STP-PT-029-1

EXTERNAL PRESSURE DESIGN IN CREEP RANGE

ASME STANDARDS TECHNOLOGY, LLC

STP-PT-029-1

External Pressure Design in Creep Range

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Global Engineering & Technology, LLC

ASME STANDARDS TECHNOLOGY, LLC

Date of Issuance: July 29, 2011

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ASME Standards Technology, LLC
Three Park Avenue, New York, NY 10016-5990

ISBN No. 978-0-7918-3398-8

Summary of Changes

July 29, 2011

STP-PT-029-1

External Pressure Design in Creep Range

The following changes have been made to the first revision of STP-PT-029.

Rev. 1 Page	*Location*	*Change*
vi-vii	Table of Contents	Updated to reflect changes
3	table 2, row 8	Corrected from “9Cr-1M0-V” to “9Cr-1Mo-V”
5	last paragraph, line 3	Replaced “k' ” with “$k'T$”
5	equation (10)	Removed “T”
6	paragraph 1	Inserted “$K' = k'T$”
6	equation (11)	Replaced “1464” with “39,300”
6	equation (10)	Removed “T”
6	equation (11)	Replaced “1464” with “39,300”
7	bullet 2	Replaced “48” with “120”
7	bullet 3	Replaced “10,000” with “100,000”
7	section 3.3.1.1, line 3	Replaced “10,000” with “100,000”
7	section 3.3.1.1, line 3	Replaced “860” with “8,600”
7	equation	Removed “100”
7	equation	Replaced “48” with “120”
7	equation	Replaced “5,880” with “94,100”
7	equation	Replaced “<” with “>”
7	paragraph 8	Replaced “5,880” with “14,200”
7	paragraph 8	Replaced “860” with “8,600”

Rev. 1 Page	*Location*	*Change*
7	section 3.3.1.2, line 3	Replaced “10,000” with “100,000”
7	section 3.3.1.2, line 3	Replaced “380” with “3,800”
8	first equation	Removed “10,000”
8	first equation	Replaced “48” with “120”
8	first equation	Replaced “515” with “5,570”
8	paragraph 3	Replaced “515” with “5,570”
8	paragraph 3	Replaced “380” with “3,800”
8	section 3.3.1.3, line 1	Replaced “40” with “20”
8	section 3.3.1.3, line 2	Replaced “300” with “3,800”
8	second equation	Removed “100,000”
8	second equation	Replaced “48” with “120”
8	second equation	Replaced “2.94” with “2.96”
8	second equation	Replaced “300” with “4,740”
8	paragraph 9	Replaced “300” with “4,740”
8	paragraph 9	Replaced “equal to” with “greater than”
8	paragraph 9	Replaced “300” with “3,800”
8	paragraph 9	Replaced “40” with “20”
11	equation (19)	Replaced “*K’* ” with “*k’* ”
11	paragraph 5	Replaced “*K’* ” with “*k’* ”
11	paragraph 5	Replaced “(2) ” with “(1)”
12	equation (20)	Removed “*T*”
12	paragraph 1	Inserted “*K’* = *k’T*”
13	bullet 4	Replaced “24.7” with “24,700”
13	paragraph 5	Removed “8”
14	paragraph 4	Removed “8”
15	bullet 3	Inserted “S = 20,700 psi at 1000°F for short time”
15	bullet 4	Inserted “S = 6,300 psi at 1000°F for 100,000 hours”
15	bullet 6	Replaced “1.0” with “1.5”
15	paragraph 4	Inserted “This stress is < 20,700 psi and”
15	last equation	Removed “100,000”
15	paragraph 7	Replaced “395 <” with “12,300 psi > 6,300 psi. Use B = 6,300 psi >”
15	paragraph 8	Removed “not”
16	paragraph 3	Replaced “low” with “high”

Rev. 1 Page	*Location*	*Change*
16	bullet 1	Inserted “The ratio e/t of the cylinder is highly arbitrary and does not have an influence on the results.”
16	bullet 1	Removed “assumed to have a very large R_o/t ratio. However, the ratio in this case is only 45.”
16	bullet 2	Replaced “get to be too conservative” with “become approximate”
16	paragraph 4	Removed “the conservative”
18	equation (29)	Replaced “K' ” with “k' ”
18	paragraph 5	Replaced “K' ” with “k' ”
18	paragraph 5	Replaced “(2) ” with “(1)”
19	equation (30)	Removed “T”
19	paragraph 1	Inserted “$K' = k'T$”
20	paragraph 4	Replaced “1.0” with “1.5”
21	first equation	Replaced “1.0” with “1.5”
21	first equation	Removed “100,000”
21	first equation	Replaced “0.55” with “16.9”
21	paragraph 2	Replaced “0.55 psi <” with “16.9 psi >”
21	paragraph 3	Replaced “inadequate” with “adequate”
21	bullet 1	Inserted “The ratio e/t of the cylinder is highly arbitrary and does not have an influence on the results.”
21	bullet 1	Removed “assumed to have a very large D_o/t ratio. However, the ratio in this case is only 144.”
21	bullet 2	Replaced “get to be too conservative” with “become approximate”
23	equation (40)	Replaced “P_a” with “σ_c”
23	equation (40)	Removed “T”
23	equation (40)	Replaced “0.588” with “0.353”

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FOREWORD

This document was developed under a research and development project that resulted from ASME Pressure Technology Codes & Standards (PTCS) committee requests to identify, prioritize and address technology gaps in current or new PTCS Codes, Standards and Guidelines. This project is one of several included for ASME fiscal year 2009 sponsorship which are intended to establish and maintain the technical relevance of ASME codes & standards products. The specific project related to this document is project 09-03 (BPVC #2) titled "External Pressure Design in Creep Range".

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ABSTRACT

At the present time the ASME Boiler and Pressure vessel code does not include rules for the design of components under external pressure and axial compressive loads in the time-dependent creep regime. A method is suggested in this report for designing components in the time-dependent creep regime. The design methodology is developed for columns and cylindrical shells under axial compression as well as cylindrical, spherical and conical shells under external pressure. An external pressure chart for 2.25Cr-1Mo steel was developed at 1000°F to demonstrate the applicability of the methods developed in this report. In addition, variable factors of safety are imbedded in the design equations in order to transition from the design factors used in the time-independent External Pressure Charts of Section II to the lower design factors specified in Section III for time-dependent, creep buckling.

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1 INTRODUCTION

The 2007 edition of the ASME Boiler and Pressure Vessel Code, Sections I and VIII, includes rules for the design of cylindrical, conical, spherical, ellipsoidal and torispherical shells subjected to external pressure. Rules are also given for the design of cylindrical and conical shells under axial compressive loads. In addition, Section VIII gives rules for the design of structural members subjected to axial compressive loads such as heat exchanger tubes. All of these rules are applicable at temperatures below the creep range of the material. The ASME code gives approximate temperatures above which creep becomes prominent as shown in Table 1. These temperature limits are approximate and serve as guidelines for design purposes and should not be thought of as absolute values for a given specified material.

Table 1 - Approximate Temperatures at Which Creep Becomes a Design Consideration for Various Materials (these temperatures may vary significantly for specific product chemistry and failure mode under consideration).

Material	Temperature (°F)	Temperature (°C)
Carbon and Low Alloy Steel	700-900	370-480
Stainless Steels	800-1000	425-535
Aluminum Alloys	300	150
Copper Alloys	300	150
Nickel Alloys	900-1100	480-595
Titanium and Zirconium Alloys	600-650	315-345

There are situations where it is beneficial for newly constructed boiler and pressure vessel components to operate at temperatures beyond the creep cut-off limits of the material. Thus, new design criteria are needed for establishing allowable compressive stress for various components. In this report, suggested procedures are presented to address the design of ASME components under compressive and external pressure loads operating at elevated temperatures in the creep range.

2 STRESS-STRAIN RELATIONSHIPS IN THE CREEP RANGE

Theoretically [1] the relationship between stress and strain in the creep range can be expressed by Norton's equation:

$$\frac{d\varepsilon}{dT} = k'\sigma^n \tag{1}$$

Where:

k' = constant

n = creep exponent which is a function of material property and temperature

$d\varepsilon/dT$ = strain rate

σ = stress

This non-linear equation is difficult to use since it must be integrated to get the strain in a given component. However, when the stress field in the component stabilizes to a constant value after redistribution and when the strain value is relatively large, then Equation (1) may be replaced by the much simpler viscoelastic equation:

$$\varepsilon = K'\sigma^n \tag{2}$$

Where:

K' = constant

ε = total strain

Equation (2) is more practical to use for solving indeterminate structures than Equation (1) and it assumes the component represented by the viscoelastic Equation (2) is loaded in the same manner and has the same boundary conditions as when represented by the creep strain rate Equation (1). It also assumes that only secondary creep strain is considered and that the total strain given by Equation (2) is approximately equal in magnitude to the creep strain rate given by Equation (1).

Equation (2) and the stated assumptions mentioned above are collectively referred to as the Stationary Stress Method or the Elastic Analog Method [2], [3]. It is applicable mainly in the secondary creep range which occurs after about 100 hours of service at elevated temperatures for materials used in ASME pressure vessels.

