = \varphi_{\alpha 2} + \varphi_{\beta 2} = \frac{\varphi_{\gamma 1}}{|u|} \tag{119}$$

The total contact ratio, ε_γ, is the ratio of the total angle of transmission to the angular pitch. It is equal to the sum of the transverse contact ratio and the overlap ratio:

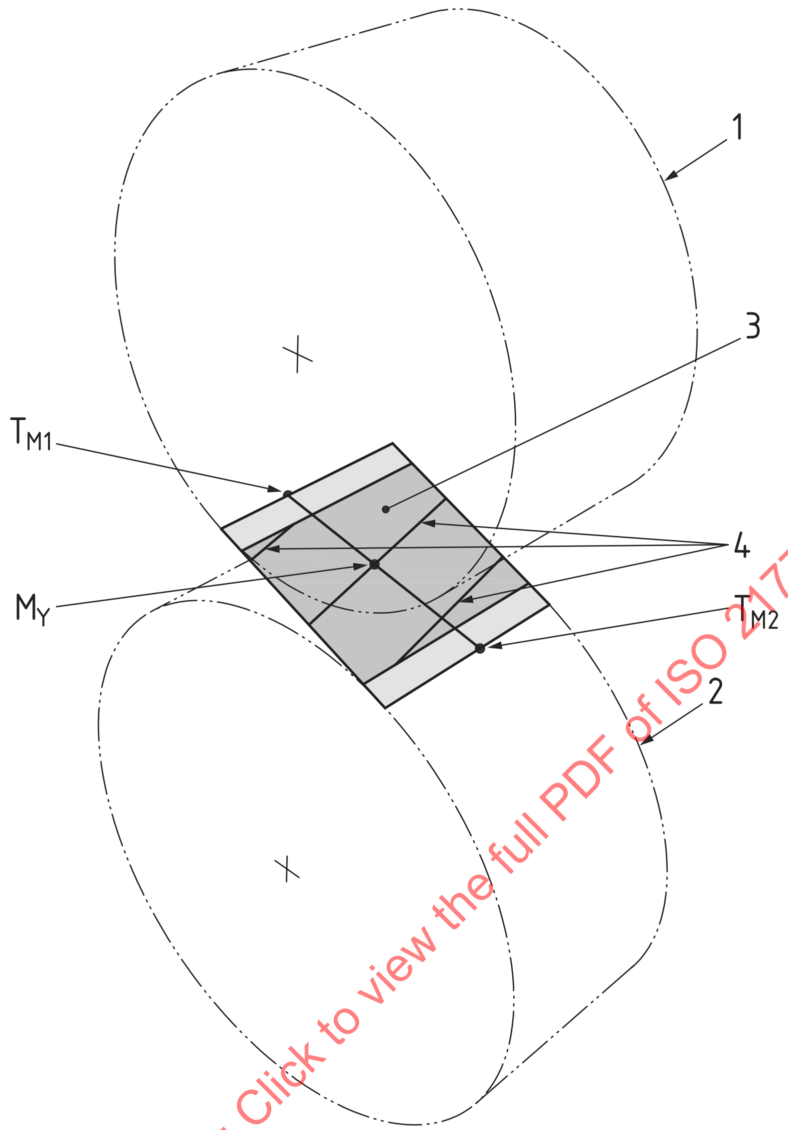
$$\varepsilon_\gamma = \frac{\varphi_{\gamma 1}}{\tau_1} = \frac{\varphi_{\gamma 2}}{\tau_2} = \varepsilon_\alpha + \varepsilon_\beta \tag{120}$$

5.5.10 Contact line and sum of the contact line lengths

The contact line is the theoretical line at an instant in time where the flanks of a tooth pair of the gear and mating gear touch; l_{max} is the maximum length of such a contact line of a flank pair.

When spur gears are in contact, the individual tooth pair contact line length remains constant. When tooth traces are not modified (e.g. by crowning), the value of l_{max} is equal to the active facewidth, b_w; see 5.5.9.2.

In the case of helical gears, the contact lines are within the zone of action and are at angle β_b to the working pitch axis, see Figure 25. The length of the contact line changes with the rolling of the flank pair. It starts as a point contact at the beginning of engagement, reaches its maximum value, l_{max}, in a working position or in a certain range of rotational angles and is subsequently reduced to point contact at the end of the engagement of the flank pair.



Key

1 base cylinder of pinion

2 base cylinder of gear wheel

3 zone of action

4 contact lines on mating helical gears

Figure 25 — Zone of action

The maximum length of a contact line, l_{max}, is given by:

$$l_{max} = \frac{g_\alpha}{\sin|\beta_b|} \quad (121)$$

or

$$l_{max} = \frac{b_w}{\cos\beta_b} \quad (122)$$

whichever is less.

NOTE 1 If $\frac{b_w}{\cos\beta_b} > \frac{g_\alpha}{\sin|\beta_b|}$, then the contact line extends across the whole active range of the transverse profile but not across the whole facewidth; while if it is less, then the contact line extends across the whole facewidth but across only part of the active range of the transverse profile.

The sum of the individual contact lines, $\sum l$, is the total length of all the contact lines which occur at the same time when the gear pair is in an instantaneous working position in the zone of action.

NOTE 2 Σl varies with contact position unless the overlap ratio is an integer.

5.6 Backlash

5.6.1 General

The backlash is the clearance between the non-working flanks of the teeth of a gear pair when the working flanks are in contact. Backlash can be defined in either a normal or a transverse section and either perpendicular to the flank, in a circumferential direction (along an arc), at a specified diameter or a radial direction.

There is a distinction between the normal backlash, j_{bn}, transverse backlash, j_{bt}, and radial backlash, j_r; see Figure 26. See ISO 21771-2 for a more in-depth discussion of backlash, including prediction and measurement of backlash.

5.6.2 Transverse backlash

Transverse backlash, j_{bt}, is the shortest distance in a transverse plane between the non-working flanks of the teeth of a gear pair when the working flanks are in contact with zero force. It can be calculated as:

$$j_{bt} = \cos\alpha_{wt} \left(\frac{\pi d_{w1}}{z_1} - s_{wt1} - s_{wt2} \right) \tag{123}$$

NOTE If mean values are used for tooth thickness, then a mean value of backlash will result. Use of minimum tooth thickness values will result in calculation of maximum backlash.

5.6.3 Circumferential backlash

The circumferential backlash, j_{wt}, is the length of the working pitch circle arc through which each of the two gears can be rotated, whilst the other is held stationary, from the point where the right flanks are in contact to the point where the left flanks are in contact. Its magnitude is given by:

$$j_{wt} = \frac{1}{\cos\alpha_{wt}\ \cos\beta_b}\ j_{bn} \tag{124}$$

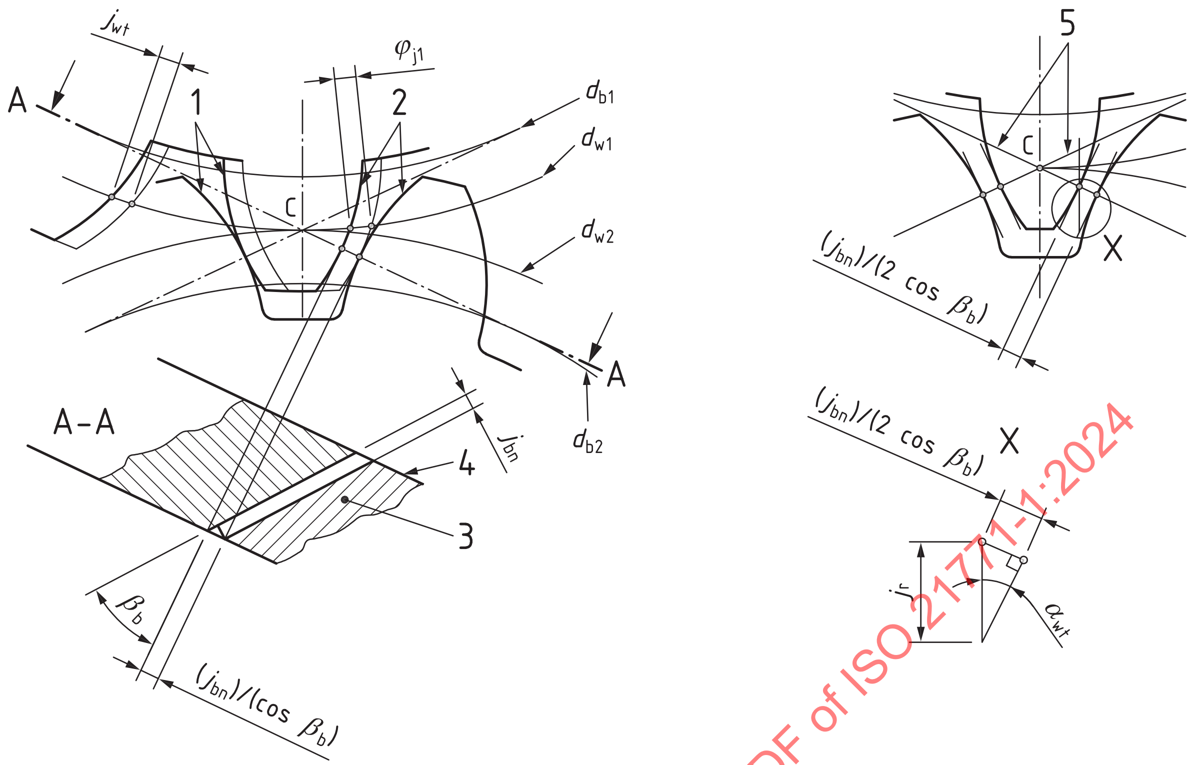
or in relation to the length of the reference circle arc by:

$$j_t = \frac{1}{\cos\beta\ \cos\alpha_n}\ j_{bn} \tag{125}$$

5.6.4 Normal backlash

Normal backlash, j_{bn}, is the shortest distance between the non-working flanks of the teeth of a gear pair when the working flanks are in contact with zero force. It is defined in the plane of action of the non-working flanks.

$$j_{bn} = j_{bt} \cos\beta_b \tag{126}$$



Key

1 working flanks

2 non-working flanks

3 plane of action non-working flanks

4 end face

5 plane of action

Figure 26 — Backlash

5.6.5 Radial backlash

The radial backlash, j_r, is the difference in the centre distance in the working condition of the gear pair and the centre distance produced if one of the gears is moved along the centre line until zero backlash engagement of the flank pairs occurs, see Figure 26 (right-hand side).

The relation between circumferential backlash, j_{wt}, and radial backlash, j_r, can be approximated by:

$$j_r = \frac{1}{2\tan\alpha_{wt}} j_{wt} \qquad (127)$$

NOTE The radial backlash can be found as the difference between the working centre distance and the tight mesh centre distance.

5.6.6 Angular backlash

The angular backlash, φ_j, is the angle of rotation through which the gear can be rotated, while the mating gear is stationary, from the point where the right flanks are in contact to the point where the left flanks are in contact, see Figure 26. The angular backlash for each gear is found from the normal backlash, j_{bn}:

$$\varphi_{j1} = \frac{2}{m_n\, z_1\, \cos\alpha_n}\, j_{bn} = \frac{2}{d_{w1}}\, j_{wt} \tag{128}$$

$$\varphi_{j2} = \frac{2}{m_n\, |z_2|\, \cos\alpha_n}\, j_{bn} = \frac{2}{d_{w2}}\, j_{wt} \tag{129}$$

5.7 Velocities and sliding conditions at the tooth flanks

5.7.1 Angular velocity

The angular velocity of a rotating gear is:

$$\omega = \frac{\pi n}{30} \tag{130}$$

The rotational speed, n, is expressed in rpm and angular velocity, ω, in radians per second.

5.7.2 Circumferential velocity

The circumferential velocity is always defined in a transverse plane, in a direction tangent to the cylinder. The circumferential velocity at a given diameter, d_y ,is:

$$v_y = \frac{\omega\, d_y}{2\,000} \tag{131}$$

The circumferential velocity at the reference diameter, d, is:

$$v = \frac{\omega\, d}{2\,000} \tag{132}$$

The circumferential velocity at the working pitch diameter, d_w, is:

$$v_w = \frac{\omega\, d_w}{2\,000} \tag{133}$$

The circumferential velocity at the base diameter, d_b, is:

$$v_b = \frac{\omega\, d_b}{2\,000} \tag{134}$$

NOTE The divisor of 2 000 is used, thus these velocities are in m/s.

5.7.3 Normal velocity

The velocity along the line of action, which is normal to the tooth profile in the transverse plane and therefore known as the normal velocity, is equal to the circumferential velocity at the base circle:

$$v_n = v_b \tag{135}$$

NOTE In a meshing involute gear pair, the circumferential velocity is the same at both base circles.

5.7.4 Rolling velocity

The rolling velocity is the velocity in a transverse plane in a direction tangent to the tooth flank at a given diameter d_y. It can be calculated by:

$$v_{ry} = \frac{\omega \rho_y}{1\,000} \quad (136)$$

where ρ_y can be found from Formula (22).

NOTE At a given point of contact in a meshing gear pair, the circumferential velocities of the two gears will have the same magnitude, and the normal velocities will have equal magnitudes. However, the angular velocities will differ and the rolling velocities will be different.

So, for the pinion, the rolling velocity is:

$$v_{ry1} = \frac{\omega_1}{1\,000} \overline{T_1 Y_1} = \frac{\omega_1 \rho_{y1}}{1\,000} \quad (137)$$

And for the gear wheel:

$$v_{ry2} = \frac{\omega_2}{1\,000} \overline{T_2 Y_2} = \frac{\omega_2 \rho_{y2}}{1\,000} \quad (138)$$

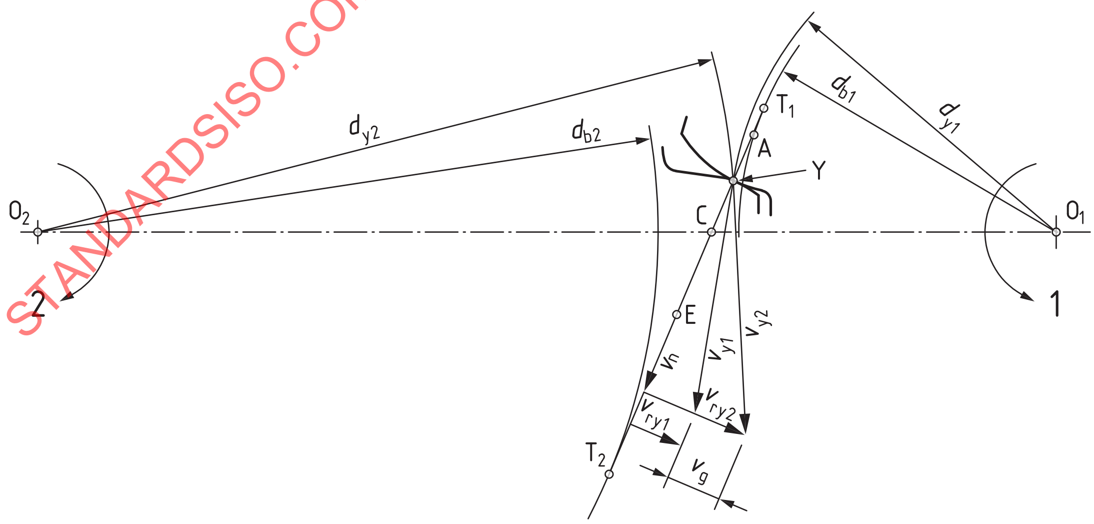
5.7.5 Sliding velocity

At a point of contact of two tooth flanks in engagement, the sliding velocity, v_g, is the difference of the velocities of the two transverse profiles in the direction of the common tangent. See Figures 27 and 28.

The sliding velocity is:

$$v_g = \left| v_{ry1} - v_{ry2} \right| = \left| \frac{\omega_1}{1\,000} \left(\frac{\rho_{y2}}{u} - \rho_{y1} \right) \right| \quad (139)$$

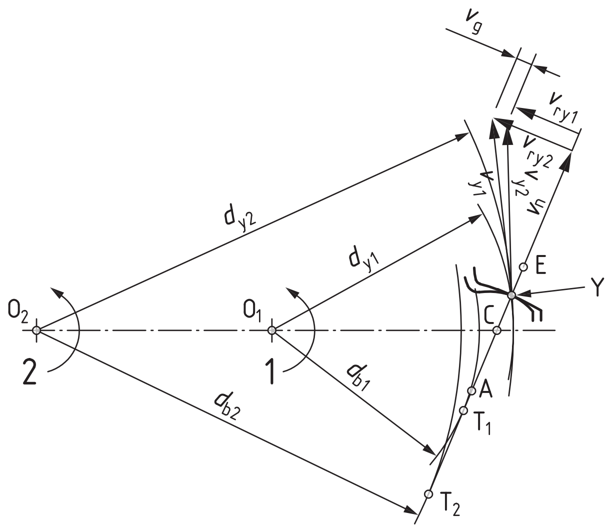
where |~| denotes absolute value and where the curvature radii, ρ_{y1} and ρ_{y2}, are to be determined using Formula (22).



Key

1 direction of rotation of driving pinion

Figure 27 — Sliding velocity, v_g, at point of contact Y on external gear pair



Key

1 direction of rotation of driving pinion

NOTE For internal gears, since the number of teeth is negative, the diameters used in calculations are negative. On drawings, the diameters are usually shown as positive numbers.

Figure 28 — Sliding velocity, v_g, at point of contact Y on internal gear pair

The distance, $g_{\alpha y}$, between Y and C is:

$$g_{\alpha y} = \left|\rho_{c1} - \rho_{y1}\right| = \left|\rho_{c2} - \rho_{y2}\right| \tag{140}$$

Hence

$$v_g = \left|\frac{\omega_1}{1\,000} g_{\alpha y} \left(1 + \frac{1}{u}\right)\right| \tag{141}$$

NOTE $g_{\alpha y}$ is always positive. Since u is positive for an external gear pair and negative for an internal gear pair, it usually follows that the sliding velocity is greater for external gear teeth than for internal gear teeth.

The sliding velocity is proportional to distance $g_{\alpha y}$ and equal to zero at the working pitch point. It reaches its maximum values at the end points A and E of the path of contact:

$$v_{gf} = \left|\frac{\omega_1}{1\,000} g_f \left(1 + \frac{1}{u}\right)\right| \tag{142}$$

$$v_{ga} = \left|\frac{\omega_1}{1\,000} g_a \left(1 + \frac{1}{u}\right)\right| \tag{143}$$

with g_f and g_a according to Formulae (96) and (97).

5.7.6 Sliding factor

The circumferential velocity of the pitch circles, v_w, is:

$$v_w = \frac{\omega_1\, d_{w1}}{2\,000} = \frac{\omega_2\, d_{w2}}{2\,000} \quad (144)$$

The sliding factor, K_g, is the ratio of sliding velocity, v_g, to the circumferential velocity, v_w, of the working pitch circles:

$$K_g = \frac{v_g}{v_w} = \left| \frac{2\, g_{\alpha y}}{d_{w1}} \left(1 + \frac{1}{u}\right) \right| \quad (145)$$

The maximum values for K_g are attained at end points A and E of the path of contact:

— at A:

$$K_{gf} = \left| \frac{2\, g_f}{d_{w1}} \left(1 + \frac{1}{u}\right) \right| \quad (146)$$

— at E:

$$K_{ga} = \left| \frac{2\, g_a}{d_{w1}} \left(1 + \frac{1}{u}\right) \right| \quad (147)$$

5.7.7 Specific sliding

The specific sliding, ζ, is the ratio of the sliding velocity to the speed of a transverse profile in the direction of the velocity tangent to the profile. The rolling velocity is equal to $\rho_y \omega$. Formula (136) yields:

$$\zeta_1 = \frac{v_{ry1} - v_{ry2}}{v_{ry1}} = 1 - \frac{\rho_{y2}}{u\, \rho_{y1}} \quad (148)$$

$$\zeta_2 = \frac{v_{ry2} - v_{ry1}}{v_{ry2}} = 1 - \frac{u\, \rho_{y1}}{\rho_{y2}} \quad (149)$$

The maximum values of ζ are reached at end points A and E of the path of contact:

— at A:

$$\zeta_{f1} = 1 - \frac{\rho_{A2}}{u\, \rho_{A1}} \quad (150)$$

— at E:

$$\zeta_{f2} = 1 - \frac{u\, \rho_{E1}}{\rho_{E2}} \quad (151)$$

using the curvature radii, ρ_A and ρ_E, according to 4.4.8 and 5.5.7.

6 Crossed axis cylindrical gear pairs

6.1 General

Crossed axis gears differ from parallel axes gears in that the contact between tooth flanks is no longer a line of contact moving in the plane of action but a point of contact between tooth flanks moving along the linear path of contact resulting from the intersection of the base planes of the pinion and the gear wheel.

For a specified crossed axis gear pair the centre distance is defined by the distance between the two axes of the gears (length of the common normal to the axes). There is a unique skew angle (angle between the projected axes of each gear in a plane perpendicular to the centre line) that results in conjugate action and the maximum backlash at that centre distance. With crossed axis gears, any change in centre distance requires a change in skew angle to maintain conjugate action. Therefore, the centre distance of crossed axes gears is handled in a different way than that used for parallel axes involute cylindrical gears.

The basic prerequisite that the pinion and the mating gear have identical basic rack tooth profiles applies. However, they do not need to have the same base helix angle and can have the same or opposite hand of helix. One of the gear pair can be a spur gear. With an internal gear, the pinion shall have a helix angle greater in magnitude than the helix angle of the internal gear.

The concepts developed for parallel axes gear pairs in 5.1 to 5.3.2 for parallel axes gears also apply to crossed axes gears.

Crossed axis gears can be classified into three types:

a) In the particular case where the two gears in a crossed axis pair have the same helix angle and the same hand, the working pitch diameters can be calculated from the specified centre distance in the same way as for parallel axes gears. From these working pitch diameters, the working helix angle (which will be the same for both gears), pressure angle, tooth thickness and backlash can be calculated.

   The skew angle is set to twice the working helix angle. Calculations specific to crossed axes gears are needed for path of contact, contact ratio, sliding conditions, and equivalent radius of curvature.

   NOTE With crossed axis gears, the ratio of the working diameters is equal to the gear ratio only in this particular case where the two gears have the same helix angle.

b) When the two gears in a crossed axis pair are operated at a working centre distance that is one half of the sum of their reference diameters, then the working diameters will equal the reference diameters.

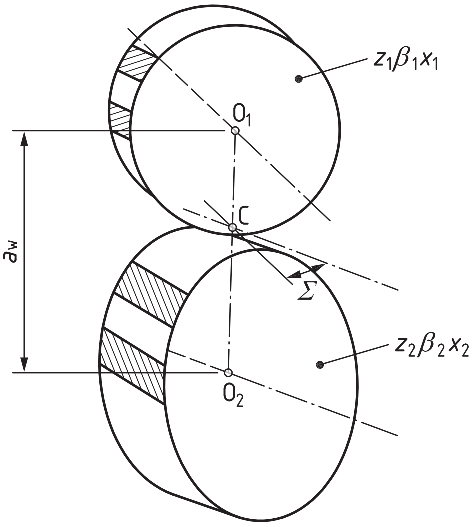
   Consequently, there will be no difference between the working and reference helix angles, pressure angles, and tooth thickness. For conjugate action, the skew angle shall be set to the sum of the helix angles at the reference diameters. Calculations specific to crossed axes gears are needed for path of contact, contact ratio, sliding conditions, and equivalent radius of curvature. Note that if the helix angles of the two gears are not equal, the ratio of the working diameters will not equal the gear ratio.

c) In the more general case, where the mating gears do not have the same helix angle and are not operated at the reference centre distance, a procedure is presented (see 6.3.12 and 6.3.13) to find the working normal pressure angle and from there find the working diameters, the working helix angles, transverse pressure angles, tooth thickness and backlash. The gear mesh geometry can first be determined for the centre distance without backlash (i.e. double flank working conditions), and then this centre distance can be used with reduced tooth thickness to create backlash.

A pinion mated with a rack is not treated as a crossed axis gear even if the axis of the pinion is not perpendicular to the transverse plane of the rack. As long as the rack and pinion have the same normal base pitch and are properly aligned, they will have line contact and be conjugate at any centre distance that has a contact ratio greater than one. If they have different helix angles, the pinion axis will not be perpendicular to the transverse plane of the rack, and there will be sliding across the facewidth. In this case, the calculations in Clause 5 can be used, but formulae for the combined radial and axial sliding are not provided. However, sliding can be closely approximated by using Formula (211) for a spur gear with 9999 teeth meshing with the pinion.

6.2 Concepts for a gear pair

The concepts presented in 5.2 for parallel axis gears also apply to crossed axis helical gears.



NOTE The hatched areas represent the tooth thickness on the working pitch cylinder.

Figure 29 — Working pitch surfaces, working pitch point and skew angle

6.3 Mating quantities

6.3.1 Gear ratio

The gear ratio, u, is defined in the same way as for parallel axes cylindrical gear pairs (see 5.3.1).

6.3.2 Driving gear, driven gear and transmission ratio

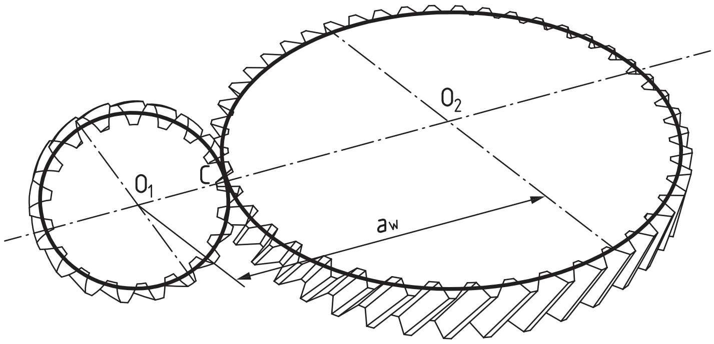
These notions are defined in the same way as for parallel axes cylindrical gear pairs (see 5.3.2).

6.3.3 Working pitch surfaces and working pitch point

The working pitch surfaces of the pinion and gear wheel are cylinders which contact together at the working pitch point C (see Figure 29); they are called working pitch cylinders defined by diameters d_{w1} and d_{w2}. The projections of the axes of rotation of the pinion and of the gear wheel in a common tangent plane to the working pitch cylinders, are crossing the working pitch point and define the skew angle Σ.

6.3.4 Line of centresa and working centre distance

The common perpendicular line which connects the two axes and crossing the working pitch point C is called the line of centres. The working centre distance, a_w, is the distance between the axes of the two gears on the line of centres, see Figure 30. It is defined by the half sum of the working pitch diameters.



NOTE 1 Half of the pinion and half of the gear wheel are shown, the portions above their respective mid planes have been removed to show the centre line.

NOTE 2 The gears can have profile shift and different generating helical angles β_1 and β_2.

Figure 30 — Line of centres and centre distance

NOTE 1 For external gears, unless the crossed axis gears have identical helix angles (same hand), the ratio of the working pitch diameters is not equal to the gear ratio.

The points O_1 and O_2 are defined by the intersection of the line of centres and the two gear axes. These points are defined as the centres of the pinion and gear wheel.

NOTE 2 For internal gears, since the number of teeth is negative, the diameters are negative, and the centre distance is negative.

6.3.5 Skew angle

In an involute crossed axes cylindrical gear pair, the helix angle on the working pitch cylinder for the pinion β_{w1} can differ from the helix angle on the working pitch cylinder for the gear β_{w2}, see 5.3.6. For theoretical conjugate action, the skew angle equals the sum of the working helix angles, see notes in 6.3.13:

$$\Sigma = \beta_{w1} + \beta_{w2} \quad (152)$$

Care should be taken with +/- signage.

6.3.6 Intermeshing racks at working pitch cylinders

It is possible to define two intermeshing racks in the common tangent plane to the working pitch cylinders: one for the pinion and one for the gear wheel (see Figure 31).

These two rack profiles do not correspond to the basic rack profile used to generate each part of the gear pair (see 6.3.9). These two racks are complementary and have the same normal profile and the same basic characteristics:

— same working normal pressure angle, α_{wn};

— same working normal pitch, p_{wn};

— therefore, they also have the same normal base pitch, p_{bn}.

It is important to notice that their working transverse pressure angles are different, as well as addendum and dedendum in order to match respectively tip and root diameters for the gear pair.

In the transverse plane of each gear, the tangent line to the working pitch circle is the working pitch line for each intermeshing rack, along which the working transverse tooth thicknesses, s_{wt1} for the pinion and

s_{wt2}, for the gear, are defined; they are linked to their respective working normal tooth thicknesses s_{wn1} and s_{wn2} by Formula (53):

$$s_{wn1} = s_{wt1} \cos \beta_{w1} \tag{153}$$

$$s_{wn2} = s_{wt2} \cos \beta_{w2} \tag{154}$$

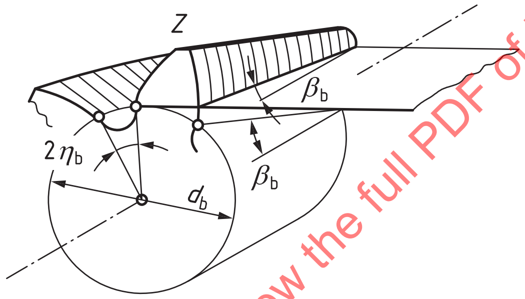
The working normal tooth thicknesses s_{wn1} and s_{wn2} are not necessarily equal.

