ASME PTB-4-2021

ASME Section VIII
Division 1
Example Problem Manual

ASME
SETTING THE STANDARD

PTB-4-2021

ASME Section VIII – Division 1 Example Problem Manual

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FOREWORD TO THE THIRD EDITION

This document is the third edition of the ASME Section VIII – Division 1 Example Problem Manual. The purpose of this third edition is to update the example problems to keep current with the changes incorporated into the 2021 edition of the ASME B&PV Code, Section VIII, Division 1. The example problems included in the second edition of the manual were based on the contents of the 2013 edition of the B&PV Code.

Known corrections to paragraph changes and references, design equations, and calculation results have been made in this third edition. Additionally, some formatting modifications were made to facilitate better use of the example manual, as applicable.

FOREWORD TO THE SECOND EDITION

This document is the second edition of the ASME Section VIII – Division 1 example problem manual. The purpose of this second edition is to update the example problems to keep current with the changes incorporated into the 2013 edition of the ASME B&PV Code, Section VIII, Division 1. The example problems included in the first edition of the manual were based on the contents of the 2010 edition of the B&PV Code. In 2011, ASME transitioned to a two year publishing cycle for the B&PV Code without the release of addenda. The release of the 2011 addenda to the 2010 edition was the last addenda published by ASME and numerous changes to the Code were since adopted.

This second edition of the example manual includes two new sections covering examples for tube–to–tubesheet welds and required markings of pressure vessel nameplates. Known corrections to design equations and results have also been made in this second edition. Additionally, some formatting modifications were made to facilitate better use of the example manual, as applicable.

FOREWORD

This document is the Division 1 Example Problem Manual. In this manual, example problems are solved using both the Division 1 and Division 2 rules. When the design rule is the same, the example problem is solved using the Division 2 rules with the Division 1 allowable stress and weld joint efficiency. With this approach, users of Division 1 will become familiar and adept at using Division 2, and this will also provide a significant training benefit to the Division 1 user in that Division 2 has been designed as the home for the common rules' initiative being undertaken by the ASME Section VIII Committee.

In 2007, ASME released a new version of the ASME B&PV Code, Section VIII, Division 2. This new version of Division 2 incorporated the latest technologies to enhance competitiveness and is structured in a way to make it more user-friendly for both users and the committees that maintain it. In addition to updating many of the design-by-analysis technologies, the design-by-rule technologies, many adopted from the Division 1 rules, were modernized. ASME has issued *ASME Section VIII – Division 2 Criteria and Commentary, PTB-1-2009* that provides background and insight into design-by-analysis and design-by-rule technologies.

The ASME Section VIII Committee is currently undertaking an effort to review and identify common rules contained in the Section VIII Division 1, Division 2, and Division 3 B&PV Codes. In this context, common rules are defined as those rules in the Section VIII, Division 1, Division 2, and Division 3 Codes that are identical and difficult to maintain because they are computationally or editorially complex, or they require frequent updating because of the introduction of new technologies. Common rules typically occur in the design-by-rule and design-by-analysis parts of the code; but also exist in material, fabrication, and examination requirements. A plan has been developed to coordinate common rules with the following objectives.

- Common rules in the Section VIII Division 1, 2, and 3 codes should be identical and updated at the same time to ensure consistency.
- Common rules will be identified and published in a single document and referenced by other documents to; promote user-friendliness, minimize volunteer time on maintenance activities, and increase volunteer time for incorporation of new technologies to keep the Section VIII codes competitive and to facilitate publication.
- Core rules for basic vessel design such as wall thickness for shells and formed heads, nozzle design, etc. will be maintained in Division 1; although different from Division 2 these rules are time-proven and should remain in Division1 because they provide sufficient design requirements for many vessels.
- ASME Section VIII Committee recognizes that Division 2 is the most technically advanced and best organized for referencing from the other Divisions and recommends that, with the exception of overpressure protection requirements, common rules identified by the committee shall reside in Division 2 and be referenced from Division 1 and Division 3, as applicable.

As a starting point for the common rules' initiative, the ASME Section VIII Committee has developed Code Case 2695 to permit the use of some the design-by-rule procedures in Division 2 to be used for Division 1 construction.

As part of the common rules' initiative, the ASME Section VIII Committee is working with ASME LLC to create separate example problem manuals for each Division. These manuals will contain problem examples that illustrate the proper use of code rules in design. The *ASME Section VIII - Division 2 Example Problem Manual, PTB-3 2009* has been completed and issued.

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PART 1

GENERAL REQUIREMENTS

1.1 Introduction

ASME B&PV Code, Section VIII, Division 1 contains mandatory requirements, specific prohibitions, and non-mandatory guidance for the design, materials, fabrication, examination, inspection, testing, and certification of pressure vessels and their associated pressure relief devices.

1.2 Scope

Example problems illustrating the use of the design-by-rule methods in ASME B&PV Code, Section VIII, Division 1 are provided in this document. Example problems are provided for most of the calculation procedures in either SI or US Customary units.

1.3 Definitions

The following definitions are used in this manual.

1) VIII-1 – ASME B&PV Code, Section VIII, Division 1, 2021
2) VIII-2 – ASME B&PV Code, Section VIII, Division 2, 2021

1.4 Organization and Use

An introduction to the example problems in this document is described in Part 2 of this document. The remaining Parts of this document contain the example problems. All paragraph references without a code designation, i.e., VIII-1 or VIII-2, see References, are to the ASME B&PV Code, Section VIII, Division 1, 2021 [1], or the ASME B&PV Code, Section VIII, Division 1, 2021 [2], respectively.

The example problems in this manual follow the design by rule methods in ASME B&PV Code, Section VIII, Division 1. Many of the example problems are also solved using ASME B&PV Code, Section VIII, Division 2 design-by-rule procedures contained in Part 4 of this Code using the allowable stress from VIII-1. In addition, where the design rules are the same, the VIII-2 format has been used in this example problem manual because of the user-friendliness of these rules.

1.5 Comparison of VIII-1 and VIII-2 Design Rules

Since many of the design rules in VIII-2 were developed using the principles of VIII-1, it is recommended that users of this manual obtain a copy of ASME PTB-1-2013 [2] that contains the VIII-2 criteria and commentary on the technical background to these rules. A comparison of the design-by-rule procedures in VIII-2 compared with VIII-1 is shown in Table E1.1.

1.6 Mandatory Appendix 46 (Supersedes ASME Code Case 2695)

In recognition of the similarities and the use of the latest technology in developing the design-by-rule part of VIII-2, ASME initially issued Code Case 2695 in September 2011 which permits the use of VIII-2 design rules with VIII-1 allowable stresses with some limitations. Code Case 2695 is shown in Table E1.2. However, as part of the ASME Task Group U-2(g), Code Case 2695 was modified and incorporated into the 2019 VIII-1, Mandatory Appendix 46. The reference to Mandatory Appendix 46 is made in the re-written VIII-1, paragraph U-2(g) which permits the use of alternative means when design rules do not exist in VIII-1.

Mandatory Appendix 46 is applicable when using VIII-2 to establish the thickness and other design details of a component for an VIII-1 pressure vessel. In addition to the traditional use of Code Case 2695, Mandatory Appendix 46 permits the use of the Part 5 Design-by-Analysis rules of VIII-2, with additional guidance and restrictions.

1.7 VIII-2 Vessel Classes

The 2017 edition of VIII-2 introduced a two–class vessel structure in an attempt to attract more users to VIII–2. The format of the two–class structure assigns a Class 1 vessel a design margin of 3.0 on the ultimate tensile strength (UTS), to be consistent with pre-2007 edition of VIII-2 philosophy, while maintaining the 2.4 design margin on UTS for a Class 2 vessel. Class 1 vessels shall use the allowable stresses published in ASME B&PV Code, Section II, Part D, Subpart 1, Table 2A or Table 2B. Class 2 vessels shall use the allowable stresses published in ASME B&PV Code, Section II, Part D, Subpart 1, Table 5A or Table 5B.

The design and construction of a Class 1 vessel is permitted under the following limitations and relaxation of rules as compared to a Class 2 vessel.

- Design margin of 3.0 on UTS,
- The User's Design Specification (UDS) requires certification only when a fatigue analysis is mandated in the UDS,
- The Manufacturer's Design Report (MDR) requires certification only when the following are performed:
    - Fatigue analysis,
    - Use of Part 5 Design-by-Analysis to determine thickness of pressure parts when design rules are not provided in Part 4 Design-by-Rule,
    - Use of Part 4.8 to design quick-actuating closures, or
    - Dynamic seismic analysis.
- Part 5 DBA methods shall not be used in lieu of Part 4 DBR, and
- All other aspects of construction including materials, fabrication, examination, and testing shall be in accordance with the applicable parts of VIII–2.

1.8 References

[1] ASME B&PV Code, Section VIII, Division 1, *Rules for Construction of Pressure Vessels*, 2021, ASME, New York, NY, 2021.

[2] ASME B&PV Code, Section VIII, Division 2, *Rules for Construction of Pressure Vessels – Alternative Rules*, 2021, ASME, New York, NY, 2021.

[3] Osage, D., *ASME Section VIII – Division 2 Criteria and Commentary*, PTB-1-2013, ASME, New York, NY, 2013.

1.9 Tables

Table E1.1 – Comparison of Design Rules Between VIII-2 and VIII-1

Paragraph in Section VIII, Division 2	Comments Pertaining to Section VIII, Division 1
4.1	General Requirements, harmonized with VIII-1, i.e. MAWP introduced, etc.
4.2	Design Rules for Welded Joints, a restrictive subset of rules in VIII-1, UG & UW
4.3	Design Rules for Shells Under Pressure, mostly new technology
4.4	Design Rules for Shells Under External Pressure and Allowable Compressive Stresses, almost identical to CC2286 with exception of stiffening ring requirements at cone-to-cylinder junctions
4.5	Design Rules for Shells Openings in Shells and Heads, new technology
4.6	Design Rules for Flat Heads, identical to UG-34
4.7	Design Rules for Spherically Dished Bolted Covers, identical to Appendix 1-6 and Appendix 14 except Soehren's stress analysis method for Type 6D Heads is included
4.8	Design Rules for Quick Actuating (Quick Opening) Closures, identical to UG-35.2
4.9	Design Rules for Braced and Stayed Surfaces, a restrictive subset of rules in paragraph UG-47(a)
4.10	Design Rules for Ligaments, identical to paragraph UG-53
4.11	Design Rules for Jacketed Vessels, a more restrictive subset of rules in Appendix 9
4.12	Design Rules for Non-circular vessels, identical to Appendix 13 but re-written for clarity
4.13	Design Rules for Layered Vessels, identical to Part ULW
4.14	Evaluation of Vessels Outside of Tolerance, new technology per API 579-1/ASME FFS-1
4.15	Design Rules for Supports and Attachments, new for VIII-2 using existing technology
4.16	Design Rules for Flanged Joints, almost identical to Appendix 2
4.17	Design Rules for Clamped Connections, identical to Appendix 24
4.18	Design Rules for Shell and Tube Heat Exchangers, identical to Part UHX
4.19	Design Rules for Bellows Expansion Joints, identical to Appendix 26
4.20	Design Rules for Flexible Shell Element Expansion Joints, identical to Appendix 5
4.21	Tube-to-Tubesheet Joint Strength, identical to UW-20

Notes:
1. During the VIII-2 re-write project, an effort was made to harmonize the design-by-rule requirements in VIII-2 with VIII-1. As shown in this table, based on this effort, the design rules in VIII-2 and VIII-1 are either identical or represent a more restrictive subset of the design rules in VIII-1.
2. In the comparison of code rules in presented in this table, the term identical is used but is difficult to achieve and maintain because of coordination of ballot items on VIII-1 and VIII-2. There may be slight differences, but the objective is to make the design rules identical. The restrictive subset of the rules in VIII-1 was introduced in VIII-2 mainly in weld details. In general, it was thought by the committee the full penetration welds should be used in most of the construction details of a VIII-2 vessel.

Table E1.2 – ASME BPV Code Case 2695

Approval Date: September 26, 2011

Code Cases will remain available for use until annulled by the applicable Standards Committee.

Case 2695
Allowing Section VIII, Division 2 Design Rules to Be Used for Section VIII, Division 1 Pressure Vessel
Section VIII, Division 1; Section VIII, Division 2

Inquiry: Under what conditions may the design-by-rule requirements in Part 4 of Section VIII, Division 2 be used to design the components for a Section VIII, Division 1 pressure vessel?

Reply: It is the opinion of the Committee that the design-by-rule requirements in Part 4 of Section VIII, Division 2 may be used to design the components for a Section VIII, Division 1 pressure vessel, provided the following conditions are met:

(a) The allowable design tensile stress shall be in accordance with UG-23 of Section VIII, Division 1.

(b) The weld joint efficiency shall be established in accordance with UW-11 and UW-12 of Section VIII, Division 1.