The value of n in Equation (2) varies between 2 and 8 for most materials and corresponding temperatures encountered in the ASME code applications. For an elastic analysis, the value of n is equal to 1.0 and the value of K' becomes $1/E$. Thus, Equation (2) reverts to the elastic equation $\varepsilon = \sigma/E$, where E is the modulus of elasticity.

The actual relationship between stress, strain and time in the creep regime is developed from test data for a given material. This is accomplished [3] by subjecting a tensile specimen to a constant load at a given temperature and then measuring the strain at various time increments. A series of strain-time lines are then plotted for various average stress levels as shown in Figure 1. The data in Figure 2 are then rearranged to plot a series of stress-strain curves, called isochronous curves, for specific times at a constant temperature as shown in Figure 2. A typical chart showing isochronous stress-strain curves for 2.25Cr -1Mo steel at 1000°F is given in Figure 3. Isochronous curves for various materials and temperatures presently accessible to the designer through ASME are listed in Table 2.

Table 2 - Temperature Range of Available Isochronous Stress-Strain Curves.

Material	Temperature Range(°F)	Temperature Range (°C)	Reference
304 Stainless Steel	800-1500	425-815	III-NH
316 Stainless Steel	800-1500	425-815	III-NH
Nickel Alloy 800H	800-1400	425-760	III-NH
Carbon Steel	750-930	400-500	Griffin [4]
SA533B & SA508B	700-1000	370-540	III CCN-499
2.25Cr-1Mo Steel	700-1200	370-650	III-NH
9Cr-1Mo-V Steel	700-1200	370-650	III-NH

The isochronous stress-strain curves provide a convenient method for calculating the values of n and K' in Equation (2) although theoretically, they are not true stress-strain curves. This can be illustrated by referring to the 10,000 hour isochronous curve in Figure 3. A stress level of 5400 psi corresponds to a strain of 0.00071, and a stress level of 8000 psi corresponds to a strain of 0.0024. Substituting these values into Equation (2) and solving for n and K' results in n = 3.1 and $K' = 1.91x10^{-15}$.

3 AXIAL COMPRESSION OF TUBES AND COLUMNS

3.1 Theoretical Derivations

Many components in boilers and pressure vessels—support legs for vessels, tubes in fixed heat exchangers and header supports in boilers, for example —are subjected to axial compressive forces. The design of these components is based on Euler's elastic buckling equation. The equation for members loaded axially in the elastic range is given by:

$$\sigma_{cr} = \frac{\pi^2 E}{(l/r)^2} \text{ but not greater than } S \quad (3)$$

Where:

a = cross sectional area of axial member

E = modulus of elasticity

I = moment of inertia of member

l = effective length of member

r = radius of gyration = $\sqrt{I/a}$

S = allowable stress in tension

σ_{cr} = critical axial buckling stress

Theoretical analysis of columns operating in the creep regime is very complicated [1] due to various factors some of which are not necessarily well known. Factors that have significant influence on creep buckling include effect of primary and secondary creep on buckling, interaction between elastic and creep buckling, and behavior of material properties with time at elevated temperatures, and initial out-of-roundness. Some of these factors were taken into consideration by Hoff [5] to obtain a buckling equation in terms of critical creep time. However, the equation is too complicated for design purposes because it involves initial imperfection terms that are not readily known by the designer. A simplified equation can be derived based on the following facts that are of particular interest to the designer.

- Columns with eccentricity will deform at any axial compressive load, regardless of load magnitude, when the column is subjected to temperatures in the creep range over a long period of time.
- Creep buckling occurs over some period of time rather than instantaneously. Thus, the design premise is based on determining a *critical* time period that is longer than the intended service of a component, rather than *critical* buckling force.
- The Euler elastic buckling equation is not applicable in the creep range due to the non-linear relationship between stress and strain.
- Creep buckling occurs at a specified time only for non-linear stress-strain behavior.
- The tangent modulus, E_t, gives a simplified equation for designing in the creep regime.
- The shape of the deformed column is assumed [6] to be sinusoidal.

Simplified column buckling equations based on the above assumptions can be derived for temperatures in the creep range. One such method, mentioned by Kraus [7], is presented in this section. Detailed evaluation of Kraus's method by Griffin showed that the results obtained theoretically may differ from the results obtained from test data unless careful consideration is given to various parameters affecting the buckling.

The buckling equation [6] for columns operating in the creep range is similar to the elastic buckling expression given by Equation (3) with the exception that tangent modulus, E_t, is substituted for Young's modulus of elasticity, E, where E_t is defined on the isochronous stress-strain curves. The background of, and justification for, this substitution is explained later in Section 4.1.2 for the inelastic axial buckling of cylindrical shells. The equation becomes:

$$\sigma_{cr} = \frac{\pi^2 E_t}{(l/r)^2} \text{ but not greater than } S \quad (4)$$

Where:

E_t = tangent modulus

The value of E_t in Equation (4) can be obtained from isochronous curves by using Norton's Equation (1) for creep. Integration of Equation (1) with respect to time yields:

$$\varepsilon - \varepsilon_0 = k'\sigma^n T \quad (5)$$

Where:

T = time

ε = strain

ε_0 = initial strain at time $T = 0$.

The tangent modulus, E_t, is defined as:

$$E_t = \frac{d\sigma}{d\varepsilon} \quad (6)$$

Differentiation of Equation (5) with respect to stress gives:

$$d\varepsilon = k'n\sigma^{n-1}T \cdot d\sigma \quad (7)$$

Combining Equations (6) and (7) results in:

$$E_t = \frac{1}{k'n\sigma^{n-1}T} \quad (8)$$

Combining this equation with Equation (4) for creep buckling gives:

$$\sigma_{cr}^n = \frac{\pi^2}{k'nT(l/r)^2} \quad (9)$$

The constant k' in Equation (9) is obtained from the Norton Equation (1). However, for many engineering problems a stationary stress condition exists and Equation (2) becomes applicable. Thus, the value of K' can be substituted for $k'T$ in Equation (9). The values of K' and n in Equation (9) are obtained from an appropriate isochronous curve in conjunction with Equation (2) as shown earlier. It is to be noted that tests referenced by Griffin indicate that results obtained from Equation (9) tend to be more accurate when the two points chosen on an isochronous curve for calculating n and K' are taken from the vicinity of the buckling strain. Otherwise, the results tend to be on the unsafe side. For design purposes, a factor of safety is added to Equation (9) and it becomes:

$$\sigma_c = \frac{1}{FS}\left[\frac{\pi^2}{K'n(l/r)^2}\right]^{1/n} \text{ but not greater than } S \quad (10)$$

Where:

FS = factor of safety

K' = $k'T$

S = the allowable tensile stress in the creep range for a given number of hours

σ_c = allowable compressive stress

The deflection of the column, versus time, is of the shape shown in Figure 4. Most of the deflection, and hence damage, occurs towards the end of time period, *T*. Thus, a low factor of safety, *FS*, is justified in Equation (10) due to the various conservative assumptions made in deriving the equation. Accordingly, a factor of safety, *FS*, which varies from 1.92 at instantaneous time to 1.2 at 100,000 hours is deemed justifiable. The 1.92 factor is based on the Steel Construction Manual criterion [8] used in VIII-1 for tube buckling in heat exchangers. The 1.2 factor is based on the conservative assumption of using the tangent modulus in Equation (10) rather than the combination of tangent and secant moduli. Thus, factor of safety may be expressed as:

$$FS = 1.92 - 0.0625 \ln T$$

Where:

T = number of operating hours. The minimum value of T is equal to 1.0 hour and the maximum value is equal to 500,000 hours.

Equation (10) can be simplified for specific design applications. For example, if the temperature is 1000°F, the material is 2.25Cr-1Mo steel and the design life is 100,000 hours, then Equation (10) for the allowable compressive stress of a column becomes:

$$\sigma_c = \frac{39,300}{(l/r)^{0.571}} \text{ but not greater than } S \quad (11)$$

3.2 Design Equations

A conservative design equation for the design of tubes and columns subjected to axial compressive loads in the creep range is Equation (10):

$$\sigma_c = \frac{1}{FS}\left[\frac{\pi^2}{K'n(l/r)^2}\right]^{1/n} \text{ but not greater than } S \quad (10)$$

Where:

$FS = 1.92 - 0.0625 \ln T$

T = number of operating hours. The minimum value of T is equal to 1.0 hour and the maximum value is equal to 500,000 hours.

For 2.25Cr-1Mo steel operating at a temperature of 1000°F and a design life of 100,000 hours, the design equation becomes:

$$\sigma_c = \frac{39,300}{(l/r)^{0.571}} \text{ but not greater than } S \quad (11)$$

3.3 Applications

The following example illustrates the application of the above axial buckling equations to a vessel support system.

3.3.1 Example 1

A pressure vessel is supported by four legs. The design data are as follows.

- Legs consist of 8-inch pipes with an outside diameter of 8.625 inches.
- Effective length of pipes is 120 inches
- Total weight of vessel = 100,000 lbs
- Design temperature = 1000°F
- Pipe material is 2.25Cr-1Mo steel.
- E = 24,700,000 psi

Determine the required pipe thickness that can safely support the load for the following time periods.