At a centre distance without backlash the sum of respective working normal tooth thicknesses s_{wn1} and s_{wn2} is equal to the working normal pitch, p_{wn}:

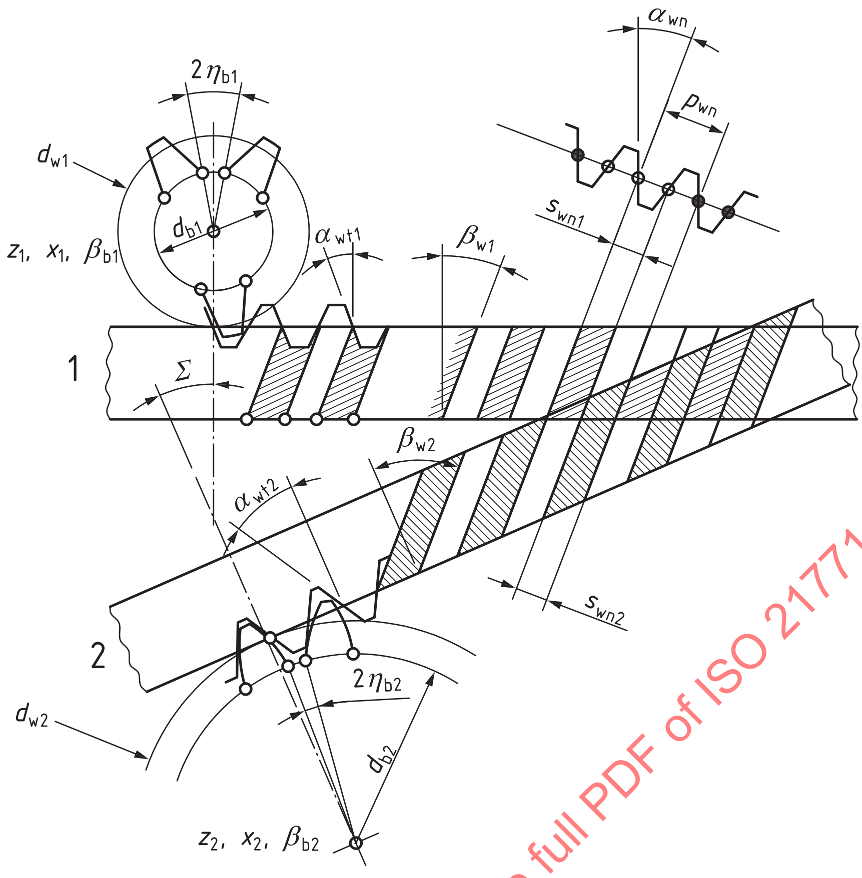
$$p_{wn} = s_{wn1} + s_{wn2} \tag{155}$$

Based on Formulae (7), (28) and (29), the working normal pitch can also be expressed by:

$$p_{wn} = \frac{d_{b1}}{\cos \alpha_{wt1}} \cdot \left(\frac{\pi}{z_1} \right) \cdot \cos \beta_{w1} = \frac{d_{b2}}{\cos \alpha_{wt2}} \cdot \left(\frac{\pi}{z_2} \right) \cdot \cos \beta_{w2} \tag{156}$$



a) Base plane of single helical gear



b) Intermeshing racks

Key

1 intermeshing rack of helical pinion

2 intermeshing rack of helical gear wheel

Figure 31 — Intermeshing of involute crossed axis helical gears

6.3.7 Working transverse pressure angles

The working transverse pressure angles are different on each intermeshing rack. They can be determined by combining Formulae (10) and (17):

For the pinion it is expressed as Formula (157):

$$\tan\alpha_{wt1} = \frac{\tan\alpha_{wn}}{\sqrt{1-\dfrac{\sin^2\beta_1\cos^2\alpha_n}{\cos^2\alpha_{wn}}}} \tag{157}$$

For the gear wheel it is expressed as Formula (158):

$$\tan\alpha_{wt2} = \frac{\tan\alpha_{wn}}{\sqrt{1-\dfrac{\sin^2\beta_2\cos^2\alpha_n}{\cos^2\alpha_{wn}}}} \tag{158}$$

6.3.8 Working transverse tooth thicknesses

The working transverse tooth thicknesses are different on each intermeshing rack.

From Formula (52), the transverse space width half angle on working pitch diameter is:

$$\eta_{wt} = \eta_b + \frac{z}{|z|} \operatorname{inv} \alpha_{wt} \tag{159}$$

As the transverse tooth thickness half angle on working pitch diameter is equal to the difference between the transverse working pitch half angle and the transverse space width half angle:

$$\psi_{wt} = \frac{\pi}{z} - \eta_{wt} = \frac{\pi}{z} - \eta_b - \frac{z}{|z|} \operatorname{inv} \alpha_{wt} \tag{160}$$

Then the transverse tooth thickness on each working pitch diameter is given by:

For the pinion it is expressed as Formula (161):

$$s_{wt1} = \psi_{w1} d_{w1} = \frac{d_{b1}}{\cos \alpha_{wt1}} \left(\frac{\pi}{z_1} - \eta_{b1} - \operatorname{inv} \alpha_{wt1} \right) \tag{161}$$

For the gear wheel it is expressed as Formula (162):

$$s_{wt2} = \psi_{w2} d_{w2} = \frac{d_{b2}}{\cos \alpha_{wt2}} \left(\frac{\pi}{z_2} - \eta_{b2} - \operatorname{inv} \alpha_{wt2} \right) \tag{162}$$

6.3.9 Basic rack profile to generate each part of the gear pair

An involute cylindrical helical gear can mesh with any rack tooth profile, defined in a transverse plane on a cylinder of diameter d_y that has:

a) the same transverse pressure angle α_{yt} as the gear, see Formula (7);

b) the same transverse base pitch p_{bt}, defined with the transverse pitch p_{yt} on d_y see Formula (31).

And consequently, having on the normal surface through a cylinder with diameter d_y :

c) the same normal pressure angle α_{yn}, defined in 4.4.4 by Formula (10):

$$\tan \alpha_{yn} = \tan \alpha_{yt1} \cos \beta_{y1} = \tan \alpha_{yt2} \cos \beta_{y2} \tag{163}$$

d) and the same normal base pitch p_{bn}, defined in 4.5.5.1 by Formula (32):

$$p_{bn} = p_{yn} \cos \alpha_{yn} = p_{bt1} \cos \beta_{b1} = p_{bt2} \cos \beta_{b2} \tag{164}$$

Considering the two last formulae on the reference cylinders (which are also the generating pitch cylinders) with diameters d_1 and d_2 of each part of the gear pair, then:

$$\tan \alpha_n = \tan \alpha_{t1} \cos \beta_1 = \tan \alpha_{t2} \cos \beta_2 \tag{165}$$

and

$$p_{bn} = p_n \cos \alpha_n = p_{bt1} \cos \beta_{b1} = p_{bt2} \cos \beta_{b2} \tag{166}$$

This means that the two gears shall be generated with the same basic rack tooth profile with a normal pressure angle α_n, a normal base pitch p_{bn} and consequently the same normal module m_n.

Base diameters are still defined by Formula (6).

6.3.10 Profile shift coefficients

To generate each part of the gear pair the basic rack profile can be shifted as defined in 4.3.10.

The profile shift coefficient is defined as x_1 for the pinion and x_2 for the gear wheel.

This means that all basic geometrical data of the pinion and gear wheel can be determined with formulae given in Clause 4.

6.3.11 Base space width half angle

Based on the parameters of the basic rack profile to generate parameters, and Formulae (51) and (52), the base transverse space width half angles are expressed by:

$$\eta_{b1} = \frac{\pi - 4\,x_1 \tan\alpha_n}{2\,z_1} - \mathrm{inv}\,\alpha_{t1} \tag{167}$$

$$\eta_{b2} = \frac{\pi - 4\,x_2 \tan\alpha_n}{2\,z_2} - \mathrm{inv}\,\alpha_{t2} \tag{168}$$

6.3.12 Working conditions at minimum centre distance

The two gears in a crossed axis pair will have different working transverse pressure angles, so for crossed axis gears Formula (77) can be restated as explained in Annex C, Formula (C.9):

$$z_1\left(\mathrm{inv}\,\alpha_{wt1} - \mathrm{inv}\,\alpha_{t1}\right) + z_2\left(\mathrm{inv}\,\alpha_{wt2} - \mathrm{inv}\,\alpha_{t2}\right) = 2\left(x_1 + x_2\right)\tan\alpha_n \tag{169}$$

Since the two gears in a crossed axis pair do have the same working normal pressure angle, using Formulae (8), (157) and (158), this can be rewritten in a form that can be solved (using iteration) for the working normal pressure angle at zero backlash (see details in Annex C):

$$\begin{aligned} & z_1\left\{\mathrm{inv}\left[\arctan\left(\frac{\tan\alpha_{wn}}{\sqrt{1-\frac{\sin^2\beta_1\cos^2\alpha_n}{\cos^2\alpha_{wn}}}}\right)\right] - \mathrm{inv}\left[\arctan\left(\frac{\tan\alpha_n}{\cos\beta_1}\right)\right]\right\} + \\ & z_2\left\{\mathrm{inv}\left[\arctan\left(\frac{\tan\alpha_{wn}}{\sqrt{1-\frac{\sin^2\beta_2\cos^2\alpha_n}{\cos^2\alpha_{wn}}}}\right)\right] - \mathrm{inv}\left[\arctan\left(\frac{\tan\alpha_n}{\cos\beta_2}\right)\right]\right\} - 2\left(x_1 + x_2\right)\tan\alpha_n = 0 \end{aligned} \tag{170}$$

The corresponding minimum distance between the axis of rotation, $a_{w\,min}$, (i.e. centre distance at zero backlash) and working pitch diameters can be obtained with (see details in Annex C):

$$a_{w\,min} = \frac{d_{w1} + d_{w2}}{2} = \frac{m_n}{2}\left(\frac{z_1}{\sqrt{\frac{\cos^2\alpha_{wn}}{\cos^2\alpha_n} - \sin^2\beta_1}}\right) + \frac{m_n}{2}\left(\frac{z_2}{\sqrt{\frac{\cos^2\alpha_{wn}}{\cos^2\alpha_n} - \sin^2\beta_2}}\right) \tag{171}$$

The skew angle for conjugate action at zero backlash (which also results in minimum centre distance) can be obtained with the transformed Formulae (17) and (152) (see details in Annex C):

$$\Sigma = \beta_{w1} + \beta_{w2} = \arcsin\left(\frac{\sin\beta_1 \cdot \cos\alpha_n}{\cos\alpha_{wn}}\right) + \arcsin\left(\frac{\sin\beta_2 \cdot \cos\alpha_n}{\cos\alpha_{wn}}\right) \tag{172}$$

6.3.13 Working conditions with backlash

If, to create backlash, the centre distance is larger than the minimum, the relationship between the defined working centre distance a_w and working transverse pressure angles, can be used to obtain the working normal pressure angle (see Annex C).

$$a_w = \frac{m_n}{2}\left\{\frac{z_1}{\sqrt{\dfrac{\cos^2\alpha_{wn}}{\cos^2\alpha_n} - \sin^2\beta_1}} + \frac{z_2}{\sqrt{\dfrac{\cos^2\alpha_{wn}}{\cos^2\alpha_n} - \sin^2\beta_2}}\right\} \tag{173}$$

Then the working helix angles and the skew angle are still obtained by Formula (172).

NOTE 1 When centre distance is changed, theoretical conjugate action will only be maintained if the skew angle is also changed.

NOTE 2 Changing tooth thickness will not change the meshing action of the gear pair. So, if a gear pair is designed for conjugate action with zero backlash, if the design centre distance and skew angle are maintained and backlash is obtained by thinning the teeth, then the gears will still be conjugate.

6.3.14 Working depth

The working depth, h_w, of a gear pair is the overlap of the tip circles of the two cylindrical gears on the line of centres, see Formula (73).

6.3.15 Tip clearance

The tip clearance, c, is the minimum distance between the tip circle of a gear and the root circle of the mating gear, see Formulae (74) and (75) and Figure 19.

6.3.16 Calculation of tip alteration coefficient k (for crossed axis gear)

It is sometimes necessary for the addendum to be altered to suit the mating conditions and the specified minimum tip clearance of the mate.

If the tip clearance c_1 (or c_2) corresponding to the basic rack tooth profile is to be retained, then the necessary tip alteration coefficient k (for both pinion and gear) can be estimated as:

$$k = \frac{1}{m_n}\left(a_w - \frac{d_1 + d_2}{2}\right) - (x_1 + x_2) = \frac{a_w}{m_n} - \frac{1}{2}\left(\frac{z_1}{\cos\beta_1} + \frac{z_2}{\cos\beta_2}\right) - (x_1 + x_2) \tag{174}$$

The tip alteration coefficient, k, calculated in this way is obtained with correct sign, that is to say negative values in the case of external gear pairs, so that the tip diameters become smaller.

The calculated values are often so small that they are cancelled out by the deeper infeed of the cutting tool which is necessary for producing the backlash and by the negative root diameter deviations so that the remaining effective tip clearance of the mate is altered to only a slight extent (or within permissible limits). See 6.3.15.

6.4 Tooth engagement

6.4.1 General

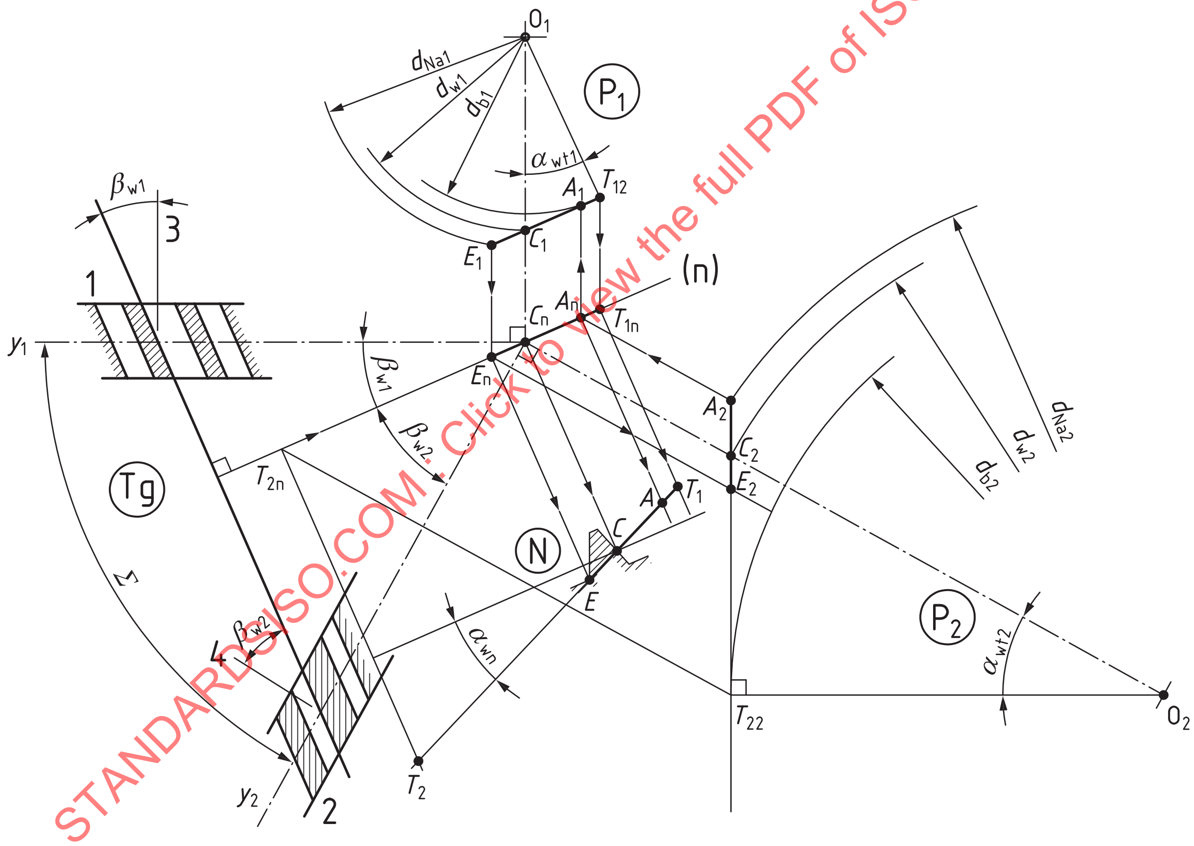
Tooth engagement refers to the meshing of a gear with its mating gear. The tooth engagement is influenced by the geometry of the gear pair. See Figure 32, which shows meshing conditions and active ranges on working flanks in transverse planes and the working normal plane.

6.4.2 Line of action, path of contact and point of contact

The parameters for the meshing of the gears are given in Figure 32, see the key to that figure for the symbols used. The active plane of action of each cylindrical gear of the crossed axis gear pair is still tangent to their base cylinders, but unlike parallel axis gears, crossed axis gears do not share a single plane of action. The working pitch point C is defined by the intersection of the centre line and the common tangent plane to the working pitch cylinders of the gears Tg. The active planes of action of the pinion and gear are tangent to their respective base cylinders and cross at point C. Their intersection defines the line of action of the gears $\overline{T_1 T_2}$ along which the point of contact between tooth flanks is moving during the gear mesh. The line of action of the gear is in the working normal plane, N.

The trace (n) of the working normal plane, N, in the common tangent plane to working pitch cylinders of gears Tg is inclined to β_{w1} and β_{w2} according to the projection of mid planes of the pinion and the mating gear. Those mid planes intersect along the centre line.

At any instant of the gear mesh between tooth flanks, the intersection of the line of action with the corresponding tooth flanks of the gear pair is a point of contact. The path of contact is the part of the line of action which is limited by the active tip diameters d_{Na1} , d_{Na2} of the pinion and the mating gear. With the rotation of the gears around their axes, the point of contact moves along the path of contact $\overline{AE}$. On tooth flanks, the points of contact are identical to the generators of flank and mating flank, see 4.4.1.



Key

1	intermeshing racks of helical pinion	2	intermeshing racks of helical gear wheel
3	parallel to axis of rotation of the pinion	4	parallel to axis of rotation of the gear wheel
Tg	tangent plane to working pitch cylinders	Y_1, Y_2	mid planes of working intermeshing racks crossing at C_n
P_1	transverse plane of the pinion	P_2	transverse plane of the gear
N	working normal plane and full length of line of action		
(n)	trace of normal plane in Tg		

Subscripts:

1 for pinion

2 for gear wheel

n projected on the trace of the working normal plane

NOTE See 5.5.6.1 for a description of points A, C, and E.

Figure 32 — Meshing conditions in transverse planes and working normal plane

For clarity, in Figure 32 the projection of the pinion and gear in their respective transverse planes are shifted from working pitch point (C_n).

The line of action, $\overline{T_1T_2}$, can be projected in transverse sections P_1 and P_2 , in $\overline{T_{11}A_1}$ and $\overline{T_{22}E_2}$. It is necessary to distinguish between the right flank line of action and the left flank line of action.

The line of action, $\overline{T_1T_2}$, is inclined to the common tangent plane Tg to the working pitch cylinders by the working normal pressure angle α_{wn} and it contacts the two base cylinders at T_1 and T_2, projected in transverse sections P_1 and P_2 to the points T_{11} and T_{22}., with transverse working pressure angles, α_{wt1} and α_{wt2} (see Figure 32). The transverse paths of contact are $\overline{A_1E_1}$ and $\overline{A_2E_2}$.

A path of contact is linked to the flank that is normal to it. Hence, one of the paths of contact is linked to the right flanks and the other to the left flanks. Point A on the path of contact is where the line of action intersects the active tip diameter of the gear wheel, d_{Na2} , and point E is where the line of contact intersects the active tip diameter of the pinion, d_{Na1} .

NOTE In Figure 32, only the line of action of the working flanks is shown in each case. Figure 32 represents only the path of contact between right flanks of pinion with right flanks of gear, the path of contact with other flanks is symmetric to the centre line in the working normal plane N.

Working pitch point C is also the point at which the two lines of action (paths of contact) for both flanks intersect.

A point of contact is defined by the intersection of the path of contact $\overline{AE}$ with the corresponding tooth flanks when the gears are in a specific position.

6.4.3 Active area of the tooth flanks, start of active profile and active tip diameters

6.4.3.1 General

The start of involute diameter (root form diameter), d_{Ff} , is the start of the involute portion of the profile. For an external gear it is the greater of the base diameter, d_b , or the diameter of the intersection of the flank with the tooth root fillet or trochoid (taking into account undercut if necessary, see Clause 10).

In a defined gear pair the active tip diameter, d_{na} , of a gear can be governed either by its tip form diameter, d_{Fa} or by the start of involute of its mate, d_{Ff} . The start of active profile (active root) diameter, d_{Nf} , can be governed either by the diameter of start of involute, d_{Ff} , or by the tip form diameter of its mate, d_{Fa} . During rotation, the active tip diameter of a gear will contact the active root diameter of the mating gear. The active area of the flank extends from the active tip diameter d_{Na} to the active root diameter, d_{Nf} , and so is dependent on the characteristics of both gears and the centre distance.

Usually, contact near the root on both gears is limited by the tip form diameter of the mating gear.

6.4.3.2 Pinion start of active profile and active tip diameters

Contact in the pinion dedendum can be limited by the tip form diameter of the gear wheel. In this case, the point A_2 in the transverse plane of the gear is projected in the working normal plane (N in Figure 32) in A which is projected in A_1 in the transverse plane of the pinion to calculate (see Annex C, C.3):

$$d_{Nf1} = \sqrt{\left[\left(d_{Fa2} \cdot \sin\alpha_{Fat2} - d_{w2} \cdot \sin\alpha_{wt2} \right) \cdot \frac{\cos\beta_{b1}}{\cos\beta_{b2}} - d_{w1} \cdot \sin\alpha_{wt1} \right]^2 + d_{b1}^2} \quad (175)$$

$$d_{Na2} = d_{Fa2} \quad (176)$$

However, if d_{Ff1} is greater than the quantity calculated for d_{Nf1}, then:

$$d_{Nf1} = d_{Ff1} \quad (177)$$

$$d_{Na2} = \sqrt{\left[\left(d_{Ff1} \cdot \sin\alpha_{Fft1} - d_{w1} \cdot \sin\alpha_{wt1} \right) \cdot \frac{\cos\beta_{b2}}{\cos\beta_{b1}} - d_{w2} \cdot \sin\alpha_{wt2} \right]^2 + d_{b2}^2} \quad (178)$$

See 9.7, 10.3 or 11.3 for d_{Ff}.

It is important to verify if the gear should be truncated to avoid gear mesh interference with the tooth root fillet of the pinion.

6.4.3.3 Gear wheel start of active profile and active tip diameters

Contact in the gear wheel dedendum can be limited by the tip form diameter of the pinion. In this case the point E_1 in the transverse plane of the pinion is projected in the working normal plane (N in Figure 32) in E which is projected in E_2 in the transverse plane of the mating gear to calculate (see Annex C, C.3):

$$d_{Nf2} = \sqrt{\left[\left(d_{Fa1} \cdot \sin\alpha_{Fat1} - d_{w1} \cdot \sin\alpha_{wt1} \right) \cdot \frac{\cos\beta_{b2}}{\cos\beta_{b1}} - d_{w2} \cdot \sin\alpha_{wt2} \right]^2 + d_{b2}^2} \quad (179)$$

$$d_{Na1} = d_{Fa1} \quad (180)$$

However, if d_{Ff2} is greater than the quantity calculated d_{Nf2}, then:

$$d_{Nf2} = d_{Ff2} \quad (181)$$

$$d_{Na1} = \sqrt{\left[\left(d_{Ff2} \cdot \sin\alpha_{Fft2} - d_{w2} \cdot \sin\alpha_{wt2} \right) \cdot \frac{\cos\beta_{b1}}{\cos\beta_{b2}} - d_{w1} \cdot \sin\alpha_{wt1} \right]^2 + d_{b1}^2} \quad (182)$$

It is important to verify if the pinion should be truncated to avoid gear mesh interference with the tooth root fillet of the gear wheel.

6.4.4 Form over dimension

The form over dimension, c_F, is the radial distance between the active root diameter and root form diameter. See 5.5.5.

6.4.5 Designations and values relating to the line of action

6.4.5.1 Special points on the line of action

Special points on the line of action are as given in 5.5.6.1.

6.4.5.2 Length of the path of contact

The length, g_α, of the path of contact (length between points A and E on the line of action) of two mating cylindrical gears are:

— in transverse plane of the pinion:

$$g_{\alpha 1}=\overline{\mathrm{A}_1\mathrm{E}_1}=\frac{1}{2}\left[\sqrt{d_{\mathrm{Na1}}^2-d_{\mathrm{b1}}^2}-\sqrt{d_{\mathrm{Nf1}}^2-d_{\mathrm{b1}}^2}\right] \tag{183}$$

— in transverse plane of the gear:

$$g_{\alpha 2}=\overline{\mathrm{A}_2\mathrm{E}_2}=\frac{1}{2}\left[\sqrt{d_{\mathrm{Na2}}^2-d_{\mathrm{b2}}^2}-\sqrt{d_{\mathrm{Nf2}}^2-d_{\mathrm{b2}}^2}\right] \tag{184}$$

— in the working normal plane of the gear, which is the full length of the path of contact:

$$g_\alpha=\overline{\mathrm{AE}}=\frac{g_{\alpha 1}}{\cos\beta_{\mathrm{b1}}}=\frac{g_{\alpha 2}}{\cos\beta_{\mathrm{b2}}} \tag{185}$$

The path of contact is divided by working pitch point C into the addendum path of contact and the dedendum path of contact. The addendum path of contact is the portion of the path of contact between the active tip diameter, d_{Na}, and the working pitch point. The dedendum path of contact goes from the working pitch point towards the root to the start of active profile diameter, d_{Nf}, see Figure 32.

In a crossed axis gear pair, when the base helix angles are not equal, the addendum path of contact of a gear is not equal to the dedendum path of contact of its mate. The lengths of the addendum paths of contact are:

$$g_{\mathrm{a1}}=\overline{\mathrm{C}_1\mathrm{E}_1}=\frac{1}{2}\left(\sqrt{d_{\mathrm{Na1}}^2-d_{\mathrm{b1}}^2}-d_{\mathrm{b1}}\tan\alpha_{\mathrm{wt1}}\right)=g_{\mathrm{f2}}\frac{\cos\beta_{\mathrm{b1}}}{\cos\beta_{\mathrm{b2}}} \tag{186}$$

$$g_{\mathrm{a2}}=\overline{\mathrm{A}_2\mathrm{C}_2}=\frac{1}{2}\left(\sqrt{d_{\mathrm{Na2}}^2-d_{\mathrm{b2}}^2}-d_{\mathrm{b2}}\tan\alpha_{\mathrm{wt2}}\right)=g_{\mathrm{f1}}\frac{\cos\beta_{\mathrm{b2}}}{\cos\beta_{\mathrm{b1}}} \tag{187}$$

The lengths of the dedendum paths of contact are:

$$g_{\mathrm{f1}}=\overline{\mathrm{A}_1\mathrm{C}_1}=\frac{1}{2}\left(d_{\mathrm{b1}}\tan\alpha_{\mathrm{wt1}}-\sqrt{d_{\mathrm{Nf1}}^2-d_{\mathrm{b1}}^2}\right)=g_{\mathrm{a2}}\frac{\cos\beta_{\mathrm{b1}}}{\cos\beta_{\mathrm{b2}}} \tag{188}$$

$$g_{\mathrm{f2}}=\overline{\mathrm{C}_2\mathrm{E}_2}=\frac{1}{2}\left(d_{\mathrm{b2}}\tan\alpha_{\mathrm{wt2}}-\sqrt{d_{\mathrm{Nf2}}^2-d_{\mathrm{b2}}^2}\right)=g_{\mathrm{a1}}\frac{\cos\beta_{\mathrm{b2}}}{\cos\beta_{\mathrm{b1}}} \tag{189}$$

The two portions of the path of contact are also known as the approach path of contact and the recess path of contact. The approach path of contact corresponds to the addendum path of contact of the driven gear (which mates with the dedendum path of contact of the driving gear). So, when the pinion is driving and the gear wheel is the driven gear, the approach path of contact is the addendum path of contact of the gear wheel and the recess path of contact is the dedendum path of contact of the gear wheel.

For the case where the gear wheel is the driver, the approach path of contact is the addendum path of contact of the pinion and the recess path of contact is the dedendum path of contact of the pinion.

6.4.6 Tooth interference

Interference conditions occur if parts of the tooth flanks or top lands of a gear come into contact with non-involute flank sections or the root on the mating gear. Additional meshing difficulties can be caused by contact by a tooth tip with a non-working flank.

NOTE Such complexities for crossed axis gears are not covered in this document.

6.4.7 Mesh geometry parameters

6.4.7.1 Transverse angle of transmission and transverse contact ratio

The transverse angle of transmission, φ_α, is the centre angle through which a gear of a gear pair rotates from start to finish of engagement of one active tooth flank transverse profile with its mating profile. The transverse angle of transmission of pinion and gear is given as follows:

$$\varphi_{\alpha 1} = \frac{2 g_{\alpha 1}}{d_{b1}} = |u|\, \varphi_{\alpha 2} \quad (190)$$

$$\varphi_{\alpha 2} = \frac{2 g_{\alpha 2}}{|d_{b2}|} = \frac{\varphi_{\alpha 1}}{|u|} \quad (191)$$

The transverse contact ratios, $\varepsilon_{\alpha 1}$ and $\varepsilon_{\alpha 2}$, are the ratio of the transverse angle of transmission, $\varphi_{\alpha 1}$ and $\varphi_{\alpha 2}$, to the angular pitch, τ_1, and τ_2, or the ratio of the path of contact to the transverse base pitches:

$$\varepsilon_{\alpha 1} = \frac{\varphi_{\alpha 1}}{\tau_1} = \frac{g_{\alpha 1}}{p_{bt1}} = \frac{g_{f1} + g_{a1}}{p_{bt1}} \quad (192)$$

$$\varepsilon_{\alpha 2} = \frac{\varphi_{\alpha 2}}{\tau_2} = \frac{g_{\alpha 2}}{p_{bt2}} = \frac{g_{f2} + g_{a2}}{p_{bt2}} \quad (193)$$

6.4.7.2 Contact ratio

The contact ratio, $\varepsilon_{\alpha n}$, in the common normal working surface is the ratio of the length of path of contact, $\overline{AE}$, to the normal base pitch:

$$\varepsilon_{\alpha n} = \frac{\overline{AE}}{p_{bn}} = \frac{g_\alpha}{p_{bn}} \quad (194)$$

6.4.7.3 Active facewidth

The active facewidth, b_w, is the overlapping section of the usable facewidths of the gear pair, see Figure 23.