(c) Material impact test exemptions shall be in accordance with the rules of Section VIII, Division 1.

(d) If the thickness of a shell section or formed head is determined using Section VIII, Division 2 design rules, the following requirements apply:

(1) For design of nozzles, any nozzle and its reinforcement attached to that shell section or formed head shall be designed in accordance with Section VIII, Division 2.

(2) For conical transitions, each of the shell elements comprising the junction and the junction itself shall be designed in accordance with Section VIII, Division 2.

(3) For material impact test exemptions, the required thickness used in the coincident ratio defined in Section VIII, Division 1 shall be calculated in accordance with Section VIII, Division 2.

(e) The fatigue analysis screening in accordance with Part 4, para. 4.1.1.4 of Section VIII, Division 2 is not required. However, it may be used when required by UG-22 of Section VIII, Division 1.

(f) The provisions shown in Part 4 of Section VIII, Division 2 to establish the design thickness and/or configuration using the design-by-analysis procedures of Part 5 of Section VIII, Division 2 are not permitted.

(g) The Design Loads and Load Case Combinations specified in Part 4, para. 4.1.5.3 of Section VIII, Division 2 are not required.

(h) The primary stress check specified in Part 4, para. 4.1.6 of Section VIII, Division 2 is not required.

(i) Weld joint details shall be in accordance with Part 4, para. 4.2 of Section VIII, Division 2 with the exclusion of Category E welds.

(j) The fabrication tolerances specified in Part 4, paras. 4.3 and 4.4 of Section VIII, Division 2 shall be satisfied. The provision of evaluation of vessels outside of tolerance per Part 4, para. 4.14 of Section VIII, Division 2 is not permitted.

(k) The vessel and vessel components designed using these rules shall be noted on the Manufacturer's Data Report.

(l) All other requirements for construction shall comply with Section VIII, Division 1.

(m) This Case number shall be shown on the Manufacturer's Data Report.

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PART 2

EXAMPLE PROBLEM DESCRIPTIONS

2.1 General

Example problems are provided for;

- Part 3 – Materials Requirements
- Part 4 – Design by Rule Requirements parts in Section VIII, Division 1
- Part 5 – Design by Analysis
- Part 6 – Fabrication Requirements
- Part 7 – Examination Requirements
- Part 8 – Pressure Testing Requirements

A summary of the example problems provided is contained in Table E2.1

2.2 Example Problem Format

In all of the example problems, with the exception of tubesheet design rules in paragraph 4.18, the code equations are shown with symbols and with substituted numerical values to fully illustrate the use of the code rules. Because of the complexity of the tubesheet rules, only the results for each step in the calculation producer is shown.

If the design rules in VIII-1 are the same as those in VIII-2, the example problems are typically solved using the procedures given in VIII-2 because of the structured format of the rules, i.e., a step-by-step procedure is provided. When this is done, the paragraphs containing rules are shown for both VIII-1 and VIII-2.

2.3 Calculation Precision

The calculation precision used in the example problems is intended for demonstration proposes only; any intended precision is not implied. In general, the calculation precision should be equivalent to that obtained by computer implementation, rounding of calculations should only be done on the final results.

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PART 3

MATERIALS REQUIREMENTS

3.1 Commentary on Rules to Establish the Minimum Design Metal Temperature (MDMT)

Requirements for low temperature operation for vessels and vessel parts constructed of carbon and low alloy steels are provided in paragraphs UCS-66, UCS-67, and UCS-68. The organization of the requirements is as follows:

Paragraph UCS-66 – provides rules for exemption of impact test requirements for carbon and low alloy steel base material listed in Part UCS.

Paragraph UCS-67 – provides rules for exemption of impact test requirements for welding procedures.

Paragraph UCS-68 – provides supplemental design rules for carbon and low alloy steels with regard to Weld Joint Categories, Joint Types, post weld heat treatment requirements, and allowable stress values.

Paragraph UCS-66(a) provides impact test exemption rules based on a combination of material specification, governing thickness, and required MDMT using exemption curves. The rules are applicable to individual components and welded assemblies comprised of two or more components with a governing thickness. Welded, nonwelded, and cast components are covered with limitation of the exemption rules based on thickness.

Paragraph UCS-66(b) provides for an additional reduction of temperature for impact test exemption based on a temperature reduction curve and a coincident ratio defined simply as the required thickness to the nominal thickness. The coincident ratio can also be determined using a pressure or stress ratio, i.e., design pressure to MAWP or calculated stress to allowable stress.

There are significant changes to paragraphs UCS-66(a), UCS-66(b), and UCS-66(c) in the 2021 edition of VIII–1. A brief summary of these changes follows.

1) Paragraph UCS-66(a) – The material classifications (Impact Test Exemption Curves) found in Figure UCS-66 for A/SA-105 flanges have been changed to address the concern of Charpy Impact Test energy values when subject to ambient and low temperatures. The changes are made to the NOTES of Figure UCS-66 and are summarized as follows.

   - Curve A applies to A/SA-105 forged flanges supplied in the as-forged condition, and
   - Curve B applies to A/SA-105 flanges produced to fine grain practice and normalized, normalized and tempered, or quenched and tempered after forging.

   VIII-1 does not provide specific guidance to the User as to what grain size constitutes a fine grain practice for an A/SA-105 forging specification. However, General Note (e)(2) of Figure UCS-66 states fine grain practice is defined as the procedure necessary to obtain a fine austenitic grain size as described in SA-20.

2) Paragraph UCS-66(b) – The use of this paragraph is no longer applicable to bolts and nuts. The explicit reference to exclude bolts and nuts in this paragraph was added as a result of the removal of the reference to bolts and nuts from paragraph UCS-66(b)(1)(-b).

3) Paragraph UCS-66(b)(1)(-c) – The option to reduce an MDMT for a flange when the MDMT is established based on paragraph UCS-66(c) was removed (see 5) below).

   - Paragraph UCS-66(b) provides a basis to have a colder MDMT than that derived from the Impact Test Exemption Curves utilized in UCS-66(a) when the coincident ratio, defined in Figure UCS-66.1, is less than one.
   - Paragraph UCS-66(b)(1)(-a) provides guidance on determining the coincident ratio, via Figure UCS-66.1 and Figure UCS-66.2, for components stressed in general primary membrane tensile stress.
   - Paragraph UCS-66(b)(1)(-b) applies to components not stressed in general primary membrane tensile stress and provides guidance on determining the coincident ratio based upon the maximum design pressure to maximum allowable pressure of the component.
   - Paragraph UCS-66(b)(1)(-c) applies to flanges, attached by welding and provides the option to determine the coincident ratio using the nozzle neck or shell to which the flange is attached, via paragraph UCS-66(b)(1)(-a), in lieu of the coincident ratio determined in paragraph UCS-66(b)(1)(-b).

4) Paragraph UCS-66(b)(1)(-d) – Provides specific guidance that longitudinal stress in the vessel due to net-section bending that results in general primary membrane tensile stress shall be considered when calculating the coincident ratio in Figure UCS-66.2.

5) Paragraph UCS-66(c) – The exemption temperature for which impact testing is not required for ferritic flanges produced to ASME B16.5 and ASME B16.47 standards has been changed. The paragraph was re-written to coincide with the changes noted in paragraph UCS-66(a) and Figure UCS-66.

   - Charpy Impact testing is not required for ferritic steel flanges when produced to fine grain practice and supplied in the heat-treated condition (normalized, normalized and tempered, or quenched and tempered after forging) when used at design temperatures no colder than –20°F.
   - Charpy Impact testing is not required for ferritic steel flanges supplied in the as-forged condition when used at temperatures no colder than 0°F.

The following logic diagrams, shown in Figure E3.1.1, Figure E3.1.2, and Figure E3.1.3, were developed to help provide guidance to the user/designated agent/Manufacturer for determining the impact test exemption rules of paragraphs UCS-66(a) and UCS-66(b).

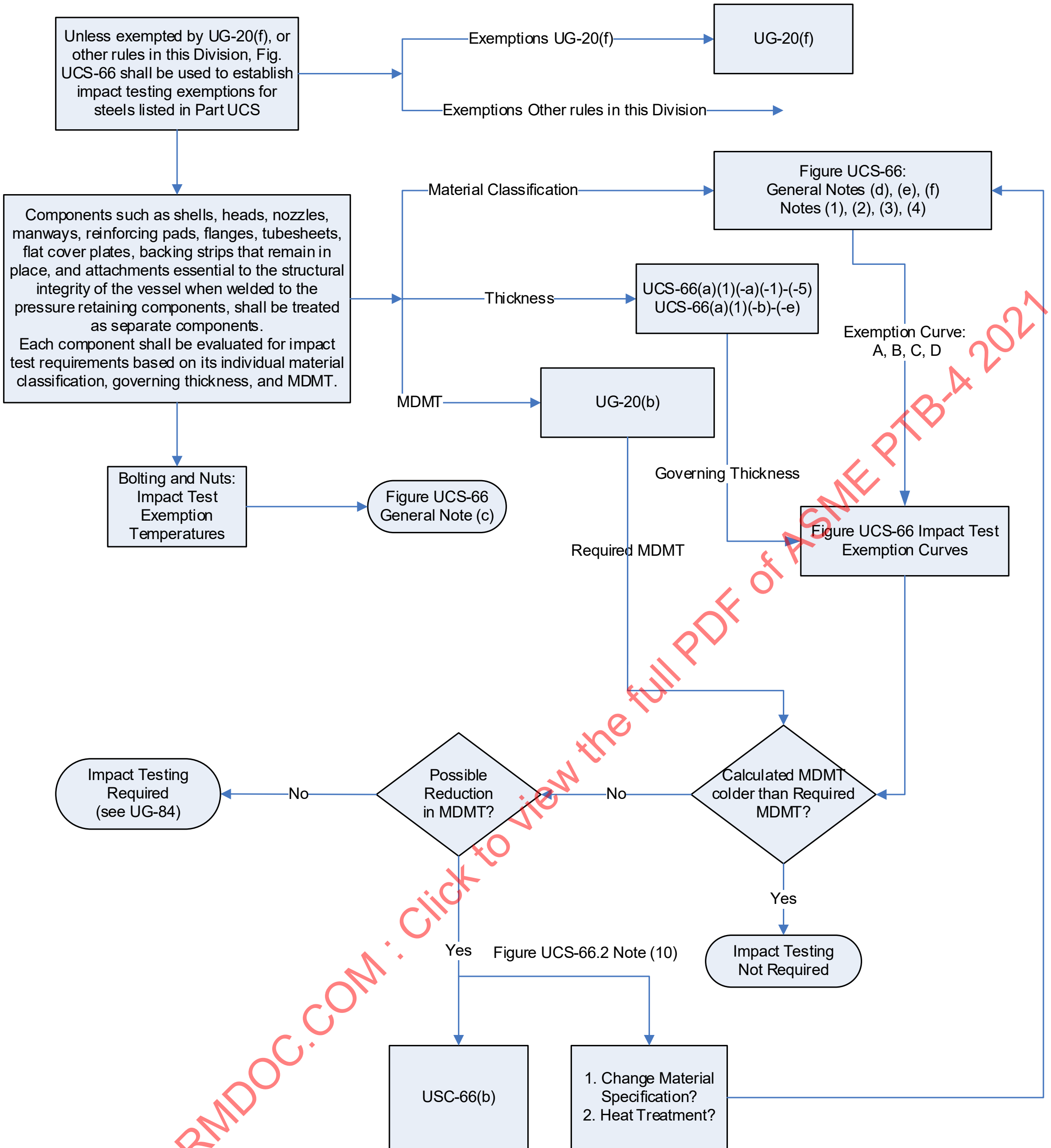

Figure E3.1.1 – Logic Diagram for UCS-66(a)