- 100 hours (short duration). S = 14,200 psi
- 10,000 hours (a little over one year). S = 8700 psi
- 100,000 hours (a little over 11 years). S = 6300 psi

3.3.1.1 100 hour duration solution

Try 8 inch Sch 5 pipe.

Thickness = 0.109 inch, area = 2.91 inches^2, radius of gyration = 3.01 inches,

Actual stress = (100,000/4)/2.91 = 8,600 psi

The isochronous chart in Figure 3 shows that the curve for 100 hours is elastic with a proportional limit well higher than the allowable stress of 14,200 psi. Hence, n = 1.00 and $K' = 1/E$ = 4.049x10^{-8}.

From Equation (10):

$$FS = 1.92 - 0.0625\ln(100) = 1.63$$

$$\sigma_c = \frac{1}{1.63}\left[\frac{\pi^2}{(4.049\times10^{-8})(1.0)(120/3.01)^2}\right]^{1/1} = 94{,}100 \text{ psi} > 14{,}200 \text{ psi}$$

Thus the allowable stress is 14,200 psi which is greater than the actual stress of 8,600 psi. **Use 8 inch Sch 5 pipe.**

3.3.1.2 10,000 hour duration solution

Try 8 inch Sch 20 pipe.

Thickness = 0.25inch, area = 6.57 inches^2, radius of gyration = 2.96 inches.

Actual stress = (100,000/4)/6.57 = 3,800 psi

The isochronous chart in Figure 3 shows that the curve is inelastic for 10,000 hours with a limit on stress of 8000 psi. Two points, A and B, are chosen on the 10,000 hour curve. The stress and corresponding strain values at these two points are:

ε_A = 0.00071 at σ_A = 5400 psi

ε_B = 0.0024 at σ_B = 8000 psi

Substituting these values into Equation (2) and solving for n and K' results in

$n \approx 3.1$ and $K' = 1.91\text{x}10^{-15}$.

The isochronous stress-strain curves in Figure 3 are based on average values. The quantities n and K' calculated above are based on these average values rather than the minimum values which are usually taken as 80% of those in Figure 3. This was done for convenience since the average and minimum curves have essentially the same shape.

From Equation (10):

$$FS = 1.92 - 0.0625\ln(10{,}000) = 1.34$$

$$\sigma_c = \frac{1}{1.34}\left[\frac{\pi^2}{(1.91\times10^{-15})(3.1)(120/2.96)^2}\right]^{1/3.1} \approx 5{,}570 \text{ psi} < 8{,}700 \text{ psi}$$

Thus the allowable stress is 5,570 psi which is greater than the actual stress of 3,800 psi. **Use 8 inch Sch 20 pipe.**

3.3.1.3 100,000 hour duration solution

Try 8 inch Sch 20 pipe.

Actual stress = 3,800 psi

The isochronous chart in Figure 3 shows that the curve is inelastic for 100,000 hours with a limit on stress of 8000 psi. Two points, *C* and *D*, are chosen on the 100,000 hour curve. The stress and corresponding strain values at these two points are:

ε_C = 0.00050 at σ_C = 4000 psi

ε_D = 0.0017 at σ_D = 5700 psi

Substituting these values into Equation (2) and solving for n and K' results in

$n \approx 3.5$ and $K' = 1.24 \text{ x } 10^{-16}$.

Here again the quantities n and K' calculated above are based on these average values from Figure 3 rather than the minimum values which are usually taken as 80% of those in Figure 3. This was done for convenience since the average and minimum curves have essentially the same shape.

From Equation (10):

$$FS = 1.92 - 0.0625\ln(100{,}000) = 1.20$$

$$\sigma_c = \frac{1}{1.2}\left[\frac{\pi^2}{(1.24\times10^{-16})(3.5)(120/2.96)^2}\right]^{1/3.5} \approx 4{,}740 \text{ psi} < 6{,}300 \text{ psi}$$

Thus the allowable stress is 4,740 psi which is greater than the actual stress of 3,800 psi. **Use 8 inch Sch 20 pipe.**

It should be noted that, in this case, the simpler Equation (11) could have been used.

4 AXIAL COMPRESSION OF CYLINDRICAL SHELLS

4.1 Theoretical Equations Below the Creep Range

Cylindrical shells and vessel skirts are routinely subjected to compressive stress due to applied loads such as dead and contents weight, wind loads and earthquake forces. Euler's buckling Equation (3) does not apply to the majority of skirt and vessel cases due to their small l/r ratio. Instead, the prevalent buckling mode is axisymmetric or the diamond pattern depending on geometry.

4.1.1 Elastic Buckling

The applicable critical in-plane buckling equation for long cylindrical shells [9] under axial compression used by the ASME codes is given by:

$$\sigma_{crL} = 0.63E\frac{t}{R_o} \tag{12}$$

Where:

E = Young's modulus of elasticity

R_o = outside radius

t = thickness of cylinder

σ_{crL} = buckling stress in a cylindrical shell due to axial loads

It should be noted that Equation (12) is independent of the length of cylinder. The critical elastic strain is expressed as:

$$\varepsilon_{crL} = \frac{\sigma_{crL}}{E} = \frac{0.63}{R_o/t} \tag{13}$$

Where:

ε_{crL} = buckling strain in a cylindrical shell due to axial loads

Experimental data have shown [10] that buckling may occur at a value of one-tenth of that calculated from Equation (12) for large R/t ratios. Accordingly, the design limits provided in the ASME Section II External Pressure Charts are based on the classical elastic buckling theory with modifications to take into account the effects of geometrical imperfections such as out-of-roundness and eccentric loading. A knock-down factor, α , of 5.0 is used by ASME for the effect of geometrical imperfections on axial compression. In addition, a design factor, *DF*, of 2.0 is also added to take into account such items as reduced modulus in the inelastic range and variation of material properties that cause tests to deviate from theory. The knock-down and design factors are used by ASME to determine allowable compressive stress in a cylinder as follows:

$$A = \frac{0.63}{\alpha\left(R_0/t\right)} \tag{14}$$

$$B = \frac{AE}{DF} \tag{15}$$

Where:

A = a non-dimensional factor based on geometry

B = allowable compressive stress.

Using a knock-down factor of $\alpha = 5.0$ and a design factor $DF = 2.0$, Equations (14) and (15) become:

$$A = \frac{0.125}{R_o/t} \tag{16}$$

and

$$B = \frac{AE}{2} \text{ but not greater than } S \tag{17}$$

4.1.2 Inelastic Buckling

Equation (17) is only applicable in the elastic region where E is constant for a given material and temperature. For inelastic buckling, the classical equations for elastic buckling stress are used with the elastic modulus replaced by a reduced modulus to reflect inelastic (non-linear) behavior of the material beyond the elastic limit. Gerard [11] and others have derived these reduced moduli in terms of the secant and tangent moduli for the material. These equations, and the approximations used in their derivations, are summarized by Griffin [4]. For the *column buckling* mode, Shanley [12], using the no-strain reversal model, concludes that the appropriate reduced modulus is simply the tangent modulus. For a *cylinder under axial compression*, the reduced modulus is a function of both the secant and tangent moduli. However, beyond the yield point, where buckling generally occurs, the tangent modulus is much smaller than the secant modulus so that the reduced modulus can be approximated to the first order by the tangent modulus. The critical stress for a thin-walled *cylinder under axial compression* is given by:

$$\sigma_c = \frac{t}{R_m}\left[\frac{E_s E_t}{3\left(1-\mu^2\right)}\right]^{1/2} \tag{18}$$

Where:

R_m = mean radius

t = wall thickness

E_t = tangent modulus, $d\sigma/d\varepsilon$

E_s = secant modulus, σ/ε

E = Young's modulus of elasticity

ν = Poisson's ratio

$\mu = \frac{1}{2} - \left(\frac{1}{2} - \nu\right)\frac{E_s}{E}$

In the elastic range, where $E_t = E_s = E$, Equation (18) reduces to Equation (12) which is the classical formula for critical stress used in derivation of the ASME Section II External Pressure Charts. To simplify calculation of the inelastic critical stress, E_s, it is conservatively approximated by E_t, and this formula reduces to the critical stress used in derivation of the External Pressure Charts beyond the elastic range, i.e., the reduced modulus is the tangent modulus. This is accomplished by using typical stress-strain curves for a specified material at various temperatures. Tangent moduli curves are then obtained from the given stress-strain curves. These new curves are then plotted in charts referred to by ASME as external pressure charts as explained in Appendix 3 of Section II-D of the ASME code.

One such chart is shown in Figure 5 for 2.25Cr-1Mo steel. The chart is limited to 900°F, which is the cut-off temperature where creep becomes prevalent for low carbon steels. At the present time, there is no specific external pressure chart for 2.25Cr-1Mo steel. The *A* value in Figure 5 is a non-dimensional factor while the *B* value is the allowable compressive stress based on elastic and tangent moduli of the stress-strain curve. The value of *B* on the right hand side of the curves cannot exceed one half of the yield stress of the material at the given temperature.

The design procedure consists of calculating Factor *A* from Equation (16) and then obtaining the allowable compressive stress *B* from either Equation (17) or the external pressure chart, depending on whether or not the Factor *A* falls in the elastic or plastic region of the chart.