6.4.7.4 Active facewidth of pinion and gear

In order to guarantee that the gear pair is able to work on these two flanks, the path of contact $\overline{AE}$ shall be covered by the complete facewidth of each gear, so this means that:

For the pinion:

$$b_1 \geq 2 \max(g_{a1}, g_{f1}) \cos\alpha_{wt1} \tan\beta_{w1} \quad (195)$$

For the gear:

$$b_2 \geq 2 \max(g_{a2}, g_{f2}) \cos\alpha_{wt2} \tan\beta_{w2} \quad (196)$$

NOTE max is a function that returns the maximum value of the arguments.

6.5 Backlash

6.5.1 General

The backlash is the clearance between the non-working flanks of the teeth of a gear pair when the working flanks are in contact.

There is a distinction between the normal backlash, j_{bn} and the circumferential backlashes, j_{wt1} and j_{wt2}.

6.5.2 Normal base backlash

Normal base backlash, j_{bn} is the shortest distance between the non-working flanks of the teeth of a gear pair when the working flanks are in contact with zero force. It is defined on the path of contact of the non-working flanks.

In the working normal plane, the working normal tooth thicknesses s_{wn1} and s_{wn2} can be determined on the working pitch cylinders with Formulae (43) and (53) as follows:

$$s_{wn1} = d_{w1}\left[\frac{\pi + 4x_1 \tan\alpha_n}{2z_1} + \mathrm{inv}\,\alpha_{t1} - \mathrm{inv}\,\alpha_{wt1}\right]\cos\beta_{w1} \tag{197}$$

$$s_{wn2} = d_{w2}\left[\frac{\pi + 4x_2 \tan\alpha_n}{2z_2} + \mathrm{inv}\,\alpha_{t2} - \mathrm{inv}\,\alpha_{wt2}\right]\cos\beta_{w2} \tag{198}$$

Then the working normal backlash j_{wn} (along a normal line (n) in the common tangent plane to the working pitch cylinders in Figure 32) is obtained by the difference between the working normal pitch p_{wn} and the sum of normal tooth thicknesses s_{wn1} and s_{wn2} :

$$j_{wn} = p_{wn} - (s_{wn1} + s_{wn2}) \tag{199}$$

with:

$$p_{wn} = \frac{\pi \cdot d_{w1}}{z_1}\cos\beta_{w1} = \frac{\pi \cdot d_{w2}}{z_2}\cos\beta_{w2} \tag{200}$$

Then along the path of contact, the normal base backlash, j_{bn} is:

$$j_{bn} = j_{wn} \cdot \cos\alpha_{wn} \tag{201}$$

6.5.3 Backlash angle and circumferential backlash

See 5.6.6 for angular backlash. The circumferential backlashes, j_{wt1} and j_{wt2} are the length of the working pitch circle arc through which each of the two gears can be rotated, while the other is held stationary, from the point where the right flanks are in contact to the point where the left flanks are in contact. Its magnitude is given by:

$$j_{wt1} = \frac{1}{\cos\alpha_{wt1}\ \cos\beta_{b2}}\ j_{bn} \tag{202}$$

$$j_{wt2} = \frac{1}{\cos\alpha_{wt2}\ \cos\beta_{b2}}\ j_{bn} \tag{203}$$

or in relation to the length of the reference circle arc by:

$$j_{t1} = \frac{1}{\cos\beta_1\ \cos\alpha_n}\ j_{bn} \tag{204}$$

$$j_{t2} = \frac{1}{\cos\beta_2\ \cos\alpha_n}\ j_{bn} \tag{205}$$

6.6 Sliding conditions at the tooth flanks

6.6.1 General

As there is no easy way to work in a transverse plane in 2D, 6.6 is developed in 3D.

Considering the transverse planes of pinion and gear crossing the working pitch point C, it is possible to define two system coordinates: (C, X_1, Y_1, Z_1) for the pinion and (C, X_2, Y_2, Z_2) for the gear (see Figures 33 and 34). Axes CZ_1 and CZ_2 are located along centre distance $0_1 0_2$, CX_1 is parallel to the axis of pinion, CX_2 is parallel to the axis of gear and the angle between axis CX_2 and CX_1, as well axis CY_2 and CY_1 is equal to axis angle Σ.

For simplicity, all calculations are done relative to pinion. This means according to the system coordinate: (C, X_1, Y_1, Z_1).

6.6.2 Angular velocities vectors

In (C, X_1, Y_1, Z_1), the angular velocity vectors are defined as follows:

For the pinion:

$$\overrightarrow{\omega_1} = \begin{pmatrix} \omega_1 \\ 0 \\ -0{,}5 d_{w1} \end{pmatrix} \tag{206}$$

For the gear:

$$\overrightarrow{\omega_2} = \begin{pmatrix} -\omega_2 \cos\Sigma \\ -\omega_2 \sin\Sigma \\ 0{,}5 d_{w2} \end{pmatrix} = \begin{pmatrix} -\dfrac{\omega_1}{u}\dfrac{z_2}{|z_2|}\cos\Sigma \\ -\dfrac{\omega_1}{u}\dfrac{z_1}{|z_2|}\sin\Sigma \\ 0{,}5 d_{w2} \end{pmatrix} \tag{207}$$

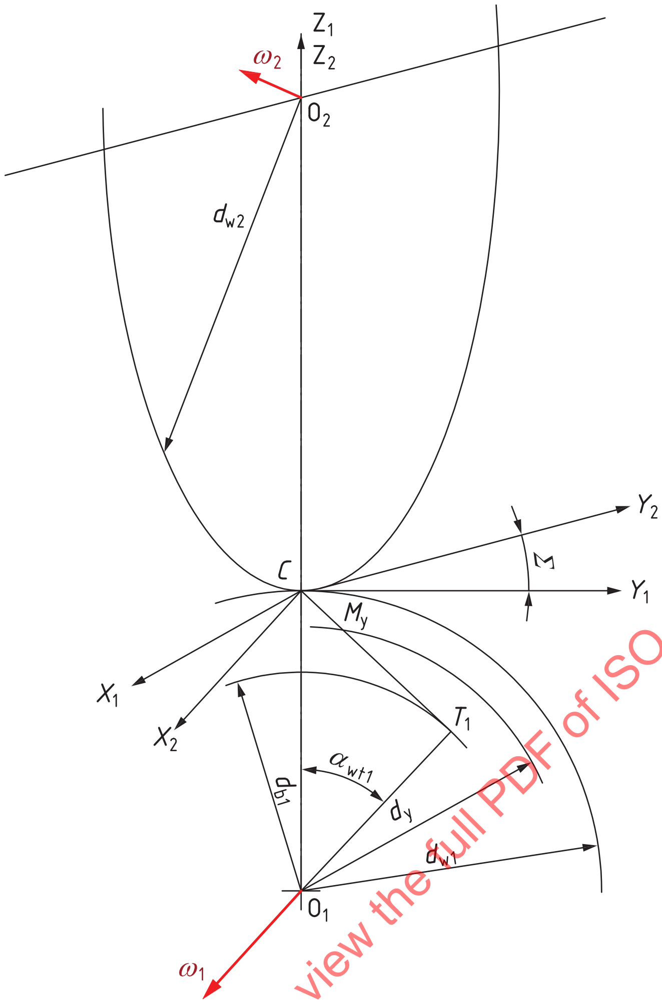


Figure 33 — Coordinate system to define velocities

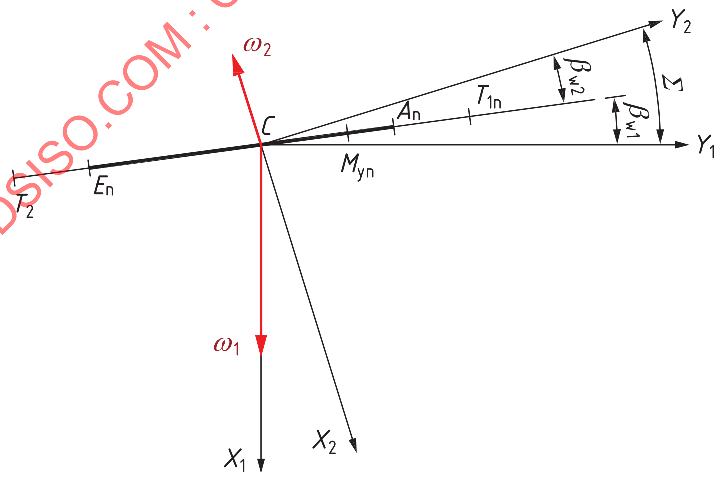


Figure 34 — Coordinate system to define velocities in working pitch plane

6.6.3 Velocity vectors of a point of contact along path of contact

Considering a point of contact M_y along the path of contact AE.

For the pinion, the coordinates of M_y can be defined by vector CM_y:

In (C, X_1, Y_1, Z_1),

$$\overrightarrow{CM_y} = \begin{pmatrix} -(\rho_{M1} - \rho_{C1}) \cdot \sin\beta_{b1} \\ -(\rho_{M1} - \rho_{C1}) \cdot \cos\beta_{b1} \cdot \cos\alpha_{wn} \\ (\rho_{M1} - \rho_{C1}) \cdot \cos\beta_{b1} \cdot \sin\alpha_{wn} \end{pmatrix} \tag{208}$$

Then the velocity of this point of the pinion is defined by the vector product in m/s:

$$\overrightarrow{v_{M1}} = \frac{\overrightarrow{\omega_1}}{1\,000} \times \overrightarrow{CM_y} \tag{209}$$

Then the velocity of this point of the gear is defined by the vector product in m/s:

$$\overrightarrow{v_{M2}} = \frac{\overrightarrow{\omega_2}}{1\,000} \times \overrightarrow{CM_y} \tag{210}$$

6.6.4 Sliding velocity

The sliding velocity is given by the difference:

$$\overrightarrow{v_{Mg}} = \overrightarrow{v_{M1}} - \overrightarrow{v_{M2}} = \frac{\left(\overrightarrow{\omega_1} - \overrightarrow{\omega_2}\right)}{1\,000} \times \overrightarrow{CM_y} \tag{211}$$

7 Principal radii of curvature of the tooth flanks

7.1 General

This clause gives the general case for crossed axes gears and then is extended to parallel axes gears for which the skew angle is zero.

In any point of the active tooth flank surface of a gear, two radii of curvature can be defined in two perpendicular normal planes to the tangent plane of the surface, called principal radii of curvature; one is minimum, the other is maximum (see 4.9). These two perpendicular normal planes are called principal planes of curvature (see keys 4 and 7 on Figure 14). The first principal direction of curvature (the smallest) is always perpendicular to the second principal direction of curvature, which is a straight line: the first radius of curvature is always equal to a finite value (see 4.9) and the second principal radius of curvature (the greatest) is infinite. In a cylindrical involute gear, the first principal plane of curvature corresponds to the plane normal to the base plane of the generator. Then:

a) For a crossed axes helical gear pair, according to 4.9, in the base plane of each gear it corresponds to principal directions $\overrightarrow{T_1}$ and $\overrightarrow{T_2}$, see Figure 35.

   For any point of contact M_y along the path of contact the first principal radius of curvature in the working normal plane (N), equivalent to ρ_{yn} in Figure 14, can be defined by:

   For the pinion:

$$\overline{T_1M_y} = \rho_{M1} = \left(\frac{1}{2}\sqrt{d_{y1}^2 - d_{b1}^2}\right)\frac{1}{\cos\beta_{b1}} = \frac{d_{b1}\tan\alpha_{yt1}}{2\cos\beta_{b1}} \tag{212}$$

   And for the mating gear:

$$\overline{T_2M_y} = \rho_{yn2} = \left(\frac{1}{2}\sqrt{d_{y2}^2 - d_{b2}^2}\right)\frac{1}{\cos\beta_{b2}} = \overline{T_1T_2} - \rho_{yn1} \tag{213}$$

   The following segments of the lines of action give the first principal radii of curvature of the tooth flanks in the working normal plane (see Figure 32) are:

$$\overline{T_1C} = \rho_{C1} = \left(\frac{1}{2}\sqrt{d_{w1}^2 - d_{b1}^2}\right)\frac{1}{\cos\beta_{b1}} = \frac{d_{b1}\cdot\tan\alpha_{wt1}}{2\cdot\cos\beta_{b1}} \quad (214)$$

$$\overline{T_2C} = \rho_{C2} = \left(\frac{1}{2}\frac{z_2}{|z_2|}\sqrt{d_{w2}^2 - d_{b2}^2}\right)\frac{1}{\cos\beta_{b2}} = \frac{d_{b2}\tan\alpha_{wt2}}{2\cdot\cos\beta_{b2}} \quad (215)$$

$$\overline{T_1T_2} = \rho_{C1} + \rho_{C2} = \rho_{A1} + \rho_{A2} = \rho_{E1} + \rho_{E2} \quad (216)$$

$$\overline{T_2A} = \rho_{A2} = \left(\frac{1}{2}\frac{z_2}{|z_2|}\sqrt{d_{Na2}^2 - d_{b2}^2}\right)\frac{1}{\cos\beta_{b2}} \quad (217)$$

$$\overline{T_1E} = \rho_{E1} = \left(\frac{1}{2}\sqrt{d_{Na1}^2 - d_{b1}^2}\right)\frac{1}{\cos\beta_{b1}} \quad (218)$$

$$\overline{T_1B} = \rho_{B1} = \rho_{E1} - p_{bn} \quad (219)$$

$$\overline{T_2D} = \rho_{D2} = \rho_{A2} - p_{bn} \quad (220)$$

b) For a parallel axes helical gear pair there is a common base plane to both gears that contains the plane of action, the zone of action and lines of contact (see Figure 25). Therefore, principal directions $\overrightarrow{T_1}$ for the pinion and $\overrightarrow{T_2}$ for the gear are identical at all points of contact. In complement to 5.5.7, covering radii of curvature in a transverse plane at particular points on the action line for any point, M_y, on any line of contact, the principal radii of curvature can be derived:

For the pinion, ρ_{yn1} (equivalent to ρ_{yn} in Figure 14 and with $\beta_{b1} = -\beta_{b2} = \beta_b$ for parallel axis gears):

$$\overline{T_{M1}M_y} = \rho_{yn1} = \left(\frac{1}{2}\sqrt{d_{y1}^2 - d_{b1}^2}\right)\frac{1}{\cos\beta_b} = \frac{d_{b1}\cdot\tan\alpha_{yt1}}{2\cdot\cos\beta_b} \quad (221)$$

For the gear, ρ_{yn2}, (equivalent to ρ_{yn} in Figure 14):

$$\overline{T_{M2}M_y} = \rho_{yn2} = \overline{T_{M1}T_{M2}} - \rho_{yn1} = a_w\frac{\sin\alpha_{wt}}{\cos\beta_b} - \rho_{yn1} \quad (222)$$

With points T_{M1} and T_{M2} as the intersection between the lines where the plane of action is tangent with base circles and the normal to contact line crossing at point M_y (see Figure 25).

For an internal gear, $\overline{T_{M2}M_y} = \rho_{yn2}$ is negative.

For any point, M_y, on any line of contact, these principal radii of curvature (see Figure 25):

— are along a normal to a line of contact in the plane of action,

— correspond to the minimum distances between point M_y and the tangents of each base cylinder with the plane of action.

7.2 Angle between the principal radius of curvature at point of contact

According to 4.9, at any point M_y of an involute tooth flank, two perpendicular planes to the tangent plane to the flank, containing the principal directions of curvature defined by $\vec{B}$ and $\vec{T}$ vectors, the radius of curvature are extremum: one is minimum, the other is maximum (infinite). The principal direction of curvature defined by $\vec{B}$ vector along a virtual line of contact in the base plane, is always tilted by a constant angle δ_w according to working pitch plane (here defined by the working pitch line for each intermeshing rack).

The two second principal directions of curvatures of the pinion and the mating gear have respectively a tilt angle δ_{w1} and δ_{w2} with the common working pitch planes of intermeshing racks.

$$\tan\delta_{w1} = \sin\alpha_{wn} \cdot \tan\beta_{w1} \tag{223}$$

$$\tan\delta_{w2} = \sin\alpha_{wn} \cdot \tan\beta_{w2} \tag{224}$$

NOTE For a parallel axes helical gear pair, $\beta_{w1} = -\beta_{w2}$ so $\delta_{w1} = -\delta_{w2}$.

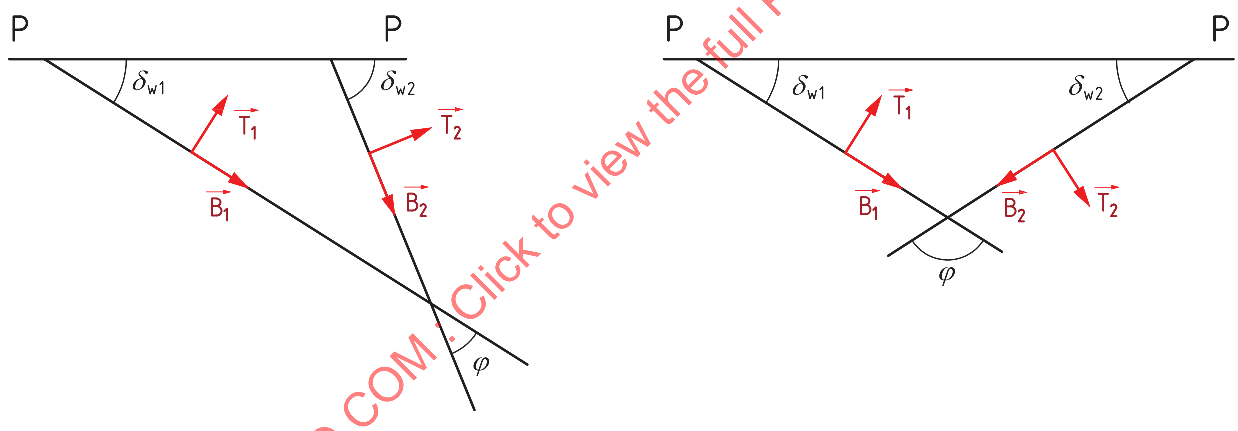
7.3 Particularities for crossed axis helical involute gear

— The tilt angles δ_{w1} and δ_{w2} are constant for any virtual line of contact, and so for any point of contact, M_y, along the path of contact AE.

— At the working pitch point, tilt angles are in the same plane, which is the tangent plane to both flanks of each gear tooth pair in contact.

— It results that the angle between the two first (or second) principal directions of curvatures of the pinion and the mating gear is also constant.

— Considering the tangent plane to both flanks of each gear tooth pair in contact, a triangle can be defined composed by the two directions of virtual line of contact $\overrightarrow{B_1}$, $\overrightarrow{B_2}$ and the trace of the common working pitch planes of intermeshing racks PP (see Figure 35).



a) Gears have same hand of helix **b) Gears have different hand of helix**

Figure 35 — Directions of second principal radius of curvature of pinion and gear flanks in contact

Figure 35 a) is the case when helix angles of pinion and gear have the same sign, Figure 35 b) is when helix angles are opposite; in this last case δ_{w1} and δ_{w2} also have opposite sign.

The angle φ_w between first (or second) principal directions of curvatures is constant and calculated with Formula (225) or Formula (226):

if helix angles of pinion and gear have the same sign:

$$\varphi_w = \delta_{w2} - \delta_{w1} \tag{225}$$

if helix angles of pinion and gear have opposite sign:

$$\varphi_w = \pi - \delta_{w2} - \delta_{w1} \tag{226}$$

NOTE For a parallel axis helical gear pair, $\varphi_w = 0$.

7.4 Equivalent radii of curvature at point of contact

In the neighbourhood of the contact point between two surfaces, the distance between them can be expressed as an ellipsoid. The first and second principal radii of this ellipsoid are called equivalent radii. The calculation starts with the general case but changes then to involute helical surfaces where one radius is infinite.

From 7.1 and 4.9, the contact between the two flanks at any point of contact M_y along the path of contact AE is equivalent to the contact of two cylinders of radius ρ_{yn1} and ρ_{yn2} tilted by angle φ.

Based on the theory of geometry of surfaces defined by Euler, those two surfaces in contact can be replaced by an ellipsoid with equivalent radius R' and R'' (see Annex D). These equivalent radii are those for a surface that has the same distances (offsets) from a plane tangent to contact point M_y as those between the two gear flanks in contact at point M_y (see Figure 25).

$$\Sigma_R = \frac{1}{R'} + \frac{1}{R''} = \frac{1}{R_1'} + \frac{1}{R_1''} + \frac{1}{R_2'} + \frac{1}{R_2''} \tag{227}$$

$$\Delta = \sqrt{\Delta_1^2 + \Delta_2^2 + 2\Delta_1 \Delta_2 \cos(2\varphi)} \tag{228}$$

$$\Delta_1 = \frac{1}{R_1'} - \frac{1}{R_1''} \text{ and } \Delta_2 = \frac{1}{R_2'} - \frac{1}{R_2''} \tag{229}$$

$$R'' \geq R' \geq 0 \tag{230}$$

For involute helical surfaces:

$$R_1' = \rho_{yn1} \quad , \quad R_2' = \rho_{yn2} \quad , \quad R_1'' = R_2'' = \infty \tag{231}$$

so:

$$\Delta = \sqrt{\left(\frac{1}{\rho_{yn1}}\right)^2 + \left(\frac{1}{\rho_{yn2}}\right)^2 + 2\frac{1}{\rho_{yn1}}\frac{1}{\rho_{yn2}}\cos(2\varphi)} \tag{232}$$

$$\Sigma_R = \frac{1}{\rho_{yn1}} + \frac{1}{\rho_{yn2}} \tag{233}$$

The first equivalent principal radius of curvature is:

$$\frac{1}{R_M'} = \frac{\Sigma_R + \Delta}{2} \tag{234}$$

The second equivalent principal radius of curvature is:

$$\frac{1}{R_M''} = \frac{\Sigma_R - \Delta}{2} \tag{235}$$

The angle θ_1 between the direction of the first principal curvature of the pinion and the direction of the 1st principal curvature of the gear wheel is given by:

$$\theta_1 = \theta + \frac{\varphi_w}{2} \tag{236}$$

with:

$$\tan 2\theta = \frac{\Delta_1 - \Delta_2}{\Delta_1 + \Delta_2}\tan\varphi_w = \frac{\rho_{yn2} - \rho_{yn1}}{\rho_{yn2} + \rho_{yn1}}\tan\varphi_w = \frac{\left(\overline{T_1T_2} - \rho_{yn1}\right) - \rho_{yn1}}{\overline{T_1T_2}}\tan\varphi_w = \left(1 - \frac{2\cdot\rho_{yn1}}{\overline{T_1T_2}}\right)\tan\varphi_w \tag{237}$$

NOTE For a parallel axes helical gear pair $\varphi = 0$ so $\theta = 0$. Therefore, the equivalent radius of curvature at point M_y is given by $R_{eqM} = \left(\frac{1}{\rho_{yn1}} + \frac{1}{\rho_{yn2}}\right)^{-1}$.

8 Tooth flank modifications

8.1 General

Tooth flank modifications are desired alterations to the tooth flank face compared with the nominal involute flank (see 3.1.3). Superimposing the modifications on the nominal involute flank produces the design tooth flank. The modifications can be defined by characteristic profiles of the tooth flank or in relation to the whole gear flank. Modification depths are always given in the transverse section and normal to the involute.

NOTE In Figures 37 to 42 and Figure 44, on the right side of the figure, a sketch of the design profile or design helix is shown. See ISO 1328-1 for explanation of design profile.

8.2 Tooth modifications which restrict the usable flank

8.2.1 Pre-finish flank undercut

A pre-finish relief at the tooth root fillet can be planned. The teeth are generated with a controlled undercut of the tooth flank in the area of the root [e.g. using a protuberance tool, see Figure 49 b)]. The magnitude of the undercut in the transverse plane, q_{Fs}, is the greatest distance between the tooth root fillet and the involute imagined as extended, see Figure 36. Below the base circle, the datum is a radial line extended from the involute profile.

NOTE q_{Fs} is determined in a transverse plane using the tooth root fillet points formulae in 10.1; but it results from generation by the tooth tip corner radius of counterpart rack tooth profile on a normal surface.

8.2.2 Tip corner chamfering, tip corner rounding

Tip corner chamfering and tip corner rounding are reliefs of the transverse profile which restrict the usable area of the tooth flank in the addendum. It results from a defined tip form diameter, d_{Fa}, that is lower than the tip diameter, d_a, giving the height of the tip corner chamfering or tip corner rounding, h_K:

$$h_K = \frac{d_a - d_{Fa}}{2} \tag{238}$$

In both cases the residual transverse tooth tip thickness, s_{aK}, is given as the dimension of this modification, see Figure 36, and is different for chamfering and rounding.

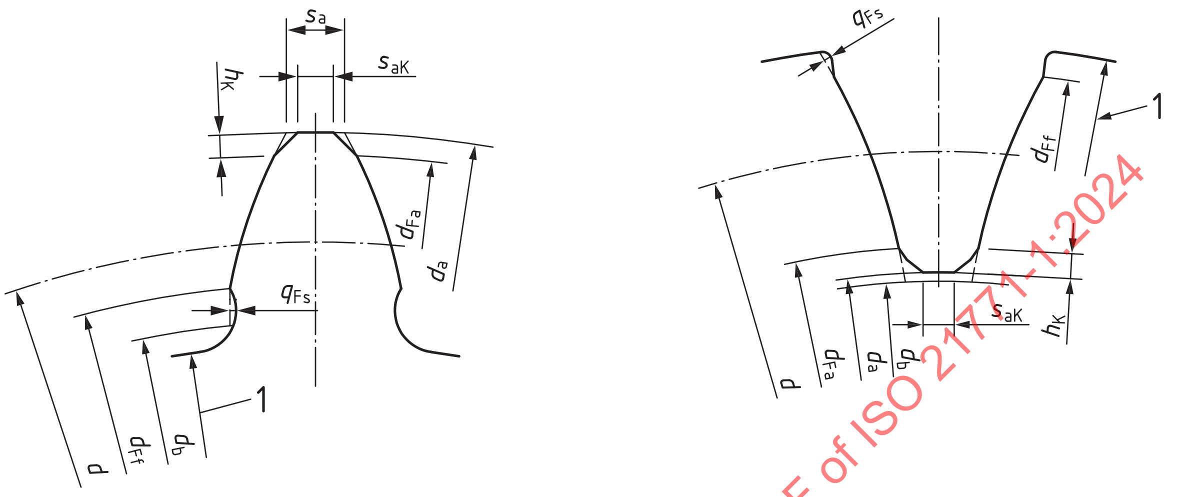
In the case of tip corner chamfering, the chamfer arising through removal of the tooth tip corner is generated along with the involute of the tooth flank and so is an involute generated with the normal pressure chamfering angle, α_{KP0}, defined on the counterpart rack tooth profile [see Figure 49 a)] and generated from:

— the chamfering base diameter:

$$d_{bK} = d \cos \alpha_{KP0t} \tag{239}$$

— with the transverse chamfering pressure angle (see Figure 49):

$$\tan \alpha_{KP0t} = \frac{\tan \alpha_{KP0}}{\cos \beta} \tag{240}$$



Key

1 d_f or d_{fE}

NOTE For internal gears, since the number of teeth is negative, the diameters used in calculations are negative. On drawings, the diameters are usually shown as positive numbers.

Figure 36 — Cylindrical gear with undercut and tip corner chamfering

α_{KP0t} shall be significantly higher than the normal pressure angle α_n in order to reach the tooth tip thickness s_{aK}. The dedendum form of the counterpart rack tooth profile can be defined by:

$$h_{FfP0} = 0{,}5(d_{Fa} \sin \alpha_{Fat} - d \sin \alpha_t) \sin \alpha_t - x_E m_n \tag{241}$$

— The residual tooth tip thickness with tip corner chamfering is:

$$s_{aK} = s_{at} - d_a \left(\operatorname{inv} \left(\arccos \frac{d_{bK}}{d_{Fa}} \right) - \operatorname{inv} \left(\arccos \frac{d_{bK}}{d_a} \right) \right) \tag{242}$$

In the case of tooth tip corner rounding, this corner is radiused in the normal plane of the counterpart rack tooth profile. Process generation is generating an approximately circular arc form (not a pure radius) at the tooth tip.

— The rounding radius of the counterpart rack tooth profile is:

$$\rho_{fP0} = \frac{h_K}{1 - \sin \alpha_t} \tag{243}$$

The residual tooth tip thickness with tooth tip corner rounding is more complex to determine and is not covered in this document.