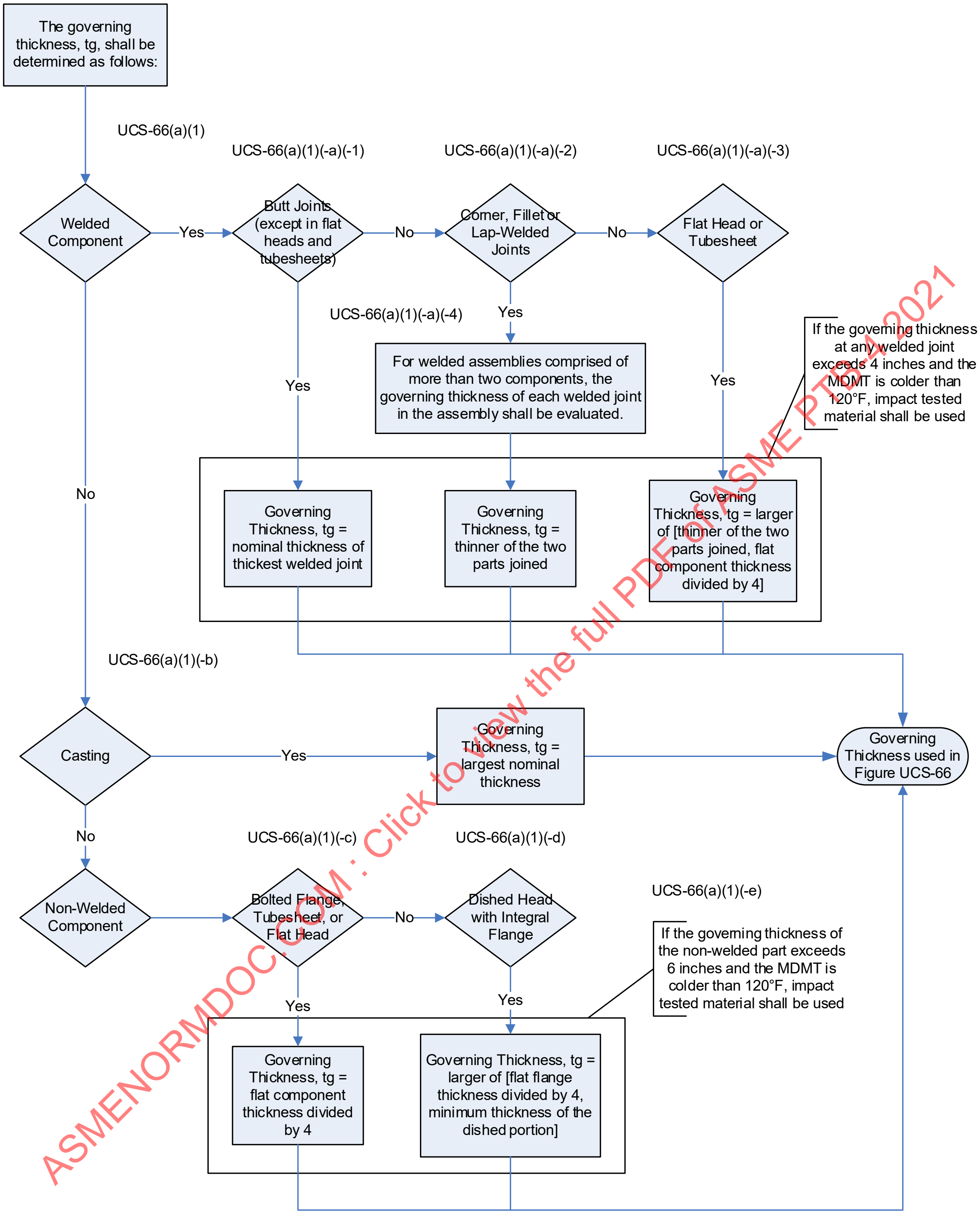

Figure E3.1.2 – Logic Diagram for UCS-66(a)(1)

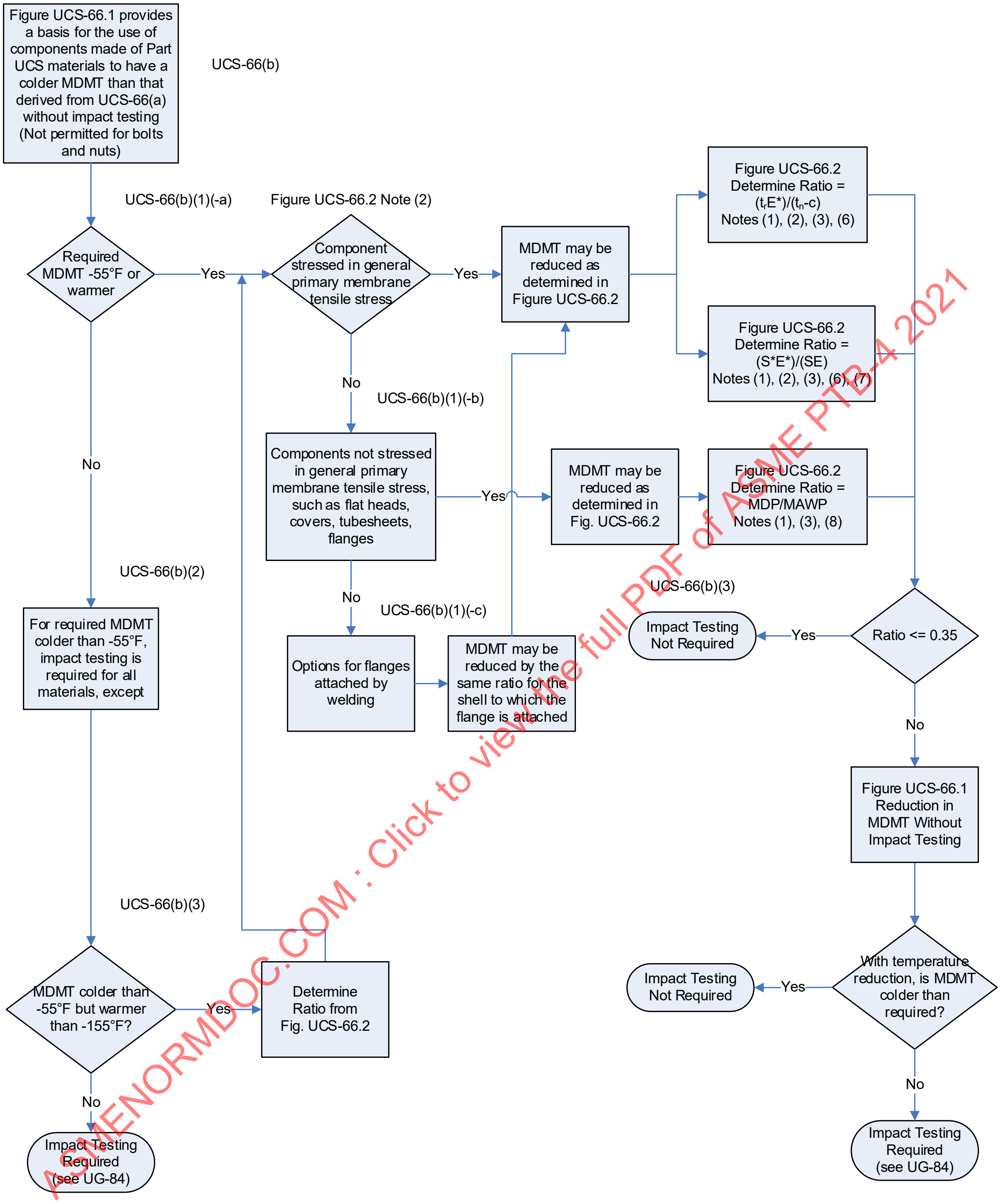

Figure E3.1.3 – Logic Diagram for UCS-66(b)

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3.2 Example E3.1 – Use of MDMT Exemptions Curves

Determine if Impact Testing is required for the proposed shell section. The shell is cylindrical with all Category A joints made up of Type 1 butt welds which have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Nominal Thickness = $1.8125\ in$
- PWHT = Yes
- MDMT = $-20°F$
- Corrosion Allowance = $0.125\ in$

In accordance with paragraph UCS-66(a), the procedure that is used to establish impact testing exemptions is shown below.

Paragraph UCS-66(a): unless exempted by the rules of UG-20(f) or other rules of this Division, Fig. UCS-66 shall be used to establish impact testing exemptions for steels listed in Part UCS. When Fig. UCS-66 is used, impact testing is required for a combination of Minimum Design Metal Temperature (MDMT) and thickness which is below the curve assigned to the subject material. If a MDMT and thickness combination is on or above the curve, impact testing is not required by the rules of this Division.

a) STEP 1 – From the Notes of Figure UCS-66, the appropriate impact test exemption curve for the material specification $SA-516,\ Grade\ 70,\ Normalized$ is designated a Curve D material.

b) STEP 2 – The governing thickness to be used in Figure UCS-66 is determined from paragraph UCS-66(a)(1)(-a)(-1) through (-a)(-5) based upon if the component under consideration is a welded part, casting, flat non-welded part, or a dished non-welded part. In this example, the cylindrical shell is a welded part attached by a butt joint and the governing thickness is equal to the nominal thickness of the thickest welded joint, see Figure UCS-66.3.

$$t_g = 1.8125\ in$$

c) STEP 3 – The required MDMT is determined from paragraph UG-20(b) and is stated in the vessel data above as $-20°F$.

d) STEP 4 – Interpreting the value of MDMT from Figure UCS-66 is performed as follows. Enter the figure along the abscissa with a governing thickness of $t_g = 1.8125\ in$ and project upward until an intersection with the Curve D material is achieved. Project this point left to the ordinate and interpret the MDMT. This results in an approximate value of $MDMT = -7°F$. Another approach to determine the MDMT with more consistency can be achieved by using the tabular values found in Table UCS-66. Linear interpolation between thicknesses shown in the table is permitted. For a $t_g = 1.8125\ in$ and a Curve D material the following value for MDMT is determined.

$$MDMT = -7°F$$

Since the calculated MDMT of $-7°F$ is warmer than the required MDMT of $-20°F$, impact testing is required using only the rules in paragraph UCS-66(a). However, impact testing may still be avoided by applying the rules of paragraph UCS-66(b) and other noted impact test exemptions referenced in paragraph UCS-66.

Additionally, paragraph UCS-68(c) permits a $30°F$ reduction in impact testing exemption temperature from that determined in Figure UCS-66 if the component is subject to a Postweld Heat Treatment (PWHT) when not otherwise a requirement of this Division. Although the vessel under consideration in this example was subject to PWHT, it was done so because the nominal thickness was in excess of that permitted without PWHT per paragraph UCS-56. Therefore, the $30°F$ reduction in impact testing temperature is not permitted.

3.3 Example E3.2 – Use of MDMT Exemption Curves with Stress Reduction

Determine if impact testing is required for the proposed shell section in E3.1. The shell is cylindrical with all Category A joints made up of Type 1 butt welds which have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psi\ @\ 300°F$
- Inside Diameter = $150\ in$
- Nominal Thickness = $1.8125\ in$
- PWHT = Yes
- MDMT = $-20°F$
- Weld Joint Efficiency = 1.0
- Corrosion Allowance = $0.125\ in$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Allowable Stress at Design Temperature = $20000\ psi$

In accordance with paragraph UCS-66(b), the procedure that is used to determine the exemption from impact testing based on a coincident thickness ratio is shown below.

Paragraph UCS-66(b): when the coincident ratio defined in Figure UCS-66.1 is less than one, Figure UCS-66.1 provides a basis for the use of components made of Part UCS material to have a colder MDMT than that derived from paragraph UCS-66(a) without impact testing.

Paragraph UCS-66(b)(1)(-a): for such components, and for a required MDMT of $-55°F$ and warmer, the MDMT without impact testing determined in paragraph UCS-66(a) for the given material and thickness may be reduced as determined from Figure UCS-66.2. If the resulting temperature is colder than the required MDMT, impact testing of the material is not required.

a) STEP 1 – The appropriate impact test exemption curve for the material specification $SA-516,\ Grade\ 70,\ Normalized$ from the Notes of Figure UCS-66, was found to be Curve D.

b) STEP 2 – The governing thickness t_g to be used in Figure UCS-66, for the welded part under consideration, was found to be $t_g = 1.8125\ in$.

c) STEP 3 – The required MDMT is determined from paragraph UG-20(b) and is stated in the vessel data above as $-20°F$.

d) STEP 4 – Interpreting the value of MDMT from Figure UCS-66 or Table UCS-66, $MDMT = -7°F$.

e) STEP 5 – Based on the design loading conditions at the MDMT, determine the ratio, R_{ts}, using the thickness basis from Figure UCS-66.2.

$$R_{ts} = \frac{t_r E^*}{t_n - CA}$$

where, t_r is the required thickness of the cylindrical shell at the specified MDMT of $-20°F$, using paragraph UG-27(c)(1).

$$t_r = \frac{PR}{SE-0.6P} = \frac{356(75.125)}{(20000(1.0)-0.6(356))} = 1.3517$$

$$where,$$

$$R = \frac{D}{2} + Corrosion\ \ Allowance = \frac{150.0}{2} + 0.125 = 75.125\ in$$

The variables E^*, t_n, and CA are defined as follows:

$$E^* = \max[E,\ 0.80] = \max[1.0, 0.8] = 1.0 \qquad \rightarrow Figure\ UCS-66.2,\ Note\ 3$$

$$t_n = 1.8125\ in$$

$$CA = 0.125\ in$$

Therefore,

$$R_{ts} = \frac{t_r E^*}{t_n - CA} = \frac{1.3517(1.0)}{1.8125-0.125} = 0.8010$$

f) STEP 6 – Interpreting the value of the temperature reduction, T_R from Figure UCS-66.1 is performed as follows. Enter the figure along the ordinate with a value of $R_{ts} = 0.8010$, project horizontally until an intersection with the provided curve is achieved. Project this point downward to the abscissa and interpret T_R. Resulting in an approximate value of $T_R = 20°F$.

g) STEP 7 – The final adjusted value of the MDMT is determined as follows.

$$MDMT = MDMT_{STEP3} - T_R = -7°F - 20°F = -27°F$$

Since the final value of MDMT is colder than the proposed MDMT, impact testing is not required.