4.2 Theoretical Equations in the Creep Range

The theoretical equations for the axial buckling of cylindrical shells in the creep range are extremely cumbersome to solve [13] due to the fact that numerous variables in the equations are unknown to the designer such as initial out-of-roundness, final out-of-roundness, time duration and changes in modulus of elasticity and yield stress due to creep. These variables coupled with known variables such as length, diameter, thickness and buckling modes makes a closed-form solution of the buckling equations very difficult to obtain. In 1968, Hoff [14], [5] derived a creep buckling equation for a cylindrical shell under axial compression in the creep range using the following assumptions.

- Norton's Equation (1) is applicable.
- The length of the cylinder is assumed to be much larger than the radius.
- The derivations are based on very thin walled cylinders with high Do/t ratios.
- The first buckling mode is elliptical in shape (ring buckling).
- The shell is assumed to have an initial out-of-roundness.
- A representative value of $n = 3.0$ was used to obtained a simplified closed form solution.

Using static and compatibility equations, Hoff came up with ten differential equations with ten unknowns. By various substitutions and simplifications, he combined the ten equations into one equation for a long cylinder subjected to a primary buckling mode of an elliptical shape and a value of $n = 3.0$ as an average number. The derived equation [15]becomes:

$$\sigma_{crL} = \left[\frac{0.588\left(0.6t/R_o\right)}{k'T} \ln\left(\frac{1.167}{e/t}\right)\right]^{1/3} \qquad (19)$$

Where:

e = initial deviation of cylinder from true shape. The value of e is expressed as a fraction of thickness.

k' = constant from Norton's Equation (1)

R_o = outside radius of shell

t = shell thickness

T = time in hours

σ_{crL} = buckling stress in a cylindrical shell due to axial loads.

Equation (19) can be rewritten in terms of design stress as:

$$B = \frac{1}{FS}\left[\frac{0.35}{K'\left(R_o/t\right)}\ln\left(\frac{1.167}{e/t}\right)\right]^{1/3} \tag{20}$$

Where:

B = allowable compressive stress

FS = factor of safety.

$K' = k'T$

4.3 Approximate Method Using Isochronous Stress-Strain Curves in the Creep Range

The creep buckling theory used in this publication is based on a classical stability theory developed by Gerard [16], [17]. Using the Rabotnov-Shesterikov stability criterion for the buckling of perfect shells, Gerard showed that the strain-rate dependent tangent and secant moduli for a fixed stress state in the creep buckling problem are directly analogous to the tangent and secant moduli associated with increasing stress in the time-independent, inelastic buckling problem. Thus, for initially perfect shells, the time-independent, inelastic buckling solutions may be used directly for creep buckling analysis if the tangent and secant moduli are defined as strain-rate dependent, i.e., are found from the appropriate constant-strain-rate stress-strain relations. This requires a complete set of constant-strain-rate stress-strain plots for each temperature, which are not generally available for the materials of interest.

The Gerard method is simplified here by replacing the constant-strain-rate stress-strain plots with isochronous stress-strain plots for determination of tangent and secant moduli. The use of isochronous plots was first suggested by Shanley [18] and later used by others including Griffin [19]. Isochronous stress-strain plots can be obtained directly from standard creep tests run at constant load, and have been developed for a number of materials of interest in elevated-temperature design as discussed in Section 2 and Table 2. Thus, the equations for time-independent, inelastic buckling can be applied directly for creep buckling using the appropriate isochronous stress-strain plot, treating it as though it was a short-time, uniaxial stress-strain plot. Although the tangent modulus at a point on the isochronous stress-strain plot is somewhat larger than on the intersecting constant-strain-rate plot, Shanley [18] has shown that the two approaches are in close agreement.

Comparisons between theory and tests show that the isochronous stress-strain approach in conjunction with Gerard's stability theory provides a reliable assessment of creep buckling loads and times. Gerard and Papirno [20] found excellent agreement between results using isochronous stress-strain curves and test results for the buckling of axially compressed columns of various aluminum alloys, titanium alloys and 17-7PH stainless steel. In fact, results obtained using the isochronous approach were in better agreement with test data than those obtained using the theoretically more correct constant-strain-rate stress-strain curves. Howl and Moore [21] found excellent agreement between tests on externally pressurized stainless steel tubes at 750°C and a theory using the isochronous approach in conjunction with the reduced modulus theory of Timoshenko and Gere [22] for inelastic buckling. Ohya [23] compared analysis results obtained by the method used here with results of several other methods and with creep buckling test results on axially compressed cylindrical shells with four different radius-to-thickness ratios made of aluminum alloy 2024-T4. He found that the method used here provides the most reasonable lower bound to the test data. He also concluded that the simplified approach provided reasonable results for both the axisymmetric and asymmetric buckling modes. More recently, Livingston [24] reviewed test data on creep buckling of cylinders under axial compression and external pressure, specifically to verify the simplified approach used here for development of Code design limits. She compared theoretical results obtained by Samuelson

[25] and Ohya [23] for axially compressed aluminum cylinders with five different radius-to-thickness ratios and to test results obtained by Kaupa [26] for cylinders under external pressure made of Alloy 800. The approach used here based on isochronous stress-strain curves shows varying degrees of conservatism, perhaps excessive based on the Alloy 800 tests, especially for longer buckling times, but she concludes that the method provides conservative, yet reasonable, results.

The critical buckling stresses calculated using the Gerard stability criterion are for geometrically perfect shells. They must be reduced to account for the effects of geometrical imperfections and uncertainty in material properties.

An external pressure chart can be developed from any given isochronous stress-strain chart. Each of the isochronous stress-strain curves for a given time period can be analyzed and converted to an external pressure stress-strain curve using the tangent modulus criterion discussed above. This tedious but straightforward procedure was utilized by the authors to obtain an external pressure chart using, as a basis, the isochronous stress-strain chart in Figure 3 for the 2.25Cr-1Mo steel at 1000°F. The following assumptions were made in developing the external pressure chart at 1000°F.

- The hot tensile stress-strain curve of Figure 3 is normalized to the minimum yield stress given in II-D, Table Y-1 as 23.7 ksi.
- The isochronous stress-strain curves in Figure 3 are based on average data. They were redrawn based on an 80% factor on plastic strain to obtain minimum values, i.e., the individual points on any given curve were lowered in a direction parallel to the Young's modulus of elasticity for that temperature.
- The yield stress is taken from the 0.2% offset of the values obtained in step 1, above.
- Young's modulus of elasticity in Figure 3 was obtained from II-D as 24,700 ksi.
- The proportional limit was taken as 60% of the yield stress in step 2, above.

An external pressure chart was constructed based on the above assumptions and parts of the general criteria in Appendix 3 of ASME II-D. The external pressure chart is shown in Figure 6. The design procedure for axial compression in the creep range using Figure 6 is as follows.

- Calculate Factor *A*:

$$A = \frac{0.125}{R_o/t} K_1 \tag{21}$$

Where:

$$K_1 = \frac{5}{5 - 0.304 \ln T}$$

- Calculate the allowable axial compressive stress from either Figure 6 or Equation (22) as applicable:

$$B = \frac{AE}{2} \tag{22}$$

Where:

T = number of design hours. The minimum value of T is equal to 1.0 hour and the maximum value is equal to 500,000 hours.

The quantity K_1 in equation from Equation (21) adjusts the knock-down factor, α, that is embedded in Equation (21) from a value of 5.0 at the instantaneous average hot tensile curve (shown in Figure 3) down to a value of 1.5 at 100,000 hours. This reduction is justified due to the conservative assumptions made in deriving Equations (21) and (22) as mentioned above. This reduces the total knock-down/design factor from 10 for time-independent buckling to 3 for time-dependent creep buckling at 100,000 hours in a smooth transition. This follows the general approach used in III, Subsection NH, to recognize that creep buckling is less sensitive to initial imperfections with increased buckling times. In Subsection NH, it is required that the critical stress calculation include the effects of initial imperfections. A design factor of 3 is applied for time-independent buckling whereas the factor is reduced to 1.5 for time-dependent buckling. The 1.5 value was derived based on a design factor of 10 on time which was common practice where creep is involved. For a material with an exponent of 3 in the creep Equation (1), a factor of 10 on time equates to a factor of about 1.5 on load. However, this creates a discontinuity in going from time-independent to time-dependent buckling, which we propose to avoid here by making the knock-down factor K_1 a function of time, reducing it to 1.5 at 100,000 hours. This, combined with a design factor of 2 imbedded in factor *B*, gives a total knock-down/design factor of 3 at 100,000 hours.

By comparison with the requirements of Subsection NH, this reduces the knock-down factor for imperfections to 2. This reduction is justified based on the definition of creep buckling used here and the conservatism inherent in the use of isochronous stress-strain relations in place of the constant-strain-rate stress-strain curves. The rationale is described by [4]. In fact, there is a strong consensus among those who have compared the Gerard theory using isochronous stress-strain curves to test data that it is sufficiently conservative to accommodate imperfections in the form of initial ovalities up to 2% without a knock-down factor. However, since there can be other types of imperfections, it seems prudent to us to maintain a minimum knock-down factor of 2 for the long term.