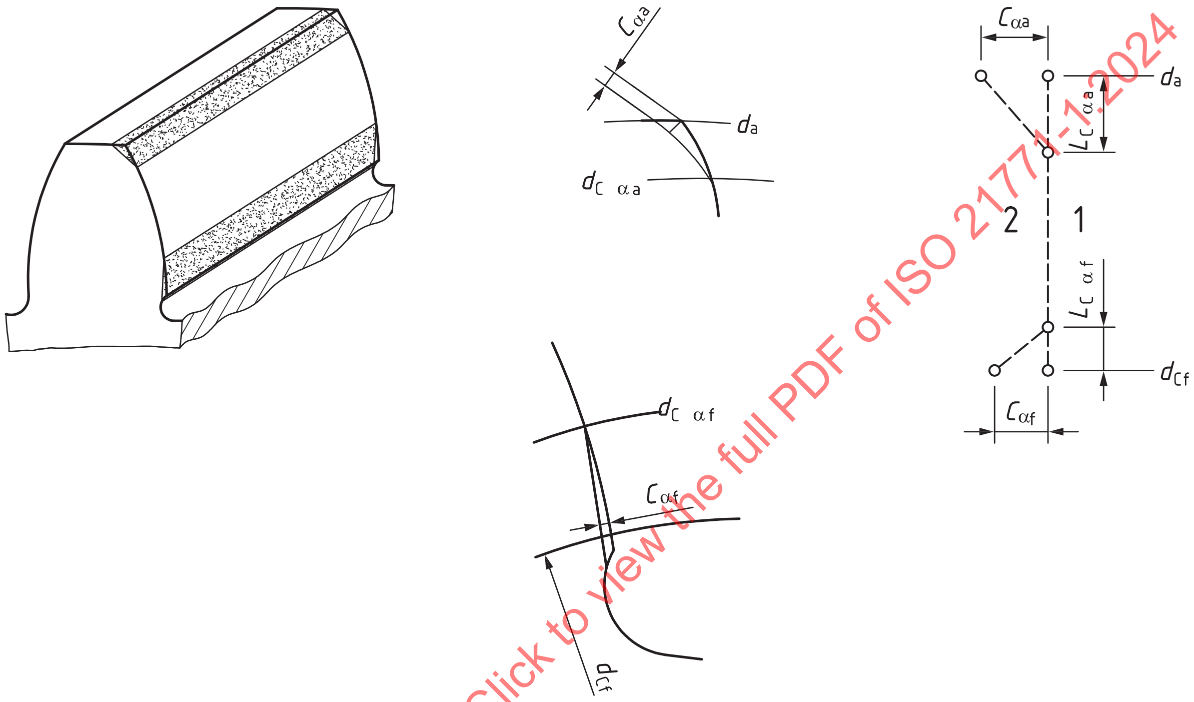
8.3 Transverse profile modifications

8.3.1 General

Additional information on modifications is available in ISO 1328-1.

8.3.2 Tip and root relief, $C_{\alpha a}$ $C_{\alpha f}$

Tip and root reliefs are the continuously increasing reliefs of the transverse profile of the nominal involute flank from defined points in each case (diameter, length of roll, roll angle) in the direction of the tip or root (mostly involute). See Figure 37 c) which shows the modification from a nominal involute.



a) View of tooth flank b) Amount of modifications c) Modification on line of action

Key

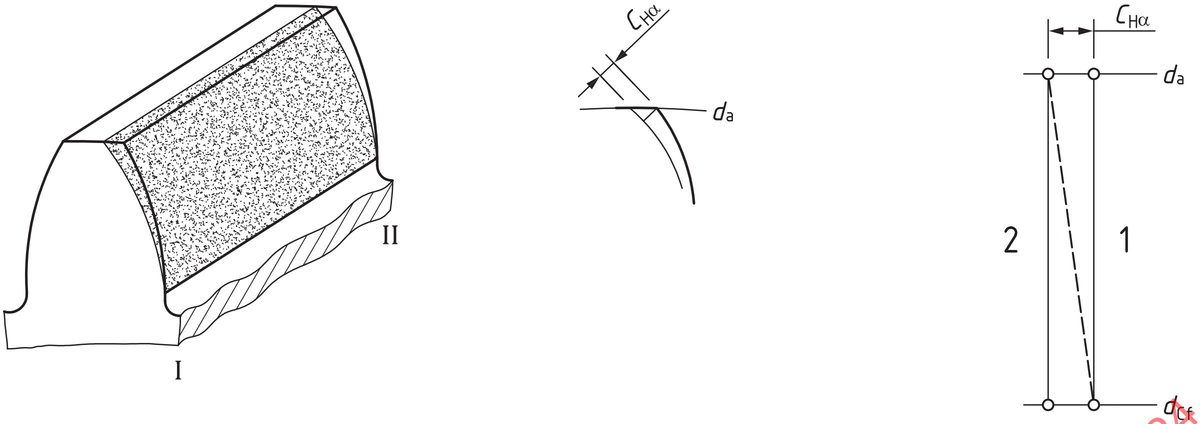
1 tooth space

2 tooth

Figure 37 — Tip and root relief

8.3.3 Transverse profile slope modification, $C_{H\alpha}$

This is similarly defined as for tip or root relief, except that transverse profile slope modification $C_{H\alpha}$ extends over the whole length of the transverse profile. The slope modification is deemed to be positive when the mean profile line shows a decrease in material towards the tooth tip, relative to the involute. A negative slope modification removes material towards the tooth root. See Figure 38 c), which shows the modification from a nominal involute.



a) View of tooth flank　　b) Amount of modifications　　c) Modification on line of action

Key

1 tooth space

2 tooth

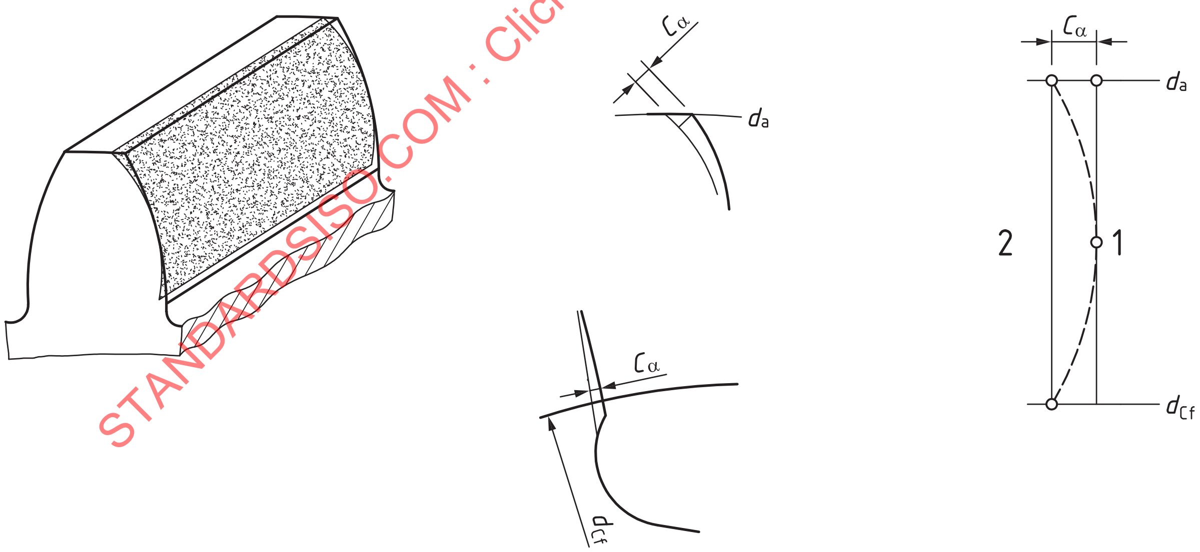
Figure 38 — Transverse profile slope modification

8.3.4 Profile crowning (barrelling), C_α

Profile crowning is the continuously increasing relief of the transverse profile from a specified line or helix in the direction of the tip and root of the gear teeth. See Figure 39 c), which shows the modification from a nominal involute.

Profile crowning is generally defined with respect to the centre of the length of roll of the usable flank and has a parabolic form passing though the points defined by C_α .

NOTE The specification of the origin of crowning can be by diameter, length of roll or roll angle.



a) View of tooth flank　　b) Amount of modifications　　c) Modification on line of action

Key

1 tooth space

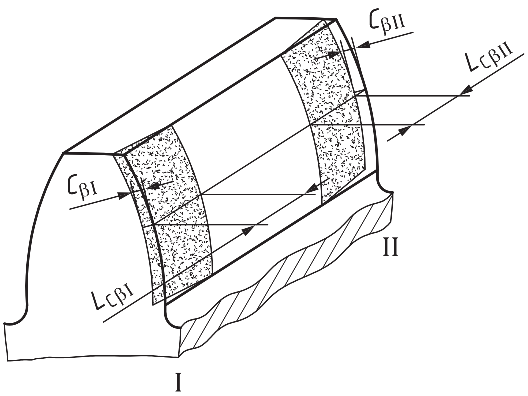
2 tooth

Figure 39 — Profile crowning

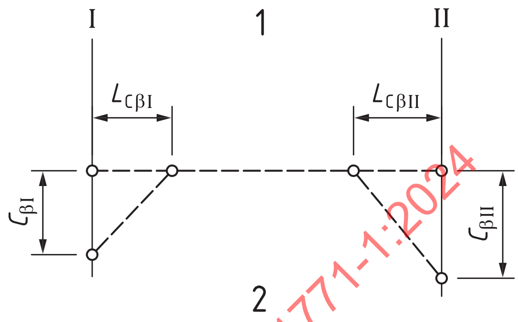
8.4 Flank line (helix) modifications

8.4.1 Flank line end relief, $C_{H\beta}$

Flank line end reliefs are continuously increasing reliefs of the flank line from defined points of the nominal involute flank in each case in the direction of both ends of the tooth (linear or parabolic). See Figure 40 b), which shows the modification from a nominal helix. The modification is normal to the involute.



a) View of tooth flank



b) Modification on facewidth

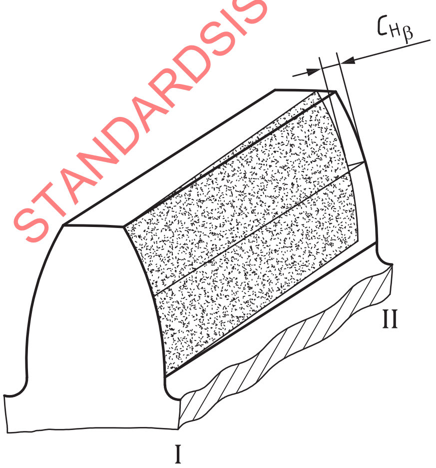
Key

$L_{C\beta I}$	length (datum face)
$C_{\beta I}$	amount of end relief (datum face)
$L_{C\beta II}$	length (non-datum face)
$C_{\beta II}$	amount of end relief (non-datum face)
1	tooth space
2	tooth

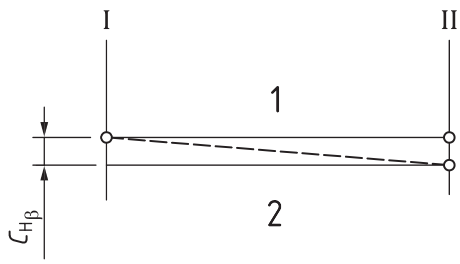
Figure 40 — Flank line end relief

8.4.2 Helix angle modification, $C_{H\beta}$

This is similarly defined as for end relief, but $L_{C\beta I}$ or $L_{C\beta II}$ extends across the whole facewidth. It is not necessarily linear. See Figure 41 b), which shows the modification from a nominal helix. The modification is normal to the involute.



a) View of tooth flank



b) Modification on facewidth

Key

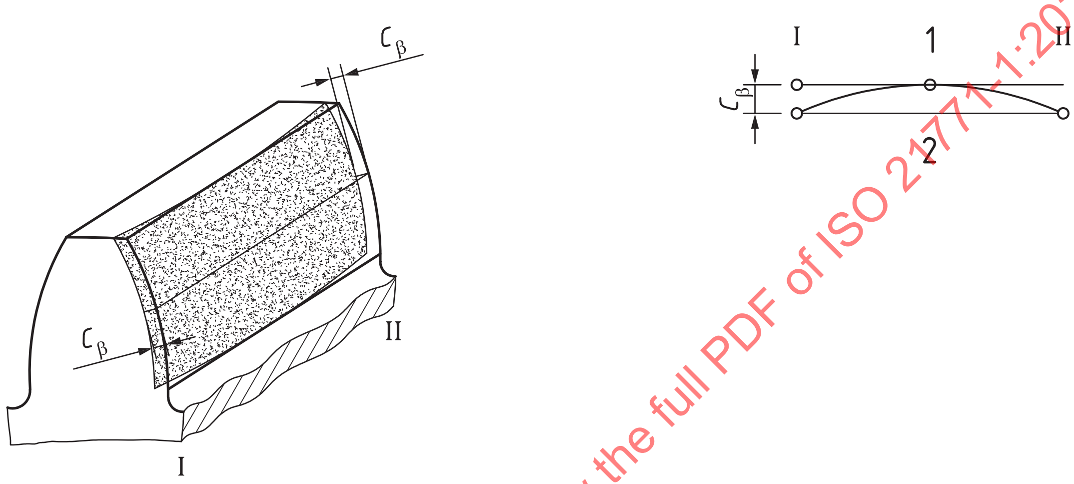
1 tooth space 2 tooth

$C_{H\beta}$ amount of flank line slope modification

Figure 41 — Helix angle modification

8.4.3 Flank line (helix) crowning, C_{β}

Flank line crowning is the continuously increasing relief of the flank line from a defined point of the nominal involute flank, symmetrically in the direction of both ends of the tooth (arc-shaped or parabolic). See Figure 42 b), which shows the modification from a nominal helix. The modification is normal to the involute.



a) View of tooth flank b) Modification on facewidth

Key

1 tooth space 2 tooth

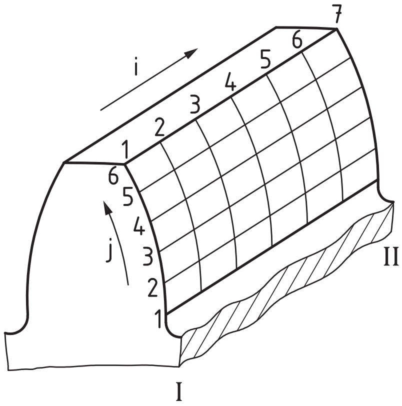
C_{β} amount of the flank line crowning

Figure 42 — Flank line crowning

8.5 Flank face modifications

8.5.1 Topographical modifications

The desired modification from the unmodified involute helicoid is determined in relation to each point of intersection on a grid laid over the tooth flank of the nominal involute flank. See Figure 43. The modification shown is theoretical and measurement requirements are not considered.



Key

i index number of transverse section

j index number of flank line

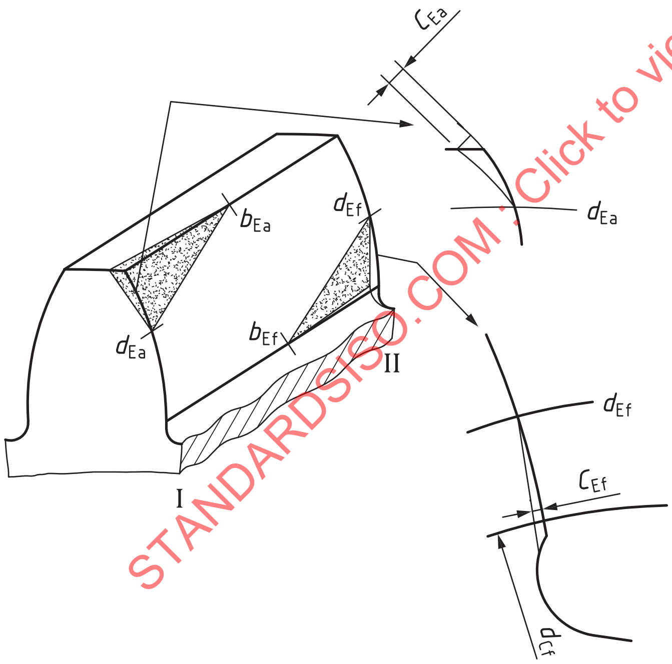
$C_{i,j}$ amount of modification on point (i,j)

NOTE If necessary, interpolated points between the defined points (i,j) of a grid can be generated.

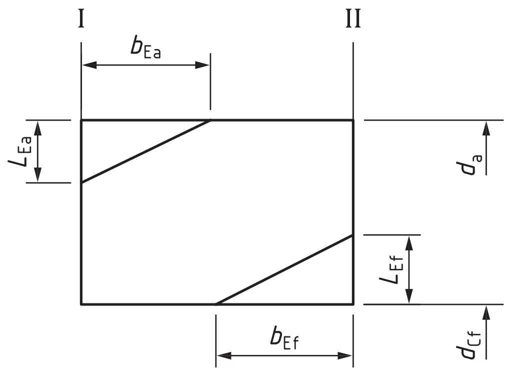
Figure 43 — Topographical modification

8.5.2 Triangular end relief

Triangular end reliefs are continuously increasing reliefs of the tooth flanks generally perpendicular to the generators of the nominal involute flank (along the lines of contact) from a defined roll angle in the direction of the start or end of roll on the tooth flank. The modification is normal to the involute. See Figure 44.



a) View of tooth flank



b) Modification on flank

Figure 44 — Triangular end relief

8.5.3 Flank twist

Twist is an effect on a flank described as a rotation of the transverse profile along a helix. There is a distinction between twist of the transverse profile, S_α , and of the flank line, S_β . If not otherwise defined, it changes linearly from the beginning to the end of the useable flank. The sign of the flank twist is very important but is not defined here. See Figure 45.

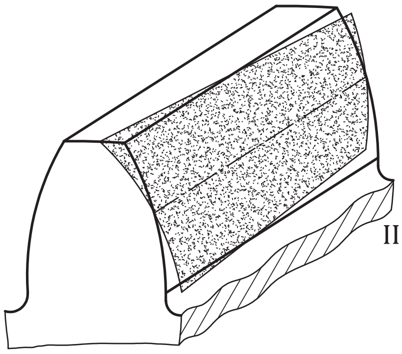


Figure 45 — Flank twist

Twist of transverse profile, S_α :

$$|S_\alpha| = |C_{\mathrm{H}\alpha 1} - C_{\mathrm{H}\alpha \mathrm{II}}| \quad (244)$$

The sign of the twist near face I shall be different from the sign of the twist near face II.

Twist around a flank line, S_β :

$$|S_\beta| = |C_{\mathrm{H}\beta\mathrm{Na}} - C_{\mathrm{H}\beta\mathrm{Nf}}| \quad (245)$$

The sign of the twist near the tip shall be different from the sign of the twist near the root.

8.6 Descriptions of modifications by functions

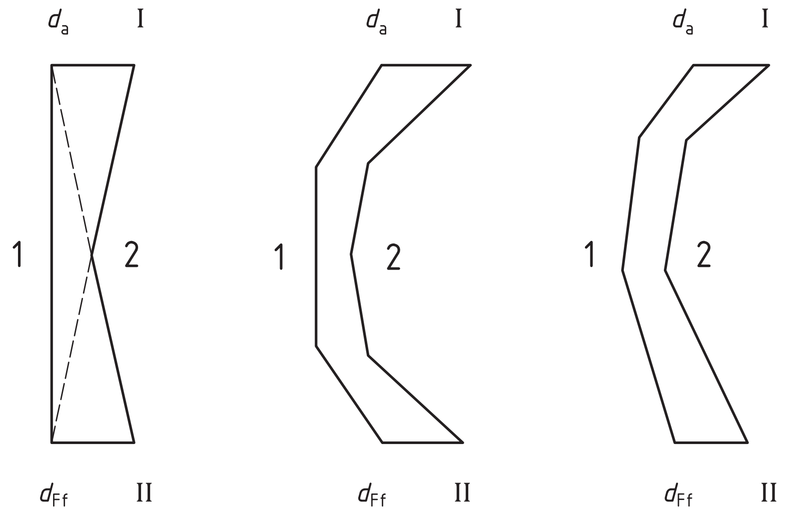
Modifications of the profile can be given as functions of the diameter, d_y , or the corresponding lengths of roll or roll angles, and modifications of the flank lines as functions of the axial distance from the start of the usable facewidth in the direction of the non-datum face. The combination of both functional relationships describes the modification of the whole flank surface.

— Modification of the profile: $C_\mathrm{ay} = f(d_\mathrm{y})$; alternatively, $C_\mathrm{ay} = f(L_\mathrm{y})$ or $C_\mathrm{ay} = f(\xi_\mathrm{y})$.

— Modification of the flank line: $C_{\beta\mathrm{y}} = f(b_\mathrm{Fy})$.

— Modification of the flank surface: $C_{\Sigma\mathrm{y}} = f(d_\mathrm{y}, b_\mathrm{Fy})$; alternatively, $C_{\Sigma\mathrm{y}} = f(L_\mathrm{y}, b_\mathrm{Fy})$ or $C_{\Sigma\mathrm{y}} = f(\xi_\mathrm{y}, b_\mathrm{Fy})$.

— Definition of tooth flank modification by tolerance fields.



Key

1 tooth space

2 tooth

I datum face (helix *K* diagram)

II non-datum face (helix *K* diagram)

NOTE Profile *K* diagrams are bounded by the tip diameter (d_a) and root form diameters (d_{Ff}), whereas helix *K* diagrams are bounded by faces I and II.

Figure 46 — *K*-diagram (examples)

Graphically, it is usual to show tooth surface modifications as deviations from the exact involute helicoid with respect to roll length for radial deviations [as shown in Figures 37 through 39, subfigures c)] and position across the facewidth for lead modifications (as in Figure 40 through 42). For intentional deviations which vary in the radial/axial direction from the exact involute geometry in any defined transverse plane, these deviations may be defined by a design profile, see ISO 1328-1. Alternatively, they may be defined by tolerance fields, generally called "K" charts, which show the range of acceptable measured values for profile or helix, see Figure 46. Such diagrams are not necessarily bounded by straight lines.

9 Geometrical limits

9.1 General

In this clause, the finished state at the conclusion of all manufacturing operations is examined. The basic concepts presented in Clause 4 are expanded to include the effect of such items as tooth thinning for backlash and manufacturing tolerances. The effects of manufacturing tolerances are both direct, such as tooth thickness tolerance, and indirect, such as the change in the axis related tooth thickness as a result of runout or profile slope deviation. The classification of tolerances is not covered in this document, see ISO 1328-1 and ISO 1328-2.

In manufacturing a cylindrical gear with involute teeth using a generating process, (e.g. using a hob, pinion-type cutter, rack-shaped cutter, grinding wheel, or grinding worm) the same concepts and the corresponding formulae which apply to a cylindrical gear pair shall apply to the paired work piece and generating tool (if $\alpha_{P0} = \alpha_P = \alpha_n$ i.e. pressure angle of the tool is equal to the pressure angle of the cylindrical gear basic rack tooth profile, see Clauses 5 and 6). When producing a cylindrical gear with involute teeth by means of non-generating methods (forging, forming, form cutting), the enveloping surface produced by the geometry of the tool and its motions are mapped directly onto the work piece.

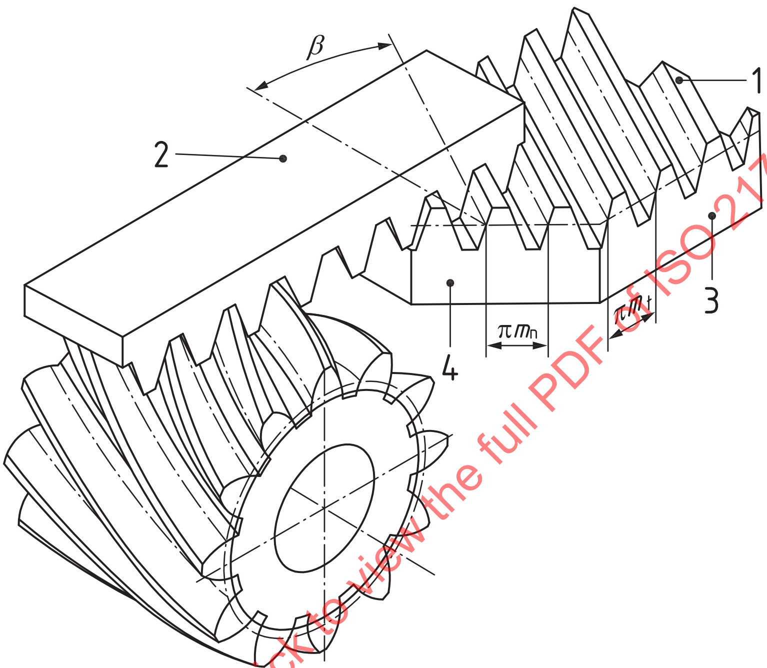
If bottom land, tooth root rounding and involute helicoid are machine-finished using the same tool, only the working cycle using this tool is of importance for the dimensions of the tooth parameters. Otherwise, the teeth produced (and their reference rack) are the sum of separate processes which produce the final dimensions of the root and usable flank surfaces including modifications in each case. The total result of the working cycles can be represented by a single hypothetical tool, the counterpart rack (see Figure 47).

NOTE The additional movements of the tool to produce the flank crowning and other modifications are not covered by the description of the counterpart rack tool.

9.2 Counterpart rack tooth profile and rack tool profile

9.2.1 Counterpart rack tooth profile

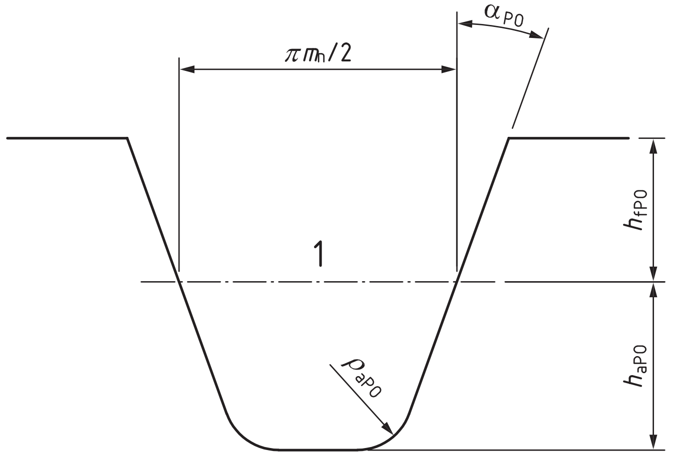
The counterpart rack tooth profile is the complement to the basic rack tooth profile enclosing the bottom land, see Figures 47, 48 and 49.



Key

1 basic rack
2 counterpart rack
3 transverse plane
4 normal plane

Figure 47 — Basic rack and counterpart rack

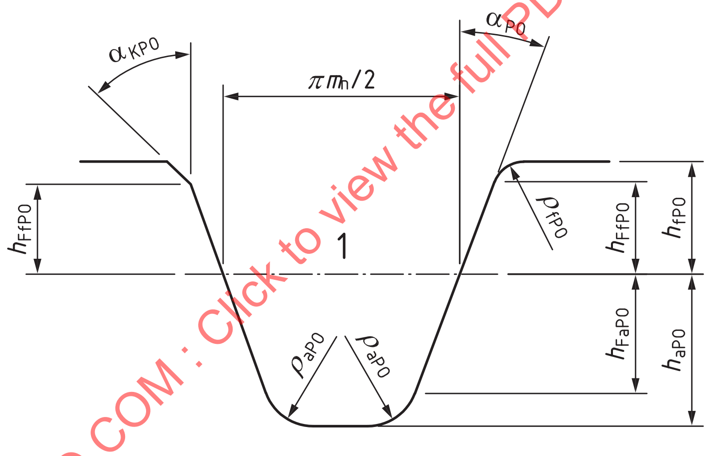


Key

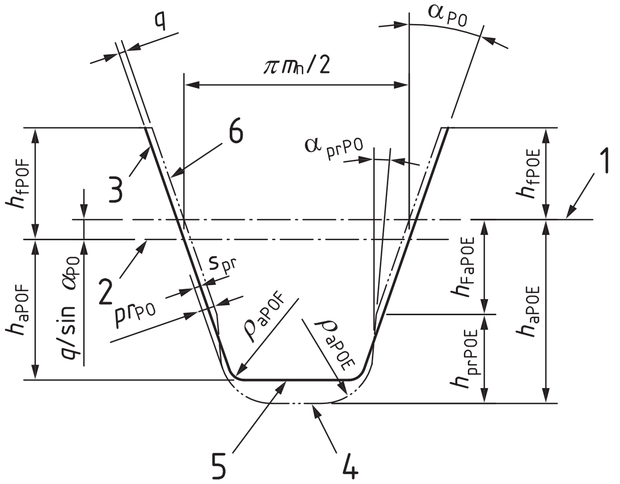
1 datum line of counterpart rack

Figure 48 — Counterpart rack tooth profile

Figure 49 b) shows the additional parameters for the counterpart rack profile with protuberance. It shows the counterpart rack tooth profile with protuberance for roughing (subscript E) superposed with the finishing counterpart rack tooth profile without protuberance (subscript F).



a) Counterpart rack tooth profile with tip chamfer or rounding



b) Counterpart rack tooth profile with protuberance

Key

1 datum line of roughing tool
2 datum line of finishing tool
3 finishing tool rack
4 tip of roughing tool
5 tip of finishing tool
6 roughing tool rack

NOTE
Items 6 and 4 make up the roughing tool.
Items 3 and 5 make up the finishing tool.
Items 3 and 4 together make up the hypothetical tool.

Figure 49 — Modified counterpart rack tooth profiles

The protuberance is defined by two main parameters:

— pr_{P0}: the protuberance;

— α_{apr0}: the protuberance angle.

Then, the gear is defined in two steps:

— Step 1: corresponding at roughing profile (subscript E) and dotted line.

— Step 2: corresponding at finishing profile (subscript F) and solid line.

At roughing step, the protuberance tool is creating:

a) a controlled undercut at the tooth root fillet;

b) the root diameter with tip profile, see Figure 49 b) key 4;

c) an eventual material allowance on involute flank (if q>0) for finish machining per flank. Then the generating profile shift, x_{E}, at roughing is:

$$x_{\text{E}} = x + \frac{q}{m_{\text{n}} \sin \alpha_{\text{P0}}} + \frac{E_{\text{sn}}}{2 m_{\text{n}} \tan \alpha_{\text{n}}} \quad (246)$$

d) or the nominal involute profile if q=0.

At finishing step, when existing, the material allowance is removed on the involute flanks. The fillet as well as the root cylinder are not changed as the finishing tool has a lower addendum ($h_{ap0F} < h_{ap0E}$).