3.4 Example E3.3 – Determine the MDMT for a Nozzle-to-Shell Welded Assembly

Determine if impact testing is required for the proposed nozzle assembly comprised of a shell and integrally reinforced nozzle. The shell is cylindrical with all Category A joints made up of Type 1 butt welds which have been 100% radiographically examined. The nozzle parameters used in the design procedure is shown in Figure E3.3.1.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psi\ @\ 300°F$
- Inside Diameter = $150\ in$
- Nominal Thickness = $1.8125\ in$
- PWHT = Yes
- MDMT = $-20°F$
- Weld Joint Efficiency = 1.0
- Corrosion Allowance = $0.125\ in$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Allowable Stress at Design Temperature = $20000\ psi$

Nozzle:

- Material = $SA-105$
- Outside Diameter = $25.5\ in$
- Thickness = $4.75\ in$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Allowable Stress at Design Temperature = $20000\ psi$

The nozzle is inserted through the shell, i.e., set–in type nozzle.

In accordance with paragraph UCS-66(a)(1)(-a)(-4), the procedure that is used to establish the governing thickness, t_g, is shown below.

Paragraph UCS-66(a)(1)(-a)(-4): for welded assemblies comprised of more than two components (e.g., nozzle-to shell joint with reinforcing pad), the governing thickness and permissible minimum design metal temperature of each of the individual welded joints of the assembly shall be determined, and the warmest of the minimum design metal temperatures shall be used as the permissible minimum design metal temperature of the welded assembly. See Figure UCS-66.3 Sketch (g) and Figure E3.3.1 of this example.

a) STEP 1 – The appropriate impact test exemption curve for the cylindrical shell material specification $SA-516,\ Grade\ 70,\ Normalized$ from the Notes of Figure UCS-66, was found to be Curve D. Similarly, the appropriate impact test exemption curve for the integrally reinforced nozzle material specification $SA-105$ from the Notes of Figure UCS-66, was found to be Curve B.

b) STEP 2 – The governing thickness of the full penetration corner joint, t_{g1} to be used in Figure UCS-66, for the welded joint under consideration, was determined per Figure UCS-66.3 Sketch (g).

$$t_{g1} = \min\left[t_A,\, t_C\right] = \min\left[1.8125,\, 4.75\right] = 1.8125\ in$$

where,

$$t_A = Shell\ thickness,\ 1.8125\ in$$
$$t_C = Nozzle\ thickness,\ 4.75\ in$$

c) STEP 3 – The required MDMT is determined from paragraph UG-20(b) and is stated in the vessel data above as $-20°F$.

d) STEP 4 – Interpreting the value of MDMT from Figure UCS-66 for the welded joint requires that both the shell and nozzle material be evaluated, and the warmest minimum design metal temperature shall be used for the assembly. The procedure is performed as follows.

For the cylindrical shell material, $SA-516,\ Grade\ 70,\ Normalized$: Enter the figure along the abscissa with a governing thickness of $t_g = 1.8125\ in$ and project upward until an intersection with the Curve D material is achieved. Project this point left to the ordinate and interpret the MDMT. This results in an approximate value of $MDMT = -7°F$. Another approach to determine the MDMT with more consistency can be achieved by using the tabular values found in Table UCS-66. Linear interpolation between thicknesses shown in the table is permitted. For a $t_g = 1.8125\ in$ and a Curve D material the following value for MDMT is determined.

$$MDMT_{curve\,D} = -7°F$$

For the nozzle material, $SA-105$: Enter the figure along the abscissa with a governing thickness of $t_g = 1.8125\ in$ and project upward until an intersection with the Curve B material is achieved. Project this point left to the ordinate and interpret the MDMT. This results in an approximate value of $MDMT = 59°F$. Similarly, a more accurate value for MDMT can be achieved by using the tabular values found in Table UCS-66. Linear interpolation between thicknesses shown in the table is permitted. For a $t_g = 1.8125\ in$ and a Curve B material the following value for MDMT is determined.

$$MDMT_{curve\,B} = 59°F$$

Therefore, the nozzle assembly minimum design metal temperature is determined as follows.

$$MDMT_{assembly} = Warmest\left[MDMT_{curve\,D},\ MDMT_{curve\,B}\right]$$
$$MDMT_{assembly} = Warmest\left[-7,\ 59\right]$$
$$MDMT_{assembly} = 59°F$$

Applying paragraph UCS-66(b): when the coincident ratio defined in Figure UCS-66.1 is less than one, Figure UCS-66.1 provides a basis for the use of components made of Part UCS material to have a colder MDMT than that derived from paragraph UCS-66(a) without impact testing.

e) STEP 5 – Based on the design loading conditions at the MDMT, determine the ratio, R_{ts}, using the thickness basis from Figure UCS-66.2.

Commentary: VIII-1 does not provide explicit guidance as to which component of a welded assembly shall R_{ts} be based upon. This example provides one possible method of satisfying the requirement and is consistent with ASME Interpretation VIII-1-01-37.

For a welded assembly, the value of R_{ts} is calculated based upon the assembly's component with the governing thickness. In this example the governing thickness of the assembly was based on the cylindrical shell.

$$R_{ts} = \frac{t_r E^*}{t_n - CA}$$

Where, t_r is the required thickness of the cylindrical shell at the specified MDMT of $-20°F$, using paragraph UG-27(c)(1).

$$t_r = \frac{PR}{SE - 0.6P} = \frac{356(75.125)}{(20000(1.0) - 0.6(356))} = 1.3517$$

$$where,$$

$$R = \frac{D}{2} + Corrosion\ \ Allowance = \frac{150.0}{2} + 0.125 = 75.125\ in$$

The variables E^*, t_n, and CA are defined as follows:

$$E^* = \max[E,\ 0.80] = \max[1.0, 0.8] = 1.0 \qquad \rightarrow Figure\ UCS-66.2,\ Note\ 3$$
$$t_n = 1.8125\ in$$
$$CA = 0.125\ in$$

Therefore,

$$R_{ts} = \frac{t_r E^*}{t_n - CA} = \frac{1.3517(1.0)}{1.8125 - 0.125} = 0.8010$$

f) STEP 6 – Interpreting the value of the temperature reduction, T_R from Figure UCS-66.1 is performed as follows. Enter the figure along the ordinate with a value of $R_{ts} = 0.8010$, project horizontally until an intersection with the provided curve is achieved. Project this point downward to the abscissa and interpret T_R. This results in an approximate value of $T_R = 20°F$

g) STEP 7 – The final adjusted MDMT of the assembly is determined as follows.

$$MDMT_{assembly} = MDMT_{STEP4} - T_R = 59°F - 20°F = 39°F$$

Since the final adjusted MDMT of the assembly is warmer than the proposed MDMT, impact testing of the nozzle forging is required.

An MDMT colder than the determined in this example would be possible if the nozzle forging were fabricated from a material specification that includes the provisions of impact testing, such as SA-350. See UCS-66(g) and General Note (c) of Figure UG-84.

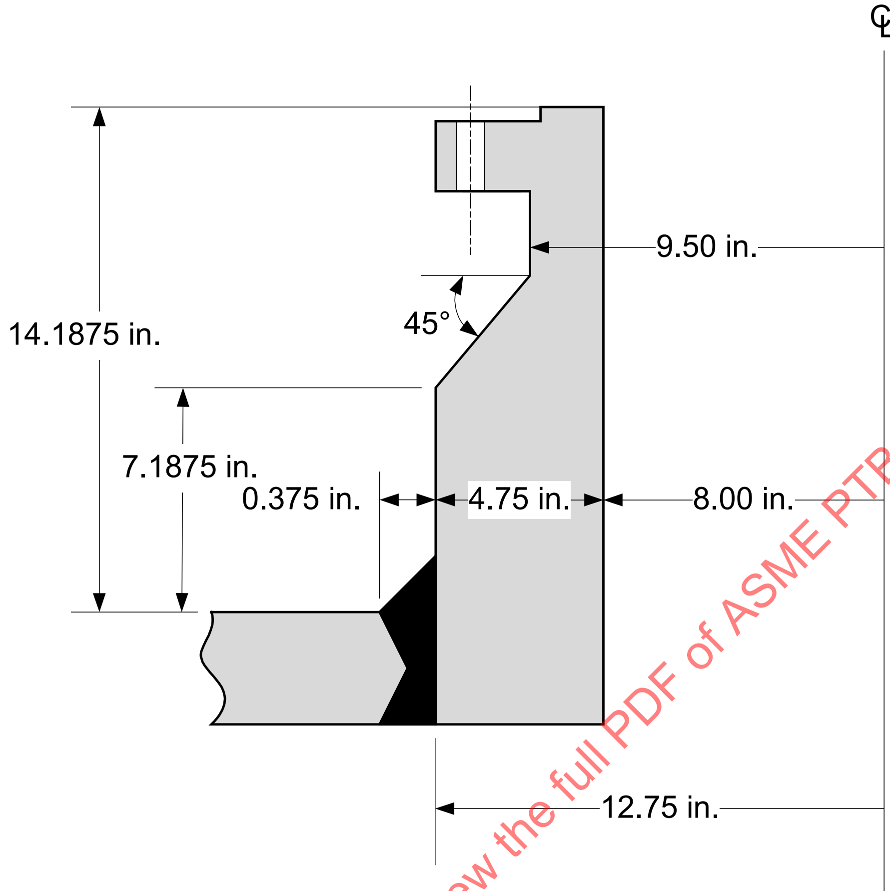


Figure E3.3.1 – Nozzle-Shell Detail

PART 4

DESIGN BY RULE REQUIREMENTS

4.1 General Requirements

4.1.1 Example E4.1.1 – Review of General Requirements for a Vessel Design

a) General Requirements

An engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with ASME B&PV Code, Section VIII, Division 1 (VIII-1). Mandatory Appendix 46 allows the use of ASME B&PV Code, Section VIII, Division 2 (VIII-2) design rules for VIII-1 pressure vessels, to take advantage of the updated design rules. The vessel in question is to be constructed of carbon steel with a corrosion allowance and a design pressure and temperature of 1650 psig at 300°F. As part of developing the design specification, the following items need to be evaluated.

b) Introduction

The scope of VIII-1 has been established to identify the components and parameters considered in formulating the rules given in VIII-1 as presented in U-1(a) through U-1(j). The user of the vessel shall establish the design requirements for pressure vessels, taking into consideration factors associated with normal operation, startup and shutdown, and abnormal conditions which may become a governing design consideration in accordance with U-2(a).

1) The design temperature shall be established in accordance with UG-20.

2) The design pressure shall be established in accordance with UG-21.

3) The design loads shall be established in accordance with UG-22.

c) Material Requirements

General material requirements as well as specific requirements based on product form and process service shall be in accordance with UG-4 through UG-15.

d) Minimum Thickness Requirements

Based on product form and process service, the parts of the vessel must meet the minimum thickness requirements presented in UG-16.

e) Corrosion Allowance in Design Equations

The equations used in the design-by-rule procedures of VIII-1 are performed in a corroded condition. The term corrosion allowance is representative of loss of metal due to corrosion, erosion, mechanical abrasion, or other environmental effects (see UG-25).

f) Design Basis

1) The pressure used in the design of a vessel component together with the coincident design metal temperature must be specified. Where applicable, the pressure resulting from static head shall be included in addition to the specified design pressure (see UG-21).

2) The specified design temperature shall not be less than the mean metal temperature expected coincidentally with the corresponding maximum pressure (see UG-20).

3) A minimum design metal temperature shall be determined and shall consider the coldest operating temperature, operational upsets, auto refrigeration, atmospheric temperature, and any other source of cooling.

4) All applicable loads shall be considered in the design to determine the minimum required wall thickness for a vessel part, see UG-22.

g) Design Allowable Stress

Specifications for all materials of construction and allowable design stresses are determined in accordance with UG-23.

4.1.2 Example E4.1.2 – Required Wall Thickness of a Hemispherical Head

Determine the required thickness for a hemispherical head at the bottom of a vertical vessel considering the following design conditions. All Category A joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $1650\ psig @ 300°F$
- Liquid Head = $60\ ft$
- Liquid Specific Gravity = 0.89
- Inside Diameter = $96.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0

The design pressure used to establish the wall thickness must be adjusted for the liquid head in accordance with paragraph UG-21.

$$Design\ Pressure = Specified\ Design\ Pressure + \gamma\rho h$$

$$Design\ Pressure = 1650 + \frac{0.89(62.4)(60)}{144} = 1673.140\ psig$$

Section VIII, Division 1 Solution

In accordance with UG-32(f), determine the required thickness of the bottom hemispherical head.

$$t = \frac{PL}{2SE - 0.2P}$$

$$L = \frac{96.0 + 2(Corrosion\ Allowance)}{2} = \frac{96.0 + 2(0.125)}{2} = 48.125\ in$$

$$t = \frac{1673.14(48.125)}{2(20000)(1.0) - 0.2(1673.14)} = 2.03\ in$$

$$t = 2.03 + Corrosion\ Allowance = 2.03 + 0.125 = 2.155\ in$$

The required thickness of the bottom head is 2.155 in.