4.4 Design Equations

A recommended procedure for calculating the allowable compressive stress in a cylindrical shell operating in the creep range is:

- Calculate Factor *A* from:

$$A = \frac{0.125}{R_o/t}K_1 \tag{21}$$

- Calculate the allowable axial compressive stress from either Figure 6 or Equation (22) as applicable:

$$B = \frac{AE}{2} \text{ but not greater than } S \tag{22}$$

Where:

$$K_1 = \frac{5}{5 - 0.304\ln T}$$

T = number of operating hours. The minimum value of T is equal to 1.0 hour and the maximum value is 500,000 hours.

4.5 Applications

4.5.1 Example 2

A hydrotreater with a thickness of 9.0 inches is supported by a skirt with the following design data.

- Outside diameter of the skirt is 180 inches
- Thickness of skirt is 2.0 inches

- Total weight of operating vessel = 2,000,000 lbs.
- Design temperature of skirt = 1000°F
- Skirt material is 2.25Cr-1Mo steel S = 20,700 psi at 1000°F for short time
- *E* = 24,700,000 psi S = 6,300 psi at 1000°F for 100,000 hours

Determine the adequacy of the skirt for the following time periods.

- Short time using Figure 6.
- 100,000 hours (a little over 11 years) using Equation (20) with *FS* = 1.5
- 100,000 hours using Figure 6

Solution:

4.5.1.1 Short time solution using Figure 6

Actual stress $\sigma = \dfrac{2,000,000}{\pi(180)(2)} = 1,770$ psi

From Equation (21):

$$K_1 = \frac{5}{5 - 0.3048\ln(1.0)} = 1.0$$

$$A = \frac{0.125}{90/2}(1.0) = 0.0028$$

From Figure 6,

B = 9000 psi. This stress is < 20,700 psi and > 1770 psi. Thus, the skirt is adequate.

It is of interest to note that the *B* value of 9000 psi obtained from Figure 6 at 1000°F is larger than the *B* value of 8000 psi obtained from Figure 5 for 900°F. This difference is due to the fact that Figure 6 is specifically constructed for 2.25Cr-1Mo steel while Figure 5 is a generic chart originally constructed for low carbon steel but is allowed by ASME to be used for 2.25Cr-1Mo steels.

4.5.1.2 100,000 hour solution using Equation (20) with *FS* = 1.0

Equation (20) is based on n = 3.0. From Figure 3 with σ = 4000 psi, a value of ε = 0.00075 is obtained. The value of K' is calculated from Equation (2) as 1.17 x 10^{-14}. Using a value of e/t of 0.0001 to simulate a perfectly round cylinder, Equation (20) yields:

$$B = \frac{1}{1.0}\left[\frac{0.35}{(1.17\times10^{-14})(90/2)}\ln\left(\frac{1.167}{0.0001}\right)\right]^{1/3}$$

B = 12,300 psi > 6,300 psi. Use B = 6,300 psi > 1770 psi.

Thus, the skirt is adequate based on this method.

4.5.1.3 100,000 hour solution using Figure 6

From Equation (21):

$$K_1 = \frac{5}{5 - 0.3048\ln(100,000)} = 3.35$$

$$A = 0.0028(3.35)$$

From Figure 6, B = 2400 psi > 1770 psi.

Thus, the skirt is adequate.

The results in 4.5.1.2 and 4.5.1.3 above show a great disparity. The high allowable stress value obtained from Equation (20) is due to many factors such as:

- The ratio e/t of the cylinder is highly arbitrary and does not have an influence on the results.
- Equation (20) assumes the stress-strain interaction to follow Norton's Equation (2) with a value of n = 3.0. However, the value of n in this case is larger than 3 and the results of Equation (20) become approximate.
- The strain value of 0.0094 corresponds to a stress of 3300 psi at 100,000 hours. This stress is clearly in the plastic region of the external pressure chart well beyond the elastic limit. Equation (20) does not take into consideration stresses in the plastic region nor does it consider primary creep. In fact, Equation (20) is not applicable in this case for strain values greater than 0.00013. This strain value corresponds to an R_o/t ratio of 960 from Equation (21).

Based on the above, it can be stated that the external pressure chart developed from the isochronous curves is more applicable for pressure vessel components than the theoretical Equation (20). However, Equation (20) may be used as a last resort when an isochronous chart is available without the existence of a corresponding external pressure chart.

5 EXTERNAL PRESSURE ON CYLINDRICAL SHELLS

5.1 Theoretical Equations Below the Creep Range

The equations for the lateral buckling of cylindrical shells at temperatures below the creep range used by ASME are based on Sturm's work [9]. Sturm's equation is expressed as:

$$P_{cr} = \kappa E \left(\frac{t}{D_o} \right)^3 \tag{23}$$

Where:

D_o = outside diameter

E = modulus of elasticity

P_{cr} = buckling external pressure

t = thickness

κ = buckling factor that is a function of various parameters such as radius, thickness, length, Poisson's ratio, applied pressure and number of buckling lobes. Figure 7 [27] shows a plot of κ for a simply supported cylindrical shell with external pressure applied on the sides and the ends of the cylinder and a Poisson's ratio of 0.3.

For elastic buckling, Equation (23) can be expressed in terms of critical strain by defining:

$$\varepsilon_{crE} = \frac{\sigma_{cr}}{E} \tag{24}$$

and

$$\sigma_{crE} = \frac{P_{cr}\left(D_o/t\right)}{2} \tag{25}$$

Where:

ε_{crE} = critical circumferential buckling strain

σ_{crE} = critical circumferential buckling stress

Substituting Equations (23) and (25) into Equation (24) results in:

$$\varepsilon_{cr} = \frac{\kappa}{2\left(D_o/t\right)^2} \tag{26}$$

A plot of Equation (26) is shown in Figure 8. The D_o/t lines in Figure 8 are approximate, "smoothed" envelopes of those shown in Figure 7. The ASME procedure for calculating allowable external pressure consists of calculating D_o/t and L/D_o values of a cylinder and then using Figure 8 to obtain Factor A. From this factor, a value of B is obtained from Figure 5. The justification for constructing and using Figure 5 was explained in Section 4.1.2. The allowable external pressure is calculated from:

$$P_a = \frac{4B}{3\left(D_o/t\right)} \tag{27}$$

If the value of A falls to the left of the lines in Figure 5, then the allowable external pressure is calculated from the following equation:

$$P_a = \frac{2AE}{D_o/t} \tag{28}$$

Experimental data have shown [10] that buckling may occur at a slightly lower value than that calculated from Equation (23). Accordingly, the ASME code uses a knock-down factor of $\alpha = 1.0$ and a design factor $DF = 3.0$.

5.2 Theoretical Equations in the Creep Range

Buckling of cylindrical shells due to external pressure in the creep range is influenced by numerous variables such as initial out-of-roundness, final out-of-roundness, time duration and changes in modulus of elasticity and yield stress due to creep. These variables, coupled with known variables such as length, diameter, thickness and buckling modes, make a closed-form solution of the buckling equations very difficult to obtain, if not impossible. Hoff [28] derived a creep buckling equation for a cylindrical shell under external pressure in the creep range. Some of his assumptions are:

- Norton's Equation (1) is applicable
- The length of the cylinder is assumed to be much larger than the radius
- The first buckling mode is elliptical in shape
- The shell is assumed to have a slight initial out-of-roundness
- A representative value of $n = 3.0$ was used to obtained a simplified closed form solution.

Hoff came up with a number of lengthy differential equations using static and compatibility equations. By various substitutions and simplifications, he combined the ten equations into one equation for a long cylinder subjected to a primary buckling mode of an elliptical shape and a value of $n = 3.0$ as an average number. The derived equation is:

$$P_{cr} = \left[\frac{\ln\left(1 + 4.5/X^2\right)}{24k'T\left(R_o/t\right)^5}\right]^{1/3} \tag{29}$$

Where:

e = initial deviation of cylinder from true shape. The value of e is expressed as a fraction of thickness.

k' = constant from Norton's Equation (1)

P_{cr} = critical external pressure

R_o = outside radius of shell

t = shell thickness

T = time in hours

$X = 3.5(e/t)R_o$

Equation (28) can be rewritten in terms of design pressure as:

$$P_a = \frac{1}{FS}\left[\frac{\ln\left(1+4.5/X^2\right)}{24K'\left(R_o/t\right)^5}\right]^{1/3} \tag{30}$$

Where:

P_a = allowable external pressure

FS = factor of safety

$K' = k'T$

5.3 Approximate Method Using Isochronous Stress-Strain Curves

An alternate method of designing cylindrical shells subjected to external pressure is to use the isochronous curves of Section III-NH to simulate the effect of creep and time on buckling as explained earlier for axial compression. This procedure is used to determine the allowable external pressure in ASME components in the creep range. The procedure consists of calculating D_o/t and L/D_o values of a cylinder and then using Figure 8 to obtain Factor *A*. The *B* value is obtained from Figure 6 and the allowable external pressure is calculated from:

$$P_a = \frac{4BK_2}{3\left(D_o/t\right)} \tag{31}$$

Where:

$$K_2 = \frac{3}{3-0.1303\ln T}$$

and

T = number of operating hours. The minimum value of T is equal to 1.0 hour and the maximum value is equal to 500,000 hours.