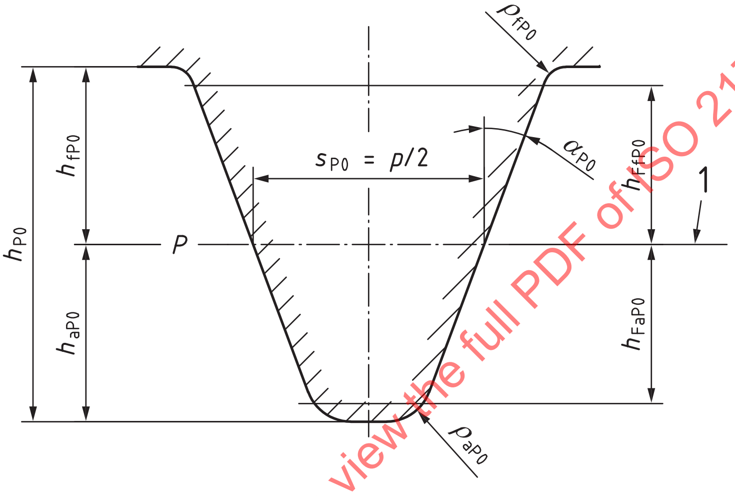
NOTE The counterpart rack tooth profile with protuberance can also generate tip chamfering or tip rounding as in Figure 49 a).

9.2.2 Rack tool profile

From the counterpart rack tooth profile, the rack tooth profiles generating the gear tooth can be obtained as follows.

The usual rack tool profile corresponding to Figure 49 a) is defined in Figure 50. It usually has a tooth root radius ρ_{fP0} to give a clearance between the tooth tip of the generated gear and the root of the rack tool.

In case semi-toping (or topping) for chamfering the rack tool profile, it also has a secondary profile with the semi-topping pressure angle α_{KP0} to generate the chamfer - see left part of Figure 49 a).



Key

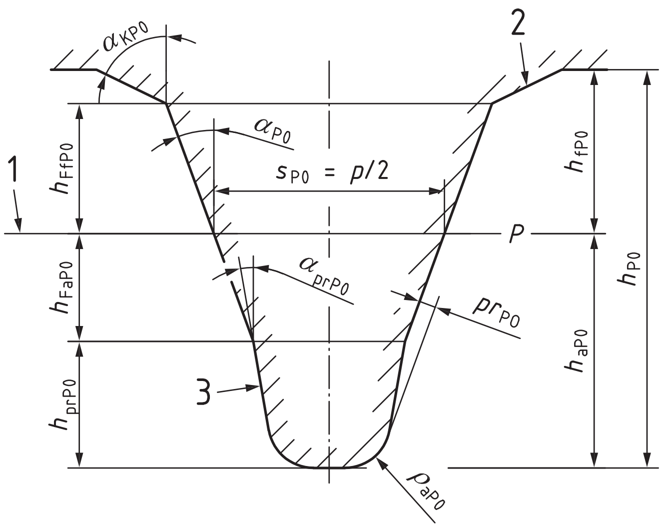
1 datum line of the rack tool

Figure 50 — Usual rack tool profile without chamfering

When using protuberance for rough machining the rack tool profile is defined in Figure 52, and the teeth are generated in two steps.

a) The protuberance area of the tool generates the fillet with a controlled undercut to create a sufficient gap in order that the tooth tip corner or tip radius of the finishing tool (disc of flat grinding wheel, shaving cutter) will not touch the initial generated fillet at roughing taking into account potential deformation due to heat treatment between roughing and finishing. The protuberance rack tool profile corresponding to right part of Figure 49 b). The semi-topping pressure angle α_{KP0} as shown on the Figure 49 a) is optional.

b) At finishing, only the main involute profile is generated on the teeth. The equivalent rack tool profile is defined without protuberance and without optional semi-topping as in Figure 50 with lower values of h_{aP0} and ρ_{aP0}. They are corresponding to parameters with subscript F on the left part of Figure 49 b).



Key

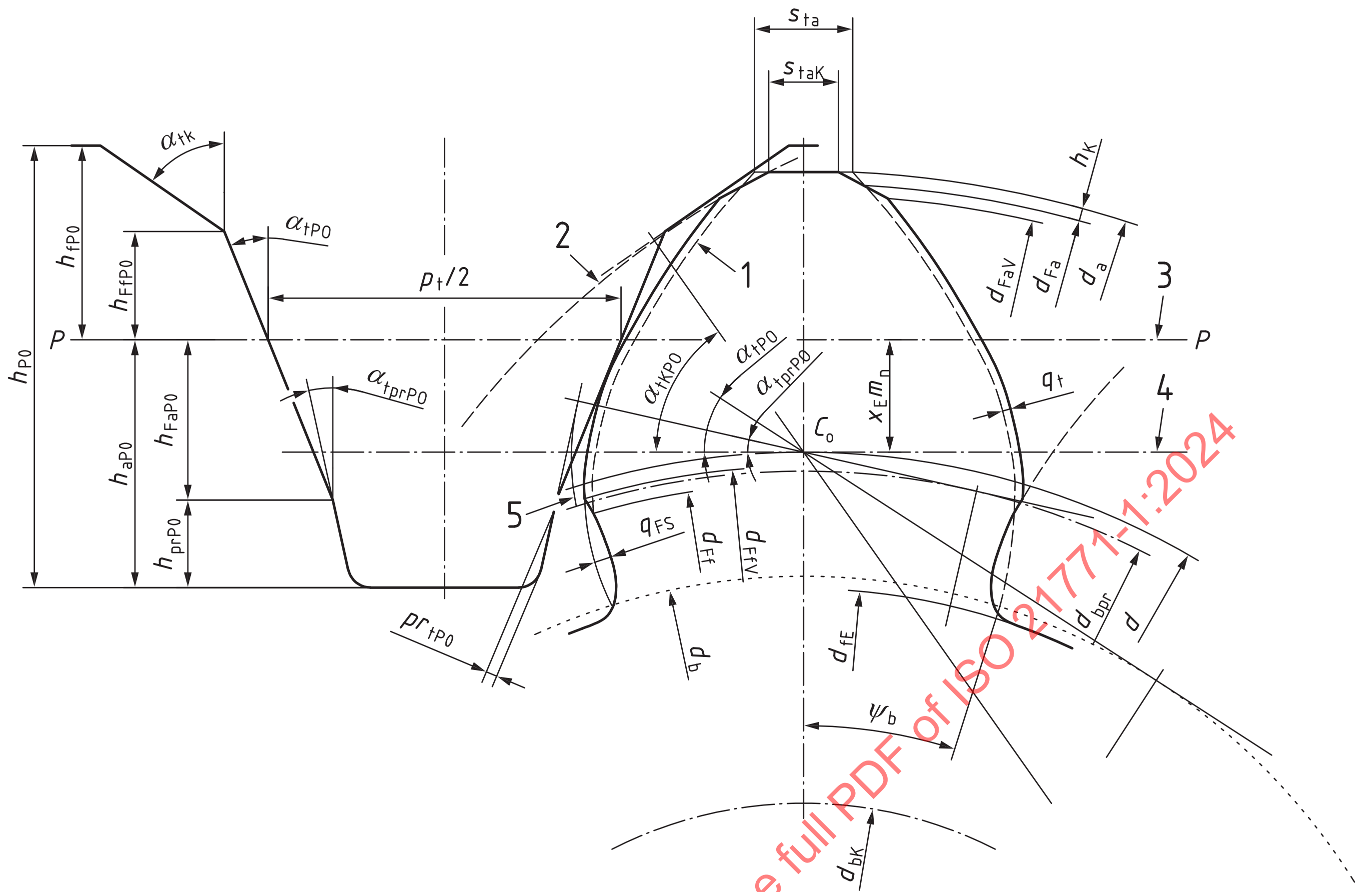
1 datum line of the rack tool

2 secondary profile for topping or semi-topping

3 protuberance tool area

NOTE "h_{FaP0}", "h_{aP0}" and "h_{prP0}" of Figure 51 correspond to "h_{FaP0E}", "h_{aP0E}" and "h_{prP0E}" of Figure 49 b).

Figure 51 — Rack tool profile with protuberance for roughing



Key

1 finished involute profile

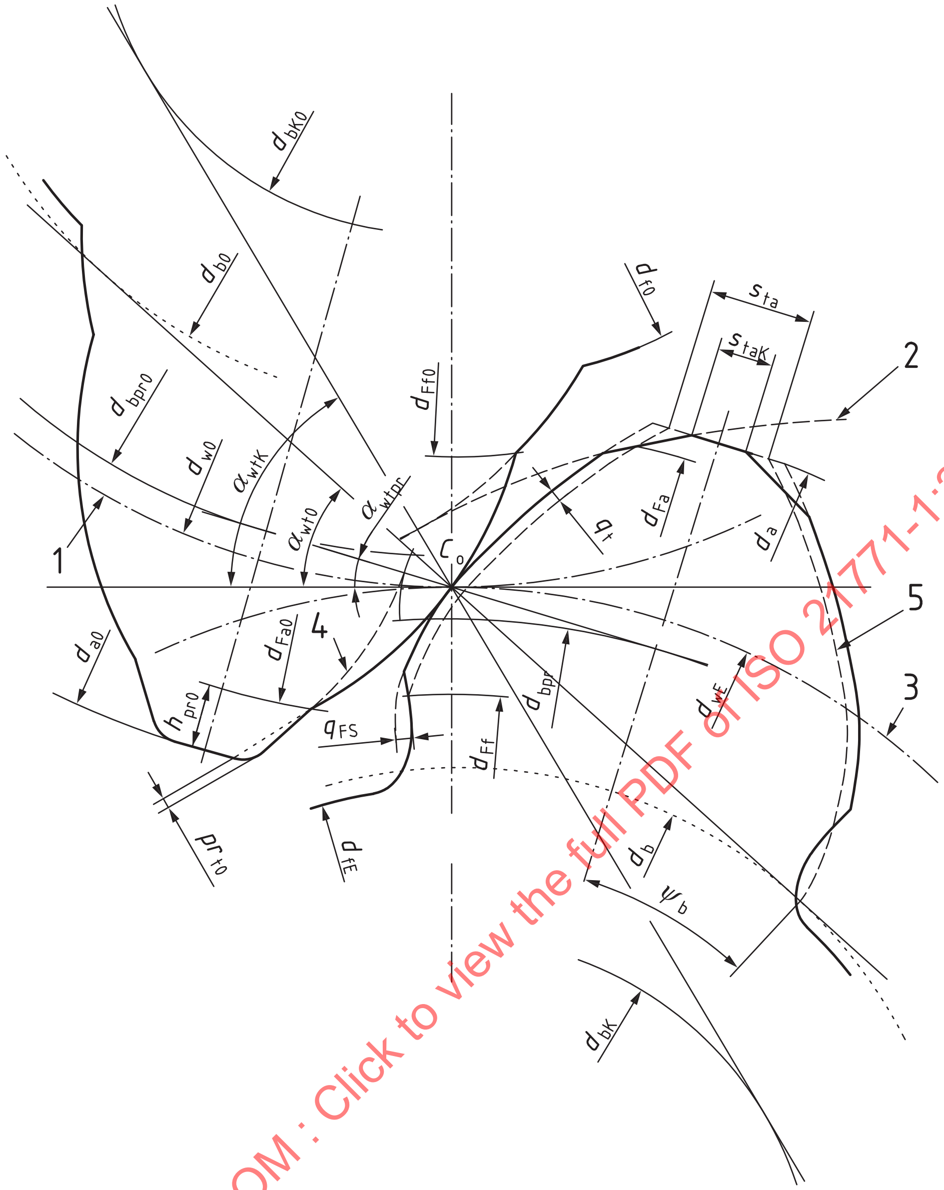
2 tip chamfer or semi-topping secondary involute

3 datum line of the rack tool at roughing

4 trace of working pitch surface of the rack tool at roughing

5 reference diameter of the generated gear (working pitch surface at roughing)

Figure 52 — Generation of a gear in two steps with protuberance rack tool in a transverse plane



Key

1 trace of working pitch cylinder of pinion-type cutter at generation
2 tip chamfer or semi-topping secondary involute
3 trace of working pitch cylinders of generated gear at generation
4 involute of protuberance portion of pinion type cutter
5 finished involute profile

Figure 53 — Generation of a gear in two steps with protuberance pinion-type cutter in a transverse plane

When using a pinion-type cutter, the same principles as for generation with rack tool profile are applicable, but the pinion-type cutter and the generated gear shall be considered as a gear pair. The pinion-type cutter and the generated gear have their own profile shifts that shall be considered to determine working parameters (centre distance and working pressure angle) during generation as in an involute cylindrical gear pair (see Clause 5 and Figure 53). It is important to notice that working parameters at generation for the generated gear are usually different from working parameters when meshing with the conjugate gear.

9.3 Machining allowance

A roughing gear-cutting tool leaves the machining allowance, q , for the subsequent finish gear cutting on the flank of the cylindrical gear. The machining allowance is defined normal to the tooth surface. The normal tooth thickness produced by the roughing gear-cutting tool on the cylindrical gear is thus $2q / \cos\alpha_n$ greater than the normal tooth thickness, s_n , produced by the finish gear-cutting tool. In practice, the machining allowance ranges from q_{min} to q_{max} . See Figure 49 b).

Machining allowance shall be considered to determine root diameter considering the allowance as an increase of profile shift coefficient (see 9.2).

When chamfering or tip rounding exist at roughing cutting, gear machining allowance will be considered in the same way as for root diameter.

9.4 Limits of normal tooth thickness

The corresponding maximum and minimum limits of normal tooth thickness (s_{ns} and s_{ni}) to be required on the generated teeth are obtained by determining (upper and lower) normal tooth thickness limit deviations (E_{sns} and E_{sni}) on the reference cylinder in the normal surface. See Figure 54.

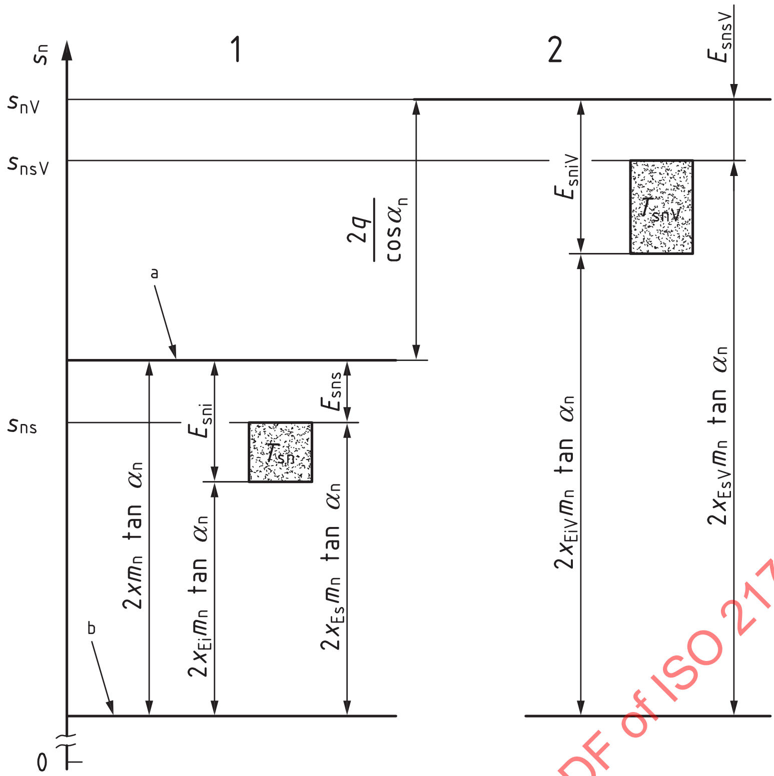
$$s_{ns} = s_n + E_{sns} \tag{247}$$

$$s_{ni} = s_n + E_{sni} \tag{248}$$

NOTE The upper and lower limit deviation for normal tooth thickness at the reference cylinder are the adjustments from nominal tooth thickness (basic size) to achieve the desired manfactured tooth thickness. They are sometimes referred to as allowances.

A negative normal tooth thickness limit deviation reduces the normal tooth thickness and increases the space width compared to the nominal dimensions, determining the contribution of the normal tooth thickness to the backlash, j_{bn} (see 5.6.4).

In addition to the finish machining tolerances, the backlash is also affected by elemental tolerances such as profile, helix and runout. The elemental tolerances will increase the tolerance band of functional normal tooth thickness; this will reduce the minimum backlash and increase the maximum backlash. (See Figure 26.) Therefore, a complete analysis of the normal tooth thickness for the purpose of determining the backlash shall include all the elemental tolerances that affect the functional normal tooth thickness. The relative importance of the different tolerances depends not only on those tolerances but also on the measuring methods used. See ISO 21771-2 for additional information.



Key

1 finishing

2 pre-machined

s_{nsV} upper normal thickness of pre-machined tooth

s_{ns} upper normal thickness of finished tooth with backlash and x>0

[a] s_n where x > 0 and j_{bn} =0.

[b] s_n where x = 0.

s_{nV} normal thickness of pre-machined tooth at zero backlash

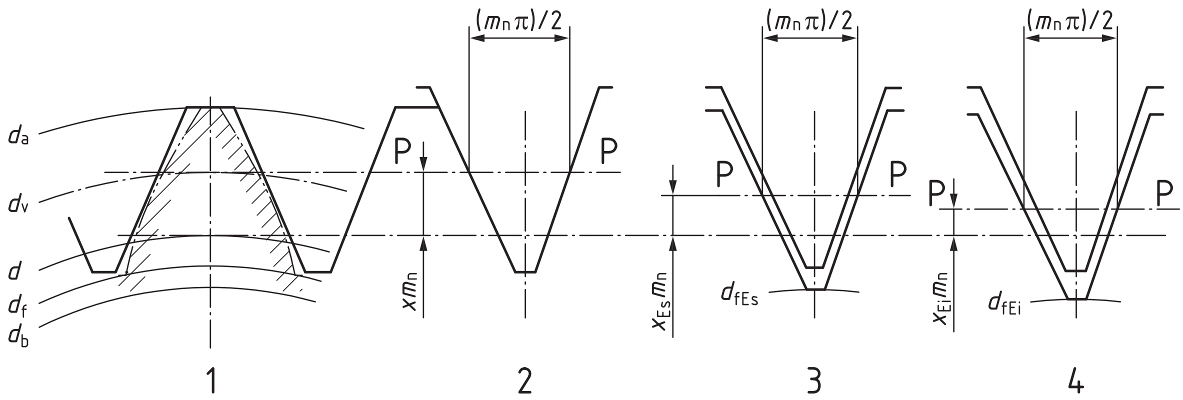
Figure 54 — Diagram of dimensioning of normal tooth thickness with positive profile shift

9.5 Generating profile shift, generating profile shift coefficient

The generating profile shift, $x_E m_n$, of a gear with involute teeth is the distance between the datum line of the counterpart rack tooth profile and the reference cylinder of the gear, see Figure 55.

The generating profile shift takes account of the predetermined upper and lower normal tooth thickness limit deviations, E_{sns} and E_{sni} (see 9.4) and, if necessary, the machining allowance q , provided for the finish-machining of a gear.

The permissible maximum and minimum normal tooth thickness inspection dimensions can be determined by calculation using x_E in the formulae given in ISO 21771-2. The cases of application shall be identified by corresponding additional subscripts.



Key

P-P datum line of basic rack
1 reference rack
2 counterpart rack
3 counterpart rack at upper tooth thickness limit
4 counterpart rack at lower tooth thickness limit

Figure 55 — Generating profile shift, $x_E m_n$ — Example: external gear, $x > 0$

Taking account of the relations shown in Figure 54, it follows for roughing with normal tooth thickness limit deviations (E_{snsV} and E_{sniV}) and a machining allowance, q, that:

$$x_{EsV} m_n = x m_n + \frac{q}{\sin\alpha_n} + \frac{E_{snsV}}{2\tan\alpha_n} \quad (249)$$

$$x_{EiV} m_n = x m_n + \frac{q}{\sin\alpha_n} + \frac{E_{sniV}}{2\tan\alpha_n} \quad (250)$$

The following applies to finish gear cutting ($q = 0$):

$$x_{Es} m_n = x m_n + \frac{E_{sns}}{2\tan\alpha_n} \quad (251)$$

$$x_{EiV} m_n = x m_n + \frac{E_{sni}}{2\tan\alpha_n} \quad (252)$$

9.6 Generated root diameter

The root diameter generated by a rack gear cutter (e.g. hob, rack-shaped cutter or grinding wheel) with the addendum h_{aP0E} is:

$$d_{fE} = d + 2 x_E m_n - 2 h_{aP0E} \quad (253)$$

The root diameter produced by a pinion-type cutter is:

$$d_{fE} = 2 a_{w0} - d_{a0} \quad (254)$$

For x_E, h_{aP0}, a_{w0} and d_{a0}, it is necessary to use the values for the process that produces the finished tooth.

9.7 Usable area of the tooth flank, tip and root form diameter

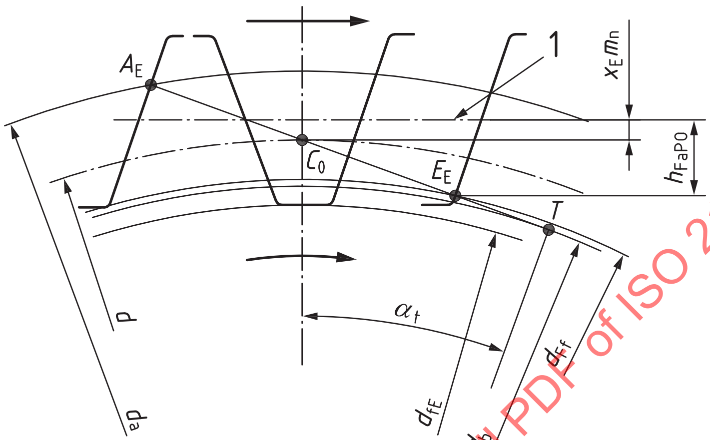
The maximum usable area of the tooth flank of a cylindrical gear is enclosed by the tip form circle (diameter d_{Fa}) and the root form circle (diameter d_{Ff}), see Figure 18 and Figure 20. These circles arise during the generation of the cylindrical gear. They are determined by the starting point A_E and the finishing point E_E

of the generating path of contact (see Figure 56) and limit the involute section of the tooth profile. With direct transition between the nominal involute helicoid and the top land of the tooth, the tip form diameter is equal to the tip diameter ($d_{Fa} = d_a$). In the case of tip chamfering, the tip form diameter and the tip diameter differ by double the height of chamfer h_K (see Figure 36):

$$d_{Fa} = d_a - 2h_K \tag{255}$$

In the case of tooth tip radiusing, see Annex A (A.2.1 for external gear or A.3.1 for internal gear) to determine tip form diameter d_{Fa}.

The root form diameter, d_{Ff}, follows from the relevant working cycle during gear cutting.



Key

1 datum line of basic rack

A_E starting point of meshing

h_{FaP0} straight part of tip flank of tool-generating profile

T contact point between generating line of action and base circle of gear

C_0 pitch point of the generating rack

E_E end point of meshing

NOTE 1 Figure is for meshing of the involute transverse profile of left flank of cylindrical gear during generation with straight flank part of tooth flank of basic generating profile.

NOTE 2 Figure 56 also covers the possible case of different pressure angles α_{P0} and α_P at the tool and the cylindrical gear basic rack tooth profile, for example, with single-tooth and single-flank tools (e.g. part generating grinding). The generating gear then has the normal pressure angle, α_{wt0}, and the generating pitch circle diameter $d_{wE} = d_b / \cos\alpha_{wt0}$ instead of α_t and d in the case of $\alpha_{P0} = \alpha_P$.

Figure 56 — Meshing of cylindrical gear during generation

In the case of tooth systems which are finish-machined using the generating method and tools whose cutter tips lie parallel to the datum line (hob, rack-shaped cutter) and have no undercut or pre-finish root relief, the following applies to an external gear:

$$d_{\mathrm{Ff}} = \sqrt{\left\{ d \sin\alpha_{\mathrm{t}} - \frac{2[h_{\mathrm{aP0}} - x_{\mathrm{E}} m_{\mathrm{n}} - \rho_{\mathrm{aP0}}(1-\sin\alpha_{\mathrm{n}})]}{\sin\alpha_{\mathrm{t}}} \right\}^2 + d_{\mathrm{b}}^2} = \sqrt{\left\{ d - 2[h_{\mathrm{ap0}} - x_{\mathrm{E}} m_{\mathrm{n}} - \rho_{\mathrm{aP0}}(1-\sin\alpha_{\mathrm{n}})] \right\}^2 + 4[h_{\mathrm{aP0}} - x_{\mathrm{E}} m_{\mathrm{n}} - \rho_{\mathrm{aP0}}(1-\sin\alpha_{\mathrm{n}})]^2 \cot^2\alpha_{\mathrm{t}}} \quad (256)$$

or, using the roll angle $\tan\alpha_{\mathrm{Ff}} = \xi_{\mathrm{Ff}}$, the following is produced:

$$d_{\mathrm{Ff}} = \frac{d_{\mathrm{b}}}{\cos\alpha_{\mathrm{Ff}}} \quad (257)$$

where α_{Ff} follows from:

$$\tan\alpha_{\mathrm{Ff}} = \xi_{\mathrm{Ff}} = \xi_{\mathrm{t}} - \frac{4\left[h_{\mathrm{aP0}} - \rho_{\mathrm{aP0}}(1-\sin\alpha_{\mathrm{n}})/m_{\mathrm{n}} - x_{\mathrm{E}}\right]\cos\beta}{z \sin 2\alpha_{\mathrm{t}}} \quad (258)$$

NOTE Formula (258) uses both transverse and normal pressure angle.

In the case of external and internal gears, which are generated by means of the generating method using a pinion-type cutter (number of teeth z_0, base diameter d_{b0}, tip form diameter d_{Fa0}, generating centre distance a_{w0}, and generating working transverse pressure angle α_{wt0}) and have no undercut or pre-finish root relief, the following applies:

$$d_{\mathrm{Ff}} = \frac{z}{|z|}\sqrt{\left(2a_{\mathrm{w0}}\sin\alpha_{\mathrm{wt0}} - \sqrt{d_{\mathrm{Fa0}}^2 - d_{\mathrm{b0}}^2}\right)^2 + d_{\mathrm{b}}^2} \quad (259)$$

or, using the roll angle $\tan\alpha_{\mathrm{Ff}} = \xi_{\mathrm{Ff}}$:

$$d_{\mathrm{Ff}} = \frac{d_{\mathrm{b}}}{\cos\alpha_{\mathrm{Ff}}} \quad (260)$$

with

$$\xi_{\mathrm{Ff}} = \frac{z_0}{z}(\xi_{\mathrm{wt0}} - \xi_{\mathrm{Fa0}}) + \xi_{\mathrm{wt0}} \quad (261)$$

$$\xi_{\mathrm{Fa0}} = \tan\left(\arccos\frac{d_{\mathrm{b0}}}{d_{\mathrm{Fa0}}}\right) \quad (262)$$

In the case of gears with undercut, the root form diameter arises from the intersection between the involute part of the tooth flank and the tooth root fillet.

9.8 Undercut

Undercut is the removal of material in the dedendum flank on external cylindrical gears. Undercutting occurs when the relative path of the tool tooth tip corner rounding cuts into the involute portion of the tooth flank during the rolling action in the generating gear pair. This undercutting can be avoided or minimized by positive profile shift.

For a cylindrical gear produced using a non-protuberance rack-shaped cutter or hob, the minimum value of the generating profile shift coefficient for zero-undercut teeth arises from:

$$x_{\mathrm{Eu}} = \frac{h_{\mathrm{FaP0}}}{m_{\mathrm{n}}} - \frac{z \sin^2 \alpha_{\mathrm{t}}}{2 \cos \beta} \tag{263}$$

When using a pinion-type cutter, x_{Eu} arises from the mating conditions of the generating gear pair.

9.9 Overcut

Overcut is tip-to-tip interference on an internal gear wheel at generation with a pinion-type cutter. It results in the removal of material in the addendum flank of an internal gear. Overcutting occurs when the relative path of the tool tooth tip corner rounding cuts into the involute portion of the tooth flank near the tooth tip during the rolling action in the generating gear pair.

At generation this interference can result from relative rotating motion between the pinion-type cutter and the generated internal gear, but also with the radial stroke motion of the pinion-cutter. Analogy with interference phenomena described in 5.5.8.3 for an internal gear pair can be used to evaluate the risk of this interference.

9.10 Minimum tooth thickness at the tip circle of a gear

In calculating minimum tooth thickness at the tip circle, the upper limit (not theoretical) tip diameter shall be used, and tip chamfering should be accounted for. Minimum tooth thickness should be limited based on material, its heat treatment and the application.

Formula (43) can be used to calculate the unmodified transverse tooth thickness at the tip.

The minimum tooth thickness, as well as the undercut, sets a lower limit on the number of teeth that it is practicable to cut in an external gear.

10 Start of involute for hob or rack type cutters

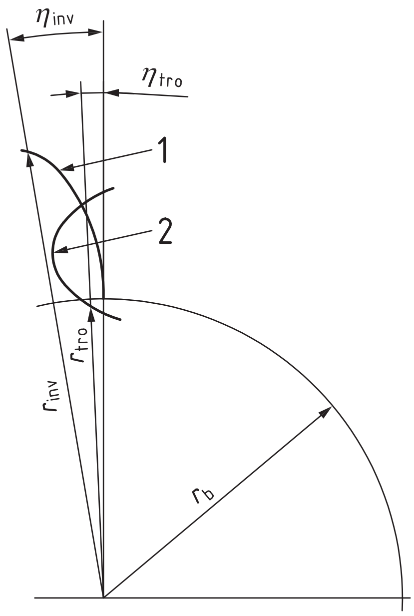
10.1 Formulae of involute and trochoid

This clause applies to machined gears except those that are generated with a pinion-type cutter, also known as a shaper cutter (see Clause 11). It accounts for backlash when the generating profile shift coefficient x_{E} is used.

The tool tooth tip corner radius is a circular radius in a normal plane of the rack tool profile (see Figures 50 and 51).

To consider machining allowance at roughing for fillet, root surface and chamfer/tip rounding, see 9.3. At finishing, the calculation of root and form diameters of the finished nominal involute shall be considered with a machining allowance equal to zero.

For the determination of the start of involute, a polar coordinate system is used (see Figure 57).