Section VIII, Division 2 Solution Using VIII-1 Allowable Stresses

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the bottom hemispherical head. Similarly, the design pressure used to establish the wall thickness must be adjusted for the liquid head in accordance with Part 4, paragraph 4.1.5.2.a as shown above.

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE} \right] - 1 \right)$$

$$D = 96.0 + 2(Corrosion\ \ Allowance) = 96.0 + 2(0.125) = 96.25\ in$$

$$t = \frac{96.25}{2}\left(\exp\left[\frac{0.5(1673.14)}{20000(1.0)} \right] - 1 \right) = 2.0557\ in$$

$$t = 1.8313 + Corrosion\ \ Allowance = 2.0557 + 0.125 = 2.1807\ in$$

The required thickness of the bottom head is $2.1807\ in$.

4.2 Welded Joints

4.2.1 Example E4.2.1 – Nondestructive Examination Requirement for Vessel Design

An engineer is tasked with preparing the design specification for a new pressure vessel that is to be constructed in accordance with ASME B&PV Code, Section VIII, Division 1 (VIII-1). Based on the process service description, anticipated design data, materials of construction, and welding process, the engineer verifies that full radiography is not required in accordance with paragraph UW-11(a) and spot radiography in accordance with paragraph UW-11(b) would be adequate. However, the savings in cost for reduced examination may be offset by the increase in materials and fabrication costs. The designer conducts a comparison for a cylindrical shell to aid in the decision for NDE requirements.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $725\ psig @ 300°F$
- Inside Diameter = $60.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- No Supplemental Loads (see UG-22)

Section VIII, Division 1 Solution

For Full RT Examination, consider the requirements for a Category A Type 1 weld in a cylindrical shell. The required wall thickness in accordance with UG-27(c)(1) is computed as shown below.

$$t = \frac{PR}{SE - 0.6P}$$

$$R = \frac{60.0 + 2(Corrosion\ \ Allowance)}{2} = \frac{60.0 + 2(0.125)}{2} = 30.125\ in$$

$$t = \frac{725(30.125)}{20000(1.0) - 0.6(725)} = 1.1163\ in$$

$$t = 1.1163 + Corrosion\ \ Allowance = 1.1163 + 0.125 = 1.2413\ in$$

Alternatively, for Spot RT Examination, the required wall thickness for a Category A Type 1 weld in accordance with UG-27(c)(1) is computed as shown below.

$$t = \frac{PR}{SE - 0.6P}$$

$$R = \frac{60.0 + 2(Corrosion\ \ Allowance)}{2} = \frac{60.0 + 2(0.125)}{2} = 30.125\ in$$

$$t = \frac{725(30.125)}{20000(0.85) - 0.6(725)} = 1.3185\ in$$

$$t = 1.3185 + Corrosion\ \ Allowance = 1.3185 + 0.125 = 1.4435\ in$$

Full RT Examination when compared to Spot RT Examination results in an approximate 14% reduction in wall thickness. Cost savings for this reduction in wall thickness will include less material and less welding, and these reductions may offset the increased examination costs. Other potential savings may include reduced shipping and reduced foundation costs.

Although the process service description, anticipated design data, materials of construction, and welding process, did not require that full radiography be performed, it should be noted that because the calculation using Spot RT Examination produces a required thickness of $1.4435\,in$, Full RT Examination would become mandatory per Table UCS-57.

Commentary – This example is intended to look only at one specific Category A weld seam and provide a comparison of the differences in vessel wall thickness and examination requirements. Other examination options are available (see paragraph UW-11(a)(5)(b)) but require examination of intersecting Category B weld seams, which is outside the scope of this example problem. If Category B weld seams were considered as well as supplemental loads, then the longitudinal stress equation in UG-27(c)(2) would be evaluated (see Endnote 20).

Section VIII, Division 2 Solution Using VIII-1 Allowable Stresses

For Full RT Examination, consider the requirements for a Category A Type 1 weld in a cylindrical shell. The required wall thickness in accordance with paragraph 4.3.3 is computed as shown below.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right)$$

$$D = 60.0 + 2\left(Corrosion\ \ Allowance\right) = 60.0 + 2\left(0.125\right) = 60.25\ in$$

$$t = \frac{60.25}{2}\left(\exp\left[\frac{725}{20000\left(1.0\right)}\right]-1\right) = 1.1121\ in$$

$$t = 1.1121 + Corrosion\ \ Allowance = 1.1121 + 0.125 = 1.2371\ in$$

Alternatively, for Spot RT Examination, the required wall thickness for a Category A Type 1 weld in accordance with paragraph 4.3.3 is computed as shown below.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right)$$

$$D = 60.0 + 2\left(Corrosion\ \ Allowance\right) = 60.0 + 2\left(0.125\right) = 60.25\ in$$

$$t = \frac{60.25}{2}\left(\exp\left[\frac{725}{20000\left(0.85\right)}\right]-1\right) = 1.3125\ in$$

$$t = 1.3125 + Corrosion\ \ Allowance = 1.3125 + 0.125 = 1.4375\ in$$

Similarly, Full RT Examination when compared to Spot RT Examination results in an approximate 14% reduction in wall thickness.

4.2.2 Example E4.2.2 – Nozzle Detail and Weld Sizing

Determine the required fillet weld size and inside corner radius of a set-in type of nozzle as shown in Figure UW-16.1(d), (Figure E4.2.2). The vessel and nozzle were designed such that their nominal thicknesses were established as follows.

Vessel Data:

- Cylinder Thickness = $0.625\ in$
- Nozzle Diameter = $NPS\ 10$
- Nozzle Thickness = $Schedule\ XS \rightarrow 0.500\ in$
- Corrosion Allowance = $0.125\ in$

Adjust variables for corrosion.

$$t_s = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$
$$t_n = 0.500 - Corrosion\ \ Allowance = 0.500 - 0.125 = 0.375\ in$$

Section VIII, Division 1 Solution

The minimum fillet weld throat dimension, t_c, is calculated as follows. See UW-16(b).

$$t_{\min} = \min\left[t_n,\ t_s,\ 0.75\ in\right] = \min\left[0.375,\ 0.500,\ 0.75\ in\right] = 0.375\ in$$
$$t_c \geq \min\left[0.7t_{\min},\ 0.25\ in\right]$$
$$t_c \geq \min\left[0.70(0.375), 0.25\ in\right]$$
$$t_c \geq 0.25\ in$$

The resulting fillet weld leg size is determined as, $t_c/0.7 = 0.357\ in$. Therefore, a fillet weld leg size of $0.357\ in$ would be acceptable.

Note: VIII-1 does not provide acceptance criterion for the inside corner radius of the exposed nozzle edge, see Figure UW-16.1(d), (Figure E4.2.2). However, paragraphs UG-76(b) and (c) reference; end of nozzles or manhole necks which are to remain unwelded in the completed vessel may be cut by methods that produce a smooth finish and exposed inside edges shall be chamfered or rounded.

Section VIII, Division 2 Solution

The reference sketch per VIII-2 is found in Table 4.2.10, Detail 4, (Figure E4.2.2). The minimum fillet weld throat dimension, t_c, is calculated as follows.

$$t_c \geq \min\left[0.7t_n,\ 0.25\ in\right]$$
$$t_n = 0.500 - Corrosion\ \ Allowance = 0.5 - 0.125 = 0.375\ in$$
$$t_c \geq \min\left[0.7(0.375), 0.25\ in\right]$$
$$t_c \geq 0.25\ in$$

The resulting fillet weld leg size is determined as, $t_c/0.7 = 0.357\ in$. Therefore, a fillet weld leg size of $0.357\ in$ would be acceptable.

The minimum inside corner radius, r_1, is calculated as follows.

$$0.125t \le r_1 \le 0.5t$$

$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$

$$0.125(0.500) \le r_1 \le 0.5(0.500)$$

$$0.0625 \le r_1 \le 0.250\ in$$

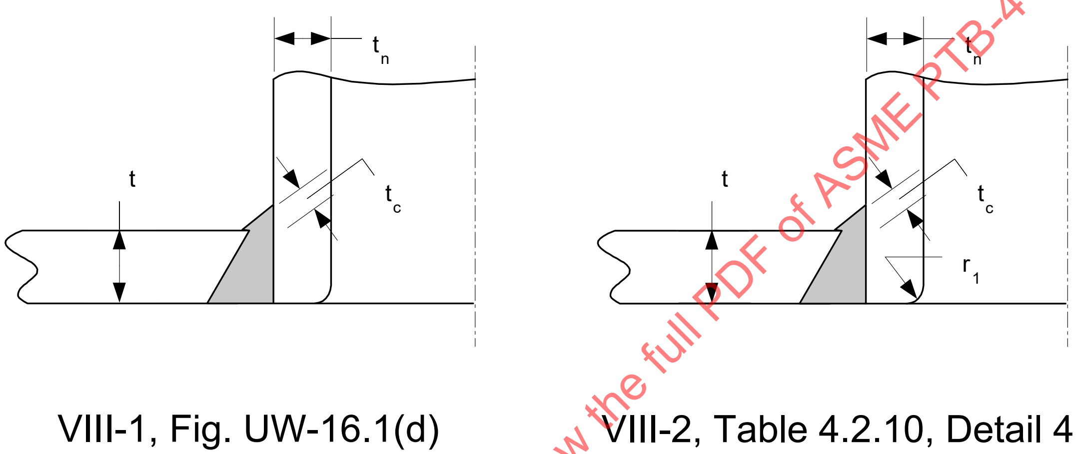


Figure E4.2.2 – Weld Details

4.2.3 Example E4.2.3 – Nozzle Detail with Reinforcement Pad and Weld Sizing

Determine the required fillet weld sizes and inside corner radius of a set-in type of nozzle with added reinforcement pad as shown in Figure UW-16.1(q), (Figure E4.2.3). The vessel and nozzle were designed such that their nominal thicknesses were established as follows.

Vessel Data:

- Cylinder Thickness = $0.625\ in$
- Nozzle Diameter = $NPS\ 10$
- Nozzle Thickness = $Schedule\ XS \rightarrow 0.500\ in$
- Reinforcement Pad Thickness = $0.625\ in$
- Corrosion Allowance = $0.125\ in$

Adjust variables for corrosion.

$$t_s = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$

$$t_n = 0.500 - Corrosion\ \ Allowance = 0.500 - 0.125 = 0.375\ in$$

$$t_e = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.0 = 0.625\ in$$

Note: The corrosion allowance specified is for internal corrosion, not external corrosion. Therefore, the corrosion allowance of the reinforcement pad thickness is set equal to zero.

Section VIII, Division 1 Solution

The minimum fillet weld throat dimension, t_c, is calculated as follows. See UW-16(b).

$$t_{\min} = \min\left[t_n,\ t_e,\ 0.75\ in\right] = \min\left[0.375,\ 0.625,\ 0.75\ in\right] = 0.375\ in$$

$$t_c \geq \min\left[0.7t_{\min},\ 0.25\ in\right]$$

$$t_c \geq \min\left[0.7(0.375), 0.25\ in\right]$$

$$t_c \geq 0.25\ in$$

The resulting fillet weld leg size is determined as, $t_c/0.7 = 0.357\ in$. Therefore, a fillet weld leg size of $0.375\ in$ would be acceptable.

The minimum fillet weld throat at the outer edge of the reinforcement pad is calculated as follows.

$$throat = 0.5t_{\min} = 0.5\min[t_s,\ t_e,\ 0.75\ in]$$

$$throat = 0.5\min[0.500,\ 0.625,\ 0.75\ in]$$

$$throat = 0.25\ in$$

The resulting fillet weld leg size is determined as, $throat/0.7 = 0.357\ in$. Therefore, a fillet weld leg size of $0.375\ in$ would be acceptable.

The minimum partial penetration weld for the reinforcement pad, t_w is calculated as follows.

$$t_w = 0.7t_{\min} = 0.7\min[t_n,\ t_e,\ 0.75\ in]$$

$$t_w = 0.7\min[0.375,\ 0.625,\ 0.75\ in]$$

$$t_w = 0.2625\ in$$

The minimum partial penetration weld for the vessel, t_w is calculated as follows.

$$t_w = 0.7t_{\min} = 0.7\min[t_s,\, t_n,\, 0.75\ in]$$

$$t_w = 0.7\min[0.500,\, 0.375,\, 0.75\ in]$$

$$t_w = 0.2625\ in$$

The user is reminded that the value of the minimum partial penetration weld for the vessel is established using corroded dimensions for the shell, t_s, and the nozzle, t_n. Since the weld is exposed to the environment contained within the vessel, the minimum partial penetration weld shall be increased by the design corrosion allowance.

$$t_{w-design} = t_w + Corrosion\ \ Allowance = 0.2625 + 0.125 = 0.3875\ in$$

Note: VIII-1 does not provide acceptance criterion for the inside corner radius of the exposed nozzle edge, see Figure UW-16.1(q), (Figure E4.2.2). However, paragraphs UG-76(b) and (c) state; end of nozzles or manhole necks which are to remain unwelded in the completed vessel may be cut by methods that produce a smooth finish and exposed inside edges shall be chamfered or rounded.