For cylinders under external pressure the knock-down factor, α, in II-D is taken as 1.0 and the design factor, *DF*, is taken as 3.0. The quantity K_2 from Equation (31) adjusts the design factor that is embedded in Equation (31) from a value of 3.0 on the instantaneous average hot tensile curve of Figure 3 to a value of 1.5 at 100,000 hours. This is consistent with the requirements of III-NH except that the design factor, 3.0, for time-independent buckling is reduced logarithmically with time to 1.5 at 100,000 hours rather than abruptly with the consideration of creep as in III-NH.

If the value of *A* falls to the left of the lines in Figure 6, then the allowable external pressure is calculated from the following equation:

$$P_{cr} = \frac{2AEK_2}{D_o/t} \tag{32}$$

5.4 Design Equations

A recommended procedure for calculating the allowable external pressure on a cylindrical shell operating in the creep range is:

- Calculate D_o/t and L/D_o values for the cylinder and then obtain Factor *A* from Figure 8.
- Using this Factor, obtain a value of *B* from Figure 6.
- Calculate the allowable external pressure from:

$$P_a = \frac{4BK_2}{3(D_o/t)} \quad (31)$$

If the value of A falls to the left of the lines in Figure 6, then the allowable external pressure is calculated from the following equation

$$P_{cr} = \frac{2AEK_2}{D_o/t} \quad (32)$$

Where:

$$K_2 = \frac{3}{3 - 0.1303 \ln T}$$

T = number of operating hours. The minimum value of T is equal to 1.0 hour and the maximum value is equal to 500,000 hours.

5.5 Applications

5.5.1 Example 3

A pressure vessel is subjected to an external pressure of 15 psi. The design data are as follows.

- Outside diameter of the shell is 12 ft.
- Effective length of shell is 30 ft.
- Shell thickness = 1.0 inch
- Shell material is 2.25Cr-1Mo steel.

Determine the allowable external pressure for the shell based on the following time periods: short time using Figure 6, 100,000 hours (a little over 11 years) using Equation (30) with FS = 1.5 and 100,000 hours using Figure 6.

5.5.1.1 Short time solution using Figure 6

L/D_o = 30/12 = 2.5, D_o/t = 144/1.0 = 144

From Figure 8, A $\approx$ 0.0003 and from Figure 6, B = 3700 psi.

From Equation (31):

$$K_2 = \frac{3}{3 - 0.1303 \ln(1.0)} = 1.0$$

$$P_a = \frac{4(3{,}700)(1.0)}{3(144)} = 34 \text{ psi} > 15 \text{ psi}$$

Thus, t = 1.0 inch is adequate

It is of interest to note that the B value of 3700 psi obtained from Figure 6 at 1000°F is larger than the B value of 3000 psi obtained from Figure 5 for 900°F. This difference is due to the fact that Figure 6 is specifically constructed for 2.25Cr-1Mo steel while Figure 5 is a generic chart originally constructed for low carbon steel but is allowed by ASME to be used for 2.25Cr-Mo steels.

5.5.1.2 100,000 hour solution using Equation (30)

From Equation (30) with $K' = 1.17 \times 10^{-14}$ and $X = 3.5(0.0001)(72) = 0.025$,

$$P_a = \frac{1}{1.5}\left[\frac{\ln\left(1+4.5/0.000625\right)}{24\left(1.17\times10^{-14}\right)\left(72/1\right)^5}\right]^{1/3} = 16.9 \text{ psi}$$

$P_a = 16.9$ psi > 15 psi

Thus, t = 1.0 inch is adequate

5.5.1.3 100,000 hour solution using Figure 6

$L/D_o = 30/12 = 2.5$, $D_o/t = 144/1.0 = 144$

From Figure 8, $A \approx 0.0003$ and from Figure 6, B = 2100 psi

From Equation (31):

$$K_2 = \frac{3}{3 - 0.1303\ln\left(100{,}000\right)} = 2.0$$

$$P_a = \frac{4\left(2{,}100\right)\left(2.0\right)}{3\left(144\right)} \approx 39 \text{ psi} > 15 \text{ psi}$$

Thus, t = 1.0 inch is adequate.

The results in 5.5.1.2 and 5.5.1.3 above show a great disparity similar to the results obtained for axial compression in Section 5. Again, the difference is due, in part, to the following factors.

- The ratio e/t of the cylinder is highly arbitrary and does have an influence on the results.
- Equation (29) assumes the stress-strain interaction to follow Norton's Equation (2) with a value of n = 3.0. However, the value of n in this case is larger than 3 and the results of Equation (29) become approximate.
- The strain value of 0.0003 corresponds to a stress of 1800 psi at 100,000 hours. This stress is clearly in the plastic region of the external pressure chart well beyond the elastic limit. Equation (29) does not take into consideration stresses in the plastic regions. In fact, Equation (29) is not applicable in this case for strain values greater than 0.00013. This strain value corresponds to a D_o/t ratio of about 300 from Figure 8.
- Equation (29) is based on cylinders with a large L/D_o ratio and elliptic buckled mode. In this case, the L/D_o equal to 2.5 and, from Figure 7, the number of buckled modes is 4. Hence, the buckling strength is much higher than that predicted by Equation (29).

Based on the above, it can be stated that, using external pressure chart, Figure 6, developed from the isochronous curves, is more accurate than the results obtained from the theoretical Equation (30).

6 EXTERNAL PRESSURE ON SPHERICAL SHELLS

6.1 Theoretical Equations Below the Creep Range

The theoretical equation for the elastic buckling stress of a spherical shell [29] is:

$$\sigma_{cr} = \frac{0.154E}{R_o/t} \tag{33}$$

or, in terms of strain:

$$\varepsilon_{cr} = \frac{0.154}{R_o/t} \tag{34}$$

ASME uses a knock-down factor, α, of approximately 1.25. Equation (34) is then rewritten as:

$$\varepsilon_{cr} = \frac{0.125}{R_o/t} \tag{35}$$

Substituting the expression $\sigma_{cr} = P_{cr}R_o/2t$, and $\sigma_{cr} = E\varepsilon_{cr}$ into Equation (35) gives:

$$P_{cr} = \frac{0.25E}{(R_o/t)^2} \tag{36}$$

Experimental data have shown [10] that buckling may occur at a value of one-half of that calculated from Equation (36). Accordingly, the ASME code uses a design factor *DF* = 4.0. Hence, the allowable external pressure in the elastic range is obtained from Equation (36) as:

$$P_{cr} = \frac{0.0625E}{(R_o/t)^2} \tag{37}$$

The ASME procedure for calculating allowable external pressure on spherical shells consists of calculating Factor *A* from:

$$A = \frac{0.125}{R_o/t} \tag{38}$$

From this factor, a value of *B* is obtained from Figure 5 and the allowable external pressure is calculated from:

$$P_a = \frac{B}{R_o/t} \tag{39}$$

If the value of *A* falls to the left of the lines in Figure 5, then the allowable external pressure is calculated from Equation (36).

It is of interest to note that the knock-down factor, α, of 1.25 and the design factor, *DF*, of 4.0 result in an overall factor of 5.0 for spherical shells.

6.2 Theoretical Equations in the Creep Range

The applicable equation in the creep range is derived by Xirouchakis [15] as:

$$\sigma_c = \frac{1}{FS}\left[\frac{0.353}{K'\left(R_o/t\right)}\ln\left(\frac{0.875}{e/t}\right)\right]^{1/3} \tag{40}$$

6.3 Approximate Method Using Isochronous Stress-Strain Curves

An alternate method of designing spherical shells subjected to external pressure is to use the isochronous curves of Section III-NH to simulate the effect of creep and time on buckling as explained earlier for axial compression. The procedure for calculating allowable external pressure on spherical shells consists of calculating Factor A from the equation:

$$A = \frac{0.125}{R_o/t} \tag{38}$$

From this factor a value of B is obtained from Figure 6 and the allowable external pressure is calculated from:

$$P_a = \frac{BK_3}{R_o/t} \tag{41}$$

Where:

$$K_3 = \frac{4}{4 - 0.1737\ln T}$$

T = number of operating hours. The minimum value of T is equal to 1.0 hour and the maximum value is equal to 500,000 hours.

The quantity K_3 adjusts the design factor, DF, that is embedded in Equation (41) from a value of 4.0 at the instantaneous average hot tensile curve in Figure 3, to a value of 2.0 at 100,000 hours. These factors are comparable to the Design Factors of 3.0 for the hot tensile curve and 1.5 for creep in III-NH since the knock-down factor for imperfections is included in Equation (41). The increase in Design Factor from 3.0 to 4.0 is justified based on the increased sensitivity of spherical shells to imperfections.

If the value of A falls to the left of the lines in Figure 6, then the allowable external pressure is calculated from the equation

$$P_a = \frac{0.0625EK_3}{\left(R_o/t\right)^2} \tag{42}$$

6.4 Design Equations

A recommended procedure for calculating allowable external pressure on spherical shells operating in the creep range is

- Calculating Factor A from the equation:

$$A = \frac{0.125}{R_o/t} \tag{38}$$

- Using this Factor, obtain a value of B from Figure 6
- Calculate the allowable external pressure from:

$$P_a = \frac{BK_3}{R_o/t} \tag{41}$$

Where:

$$K_3 = \frac{4}{4 - 0.1737\ln T}$$

T = number of operating hours. The minimum value of T is equal to 1.0 hour and the maximum value is equal to 500,000 hours.