Key

1 involute

2 trochoid

NOTE The use of η here is not related to space width half angle.

Figure 57 — Polar coordinate system for involute and trochoid

The normal rack tool profile is defined in a normal plane, see Figure 47. The normal pressure angle of the gear is always equal to the normal pressure of the normal rack tool profile $\alpha_n = \alpha_{P0}$ as well for the transverse pressure angle $\alpha_t = \alpha_{t0}$.

In the transverse plane, the projection of the tooth tip of the rack tool profile is:

— a circle for a spur gear;

— an ellipse for a helical gear (see Figure 58).

To stay general, a helical gear is used in the following developments.

The pitch line of the basic rack is defined by the line parallel to the datum line of the basic rack tangent to the reference diameter of the generated gear.

According to the law of kinematics in movement planar, for envelope curves the common normal between the profile and its generator always crosses the pitch point.

So, at any point Q of the fillet, the common normal can be determined by angle λ, which is the angle between the pitch line of the basic rack and the line joining the generated point Q and the pitch point P (see Figure 59).

For rack-type generation, the formulae of the involute and the trochoid, taking into account the elliptical tooth tip of the rack and the tool protuberance, are:

$$r_{inv} = \frac{d_y}{2} = \frac{d_b}{2}\sqrt{1+\xi_y^2} = \frac{d_b}{2}\sqrt{1+\tan^2\alpha_{yt}} \tag{264}$$

$$\eta_{\text{inv}} = \xi_{\text{y}} - \arctan \xi_{\text{y}} \tag{265}$$

where ξ_y is the intermediate parameter that is equal to $\tan \alpha_{yt}$ ($r_{inv} > r_b$), see Figure 8.

$$r_{\text{tro}}(\varphi) = \sqrt{\left(\frac{d}{2} - B(\varphi)\right)^2 + \left(B(\varphi)\frac{\cos\beta}{\tan\varphi}\right)^2} \tag{266}$$

$$\eta_{\text{tro}}(\varphi) = \theta(\varphi) + \varepsilon(\varphi) - \alpha_{\text{t}} \tag{267}$$

where

$$B(\varphi) = h_{\text{aP0}} - x\, m_{\text{n}} - \rho_{\text{aP0}} + \rho_{\text{aP0}} \sin\varphi \tag{268}$$

$$\theta(\varphi) = \tan\alpha_{\text{t}} + \frac{2}{d}\left(\rho_{\text{aP0}}\frac{\cos\varphi}{\cos\beta} - A - B(\varphi)\frac{\cos\beta}{\tan\varphi}\right) \tag{269}$$

$$A = \frac{\rho_{\text{aP0}} - s_{\text{pr}}}{\cos\alpha_{\text{n}}\ \cos\beta} + (h_{\text{aP0}} - x\, m_{\text{n}} - \rho_{\text{aP0}})\tan\alpha_{\text{t}} \tag{270}$$

$$s_{\text{pr}} = pr - q \tag{271}$$

$$\tan\varepsilon(\varphi) = \frac{B(\varphi)\cos\beta}{\left(\frac{d}{2} - B(\varphi)\right)\tan\varphi} \tag{272}$$

The trochoid/tip parameter φ is shown in Figure 58.

Each point of the trochoid can be determined with Formulae (266) and (267) in polar coordinate with polar axis crossing the start of the involute profile on the base circle, with parameter φ between α_n and $\pi/2$. For a protuberance tool, angle φ is between the protuberance pressure angle α_{pr} and $\pi/2$.

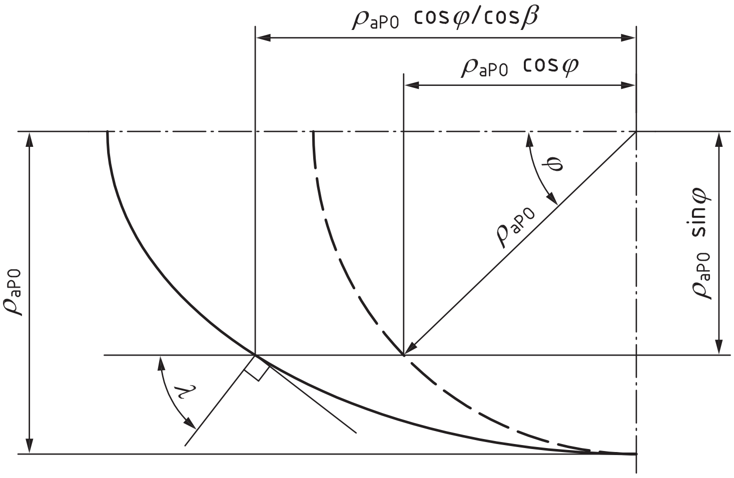
NOTE 1 To draw a trochoid with polar axis on the axis of symmetry of the tooth, an offset angle equal to the base transverse tooth thickness half angle ψ_b is added.

NOTE 2 To draw a trochoid with polar axis on the axis of symmetry of a tooth space, an offset angle equal to the base transverse space width half angle η_b is added.

10.2 Undercut conditions

For hob or rack type cutters, undercut exists if:

$$\frac{d}{2}\sin^2\alpha_{\text{t}} - [h_{\text{aP0}} - x\, m_{\text{n}} - \rho_{\text{aP0}}(1 - \sin\alpha_{\text{n}})] < 0 \tag{273}$$



NOTE The dashed line corresponds to the tooth tip corner radius in the normal plane of the rack tool profile.

The continuous line corresponds to the projected tool tooth tip corner radius in the transverse plane of the gear.

Figure 58 — Trochoid/tip parameter φ

10.3 Determination of start of involute

If undercut does not exist, the diameter to the point of start of involute is defined by the root form diameter d_{Ff}, with radius $r_{Ff} = d_{Ff}/2$ and can be calculated with:

$$r_{Ff} = r_{tro,(\varphi=\alpha_n)}$$

$$= \sqrt{\left[\frac{d}{2} - \left(h_{aP0} - x\,m_n - \rho_{aP0}\left(1 - \sin\alpha_n\right)\right)\right]^2 + \left[\frac{h_{aP0} - x\,m_n - \rho_{aP0}\left(1 - \sin\alpha_n\right)}{\tan\alpha_t}\right]^2} \quad (274)$$

If undercut does exist, the point of start of involute is located at the intersection of the involute and the trochoid, so that two conditions should be verified using polar coordinates:

$$r_{tro} = r_{inv} \quad (275)$$

$$\eta_{tro} = \eta_{inv} \quad (276)$$

Formula (275) allows ξ to be expressed as a function of φ :

$$\xi = \xi(\varphi) = \sqrt{\left(\frac{\frac{d}{2} - B(\varphi)}{\frac{d_b}{2}}\right)^2 + \left(\frac{B(\varphi)\cos\beta}{\frac{d_b}{2\tan\varphi}}\right)^2 - 1} \quad (277)$$

Formula (276) can be written as:

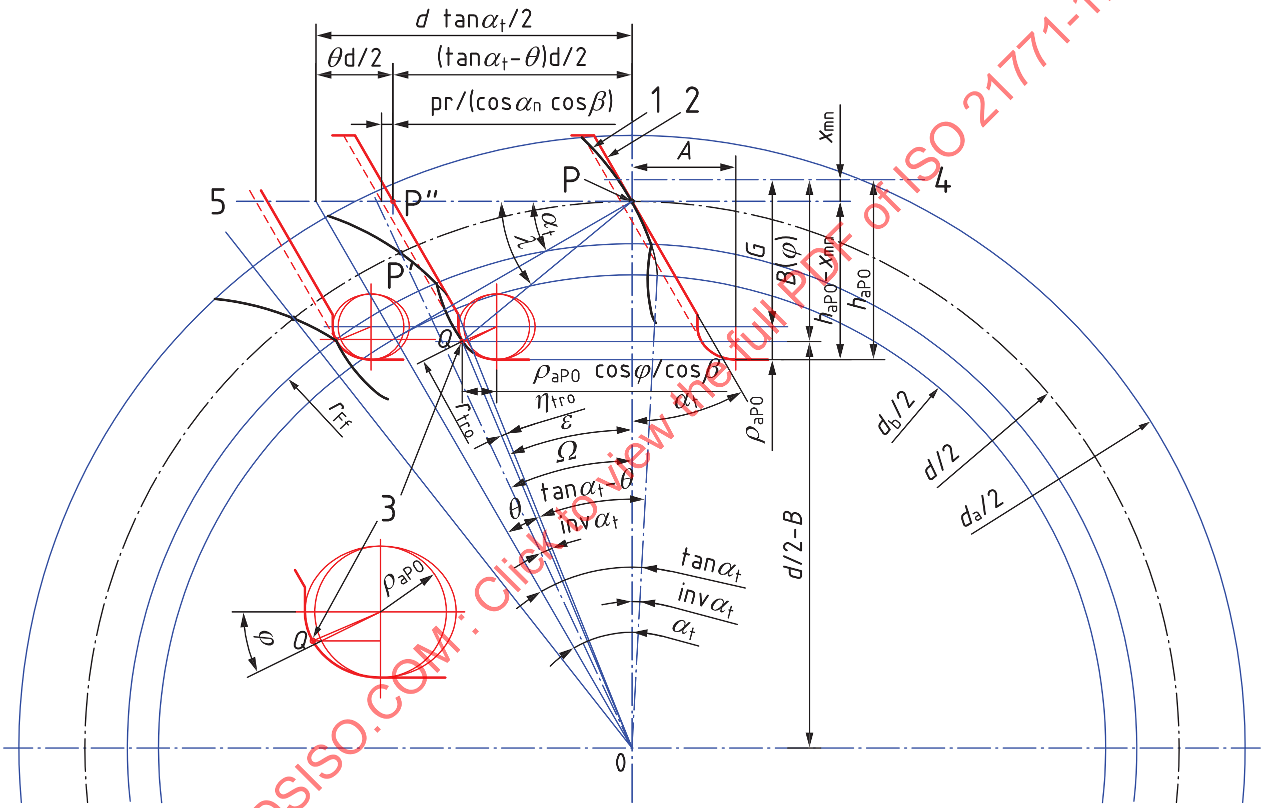
$$\theta(\varphi)+\varepsilon(\varphi)-\xi(\varphi)+\arctan\xi(\varphi)-\alpha_t = f(\varphi)=0 \quad (278)$$

Consequently, the root of $f(\varphi)$ is the value of φ at the intersection point, which allows the calculation of r_{soi} and η_{soi} according to Formulae (264) to (267).

$$r_{Ff} = r_{tro\,(\varphi)} \quad (279)$$

$f(\varphi)$ can have two solutions in between the interval $0 < \varphi < \pi/2$. However, the point of start of involute is defined by the root giving the higher value of r_{tro}, which is always the smaller root, as derived from Figure 60. The correct value of φ_{soi} can be obtained by solving Formula (278) with an iteration method, such as by Newton-Raphson, starting from a small value for φ.

See Figure 59 for a diagram of trochoid generation in the transverse plane.



Key

1 involute line

2 rack tool tooth flank

3 trochoid point

4 datum line of basic rack

5 cutting pitch line

A Position of the centre of elliptical tooth tip corner along datum line of the basic rack

G Distance between datum line to centre of tooth tip corner rounding of generating rack

P Pitch point at generation (on reference diameter of the gear)

P' Pitch point after gear rotated by angle Ω (generating point Q in the fillet)

P'' Pitch point on the generating rack after machined gear rotated by angle Ω

Q Generated point of the fillet in the relative position gear/cutting rack

Ω Angular rotation of the gear to define the relative position gear/cutting rack at Q generation

Figure 59 — Trochoid generation in transverse plane

10.4 Determination of radius of curvature of trochoid

Each point of a trochoid is generated by the envelop of the tooth tip corner radius of the rack tool profile. In the transverse plane of the gear, the tooth tip corner radius of the rack tool profile is either a portion of circle for a spur gear or a portion of an ellipse for a helical gear as shown in Figure 58.

The local radius of curvature of the ellipse (or circle), generates the local curvature of the trochoid for each relative position of the gear versus the rack tool profile.

In the general case for a helical gear, the tooth tip elliptic profile of the rack tool profile is defined according to its centre by:

$$\frac{x^2}{r_{ea}^2}+\frac{y^2}{r_{eb}^2}=1 \tag{280}$$

with the two axes of the ellipse given by:

$$r_{ea}=\rho_{aP0}\frac{\cos\phi}{\cos\beta} \tag{281}$$

$$r_{eb}=\rho_{aP0} \tag{282}$$

In any given point Y_x(x, y) of the ellipse, the radius of curvature R_c is defined by:

$$R_c=r_{ea}^2\,r_{eb}^2\left(\frac{x^2}{r_{ea}^4}+\frac{y^2}{r_{eb}^4}\right)^{3/4} \tag{283}$$

with

$$x_{tro}=r_{tro}\sin\varepsilon+A-\frac{d}{2}(\tan\alpha_t-\theta) \tag{284}$$

$$y_{tro}=r_{tro}\cos\varepsilon-\left(\frac{d}{2}-B(\varphi)+\rho_{fP}\sin\phi\right) \tag{285}$$

See Figure 59, then:

$$R_{fP}=r_{ea}^2\,r_{eb}^2\left(\frac{x_{tro}^2}{r_{ea}^4}+\frac{y_{tro}^2}{r_{eb}^4}\right)^{3/4} \tag{286}$$

To calculate the radius of curvature in the fillet with the Euler-Savary method given on Figure 60:

— the pitch point P between the cutting pitch line of the basic rack and reference diameter of the gear when generation cutting is also the cutting pitch diameter of the gear (on the instantaneous line of action);

— the generated point of the fillet Q , the point of contact between the ellipse shape of the rack tool profile and the trochoid, PQ , is the common normal to both curves;

— the radius of curvature of the basic rack profile R_{fp} at point Q , with the centre of curvature in C_{fp} ;

— the point N is at the intersection of a line perpendicular to the line of action, and a line perpendicular to the pitch line;

— then the centre of curvature of the generated trochoid/fillet at point Q is determined in $C_{tro\text{-}y}$ by the intersection of line N crossing the centre of the gear O ;

— and the radius of curvature of the trochoid/fillet $R_{tro\text{-}y}$ is the distance between Q and $C_{tro\text{-}y}$.

From Figure 60:

$$\frac{\overline{\mathrm{PC}_{\text{tro-y}}}}{\overline{\mathrm{KO}}}=\frac{\overline{\mathrm{C}_{\text{tro-y}}\mathrm{C}_{\text{fP}}}}{\overline{\mathrm{OJ}}} \tag{287}$$

with

$$\overline{\mathrm{OJ}}=\overline{\mathrm{PC}_{\text{fP}}}=\overline{\mathrm{PQ}}-R_{\text{fP}} \tag{288}$$

$$\overline{\mathrm{PC}_{\text{tro-y}}}=\overline{\mathrm{PQ}}-R_{\text{tro-y}} \tag{289}$$

$$\overline{\mathrm{C}_{\text{tro-y}}\mathrm{C}_{\text{fP}}}=R_{\text{tro-y}}-R_{\text{fP}} \tag{290}$$

then

$$\frac{\overline{\mathrm{PQ}}-R_{\text{tro-y}}}{\overline{\mathrm{KO}}}=\frac{R_{\text{tro-y}}-R_{\text{fP}}}{\overline{\mathrm{PQ}}-R_{\text{fP}}} \tag{291}$$

$$\left(\overline{\mathrm{PQ}}-R_{\text{tro-y}}\right)\left(\overline{\mathrm{PQ}}-R_{\text{fP}}\right)=\overline{\mathrm{KO}}\left(R_{\text{tro-y}}-R_{\text{fP}}\right) \tag{292}$$

So, the radius of the fillet at point Q is:

$$R_{\text{tro-y}}=\frac{\overline{\mathrm{PQ}}^{2}-R_{\text{fP}}\cdot\left(\overline{\mathrm{PQ}}+\overline{\mathrm{KO}}\right)}{\overline{\overline{\mathrm{PQ}}+\overline{\mathrm{KO}}-R_{\text{fP}}}} \tag{293}$$

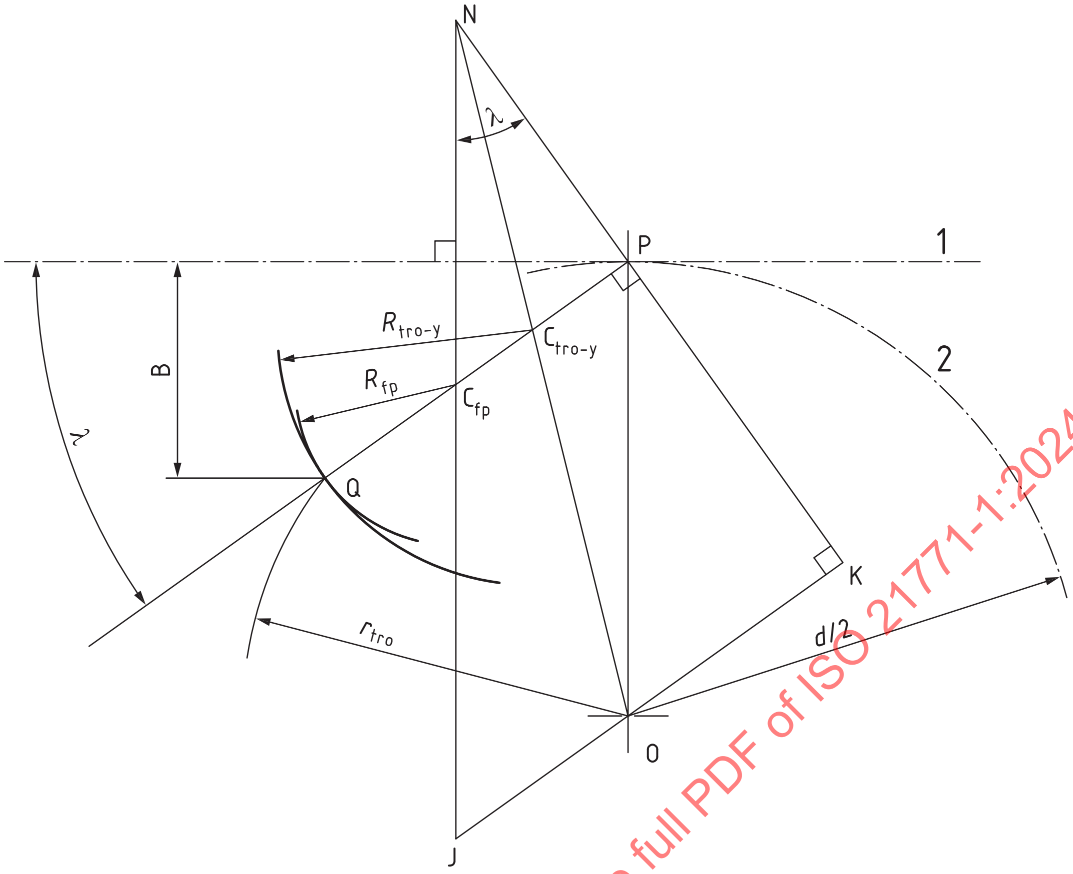
with

$$\overline{\mathrm{PQ}}=\frac{B}{\sin\lambda} \tag{294}$$

$$\overline{\mathrm{KO}}=\frac{d}{2}\sin\lambda \tag{295}$$

then

$$R_{\text{tro-y}}=\frac{\left(\dfrac{B}{\sin\lambda}\right)^{2}-R_{\text{fP}}\left(\dfrac{B}{\sin\lambda}+\dfrac{d}{2}\sin\lambda\right)}{\dfrac{B}{\sin\lambda}+\dfrac{d}{2}\sin\lambda-R_{\text{fP}}}=\frac{2B^{2}-R_{\text{fP}}\cdot\sin\lambda\left(2B+d\sin^{2}\lambda\right)}{\sin\lambda\left(2B+d\sin^{2}\lambda-2R_{\text{fP}}\sin\lambda\right)} \tag{296}$$



Key

1 cutting pitch line

2 reference diameter of the gear

B radial position of the generated point Q according to cutting pitch line

C_{fp} centre of curvature of the elliptical tooth tip corner of the cutting rack at point Q

C_{tro-y} centre of curvature of the fillet at point Q

J point of intersection between the normal to the cutting pitch line crossing C_{fp} and the line parallel to PQ

K point of intersection between the normal to PK and line ON

O centre of the generated gear

P pitch point at generation (is on reference diameter)

Q generated point of the fillet in the relative position gear/cutting rack (see Figure 58)

λ angle between the cutting pitch line and the normal to the fillet at point QB

Figure 60 — Euler -Savary geometric construction at fillet for rack type cutter

11 Start of involute calculation in case of pinion-type cutter

11.1 Geometry of pinion-type cutter

11.1.1 General

A pinion-type cutter is defined as a spur or helical pinion (see Clause 4) with the following parameters:

— z_0 number of teeth,

— m_0 normal module equal to normal module of the gear to cut, m_n

— α_{P0} normal pressure angle equal to normal pressure angle of the gear to cut, α_n

— x_0 profile shift coefficient,

— β_0 helix angle,

— h_{aP0} addendum.

And with additional parameters for the tip of the cutter:

— ρ_{aP0} tip radius defined in normal plane,

— α_{pr0} normal protuberance pressure angle (when it exists),

— pr protuberance defined normal to the profile in normal plane (when it exists).

Based on these data, Clause 4 is applicable to determine:

— d_0 reference diameter,

— d_{b0} base diameter,

— d_{a0} tip diameter.

The generated gear that is produced with a pinion-cutter is defined by:

— z number of teeth,

— x profile shift coefficient according to the tooth thickness of the gear (including machining allowance q at roughing to determine fillet form – see 9.2 and 9.3),

— q machining allowance per flank.

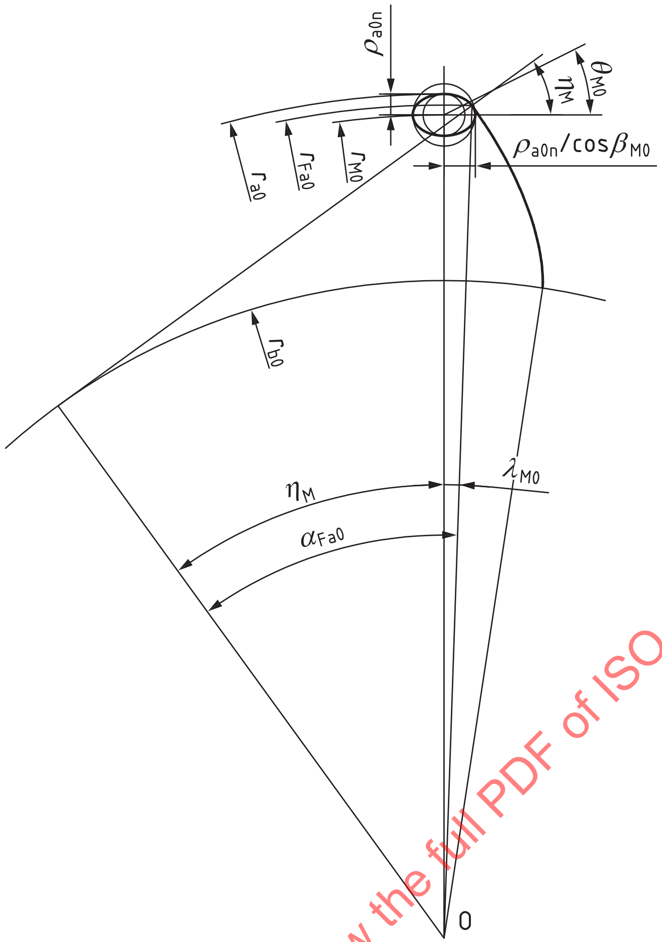


Figure 61 — Tip form of pinion-type cutter in transverse plane

11.1.2 Pinon-type cutter without protuberance

11.1.2.1 Tip form radius

A transverse plane section through a pinion-type cutter without protuberance is shown in Figure 61.

Tip form radius of the pinion-type cutter r_{Fa0} is the radius of the point on the pinion-type cutter where the involute line and the tip profile (ellipse) line of the pinion-type cutter contact on the transverse plane.

The helix angle of the cutter on the circle of radius r_{M0} (centre of the tip corner rounding) is calculated as follows.

$$\tan\beta_{M0} = \frac{r_{M0}}{r_0}\tan\beta \tag{297}$$

$$r_{M0} = r_{a0} - \rho_{a0n} \tag{298}$$

The tip form radius of the pinion-type cutter, r_{Fa0}, is:

$$r_{Fa0} = \sqrt{\left(r_{M0} + \rho_{a0n}\sin\theta_{M0}\right)^2 + \left(\frac{\rho_{a0n}}{\cos\beta_{M0}}\cos\theta_{M0}\right)^2} \tag{299}$$

$$\sin\lambda_{M0} = \frac{\rho_{a0n}}{\cos\beta_{M0}} \frac{\cos\theta_{M0}}{r_{Fa0}} \quad (300)$$

$$\cos\alpha_{Fa0} = \frac{r_{b0}}{r_{Fa0}} \quad (301)$$

$$\eta_M = \alpha_{Fa0} - \lambda_{M0} \quad (302)$$

$$\tan\eta_{M1} = \frac{\tan\theta_{M0}}{\cos\beta_{M0}} \quad (303)$$

$$f(\theta_M) = \eta_M - \eta_{M1} \quad (304)$$

It is possible to establish the value of r_{Fa0} and λ_{M0} by calculating the function $f(\theta_{M0}) = 0$ using iteration with the variable θ_{M0}.

11.1.2.2 Tooth tip corner profile

Tooth tip corner profile (ellipse) is shown in Figure 62. For more information, see B.3.3.

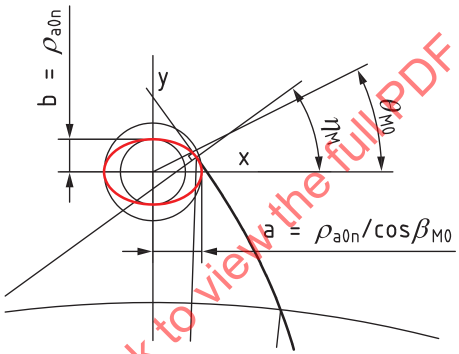


Figure 62 — Tooth tip corner profile of pinion-type cutter in transverse plane

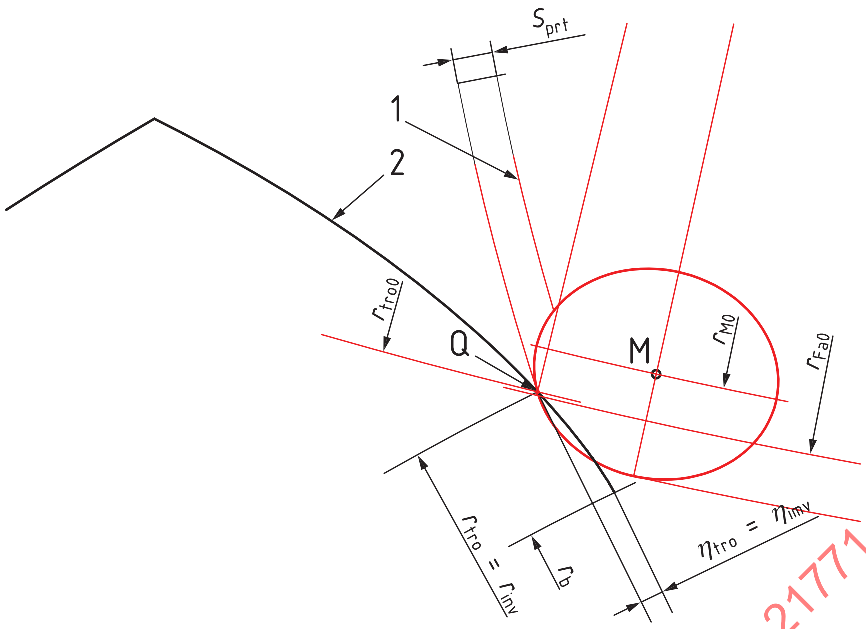
11.1.3 Pinon-type cutter with protuberance

11.1.3.1 Virtual tip form radius

A transverse plane section through a pinion-type cutter with protuberance is shown in Figure B.2.

11.1.3.2 Tooth tip corner profile

Tooth tip corner profile with protuberance is shown in Figures 63 and 64.

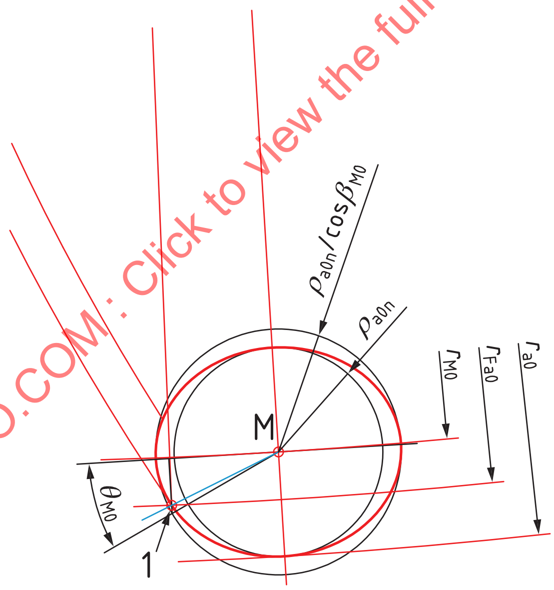


Key

1 cutter tooth flank

2 gear tooth flank

Figure 63 — Detail of the tip of pinion-type cutter



Key

1 tip form point

Figure 64 — Detail of the tip form point of pinion-type cutter in transverse plane

11.2 Undercut condition

To avoid undercut at the root of the machined gear, the rounded tooth tip of the pinion-type cutter shall not extend beyond the point defined at the tangent line of action during cutting with the base circle of the generated gear. ($\overline{TM} < a_{w0} \sin\alpha_{wt0}$ see Figure B.1 in Annex B.)