Section VIII, Division 2 Solution

The reference sketch per VIII-2 is found in Table 4.2.11, Detail 2. The minimum fillet weld throat dimension, t_c, is calculated as follows.

$$t_c \geq \min\left[0.7t_n, 6\,mm\ (0.25\ in)\right]$$

$$t_n = 0.500 - Corrosion\ \ Allowance = 0.500 - 0.125 = 0.375\ in$$

$$t_c \geq \min\left[0.7(0.375), 0.25\right]$$

$$t_c \geq 0.25\ in$$

The resulting fillet weld leg size is determined as, $t_c/0.7 = 0.357\ in$. Therefore, a fillet weld leg size of $0.375\ in$ would be acceptable.

The minimum fillet weld throat dimension, t_{f1}, is calculated as follows.

$$t_{f1} \geq \min\left[0.6t_e,\ 0.6t\right]$$

$$t_e = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.0 = 0.625\ in$$

$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$

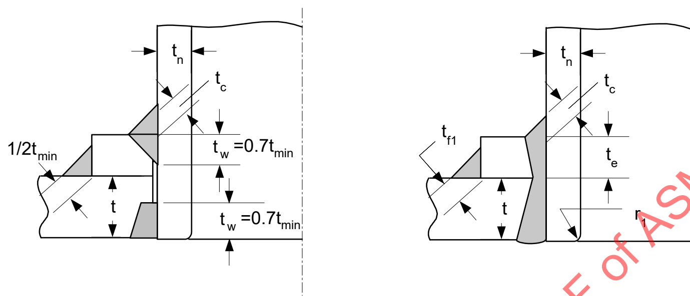
$$t_{f1} \geq \min\left[0.6(0.625), 0.6(0.500)\right]$$

$$t_{f1} \geq 0.300\ in$$

The resulting fillet weld leg size is determined as, $t_{f1}/0.7 = 0.429\ in$. Therefore, a fillet weld leg size of $0.4375\ in$ would be acceptable.

The minimum inside corner radius, r_1, is calculated as follows.

$$0.125t \le r_1 \le 0.5t$$
$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$
$$0.125(0.500) \le r_1 \le 0.5(0.500)$$
$$0.0625 \le r_1 \le 0.250\ in$$



VIII-1, Fig. UW-16.1(q) VIII-2, Table 4.2.11, Detail 2

Figure E4.2.3 – Weld Details

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4.3 Internal Design Pressure

4.3.1 Example E4.3.1 – Cylindrical Shell

Determine the required thickness for a cylindrical shell considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psig @ 300°F$
- Inside Diameter = $90.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0

Determine the inside radius and adjust for corrosion allowance.

$$D = 90.0 + 2(Corrosion\ \ Allowance) = 90.0 + 2(0.125) = 90.25$$

$$R = \frac{D}{2} = \frac{90.25\ in}{2} = 45.125\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-27(c) – The minimum thickness or maximum allowable working pressure of cylindrical shells shall be the greater thickness or lesser pressure as given by the following.

UG-27(c)(1), Circumferential Stress.

$$t_c = \frac{PR}{SE - 0.6P} = \frac{356(45.125)}{20000(1.0) - 0.6(356)} = 0.8119\ in$$

$$t_c = 0.8119 + Corrosion\ \ Allowance = 0.8119 + 0.125 = 0.9369\ in$$

UG-27(c)(2), Longitudinal Stress.

$$t_l = \frac{PR}{2SE + 0.4P} = \frac{356(45.125)}{2(20000)(1.0) + 0.4(356)} = 0.4002\ in$$

$$t_l = 0.4002 + Corrosion\ \ Allowance = 0.4002 + 0.125 = 0.5252\ in$$

Therefore, the required thickness is determined as follows.

$$t = \max[t_c,\ t_l] = \max[0.9369,\ 0.5252] = 0.9369\ in$$

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraph 4.3.3.

$$t=\frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right)=\frac{90.25}{2}\left(\exp\left[\frac{356}{20000(1.0)}\right]-1\right)=0.8104\ in$$

$$t=0.8104+Corrosion\ \ Allowance=0.8104+0.125=0.9354\ in$$

The required thickness is $0.9354\ in$.

4.3.2 Example E4.3.2 – Conical Shell

Determine the required thickness for a conical shell considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psig @ 300°F$
- Inside Diameter (Large End) = $150.0\ in$
- Inside Diameter (Small End) = $90.0\ in$
- Length of Conical Section = $78.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0

Adjust for corrosion allowance and determine the cone angle.

$$D_L = 150.0 + 2(Corrosion\ \ Allowance) = 150.0 + 2(0.125) = 150.25\ in$$

$$D_s = 90.0 + 2(Corrosion\ \ Allowance) = 150.0 + 2(0.125) = 90.25\ in$$

$$L_C = 78.0$$

$$\alpha = \arctan\left[\frac{0.5(D_L - D_S)}{L_C}\right] = \arctan\left[\frac{0.5(150.25 - 90.25)}{78.0}\right] = 21.0375\ deg$$

Section VIII, Division 1 Solution

Evaluate per UG-32(g) using the large end diameter of the conical shell.

$$t = \frac{PD}{2\cos[\alpha](SE - 0.6P)} = \frac{356(150.25)}{2\cos[21.0375](20000(1.0) - 0.6(356))} = 1.4482\ in$$

$$t = 1.4482 + Corrosion\ \ Allowance = 1.4482 + 0.125 = 1.5732\ in$$

The required thickness is 1.5732 in.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraph 4.3.4 using the large end diameter of the conical shell.

$$t = \frac{D}{2\cos[\alpha]}\left(\exp\left[\frac{P}{SE}\right] - 1\right) = \frac{150.25}{2\cos[21.0375]}\left(\exp\left[\frac{356}{20000(1.0)}\right] - 1\right) = 1.4455\ in$$

$$t = 1.4455 + Corrosion\ \ Allowance = 1.4455 + 0.125 = 1.5705\ in$$

The required thickness is 1.5705 in.

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4.3.3 Example E4.3.3 – Spherical Shell

Determine the required thickness for a spherical shell considering the following design conditions. All Category A joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-542,\ Type\,D,\ Class\ 4a$
- Design Conditions = $2080\ psig\ @ 850°F$
- Inside Diameter = $149.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $21000\ psi$
- Weld Joint Efficiency = 1.0

Section VIII, Division 1 Solution

Evaluate per UG-32(f).

$$L = \frac{D}{2} = \frac{149}{2} = 74.5\ in$$

$$t = \frac{PL}{2SE - 0.2P} = \frac{2080(74.5)}{2(21000)(1.0) - 0.2(2080)} = 3.7264\ in$$

The required thickness is $3.7264\ in.$

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraph 4.3.5.

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right] - 1\right) = \frac{149.0}{2}\left(\exp\left[\frac{0.5(2080)}{21000(1.0)}\right] - 1\right) = 3.7824\ in$$

The required thickness is $3.7824\ in.$

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4.3.4 Example E4.3.4 – Torispherical Head

Determine the Maximum Allowable Working Pressure (MAWP) for the proposed seamless torispherical head. The Category B joint joining the head to the shell is a Type 1 butt weld and has been 100% radiographically examined.

Vessel Data:

- Material = $SA-387,\ Grade\ 11,\ Class\ 1$
- Design Temperature = $650°F$
- Inside Diameter = $72.0\ in$
- Crown Radius = $72.0\ in$
- Knuckle Radius = $4.375\ in$
- Thickness = $0.625\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $17100\ psi$
- Weld Joint Efficiency = 1.0
- Modulus of Elasticity at Design Temperature = $26.55E+06\ psi$
- Yield Strength at Design Temperature = $26900\ psi$

Adjust for corrosion allowance.

$$D = 72.0 + 2(Corrosion\ \ Allowance) = 72.0 + 2(0.125) = 72.25\ in$$

$$L = 72.0 + Corrosion\ \ Allowance = 72.0 + 0.125 = 72.125\ in$$

$$r = 4.375 + Corrosion\ \ Allowance = 4.375 + 0.125 = 4.5\ in$$

$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.5\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-32(d). Note, the design equations of UG-32(d) are specific to torispherical heads in which the the knuckle radius is 6% of the inside crown radius, and the inside crown radius equals the outside diameter of the skirt. Additionally, if the ratio $t_s/L \geq 0.002$, is not satisfied, the rules of Mandatory Appendix 1-4(f) shall also be met.

$$P = \frac{SEt}{0.885L + 0.1t} = \frac{(17100)(1.0)(0.5)}{(0.885)(72.125) + 0.1(0.5)} = 132.9071\ psi$$

$$Note: \left\{\frac{t}{L} = \frac{0.5}{72.125} = 0.0069\right\} > 0.002,\ \ therefore\ the\ rules\ of\ 1\text{-}4(f)\ are\ not\ required$$

Commentary:

The design rules of Mandatory Appendix 1-4(d) are also permitted for the design of torispherical heads and are general in their presentation. The design equations are applicable to torispherical heads with variable head crown radii to knuckle radii proportions, measured as L/r. Use of the general design equations, however, introduce potential inconsistencies with the specific equation provided in UG-32(d). These noted inconsistencies do not produce a significant difference in the required thickness or calculated pressure;

however, the increased use of computer software to perform vessel design calculations exacerbates these inconsistencies and provides a source of doubt as to the intended accuracy of such numerical analyses.

Potential inconsistencies with the use of Appendix 1-4(d) include:

a) The knuckle radius is rarely, if ever, exactly equal to 6% of the inside crown radius. In practice, the knuckle radius is set equal to 6% of the inside crown radius, rounded to the nearest 1/16th, 1/8th, or 1/4th inches based upon the overall head dimensions.

b) Applicability of applying the design corrosion allowance to the crown radius, L and knuckle radius, r when determining the factor M, as defined below.

$$M = 1/4\left(3+\sqrt{\frac{L}{r}}\right)$$

The specific equations provided in UG-32(d) for torispherical heads with a 6% knuckle radius is derived from Equation (4) as follows.

$$\textit{For a 6\% knuckle radius, the value of } \frac{L}{r} = \frac{1}{0.06}$$

$$\textit{Therefore,} \qquad M = 1/4\left(3+\sqrt{\frac{L}{r}}\right) = 1/4\left(3+\sqrt{\frac{1}{0.06}}\right) = 1.7706$$

$$\textit{Resulting in,} \quad t = \frac{PLM}{2SE-0.2P} = \frac{PL(1.7706)}{2SE-0.2P} = \frac{0.885PL}{SE-0.1P}$$

$$P = \frac{2SEt}{ML+0.2t} = \frac{2SEt}{(1.7706)L+0.2t} = \frac{SEt}{0.885L+0.1t}$$

As stated in UG-16(e), "…the dimensional symbols used in all design formulas throughout this Division represent dimensions in the corroded condition". If a design corrosion allowance is applied to the crown radius, L and the knuckle radius, r when determining the factor M, then the specific equations provided in UG-32(d) for torispherical heads with a 6% knuckle radius is no longer applicable as the resulting value of M will no longer produce the coefficient of 0.885 as shown in UG-32(d).

c) Use of Table 1-4.2 to obtain the value of M in lieu of the equation as provided in b) above. With reference to the General Note which states, "Use nearest value of L/r; interpolation unnecessary" intermediate values of L/r can vary by as much as 3%.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraph 4.3.5.

a) STEP 1 – Determine, D, assume values for L, r and t (known variables from above).

$$D = 72.25 \; in$$

$$L = 72.125 \; in$$

$$r = 4.5 \; in$$

$$t = 0.5 \; in$$

b) STEP 2 – Compute the head L/D, r/D, and L/t ratios and determine if the following equations are satisfied.

$$0.7 \le \left\{ \frac{L}{D} = \frac{72.125}{72.25} = 0.9983 \right\} \le 1.0 \qquad True$$

$$\left\{ \frac{r}{D} = \frac{4.5}{72.25} = 0.0623 \right\} \ge 0.06 \qquad True$$

$$20 \le \left\{ \frac{L}{t} = \frac{72.125}{0.5} = 144.25 \right\} \le 2000 \qquad True$$

c) STEP 3 – Calculate the geometric constants β_{th}, ϕ_{th}, R_{th}:

$$\beta_{th} = \arccos\left[\frac{0.5D - r}{L - r} \right] = \arccos\left[\frac{0.5(72.25) - 4.5}{72.125 - 4.5} \right] = 1.0842 \; rad$$

$$\phi_{th} = \frac{\sqrt{Lt}}{r} = \frac{\sqrt{72.125(0.5)}}{4.5} = 1.3345 \; rad$$

Since $\phi_{th} \ge \beta_{th}$, calculate R_{th} as follows:

$$R_{th} = 0.5D = 0.5(72.25) = 36.125 \; in$$

d) STEP 4 – Compute the coefficients C_1 and C_2.