If the value of A falls to the left of the lines in Figure 6, then the allowable external pressure is calculated from the equation:

$$P_a = \frac{0.0625EK_3}{(R_o/t)^2} \tag{42}$$

6.5 Applications

The following example serves as an illustration.

6.5.1 Example 4

A pressure vessel is subjected to an external pressure of 15 psi at 1000°F. The design data are as follows.

- Outside diameter of the vessel is 12 ft.
- Spherical head thickness = 0.50 inch
- Shell material is 2.25Cr-1Mo steel

Determine the allowable external pressure for the spherical head from Figure 6 based on the following time periods: short time and 100,000 hours.

6.5.1.1 Short time solution

R_o/t = 72/0.5 = 144

From Equation (38) A = 0.125/144 = 0.00087, from Figure 6, B = 7800 psi

From Equation (41):

$$K_3 = \frac{4}{4 - 0.1737\ln(1.0)} = 1.0$$

$$P_a = \frac{7{,}100(1.0)}{144} = 49 \text{ psi} > 15 \text{ psi}$$

Thus, t = 0.5 inch is adequate.

6.5.1.2 100,000 hour solution

From Figure 6, B = 1800 psi

From Equation (41):

$$K_3 = \frac{4}{4 - 0.1737\ln(100{,}000)} = 2.0$$

$$P_a = \frac{1{,}800(2.0)}{144} \approx 25 \text{ psi} > 15 \text{ psi}$$

Thus, t = 0.5 inch is adequate.

7 EXTERNAL PRESSURE ON CONICAL SHELLS

The procedure listed in Section 5 for external pressure on cylindrical shell is applicable to conical shells with the proper selection of the effective cone length, diameter and thickness [30] is listed in the applicable ASME code section.

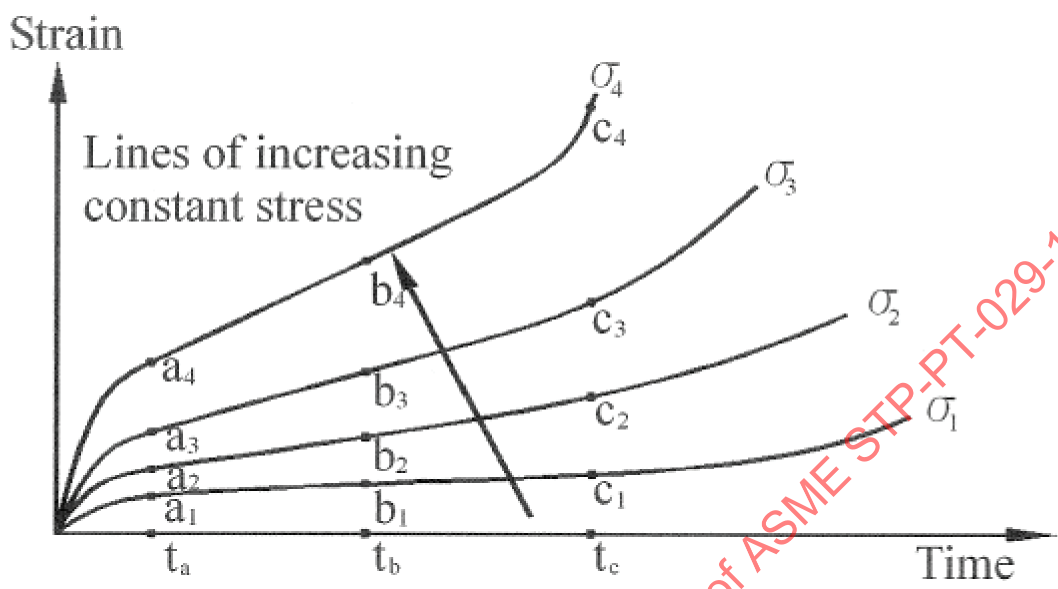

Figure 1 - Creep Curves Conventionally Plotted as Strain vs Time at Constant Stress.

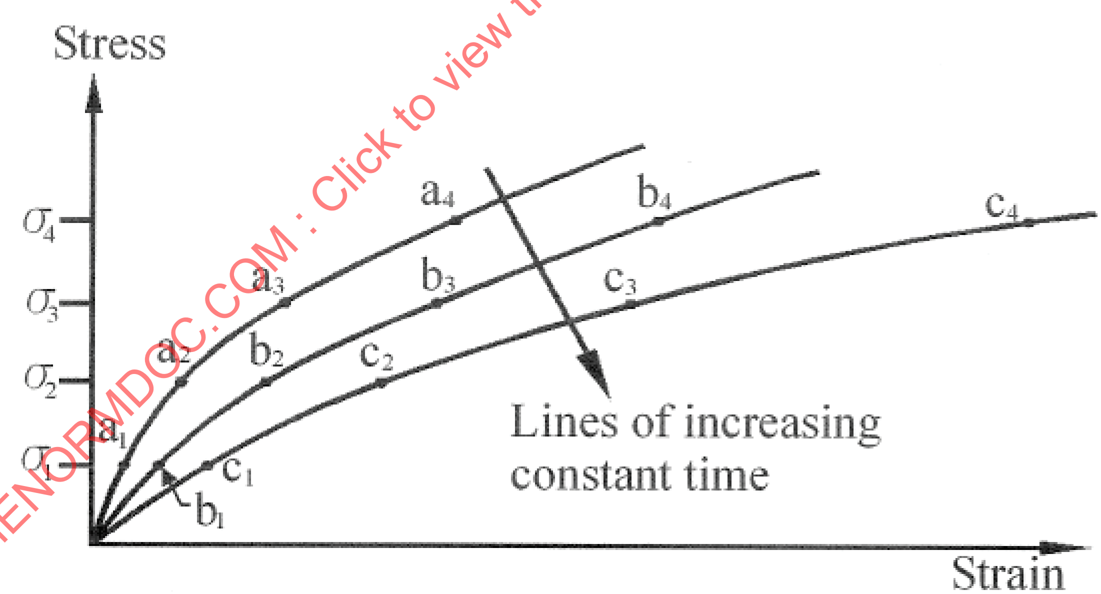

Figure 2 - Resultant Stress-Strain Curves Plotted as Stress vs Strain at Constant Time.

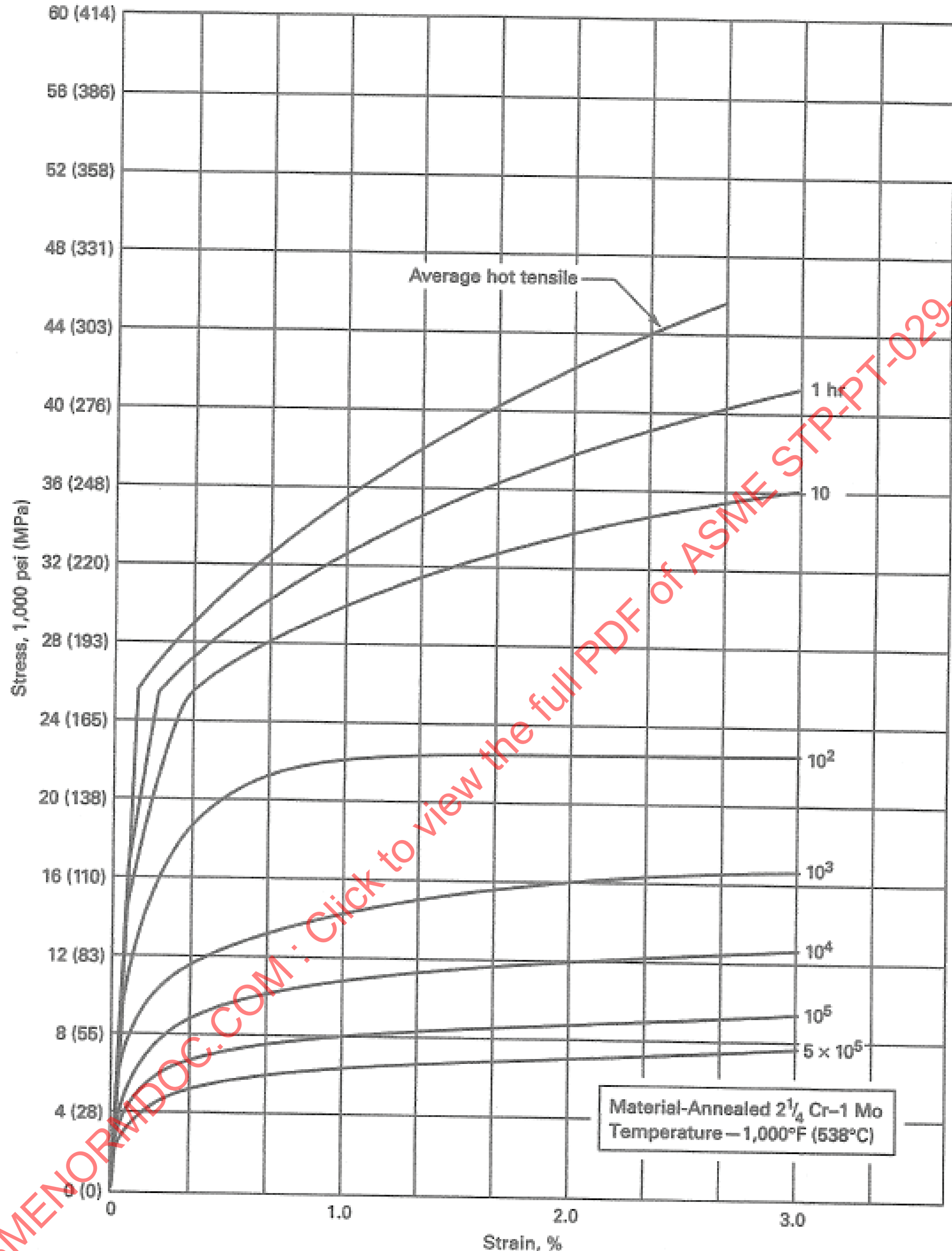

Figure 3 - Isochronous Stress-Strain Curves for 2.25Cr-1Mo Steel at 1000°F (ASME).