Undercut exists if:

$$a_{w0}\sin\alpha_{wt0}-\sqrt{r_{Fa0}^{2}-r_{b0}^{2}}<0 \quad (305)$$

The centre distance in the generating gear pair, a_{w0}, is:

$$a_{w0}=\frac{m_n}{\cos\beta}\frac{z_0+z}{2}\frac{\cos\alpha_t}{\cos\alpha_{wt0}} \quad (306)$$

The working transverse pressure angle in the generating gear pair, α_{wt0}, is:

$$\operatorname{inv}\alpha_{wt0}=2\frac{x_0+x}{z_0+z}\tan\alpha_n+\operatorname{inv}\alpha_t \quad (307)$$

NOTE The inverse involute function can be used to solve for the working pressure angle, α_{wt0}. One method for calculating the inverse involute is given in ISO 21771-2:—, Annex D.

The base radius of the pinion-type cutter, r_{b0}, is:

$$r_{b0}=\frac{m_n z_0}{2\cos\beta}\cos\alpha_t \quad (308)$$

11.3 Determination of start of involute when no undercut exists

If there is no undercut, the root form radius, r_{Ff}, is:

$$r_{Ff}=\sqrt{\left(a_{w0}\sin\alpha_{wt0}-\sqrt{r_{Fa0}^{2}-r_{b0}^{2}}\right)^{2}+r_b^{2}} \quad (309)$$

NOTE When the pinion-type cutter has no protuberance, r_{Fa0}, can be calculated with Formula (299), but when there is protuberance, see Figure B.2 in Annex B.

11.4 Start of involute when there is undercut

11.4.1 Residual fillet undercut (transverse plane)

$$s_{prt}=\frac{s_{pr}}{\cos\beta} \quad (310)$$

$$\delta_{pr0}=\frac{s_{prt}}{r_{b0}} \quad (311)$$

11.4.2 Angle centre line of gear pair to centre point of tooth tip rounding:

The angle between the centre line of a gear pair and the centre point of tooth tip rounding of pinion-type cutter is:

$$\omega_0=\omega_{F0}+\lambda_{M0} \quad (312)$$

where:

$$\omega_{F0}=\operatorname{inv}\alpha_{Fa0}-\left(\operatorname{inv}\alpha_{wt0}+\delta_{pr0}\right) \quad (313)$$

11.4.3 Point Q (tangent point of cutter tip fillet and generated gear root fillet)

See Figure 65 and Figure 66.

$$\theta_0 = \phi_0 - \omega_0 \tag{314}$$

$$\tan\lambda_0 = \frac{\tan\theta_{\mathrm{Mt}}}{\cos\beta_{\mathrm{M0}}} \tag{315}$$

$$\tan\theta_{\mathrm{Mt}} = \cos\beta_{\mathrm{M0}} \frac{r_{\mathrm{M0}} - r_{\mathrm{w0}}\cos\theta_0 + \rho_{\mathrm{a0n}}\sin\theta_{\mathrm{Mt}}}{r_{\mathrm{w0}}\sin\theta_0 + \dfrac{\rho_{\mathrm{a0n}}\cos\theta_{\mathrm{Mt}}}{\cos\beta_{\mathrm{M0}}}} \tag{316}$$

NOTE θ_{Mt} is calculated by iteration.

$$\tan\lambda_0 = \frac{r_{\mathrm{M0}} - r_{\mathrm{w0}}\cos\theta_0 + \rho_{\mathrm{a0n}}\sin\theta_{\mathrm{Mt}}}{r_{\mathrm{w0}}\sin\theta_0 + \dfrac{\rho_{\mathrm{a0n}}\cos\theta_{\mathrm{Mt}}}{\cos\beta_{\mathrm{M0}}}} \tag{317}$$

$$L_0 = \sqrt{\left(r_{\mathrm{M0}} - r_{\mathrm{w0}}\cos\theta_0 + \rho_{\mathrm{a0n}}\sin\theta_{\mathrm{Mt}}\right)^2 + \left(r_{\mathrm{w0}}\sin\theta_0 + \frac{\rho_{\mathrm{a0n}}\cos\theta_{\mathrm{Mt}}}{\cos\beta_{\mathrm{M0}}}\right)^2} \tag{318}$$

$$r_{\mathrm{tr0}} = \sqrt{r_{\mathrm{w}}^2 + L_0^2 - 2 r_{\mathrm{w}} L_0 \sin(\lambda_0 - \theta_0)} \tag{319}$$

$$\varphi = \varphi_0 \frac{z_0}{z} \tag{320}$$

$$\sin\varepsilon = \frac{L_0}{r_{\mathrm{tr0}}}\cos(\lambda_0 - \theta_0) \tag{321}$$

$$\eta_{\mathrm{tr0}} = \varepsilon + \mathrm{inv}\,\alpha_{\mathrm{wt0}} - \varphi \tag{322}$$

11.4.4 Determination of start of involute

11.4.4.1 Condition on involute line

$$r_{\mathrm{inv}} = r_{\mathrm{b}}\sqrt{1+\xi^2} \tag{323}$$

$$\eta_{\mathrm{inv}} = \xi \arctan\xi \tag{324}$$

11.4.4.2 Calculation of intersection of trochoid line and involute line

$$r_{\mathrm{inv}} = r_{\mathrm{tro}} \tag{325}$$

then:

$$\xi = \sqrt{\left(\frac{r_{\mathrm{inv}}}{r_{\mathrm{b}}}\right)^2 - 1} = \sqrt{\left(\frac{r_{\mathrm{tro}}}{r_{\mathrm{b}}}\right)^2 - 1} = \sqrt{\frac{r_{\mathrm{w}}^2 + L_0^2 - 2 r_w L_0 \sin(\lambda_0 - \theta_0)}{r_{\mathrm{b}}^2} - 1} \tag{326}$$

$$\eta_{inv} = \eta_{tro} \tag{327}$$

then:

$$\eta_{inv} - \eta_{tro} = 0 \tag{328}$$

That is,

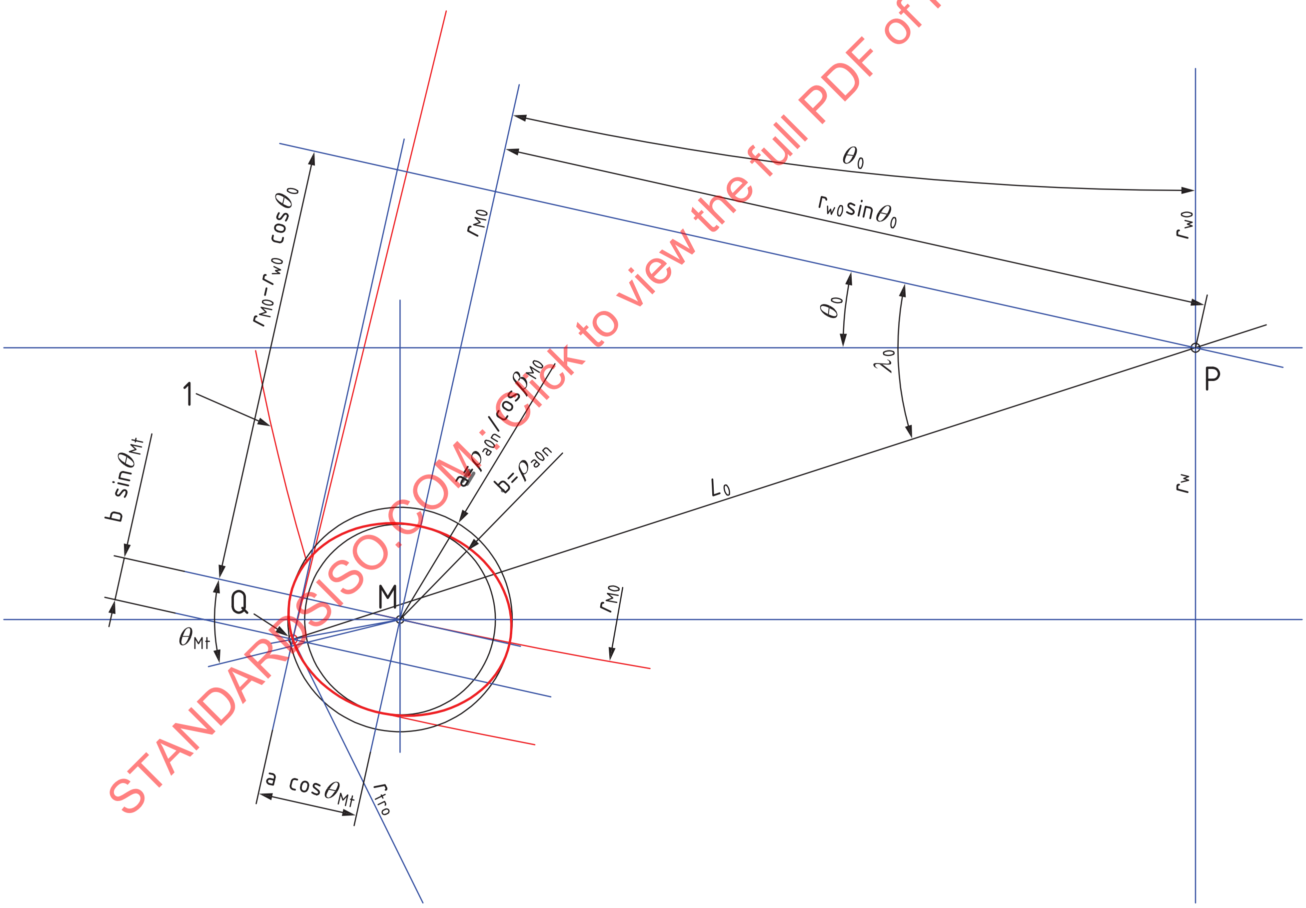
$$f(\varphi_0) = \xi(\varphi_0) - \arctan\xi(\varphi_0) - \varepsilon(\varphi_0) - \mathrm{inv}\,a_{wt0} + \varphi(\varphi_0) = 0 \tag{329}$$

By calculating the function $f(\varphi_0) = 0$ with an iteration method, since φ_0 is a variable, it is possible to find the intersection point of the trochoid line and involute line, that is "the start of involute of gear tooth flank".

11.4.4.3 Diameter of start of involute (root form diameter of generated gear)

$$d_{Ff} = 2r_{tro} \tag{330}$$

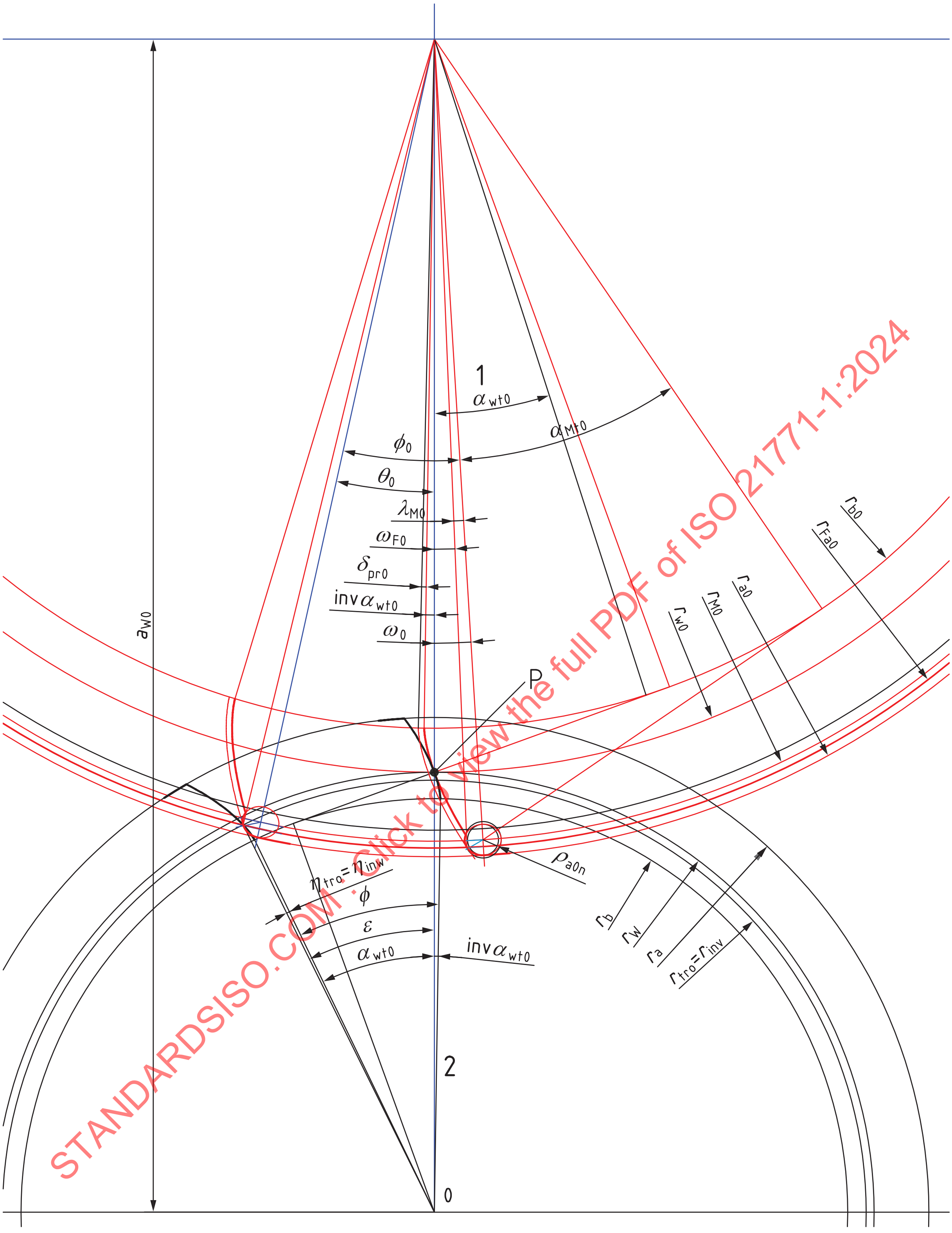
When the pinion-type cutter has protuberance, the root form diameter d_{Ff} is the maximum value of either d_{Ff} calculated by Formula (330) or $2r_{Ff}$ calculated by Formula (309).



Key

1 cutter tooth flank

Figure 65 — Tangent point between pinion-type cutter tooth tip and gear root fillet



Key

1 pinion-type cutter

2 gear

P Pitch point at generation

Q Generated point of the fillet in the relative position gear/ pinion-type cutter

ϕ Angular rotation of the gear to generate point Q

ϕ_0 Angular rotation of the pinion type-cutter to generate point Q

ω_0 Angular position of the centre of elliptical tooth tip corner to radius crossing the cutting pitch point of the pinion-type cutter

Θ_0 Intermediate angle for the calculation

Figure 66 — Trochoid generation in transverse plane

11.4.5 Determination of radius of curvature of trochoid

Each point of a trochoid is generated by the envelop of the tooth tip corner radius of the pinion-type cutter profile. In the transverse plane of the gear, the tooth tip corner radius of the pinion-type cutter profile is either a portion of a circle for a spur gear or a portion of an ellipse for a helical gear, as shown in Figure 61.

The local radius of curvature of the ellipse (or circle), generates the local curvature of the trochoid for each relative position of the gear vs the pinion-type cutter.

In the general case for a helical gear the tooth tip elliptic profile of the pinion-type cutter profile is defined according to its centre and at any given point M(x, y) of the ellipse, the radius of curvature $\overline{\mathrm{RC}_{\mathrm{yft}}}(\theta_{\mathrm{M}})$ is defined by Formula (332).

As the ellipse parameter is angle θ_{M} (see Annex B) then:

$$R_{\mathrm{fP}}(\theta_{\mathrm{M}}) = r_{\mathrm{ea}}^2 \cdot r_{\mathrm{eb}}^2 \left(\frac{[x_{\mathrm{tro}}(\theta_{\mathrm{M}})]^2}{r_{\mathrm{ea}}^4} + \frac{[y_{\mathrm{tro}}(\theta_{\mathrm{M}})]^2}{r_{\mathrm{eb}}^4} \right)^{3/4} \quad (331)$$

The radius of curvature in the fillet $\overline{\mathrm{RC}_{\mathrm{yft}}}$ can be determined by Euler-Savary method (see Figure 67):

— pitch point P between the cutting pitch circle of the pinion-type cutter (radius r_{w0}) and reference diameter of the gear (radius r_{w}) at generation centre distance a_{w0};

— generated point of the fillet M, point of contact between the ellipse shape of the pinion-type cutter profile at the tooth tip and the trochoid, PM, the instantaneous line of action, is the common normal to both profiles (which is always crossing the instantaneous centre of rotation P);

— RC_{ya0} the radius of curvature of the pinion-type cutter profile in point M, with the centre of curvature in $\mathrm{C}_{\mathrm{ya0}}$;

— drawing, at pitch point P, a line perpendicular to the line of action PM, crossing in N, the line between the centre of the pinion-type cutter O_0 and the centre of curvature $\mathrm{C}_{\mathrm{ya0}}$;

— joining point N with the centre of the gear O, the obtained point $\mathrm{C}_{\mathrm{yft}}$ on PM is the centre of curvature of the generated trochoid at point M;

— the radius of curvature of the trochoid/fillet in M is $\overline{\mathrm{RC}_{\mathrm{yft}}}$ which is the distance between M and $\mathrm{C}_{\mathrm{yft}}$.

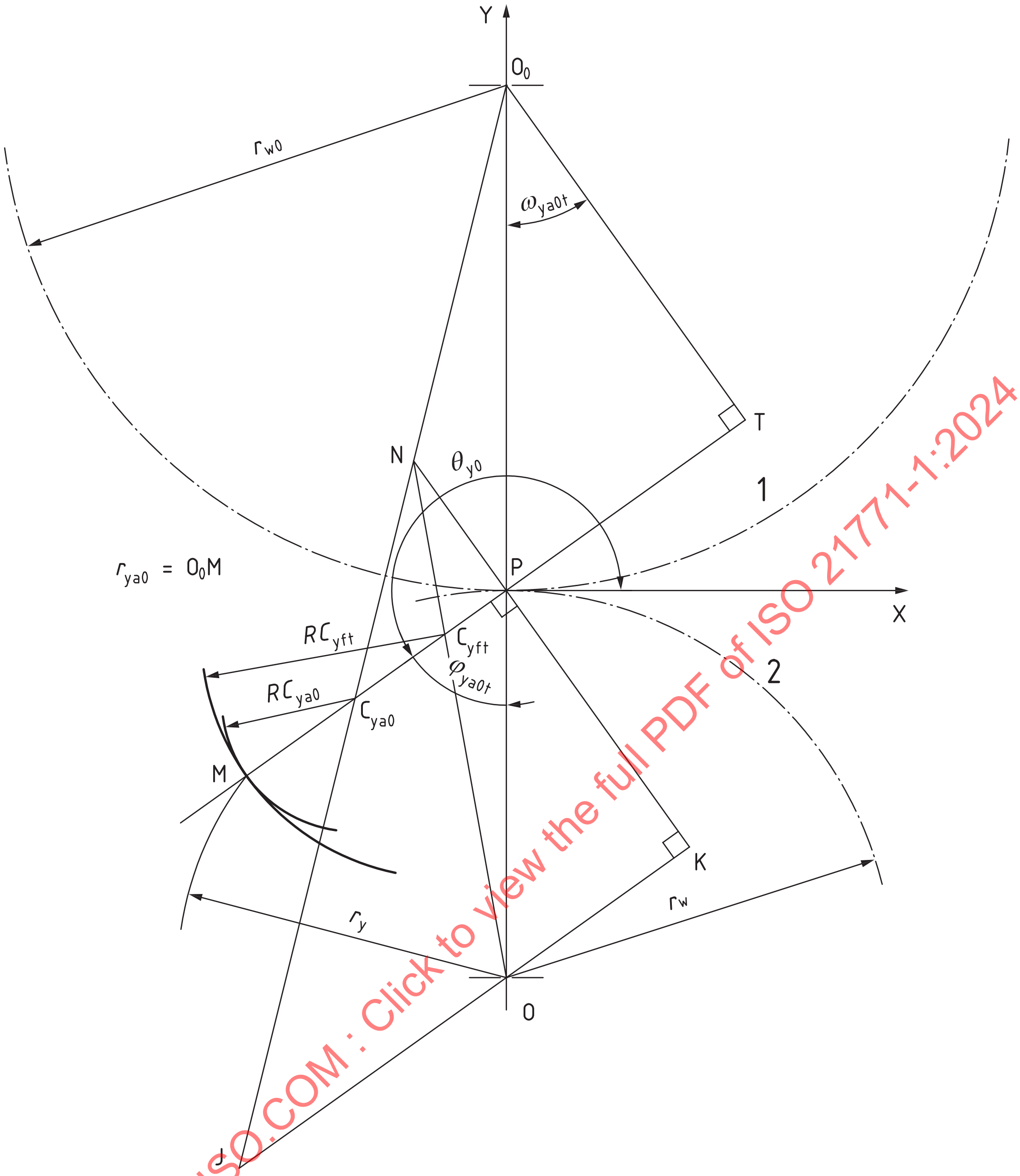
The details to determine the radius of curvature in the fillet $\overline{\mathrm{RC}_{\mathrm{yft}}}$ are given in Clause B.6 in Annex B. The radius of curvature of the trochoid $\overline{\mathrm{RC}_{\mathrm{yft}}} = \overline{\mathrm{C}_{\mathrm{yft}}\mathrm{M}}$ is obtained with:

$$\overline{\mathrm{RC}_{\mathrm{yft}}}(\theta_{\mathrm{M}}) = \overline{\mathrm{PM}} - \frac{1}{\dfrac{1}{\overline{\mathrm{PM}} - RC_{\mathrm{ya0}}(\theta_{\mathrm{M}})} + \left(\dfrac{1}{r_{\mathrm{w}}} + \dfrac{1}{r_{\mathrm{w0}}}\right)\dfrac{1}{\cos\varphi_{\mathrm{ya0t}}(\theta_{\mathrm{M}})}} \quad (332)$$

with:

$$\begin{aligned} \overline{\mathrm{PM}} = \overline{\mathrm{TM}} - \overline{\mathrm{TP}} &= r_{\mathrm{ya0}}(\theta_{\mathrm{M}})\sin\alpha_{\mathrm{ya0t}}(\theta_{\mathrm{M}}) - r_{\mathrm{w0}}\sin\omega_{\mathrm{ya0t}}(\theta_{\mathrm{M}}) \\ &= r_{\mathrm{ya0}}(\theta_{\mathrm{M}})\sin\alpha_{\mathrm{ya0t}}(\theta_{\mathrm{M}}) - r_{\mathrm{w0}}\sin\left(\alpha_{\mathrm{ya0t}}(\theta_{\mathrm{M}}) - \chi_{\mathrm{ya0t}}(\theta_{\mathrm{M}}) - \lambda_{\mathrm{ya0t}}(\theta_{\mathrm{M}})\right) \end{aligned} \quad (333)$$

$$\varphi_{\mathrm{ya0t}}(\theta_{\mathrm{M}}) = \frac{\pi}{2} - \left(\alpha_{\mathrm{ya0t}}(\theta_{\mathrm{M}}) - \chi_{\mathrm{ya0t}}(\theta_{\mathrm{M}}) - \lambda_{\mathrm{ya0t}}(\theta_{\mathrm{M}})\right) \quad (334)$$



Key

1 working pitch circle of pinion-cuter

2 working pitch circle of pinion-cuter of the generated gear

NOTE Several parameters in Figure 67 are function of θ_M as given in Formulae (332) to (334). To simplify in this figure, dependence on θ_M is not represented.

Figure 67 — Euler-Savary geometric construction at fillet for pinion-type cutter

Annex A
(informative)

Form diameters when a circular arc in a transverse plane is used for tooth tip corner radius or tooth root fillet radius

A.1 General

In some manufacturing processes, such as moulded or forged gears, it is possible to create a circular arc between the involute profile and the tip or root cylinder in the transverse plane. When creating a mould by electrical discharge machining (EDM), it is just as easy to use an arc as a trochoid.

The scope of this document does not consider effects that should be considered for EDM or moulded gears such as:

— how the spark from the EDM wire or electrode will affect the cavity shape;

— shrinkage or expansion of the material;

— tolerances.

In case of helical gears, there can be limitations to using EDM wire cutting for the cavity, thus, electrode EDM is used. The limitations should be evaluated as a function of normal module, facewidth and helix angle: These evaluations are out of the scope of this document.

The following assumptions are considered:

— the nominal geometry of the gear tooth;

— the arc of the circle is defined in the transverse plane of the gear;

— the arc is tangent to the involute profile;

— the tooth tip corner radius arc is tangent to the tip diameter, d_a ;

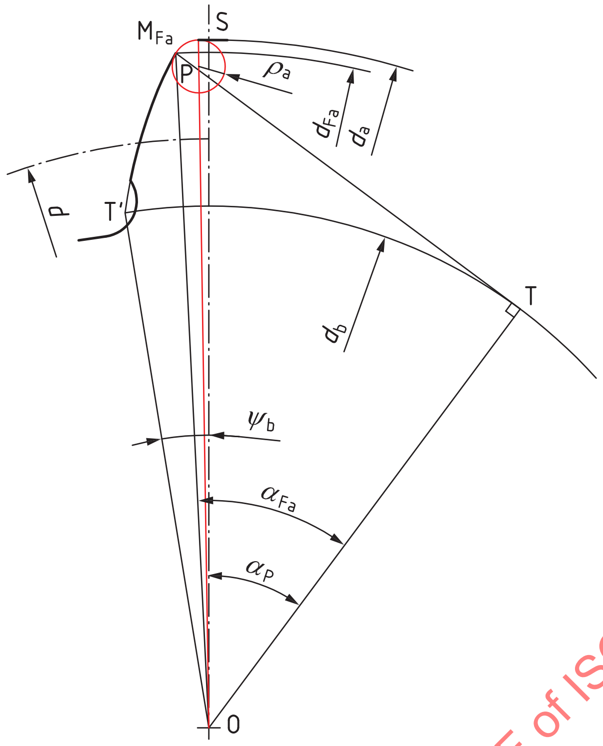
— the tooth root fillet radius arc is tangent to the root diameter, d_f .

In this annex, it is assumed that the clearance between the active tip and active root diameters of the gear pair has already been evaluated. Clearance throughout the meshing cycle should be verified again at the end of the determination of tooth tip corner or tooth root fillet radius.

A.2 External gear calculations

A.2.1 Calculation of tip form diameter d_{Fa} when tooth tip corner radius is given

Given the tip diameter, d_a , and the tooth tip corner radius, ρ_a , as well as the nominal geometry of the gear, the tip form diameter d_{Fa} can be calculated. The tip form diameter is defined by the tangency point between the tooth tip rounding circle and the involute. The tangency point and the circle centre are aligned on a common tangent to the base circle. See Figure A.1.



Key

O centre of the gear

M_{Fa} point of tangency between the involute and the tip form diameter

P centre of the arc between the involute and the tip diameter

S axis of symmetry of the tooth

T point where a line through M_{Fa} is tangent to the base circle

T' starting point of the involute profile on the base cylinder

Figure A.1 — Tooth tip corner radius for external gear

From Figure A.1, it can be written:

$$\overline{OP} = \frac{d_a}{2} - \rho_a \text{ (P is the centre of tooth tip corner circle in the transverse plane)} \quad \text{(A.1)}$$

$$\overline{TP} = \sqrt{\overline{OP}^2 - \left(\frac{d_b}{2}\right)^2} = \sqrt{\left(\frac{d_a}{2} - \rho_a\right)^2 - \left(\frac{d_b}{2}\right)^2} \quad \text{(A.2)}$$

$$\overline{TM_{Fa}} = \overline{TP} + \rho_a = \sqrt{\left(\frac{d_a}{2} - \rho_a\right)^2 - \left(\frac{d_b}{2}\right)^2} + \rho_a \quad \text{(A.3)}$$

$$d_{Fa} = 2\,\overline{OM_{Fa}} = 2\sqrt{\overline{TM_{Fa}}^2 + \left(\frac{d_b}{2}\right)^2} = 2\sqrt{\left[\sqrt{\left(\frac{d_a}{2} - \rho_a\right)^2 - \left(\frac{d_b}{2}\right)^2} + \rho_a\right]^2 + \left(\frac{d_b}{2}\right)^2} \quad \text{(A.4)}$$

$$\cos\alpha_{Fa} = \frac{d_b}{d_{Fa}} \tag{A.5}$$

The condition to avoid a pointed tooth tip is:

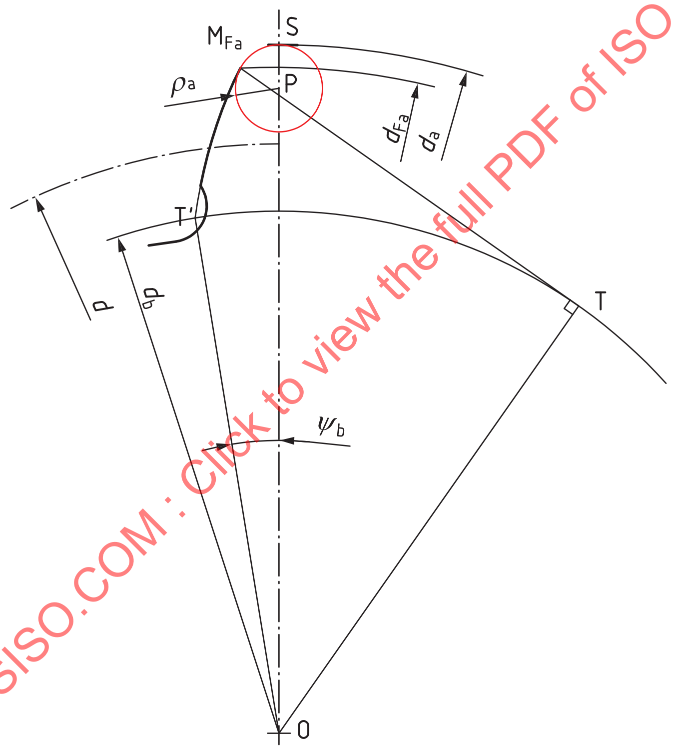
$$\widehat{T'OP} = \widehat{T'OT} + \widehat{TOP} = \frac{\overline{TM_{FA}}}{r_b} - \arctan\frac{\overline{TP}}{r_b} \le \psi_b \tag{A.6}$$

where ψ_b is the base transverse tooth thickness half angle.