Since $r/D = 0.0623 \le 0.08$, calculate C_1 and C_2 as follows:

$$C_1 = 9.31\left(\frac{r}{D} \right) - 0.086 = 9.31(0.0623) - 0.086 = 0.4940$$

$$C_2 = 1.25$$

e) STEP 5 – Calculate the value of internal pressure expected to produce elastic buckling of the knuckle, P_{eth}.

$$P_{eth} = \frac{C_1 E_T t^2}{C_2 R_{th}\left(\frac{R_{th}}{2} - r \right)} = \frac{(0.4940)(26.55E+06)(0.5)^2}{1.25(36.125)\left(\frac{36.125}{2} - 4.5 \right)} = 5353.9445 \; psi$$

f) STEP 6 – Calculate the value of internal pressure that will result in a maximum stress in the knuckle equal to the material yield strength, P_y.

$$P_y = \frac{C_3 t}{C_2 R_{th}\left(\frac{R_{th}}{2r} - 1\right)}$$

Since the allowable stress at design temperature is governed by time-independent properties, is C_3 the material yield strength at the design temperature, or $C_3 = S_y$.

$$P_y = \frac{26900(0.5)}{1.25(36.125)\left(\frac{36.125}{2(4.5)} - 1\right)} = 98.8274 \ psi$$

g) STEP 7 – Calculate the value of internal pressure expected to result in a buckling failure of the knuckle, P_{ck}.

Calculate variable G:

$$G = \frac{P_{eth}}{P_y} = \frac{5353.9445}{98.8274} = 54.1747$$

Since $G > 1.0$, calculate P_{ck} as follows:

$$P_{ck} = \left(\frac{0.77508G - 0.20354G^2 + 0.019274G^3}{1 + 0.19014G - 0.089534G^2 + 0.0093965G^3}\right)(P_y)$$

$$P_{ck} = \left(\frac{0.77508(54.1747) - 0.20354(54.1747)^2 + 0.019274(54.1747)^3}{1 + 0.19014(54.1747) - 0.089534(54.1747)^2 + 0.0093965(54.1747)^3}\right)(98.8274)$$

$$P_{ck} = 199.5671 \, psi$$

h) STEP 8 – Calculate the allowable pressure based on a buckling failure of the knuckle, P_{ak}.

$$P_{ak} = \frac{P_{ck}}{1.5} = \frac{199.5671}{1.5} = 133.0447 \ psi$$

i) STEP 9 – Calculate the allowable pressure based on rupture of the crown, P_{ac}.

$$P_{ac} = \frac{2SE}{\frac{L}{t} + 0.5} = \frac{2(17100)(1.0)}{\frac{72.125}{0.5} + 0.5} = 236.2694 \ psi$$

j) STEP 10 – Calculate the maximum allowable internal pressure, P_a.

$$P_a = \min\left[P_{ak}, P_{ac}\right] = \min\left[133.0447, 236.2694\right] = 133.0 \ psi$$

k) STEP 11 – If the allowable internal pressure computed from STEP 10 is greater than or equal to the design pressure, then the design is complete. If the allowable internal pressure computed from STEP 10 is less than the design pressure, then increase the head thickness and repeat steps 2 through 10.

The MAWP is $133.0 \ psi$.

4.3.5 Example E4.3.5 – Elliptical Head

Determine the Maximum Allowable Working Pressure (MAWP) for the proposed seamless 2:1 elliptical head. The Category B joint joining the head to the shell is a Type 1 butt weld and has been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Temperature = $300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength at Design Temperature = $33600\ psi$

Determine the elliptical head diameter to height ratio, k, and adjust for corrosion allowance.

$$k = \frac{D}{2h} = \frac{90.0}{2(22.5)} = 2.0$$

$$D = 90.0 + 2(Corrosion\ \ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

$$L = 81.0 + \ Corrosion\ \ Allowance = 81.0 + 0.125 = 81.125\ in$$

$$r = 15.3 + \ Corrosion\ \ Allowance = 15.3 + 0.125 = 15.425\ in$$

$$t = 1.125 - \ Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-32(c). Note, the design equations of UG-32(c) are specific to ellipsoidal heads in which half the minor axis (inside depth of head minus the skirt) equals one–fourth of the inside diameter of the head skirt. Additionally, if the ratio $t_s/L \geq 0.002$, is not satisfied, the rules of Mandatory Appendix 1-4(f) shall also be met.

$$P = \frac{2SEt}{D+0.2t} = \frac{2(20000)(1.0)(1.0)}{90.25+0.2(1.0)} = 442.2333\ \ psi$$

$$Note: \left\{ \frac{t}{L} = \frac{1.0}{81.125} = 0.0123 \right\} > 0.002,\ \ therefore\ the\ rules\ of\ 1\text{-}4(f)\ are\ not\ required$$

Commentary: The design rules of Mandatory Appendix 1-4(c) are also permitted for the design of ellipsoidal heads and are general in their presentation. The design equations are applicable to ellipsoidal heads with variable ratios of the major to the minor axis, measured as $D/2h$. Use of the general design equations, however, introduce potential inconsistencies with the specific equation provided in UG-32(c). These noted inconsistencies do not produce a significant difference in the required thickness or calculated pressure; however, the increased use of computer software to perform vessel design calculations exacerbates these inconsistencies and provides a source of doubt as to the intended accuracy of such numerical analyses.

Potential inconsistencies with the use of Appendix 1-4(c) include:

a) Applicability of applying the design corrosion allowance to the major head axis, D and the minor head axis, h when determining the factor K, as defined below.

$$K = 1/6\left[2 + \left(\frac{D}{2h}\right)^2\right]$$

The specific equations provided in UG-32(c) for ellipsoidal heads with a major to minor axis ratio of 2:1 is derived from Equation (1) as follows.

$$For\ a\ 2:1\ ellipsoidal\ head,\ the\ value\ of\ \frac{D}{2h} = 2$$

$$Therefore, \qquad K = 1/6\left[2 + \left(\frac{D}{2h}\right)^2\right] = 1/6\left[2 + (2)^2\right] = 1.0$$

$$Resulting\ in, \quad t = \frac{PDK}{2SE - 0.2P} = \frac{PD(1.0)}{2SE - 0.2P} = \frac{PD}{2SE - 0.2P}$$

$$P = \frac{2SEt}{KD + 0.2t} = \frac{2SEt}{(1.0)D + 0.2t} = \frac{2SEt}{D + 0.2t}$$

As stated in UG-16(e), "…the dimensional symbols used in all design formulas throughout this Division represent dimensions in the corroded condition". If a design corrosion allowance is applied to the major head axis, D and the minor head axis, h when determining the factor K, then the specific equations provided in UG-32(c) for 2:1 ellipsoidal head is no longer applicable as the resulting value of K will no longer produce the coefficient of 1.0 as shown in UG-32(c).

b) Use of Table 1-4.1 to obtain the value of K in lieu of the equation as provided in b) above. With reference to the General Note which states, "Use nearest value of $D/2h$; interpolation unnecessary" intermediate values of $D/2h$ can vary by as much as 7–8%.

<u>Section VIII, Division 2 Solution with VIII-1 Allowable Stresses</u>

Evaluate per VIII-2, paragraph 4.3.7, and paragraph 4.3.6.

Verify that the elliptical head diameter to height ratio, k, is within the established limits, permitting the use of the rules of VIII-2, paragraph 4.3.7.

$$1.7 \le \{k = 2\} \le 2.2 \qquad\qquad True$$

Determine the variables r and L using the uncorroded inside diameter, D.

$$r = D\left(\frac{0.5}{k} - 0.08\right) = 90.0\left(\frac{0.5}{2.0} - 0.08\right) = 15.3\ in$$

$$L = D(0.44k + 0.02) = 90.0(0.44(2.0) + 0.02) = 81.0\ in$$

Proceed with the design following the steps outlined in VIII-2, paragraph 4.3.6.

a) STEP 1 – Determine, D, assume values for L, r and t (determined from paragraph 4.3.7).

$$D = 90.25\ in$$
$$L = 81.125\ in$$
$$r = 15.425\ in$$
$$t = 1.0\ in$$

b) STEP 2 – Compute the head L/D, r/D, and L/t ratios and determine if the following equations are satisfied.

$$0.7 \le \left\{ \frac{L}{D} = \frac{81.125}{90.25} = 0.8989 \right\} \le 1.0 \qquad True$$

$$\left\{ \frac{r}{D} = \frac{15.425}{90.25} = 0.1709 \right\} \ge 0.06 \qquad True$$

$$20 \le \left\{ \frac{L}{t} = \frac{81.125}{1.000} = 81.125 \right\} \le 2000 \qquad True$$

c) STEP 3 – Calculate the geometric constants β_{th}, ϕ_{th}, R_{th}.

$$\beta_{th} = \arccos\left[\frac{0.5D - r}{L - r}\right] = \arccos\left[\frac{0.5(90.25) - 15.425}{81.125 - 15.425}\right] = 1.1017\ rad$$

$$\phi_{th} = \frac{\sqrt{Lt}}{r} = \frac{\sqrt{81.125(1.0)}}{15.425} = 0.5839\ rad$$

Since $\phi_{th} < \beta_{th}$, calculate R_{th} as follows:

$$R_{th} = \frac{0.5D - r}{\cos[\beta_{th} - \phi_{th}]} + r = \frac{0.5(90.25) - 15.425}{\cos[1.1017 - 0.5839]} + 15.425 = 49.6057\ in$$

d) STEP 4 – Compute the coefficients C_1 and C_2.

Since $\frac{r}{D} = 0.1709 > 0.08$, calculate C_1 and C_2 as follows:

$$C_1 = 0.692\left(\frac{r}{D}\right) + 0.605 = 0.692(0.1709) + 0.605 = 0.7233$$

$$C_2 = 1.46 - 2.6\left(\frac{r}{D}\right) = 1.46 - 2.6(0.1709) = 1.0157$$

e) STEP 5 – Calculate the value of internal pressure expected to produce elastic buckling of the knuckle, P_{eth}.

$$P_{eth} = \frac{C_1 E_T t^2}{C_2 R_{th}\left(\frac{R_{th}}{2} - r\right)} = \frac{(0.7233)(28.3E+06)(1.0)^2}{1.0157(49.6057)\left(\frac{49.6057}{2} - 15.425\right)} = 43321.6096\ psi$$

f) STEP 6 – Calculate the value of internal pressure that will result in a maximum stress in the knuckle equal to the material yield strength, P_y.

$$P_y = \frac{C_3 t}{C_2 R_{th}\left(\dfrac{R_{th}}{2r} - 1\right)}$$

Since the allowable stress at design temperature is governed by time-independent properties, is C_3 the material yield strength at the design temperature, or $C_3 = S_y$.

$$P_y = \frac{33600(1.0)}{1.0157(49.6057)\left(\dfrac{49.6057}{2(15.425)} - 1\right)} = 1096.8927\ psi$$

g) STEP 7 – Calculate the value of internal pressure expected to result in a buckling failure of the knuckle, P_{ck}.

Calculate variable G:

$$G = \frac{P_{eth}}{P_y} = \frac{43321.6096}{1096.8927} = 39.4948$$

Since $G > 1.0$, calculate P_{ck} as follows:

$$P_{ck} = \left(\frac{0.77508G - 0.20354G^2 + 0.019274G^3}{1 + 0.19014G - 0.089534G^2 + 0.0093965G^3}\right)\left(P_y\right)$$

$$P_{ck} = \left(\frac{0.77508(39.4948) - 0.20354(39.4948)^2 + 0.019274(39.4948)^3}{1 + 0.19014(39.4948) - 0.089534(39.4948)^2 + 0.0093965(39.4948)^3}\right)(1096.8927)$$

$$P_{ck} = 2206.1634\ psi$$

h) STEP 8 – Calculate the allowable pressure based on a buckling failure of the knuckle, P_{ak}.

$$P_{ak} = \frac{P_{ck}}{1.5} = \frac{2206.1634}{1.5} = 1470.8\ psi$$

i) STEP 9 – Calculate the allowable pressure based on rupture of the crown, P_{ac}.

$$P_{ac} = \frac{2SE}{\dfrac{L}{t} + 0.5} = \frac{2(20000)(1.0)}{\dfrac{81.125}{1.0} + 0.5} = 490.0459\ psi$$

j) STEP 10 – Calculate the maximum allowable internal pressure, P_a.

$$P_a = \min\left[P_{ak},\ P_{ac}\right] = \min\left[1470.8,\ 490.0459\right] = 490.0\ psi$$

k) STEP 11 – If the allowable internal pressure computed from STEP 10 is greater than or equal to the design pressure, then the design is complete. If the allowable internal pressure computed from STEP 10 is less than the design pressure, then increase the head thickness and repeat STEPs 2 through 10.