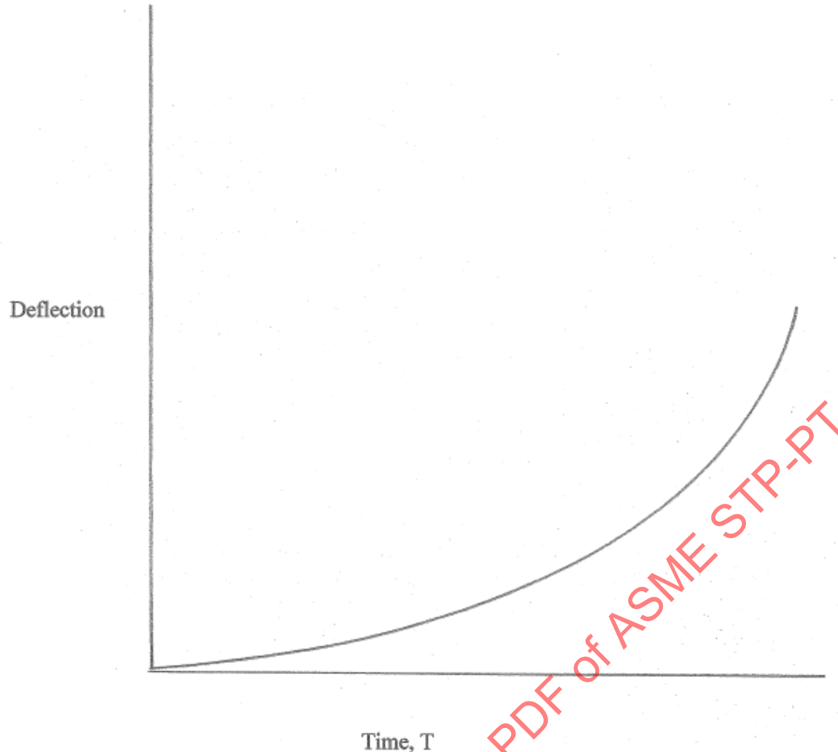

Figure 4 - Deflection in the Creep Range.

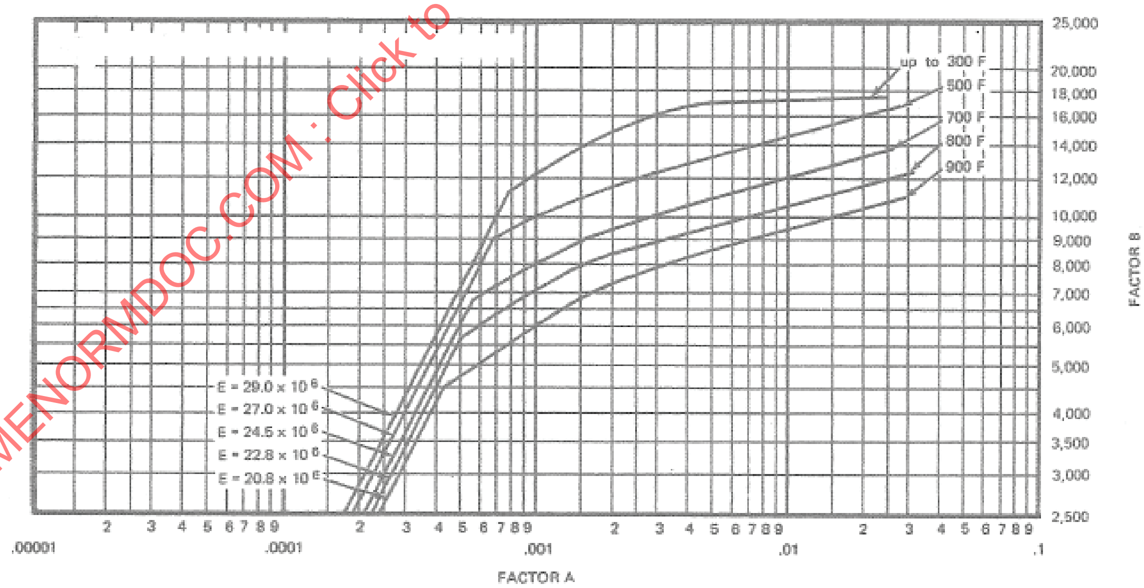

Figure 5 - External Pressure Chart for Carbon and Low-Alloy Steels with Yield Stresses of 30 ksi and Higher (ASME).

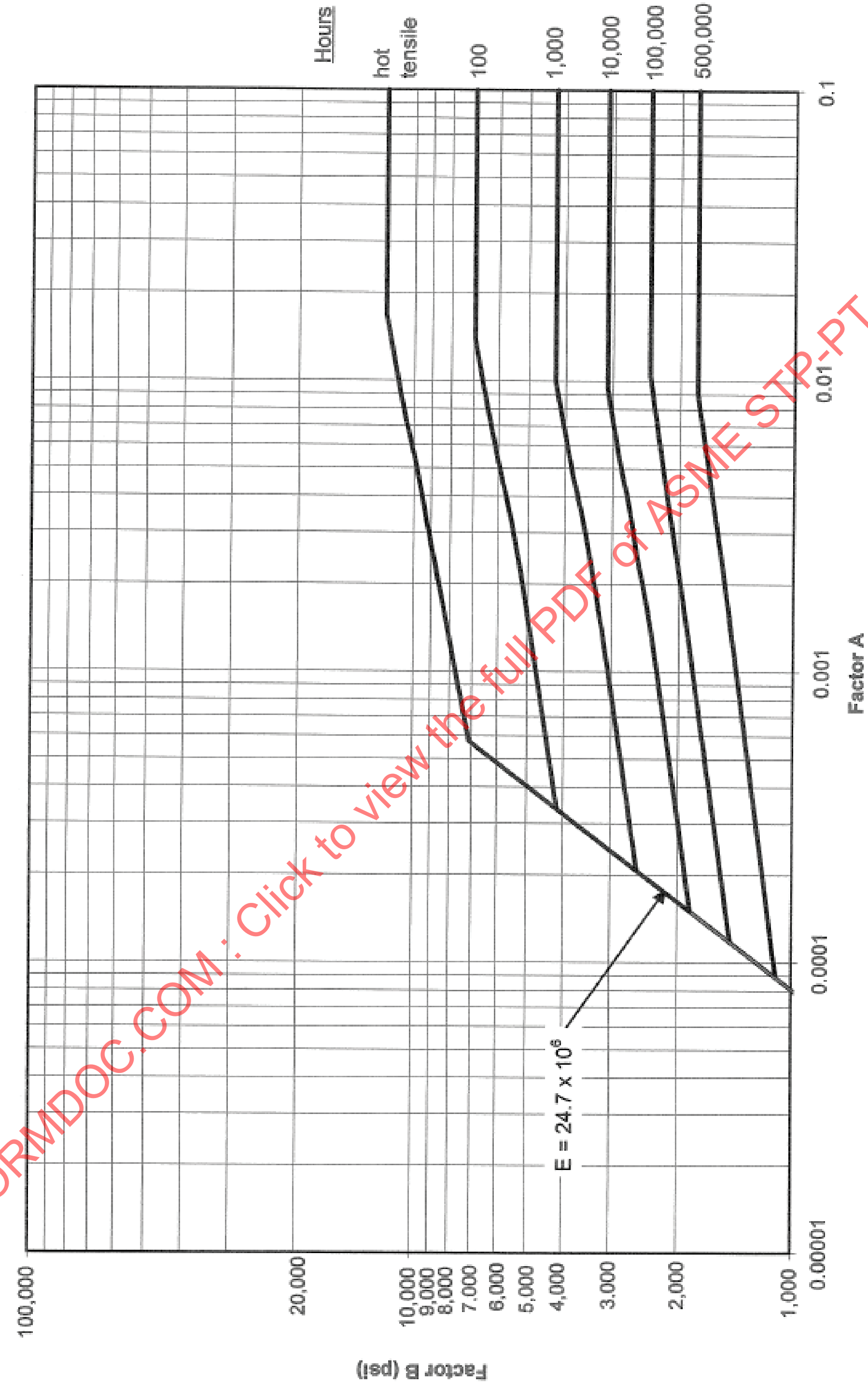

Figure 6 - External Pressure Chart for 2.25Cr-1Mo Steel at 1000°F.