A.2.2 Calculation of maximum tooth tip corner radius, ρ_{a_max}

In this calculation, tip diameter, d_a, is known. The maximum tooth tip corner radius, ρ_{a_max} is reached when the point P is on the axis of symmetry of the tooth, i.e. when the angle $\widehat{POS}$ is equal to zero (see Figures A.1 and A.2).

The value of ρ_{a_max} can be determined by iteration of the value of ρ_a, based on the development in A.2.1, up to when the condition $\widehat{T'OP} = \psi_b$ is reached.



Key

O centre of the gear

M_{Fa} point of tangency between the involute and the tip form diameter

P centre of the arc between the involute and the tip diameter

S axis of symmetry of the tooth

T point where a line through M_{Fa} is tangent to the base circle

T' starting point of the involute profile on the base cylinder

NOTE P is located on the axis of symmetry of the tooth.

Figure A.2 — Full tooth tip corner radius for external gear

NOTE A full tooth tip corner radius can significantly decrease the active height of profile and/or it can require low root diameter for the mating gear. Therefore, it is not often used.

A.2.3 Calculation of root form diameter d_{Ff} when tooth root fillet radius is given

It is assumed that the following values are known:

— root form diameter, d_{Ff};

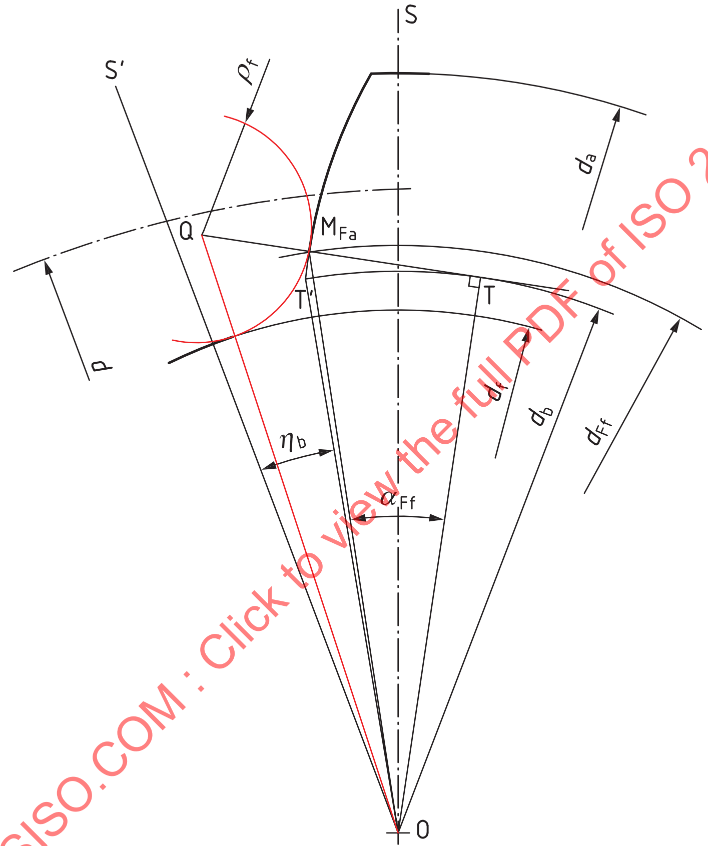
— tooth root fillet radius, ρ_f;

— nominal geometry of the gear.

The root form diameter d_{Ff} at point M_{Ff} is calculated as follows (see Figure A.3).

The centre of tooth root fillet radius Q, M_{Ff} and T are aligned on a common tangent to the base circle.

T' is the starting point of the involute profile on the base cylinder.



Key

O centre of the gear

M_{Ff} point of tangency between the involute and the root form diameter

Q centre of the arc between the involute and the root diameter

S axis of symmetry of the tooth

S' axis of symmetry of the tooth space

T point where a line through M_{Ff} is tangent to the base circle

T' starting point of the involute profile on the base cylinder

Figure A.3 — Tooth root fillet radius for external gear

From Figure A.3, it can be written:

$$\overline{OQ} = \frac{d_f}{2} + \rho_f \text{ (Q is the centre of tooth root fillet circle in the transverse plane)} \quad (A.7)$$

$$\overline{\mathrm{TQ}} = \sqrt{\overline{\mathrm{OQ}}^2 - \left(\frac{d_\mathrm{b}}{2}\right)^2} = \sqrt{\left(\frac{d_\mathrm{f}}{2} + \rho_\mathrm{f}\right)^2 - \left(\frac{d_\mathrm{b}}{2}\right)^2} \tag{A.8}$$

$$\overline{\mathrm{TM}_\mathrm{Ff}} = \overline{\mathrm{TQ}} - \rho_\mathrm{f} = \sqrt{\left(\frac{d_\mathrm{f}}{2} + \rho_\mathrm{f}\right)^2 - \left(\frac{d_\mathrm{b}}{2}\right)^2} - \rho_\mathrm{f} \tag{A.9}$$

$$d_\mathrm{Ff} = 2\,\overline{\mathrm{OM}_\mathrm{Ff}} = 2\sqrt{\overline{\mathrm{TM}_\mathrm{Ff}}^2 + \left(\frac{d_\mathrm{b}}{2}\right)^2} = 2\sqrt{\left[\sqrt{\left(\frac{d_\mathrm{f}}{2} + \rho_\mathrm{f}\right)^2 - \left(\frac{d_\mathrm{b}}{2}\right)^2} - \rho_\mathrm{f}\right]^2 + \left(\frac{d_\mathrm{b}}{2}\right)^2} \tag{A.10}$$

$$\cos\alpha_\mathrm{Ff} = \frac{d_\mathrm{b}}{d_\mathrm{Ff}} \tag{A.11}$$

The condition to avoid a pointed tooth root is:

$$\widehat{\mathrm{QOT}'} = \widehat{\mathrm{QOT}} + \widehat{\mathrm{TOT}'} = \arctan\frac{\overline{\mathrm{TQ}}}{r_\mathrm{b}} - \frac{\overline{\mathrm{TM}_\mathrm{Ff}}}{r_\mathrm{b}} \leq \eta_\mathrm{b} \tag{A.12}$$

where η_b is the base transverse space width half angle.

A.2.4 Calculation of maximum tooth root fillet radius, $\rho_{\mathrm{f\_max}}$

In this calculation, the root diameter, d_f, is known. When the point Q is on the axis of symmetry of a tooth space, i.e. when the angle $\widehat{\mathrm{QOS}}$ is equal to zero, the maximum tooth root fillet radius, $\rho_{\mathrm{f\_max}}$ is reached. See Figures A.3 and A.4.

The value of $\rho_{\mathrm{f\_max}}$ can be determined by iteration of the value of ρ_f, based on the development in A.2.3, up to when the condition $\widehat{\mathrm{QOT}'} = \eta_\mathrm{b}$ is reached.

NOTE The use of a full tooth root fillet radius reduces stress concentration in the fillet. However, a full tooth root fillet radius reduces the gear root diameter, and so enlarges the height of the teeth.

A.3 Internal gear calculations

A.3.1 Calculation of tip form diameter d_Fa when tooth tip corner radius is given

Given the tip diameter, d_a, and the tooth tip corner radius, ρ_a, as well as the nominal geometry of the gear, the tip form diameter d_Fa can be calculated. The tip form diameter d_Fa passes through point M_Fa. Points P, M_Fa and T are aligned on a common normal to the involute profile. See Figure A.4.

From Figure A.4, it can be written:

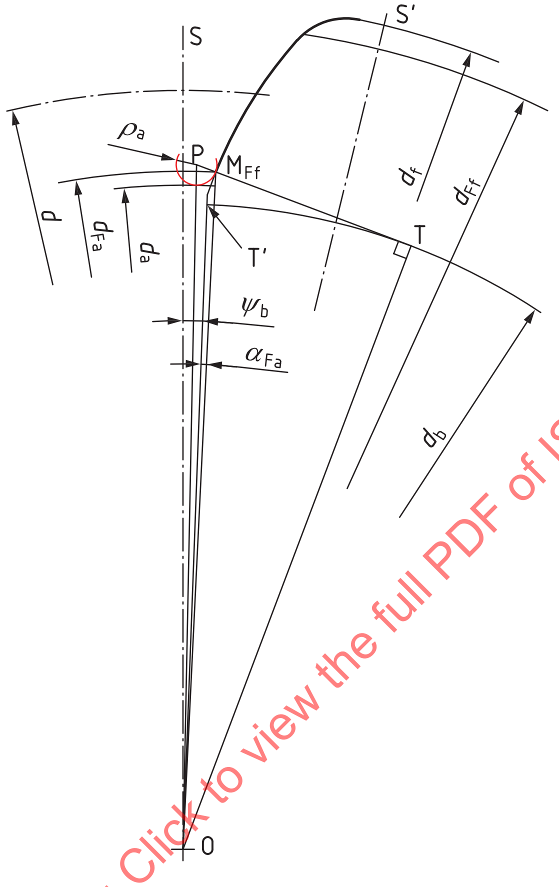
$$\overline{\mathrm{OP}} = \left|\frac{d_\mathrm{a}}{2}\right| + \rho_\mathrm{a} \quad \text{(P is the centre of tooth tip corner circle in the transverse plane)} \tag{A.13}$$

$$\overline{\mathrm{TP}} = \sqrt{\overline{\mathrm{OP}}^2 - \left(\frac{d_\mathrm{b}}{2}\right)^2} = \sqrt{\left(\left|\frac{d_\mathrm{a}}{2}\right| + \rho_\mathrm{a}\right)^2 - \left(\frac{d_\mathrm{b}}{2}\right)^2} \tag{A.14}$$

$$\overline{\mathrm{TM}_\mathrm{Fa}} = \overline{\mathrm{TP}} + \rho_\mathrm{a} = \sqrt{\left(\left|\frac{d_\mathrm{a}}{2}\right| + \rho_\mathrm{a}\right)^2 - \left(\frac{d_\mathrm{b}}{2}\right)^2} - \rho_\mathrm{a} \tag{A.15}$$

$$d_{\mathrm{Fa}} = 2\,\overline{\mathrm{OM}_{\mathrm{Fa}}} = 2\sqrt{\overline{\mathrm{TM}_{\mathrm{Fa}}}^2 + \left(\frac{d_\mathrm{b}}{2}\right)^2} = 2\sqrt{\left[\sqrt{\left(\left|\frac{d_\mathrm{a}}{2}\right| + \rho_\mathrm{a}\right)^2 - \left(\frac{d_\mathrm{b}}{2}\right)^2} - \rho_\mathrm{a}\right]^2 + \left(\frac{d_\mathrm{b}}{2}\right)^2} \tag{A.16}$$

$$\cos\alpha_{\mathrm{Fa}} = \frac{d_\mathrm{b}}{d_{\mathrm{Fa}}} \tag{A.17}$$



Key

O centre of the gear

M_{Fa} point of tangency between the involute and the tip form diameter

P centre of the arc between the involute and the tip diameter

S axis of symmetry of the tooth

S' axis of symmetry of the tooth space

T point where a line through M_{Fa} is tangent to the base circle

T' starting point of the involute profile on the base cylinder

Figure A.4 — Tooth tip corner radius for internal gear

The condition to avoid a pointed tooth tip is:

$$\widehat{\mathrm{POT}'} = \widehat{\mathrm{POT}} + \widehat{\mathrm{TOT}'} = \arctan\frac{\overline{\mathrm{TP}}}{r_\mathrm{b}} - \arctan\frac{\overline{\mathrm{TM}_{\mathrm{FA}}}}{r_\mathrm{b}} \le \psi_\mathrm{b} \tag{A.18}$$

where ψ_b is the base transverse tooth thickness half angle $\widehat{\mathrm{SOT}'}$.

A.3.2 Calculation of maximum tooth tip corner radius, ρ_{a_max}

In this calculation, tip diameter, d_a , is known. The maximum tooth tip corner radius, ρ_{a_max} is reached when the angle $\widehat{SOP}$ is equal to zero, i.e. point P on the axis of symmetry of teeth (see Figure A.4).

The value of ρ_{a_max} can be determined by iteration of the value of ρ_a , based on the development in A.3.1, up to when the condition $\widehat{POT'} = \psi_b$ is reached.

A.3.3 Calculation of root form diameter d_{Ff} when tooth root fillet radius is given

It is assumed that the following values are known:

— root form diameter, d_{Ff} ;

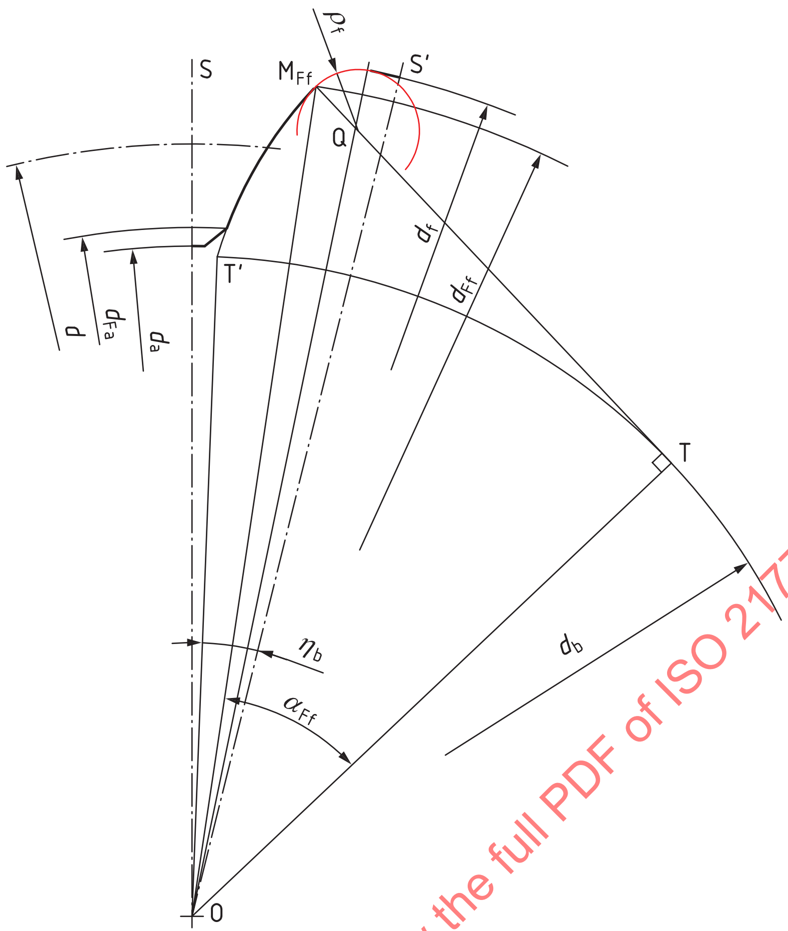
— tooth root fillet radius, ρ_f ;

— and the nominal geometry of the gear.

The root form diameter d_{Ff} at point M_{Ff} is calculated as follows (see Figure A.5).

The centre of tooth root fillet radius Q, M_{Ff} and T are aligned on a common normal to the involute profile.

T’ is the starting point of the involute profile on the base cylinder.



Key

O centre of the gear

M_{Ff} point of tangency between the involute and the root form diameter

Q centre of the arc between the involute and the root diameter

S axis of symmetry of the tooth

S' axis of symmetry of the tooth space

T point where a line through M_{Ff} is tangent to the base circle

T' starting point of the involute profile on the base cylinder

Figure A.5 — Tooth root fillet radius for an internal gear

From Figure A.5, it can be written:

$$\overline{OQ} = \left|\frac{d_f}{2}\right| - \rho_f \text{ (Q is the centre of tooth root fillet circle in the transverse plane)} \tag{A.19}$$

$$\overline{TQ} = \sqrt{\overline{OQ}^2 - \left(\frac{d_b}{2}\right)^2} = \sqrt{\left(\left|\frac{d_f}{2}\right| - \rho_f\right)^2 - \left(\frac{d_b}{2}\right)^2} \tag{A.20}$$

$$\overline{TM_{Ff}} = \overline{TQ} - \rho_f = \sqrt{\left(\left|\frac{d_f}{2}\right| - \rho_f\right)^2 - \left(\frac{d_b}{2}\right)^2} + \rho_f \tag{A.21}$$

$$d_{\mathrm{Ff}} = 2\,\overline{\mathrm{OM}_{\mathrm{Ff}}} = 2\sqrt{\overline{\mathrm{TM}_{\mathrm{Ff}}}^2 + \left(\frac{d_{\mathrm{b}}}{2}\right)^2} = 2\sqrt{\left[\sqrt{\left(\left|\frac{d_{\mathrm{f}}}{2}\right| - \rho_{\mathrm{f}}\right)^2 - \left(\frac{d_{\mathrm{b}}}{2}\right)^2} + \rho_{\mathrm{f}}\right]^2 + \left(\frac{d_{\mathrm{b}}}{2}\right)^2} \tag{A.22}$$

$$\cos\alpha_{\mathrm{Ff}} = \frac{d_{\mathrm{b}}}{d_{\mathrm{Ff}}} \tag{A.23}$$

The condition to avoid a pointed tooth root is:

$$\widehat{\mathrm{T'OQ}} = \widehat{\mathrm{TOT'}} + \widehat{\mathrm{TOQ}} = \frac{\overline{\mathrm{TM}_{\mathrm{Ff}}}}{r_{\mathrm{b}}} - \arctan\frac{\overline{\mathrm{TQ}}}{r_{\mathrm{b}}} \le \eta_{\mathrm{b}} \tag{A.24}$$

where η_{b} is the base transverse space width half angle.

NOTE The use of a full tooth root fillet radius reduces stress concentration in the fillet. However, for a given tool addendum, a full tooth root fillet radius enlarges the absolute value of the gear root diameter, and so reduces the working height of the teeth.

A.3.4 Calculation of maximum tooth root fillet radius, $\rho_{\mathrm{f\_max}}$

In this calculation, root diameter, d_{f}, is known. The maximum tooth root fillet radius, $\rho_{\mathrm{f\_max}}$, is reached when the angle $\widehat{\mathrm{QOS}}$ is equal to zero (see Figure A.5). Q is located on the axis of symmetry of a tooth space.

The value of $\rho_{\mathrm{f\_max}}$ can be determined by the iteration process on to the value of ρ_{f}, based on the development in A.3.3.

Annex B
(informative)

Tip form radius when tooth tip corner radius is defined on normal surface with pinion type cutter

B.1 General

The parameters used in this annex are defined in Clause 11.

B.2 Tangent point between tooth tip corner ellipse and involute

Figure B.1 represents the transverse plane of a helical pinion-type cutter for which:

- the tooth tip corner radius, ρ_{aP0}, is defined in a normal surface, so it is represented by an ellipse in the transverse plane with a minor axis, $r_{eb} = \rho_{aP0}$, and a major axis. $r_{ea} = \rho_{aP0}/\cos\beta$;
- the involute profile is defined from the base circle with diameter d_{b0};
- the Y axis is crossing the centre of the ellipse, M_0, and the centre of the pinion-type cutter 0.

The tip form radius goes to where the involute profile is tangent to the tooth tip corner ellipse.

An ellipse can be defined by its major axis, r_{ea}, and minor axis, r_{eb}. The coordinates for a point $M(x_M, y_M)$ on the ellipse (see Figure B.1) can be found as a function of parameter θ_M using $x_M(\theta_M) = r_{ea}\cos\theta_M$ and $y_M(\theta_M) = r_{eb}\sin\theta_M$, where θ_M is the angle to that point. The slope of the normal to the ellipse crossing at point M is:

$$s_M(\theta_M) = \frac{r_{ea}^2 y_M(\theta_M)}{r_{eb}^2 x_M(\theta_M)} = \frac{(\rho_{aP0}/\cos\beta)^2 y_M(\theta_M)}{\rho_{aP0}{}^2 x_M(\theta_M)} = \frac{y_M(\theta_M)}{\cos^2\beta\, x_M(\theta_M)} \quad \text{(B.1)}$$

The angle $\eta_{ya0t}(\theta_M)$ (see Figure B.1) is:

$$\tan\left[\eta_{ya0t}(\theta_M)\right] = s_M(\theta_M) \quad \text{(B.2)}$$

The angle $\lambda_{ya0t}(\theta_M)$ between the radius at point M and axis crossing the centre of ellipse is obtained by:

$$\tan\left[\lambda_{ya0t}(\theta_M)\right] = \frac{x_M(\theta_M)}{r_{M0} + y_M(\theta_M)} \quad \text{(B.3)}$$

Pressure angle at a point of ellipse is:

$$\alpha_{ya0t}(\theta_M) = \eta_{ya0t}(\theta_M) + \lambda_{ya0t}(\theta_M) \quad \text{(B.4)}$$

Radius at a point of ellipse is:

$$\overline{O_0 M} = r_{ya0}(\theta_M) = \sqrt{[x_M(\theta_M)]^2 + [r_{M0} + y_M(\theta_M)]^2} \quad \text{(B.5)}$$

To find the point where the tooth tip corner ellipse is also tangent to an involute profile, generated from the same base circle, the following formula can be used:

$$r_{Fa}(\theta_M) \cdot \cos\left(\alpha_{ya0t}(\theta_M)\right) - r_{b0} = 0 \quad \text{(B.6)}$$

The initial value can be chosen as $\theta_{M\_init} = \frac{\pi}{4}$. The solution is θ_{Fa0}, the value of the parametric angle of the ellipse when the ellipse is tangent to the involute.

The radius of this point can be found with the formulae in B.5.

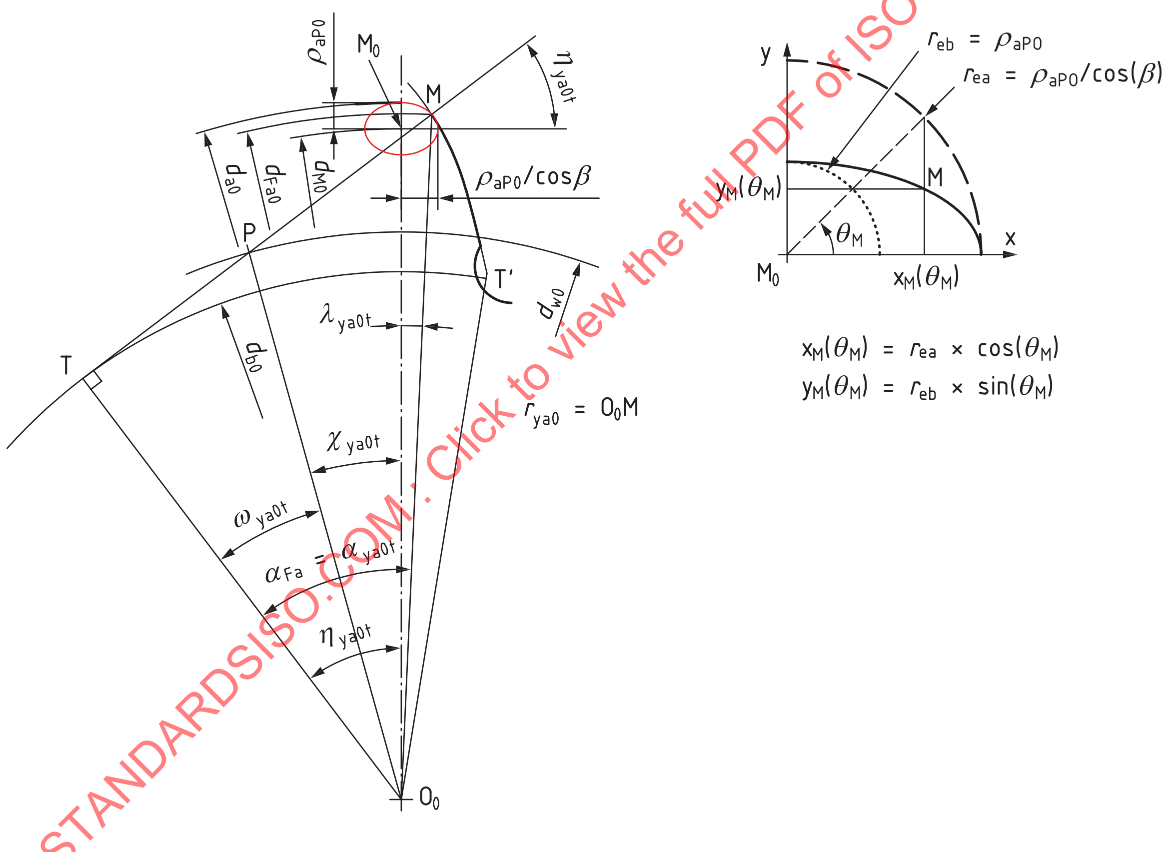


Figure B.1 — Transverse plane of a helical pinion-type cutter

If there is no protuberance, this radius is also the tip form radius of the pinion-type cutter:

$$r_{Fa0} = r_{ya0}(\theta_{Fa0}) \tag{B.7}$$

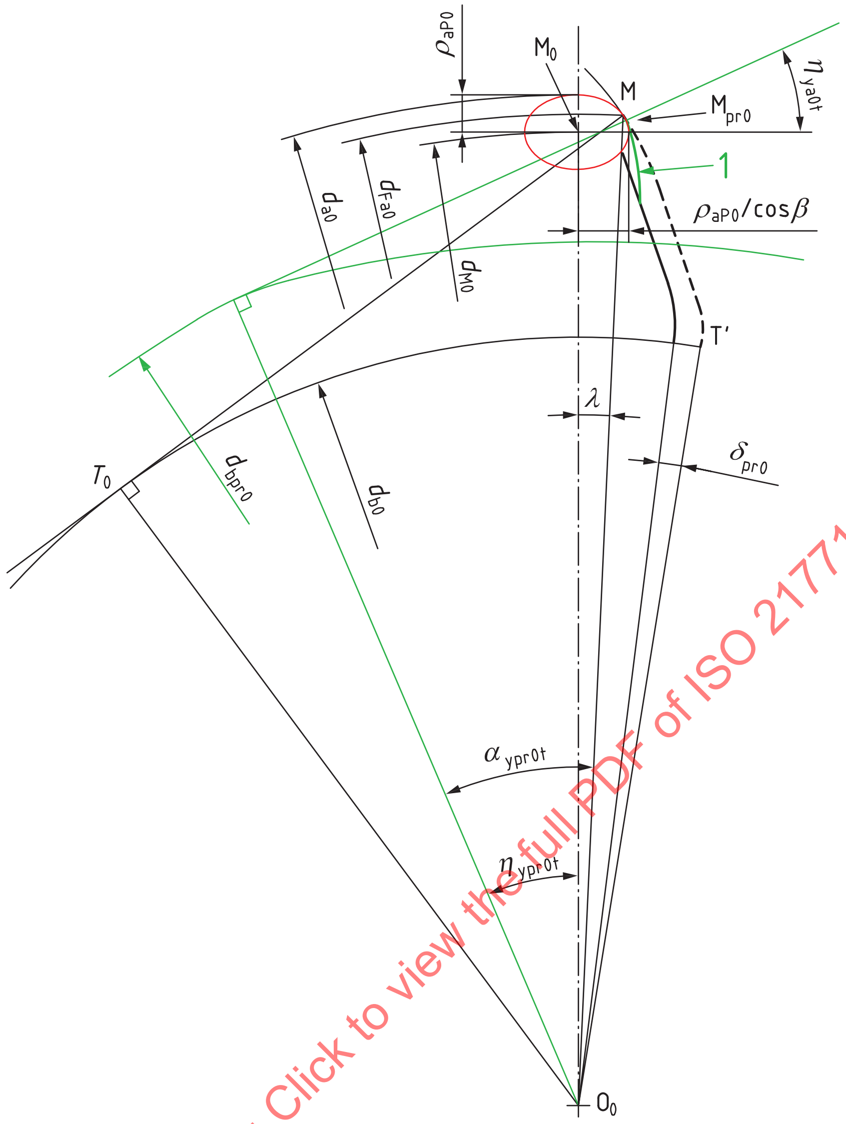
If there is protuberance:

— a secondary involute profile, key 1 on Figure B.2, with a smaller normal pressure angle α_{pr0}, in transverse plane $\alpha_{tpr0} = \arctan\left(\frac{\tan\alpha_{pr0}}{\cos\beta}\right)$ generated by the protuberance base cylinder diameter d_{bpr0} joins the tooth tip corner ellipse;

— the tip form radius of the pinion-type cutter can be determined by the same principle if there is no protuberance, by substituting the base radius of protuberance involute $r_{bpr0} = r_0 \cos\alpha_{tpr0}$ in place of r_{b0} in Formula (B.6);

— the intersection between the main involute defined by the normal pressure angle α_n (α_t, in transverse plane) generated by the base diameter d_{b0} and the secondary involute can be determined iteratively;

— the involute profile of the pinion-type cutter corresponding at finishing, is shifted from an angle δ_{b0} to angle δ_{pr0} [see Formula (311)](see Figure B.2) considering only residual fillet undercut.



Key

1 secondary involute between the main involute and tip rounding

Figure B.2 — Transverse plane of a helical pinion-type cutter with protuberance

The intersection between the main and secondary involutes can be determined iteratively.

B.3 Profile of pinion-type cutter

B.3.1 Involute profile

The coordinates of the involute profile on y-cylinder are:

$$x_{y0t}\left(r_{y0}\right) = -r_{y0} \cdot \sin(\mathrm{inv}(\alpha_{y0t}(r_{y0})) - \mathrm{inv}(\alpha_{w0t}) \tag{B.8}$$

$$y_{y0t}\left(r_{y0}\right) = r_{y0} \cdot \cos(\mathrm{inv}(\alpha_{y0t}(r_{y0})) - \mathrm{inv}(\alpha_{w0t}) - r_{w0} \tag{B.9}$$