The MAWP is $490\ psi$.

4.3.6 Example E4.3.6 – Combined Loadings and Allowable Stresses

Determine the maximum tensile stress of the proposed cylindrical shell section considering the following design conditions and specified applied loadings. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psig @ 300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0
- Axial Force = $-66152.5\ lbs$
- Net Section Bending Moment = $5.08E+06\ in-lbs$
- Torsional Moment = $0.0\ in-lbs$

Adjust variables for corrosion and determine outside dimensions.

$$D = 90.0 + 2(Corrosion\ \ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

$$R = \frac{D}{2} = 45.125\ in$$

$$t = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

$$D_o = 90.0 + 2(Uncorroded\ \ Thickness) = 90.0 + 2(1.125) = 92.25\ in$$

Section VIII, Division 1 Solution

VIII-1 does not provide rules on the loadings to be considered in the design of a vessel. However, UG-22 requires consideration of such loadings and the provisions of U-2(g) apply. This example provides one possible method of satisfying U-2(g) via Mandatory Appendix 46 as referenced in U-2(g)(1)(a); however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

This example uses VIII-2, paragraph 4.1.5.3 which provides specific requirements to account for both design loads and design load combinations used in the design of a vessel. These design loads and design load combinations (Table 4.1.1 and Table 4.1.2 of VIII-2, respectively) are shown in this example problem in Table E4.3.6.1 and Table E4.3.6.2 for reference. The load factor, Ω_P, shown in in Table 4.1.2 of VIII-2 is used to simulate the maximum anticipated operating pressure acting simultaneously with the occasional loads specified. For this example, problem, $\Omega_P = 0.9$.

Additionally, VIII-1 does not provide a procedure for the calculation of combined stresses. VIII-2, paragraph 4.3.10.2 provides a procedure, and this procedure is used in this example problem with modifications to address specific requirements of VIII-1.

In accordance with VIII-2, paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. By inspection of the results shown in Table E4.3.6.3 and Table

E4.3.6.4, Design Load Combination 5 is determined to be the governing load combination. The pressure, net section axial force, and bending moment at the location of interest for Design Load Combination are:

$$\Omega_P P + P_s = 0.9P + P_s = 320.4\ psi$$
$$F_5 = -66152.5\ lbs$$
$$M_5 = 3048000\ in - lbs$$

a) STEP 1 – Calculate the membrane stress for the cylindrical shell. Note that the circumferential membrane stress, $\sigma_{\theta m}$, is determined based on the equations in UG-27(c)(1) and the longitudinal membrane stress due to internal pressure, σ_{sm}, is determined based on the equations in UG-27(c)(2). The shear stress is computed based on the known strength of materials solution.

Note: θ is defined as the angle measured around the circumference from the direction of the applied bending moment to the point under consideration. For this example, problem, $\theta = 0.0\ deg$ to maximize the bending stress.

$$\sigma_{\theta m} = \frac{1}{E}\left(\frac{PR}{t} + 0.6P\right) = \frac{1}{1.0}\left(\frac{320.4(45.125)}{1.0} + 0.6(320.4)\right) = 14650.29\ psi$$

$$\sigma_{sm} = \frac{1}{E}\left(\left(\frac{PR}{2t} - 0.2P\right) + \frac{4F}{\pi\left(D_o^2 - D^2\right)} \pm \frac{32MD_o\cos[\theta]}{\pi\left(D_o^4 - D^4\right)}\right)$$

$$\sigma_{sm} = \frac{1}{1.0}\left(\begin{array}{l}\left(\dfrac{320.4(45.125)}{2(1.0)} - 0.2(320.4)\right) + \\ \dfrac{4(-66152.5)}{\pi\left[(92.25)^2 - (90.25)^2\right]} \pm \dfrac{32(3048000)(92.25)\cos[0.0]}{\pi\left[(92.25)^4 - (90.25)^4\right]}\end{array}\right)$$

$$\sigma_{sm} = \begin{Bmatrix} 7164.9450 + (-230.7616) + 471.1299 = 7405.3133\ psi \\ 7164.9450 + (-230.7616) - 471.1299 = 6463.0535\ psi \end{Bmatrix}$$

$$\tau = \frac{16M_t D_o}{\pi\left(D_o^4 - D^4\right)} = \frac{16(0.0)(92.25)}{\pi\left[(92.25)^4 - (90.25)^4\right]} = 0.0\ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2}\right)$$

$$\sigma_1 = \begin{Bmatrix} 0.5\left(14650.29 + 7405.3133 + \sqrt{(14650.29 - 7405.3133)^2 + 4(0.0)^2}\right) = 14650.29\ psi \\ 0.5\left(14650.29 + 6463.0535 + \sqrt{(14650.29 - 6463.0535)^2 + 4(0.0)^2}\right) = 14650.29\ psi \end{Bmatrix}$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2}\right)$$

$$\sigma_2 = \begin{Bmatrix} 0.5\left(14650.29 + 7405.3133 - \sqrt{\left(14650.29 - 7405.3133\right)^2 + 4\left(0.0\right)^2}\right) \\ 0.5\left(14650.29 + 6463.0535 - \sqrt{\left(14650.29 - 6463.0535\right)^2 + 4\left(0.0\right)^2}\right) \end{Bmatrix}$$

$$\sigma_2 = \begin{Bmatrix} 7405.3133\ psi \\ 6463.0535\ psi \end{Bmatrix}$$

$$\sigma_3 = \sigma_r = 0.0\ psi \qquad For\ stress\ on\ the\ outside\ surface$$

c) STEP 3 – At any point on the shell, the following limit shall be satisfied.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]^{0.5} \le S$$

$$\sigma_e = \begin{Bmatrix} \frac{1}{\sqrt{2}}\left[\begin{matrix}\left(14650.29 - 7405.3133\right)^2 + \\ \left(7405.3133 - 0.0\right)^2 + \left(0.0 - 14650.29\right)^2\end{matrix}\right]^{0.5} = 12687.7766\ psi \\ \frac{1}{\sqrt{2}}\left[\begin{matrix}\left(14650.29 - 6463.0535\right)^2 + \\ \left(6463.0535 - 0.0\right)^2 + \left(0.0 - 14650.29\right)^2\end{matrix}\right]^{0.5} = 12716.7783\ psi \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_e = 12687.7766\ psi \\ \sigma_e = 12716.7783\ psi \end{Bmatrix} \le \left\{S = 20000\ psi\right\} \qquad True$$

Note that VIII-2 use an equivalent stress for the acceptance criterion. A combined stress calculation in VIII-1 would be based on the maximum principal stress; therefore,

$$\max\left[\sigma_1,\ \sigma_2,\ \sigma_3\right] \le S$$

$$\left\{\max\left[\left|14650.29\right|,\ \left|7405.3133\right|,\ \left|0.0\right|\right] = 14650.29\ psi\right\} \le \left\{S = 20000\ psi\right\} \qquad True$$

Since the maximum tensile principal stress is less than the acceptance criteria, the shell section is adequately designed.

d) STEP 4 – For cylindrical and conical shells, if the meridional stress, σ_{sm} is compressive, then Equation (4.3.45) shall be satisfied where F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2 with $\lambda = 0.15$.

STEP 4 is not necessary in this example because the meridional stress, σ_{sm}, is tensile.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraph 4.3.10.

The loads transmitted to the cylindrical shell are given in the Table E4.3.6.3. Note that this table is given in terms of the load parameters shown in VIII-2, Table 4.1.1, and Table 4.1.2 (Table E4.3.6.1 and Table E4.3.6.2 of this example). The load factor, Ω_P, shown in in Table 4.1.2 of VIII-2 is used to simulate the maximum anticipated operating pressure acting simultaneously with the occasional loads specified. For this example, problem, $\Omega_P = 0.9$. As shown in Table E4.3.6.2, the acceptance criteria are that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition.

In accordance with VIII-2, paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. By inspection of the results shown in Table E4.3.6.3 and Table E4.3.6.4, Design Load Combination 5 is determined to be the governing load combination. The pressure, net section axial force, and bending moment at the location of interest for Design Load Combination 5 are as follows.

$$\Omega_P P + P_s = 0.9P + P_s = 320.4 \ psi$$
$$F_5 = -66152.5 \ lbs$$
$$M_5 = 3048000 \ in-lbs$$

Determine applicability of the rules of VIII-2, paragraph 4.3.10 based on satisfaction of the following requirements.

The section of interest is at least $2.5\sqrt{Rt}$ away from any major structural discontinuity.

$$2.5\sqrt{Rt} = 2.5\sqrt{(45.125)(1.0)} = 16.7938 \ in$$

Shear force is not applicable.

The shell R/t ratio is greater than 3.0, or:

$$\left\{R/t = \frac{45.125}{1.0} = 45.125\right\} > 3.0 \qquad True$$

a) STEP 1 – Calculate the membrane stress for the cylindrical shell. Note that the maximum bending stress occurs at $\theta = 0.0 \ deg$.

$$\sigma_{\theta m} = \frac{PD}{E(D_o - D)} = \frac{320.4(90.25)}{1.0(92.25-90.25)} = 14458.05 \ psi$$

$$\sigma_{sm} = \frac{1}{E}\left(\frac{PD^2}{D_o^2 - D^2} + \frac{4F}{\pi(D_o^2 - D^2)} \pm \frac{32MD_o\cos[\theta]}{\pi(D_o^4 - D^4)}\right)$$

$$\sigma_{sm} = \frac{1}{1.0}\left(\frac{320.4(90.25)^2}{(92.25)^2-(90.25)^2} + \frac{4(-66152.5)}{\pi\left[(92.25)^2-(90.25)^2\right]} \pm \frac{32(3048000)(92.25)\cos[0.0]}{\pi\left[(92.25)^4-(90.25)^4\right]}\right)$$

$$\sigma_{sm} = \begin{Bmatrix} 7149.8028 + (-230.7616) + 471.1299 = 7390.1711 \ psi \\ 7149.8028 + (-230.7616) - 471.1299 = 6447.9113 \ psi \end{Bmatrix}$$

$$\tau = \frac{16M_tD_o}{\pi(D_o^4 - D^4)} = \frac{16(0.0)(92.25)}{\pi\left[(92.25)^4-(90.25)^4\right]} = 0.0 \ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2}\right)$$

$$\sigma_1 = \begin{Bmatrix} 0.5\left(14458.05 + 7390.1711 + \sqrt{\left(14458.05 - 7390.1711\right)^2 + 4\left(0.0\right)^2}\right) = 14458.05\ psi \\ 0.5\left(14458.05 + 6447.9113 + \sqrt{\left(14458.05 - 6447.9113\right)^2 + 4\left(0.0\right)^2}\right) = 14458.05\ psi \end{Bmatrix}$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2}\right)$$

$$\sigma_2 = \begin{Bmatrix} 0.5\left(14458.05 + 7390.1711 - \sqrt{\left(14458.05 - 7390.1711\right)^2 + 4\left(0.0\right)^2}\right) \\ 0.5\left(14458.05 + 6447.9113 - \sqrt{\left(14458.05 - 6447.9113\right)^2 + 4\left(0.0\right)^2}\right) \end{Bmatrix}$$

$$\sigma_2 = \begin{Bmatrix} 7390.1711\ psi \\ 6447.9113\ psi \end{Bmatrix}$$

$$\sigma_3 = \sigma_r = 0.0\ psi \qquad For\ stress\ on\ the\ outside\ surface$$

c) STEP 3 – At any point on the shell, the following limit shall be satisfied.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]^{0.5} \le S$$

$$\sigma_e = \begin{Bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} \left(14458.05 - 7390.1711\right)^2 + \left(7390.1711 - 0.0\right)^2 \\ + \left(0.0 - 14458.05\right)^2 \end{bmatrix}^{0.5} = 12522.1\ psi \\ \frac{1}{\sqrt{2}}\begin{bmatrix} \left(14458.05 - 6447.9113\right)^2 + \left(6447.9113 - 0.0\right)^2 \\ + \left(0.0 - 14458.05\right)^2 \end{bmatrix}^{0.5} = 12545.4\ psi \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_e = 12522.1\ psi \\ \sigma_e = 12545.4\ psi \end{Bmatrix} \le \left\{S = 20000\ psi\right\} \qquad True$$

Since the equivalent stress is less than the acceptance criteria, the shell section is adequately designed.

d) STEP 4 – For cylindrical and conical shells, if the meridional stress, σ_{sm} is compressive, then Equation (4.3.45) shall be satisfied where F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2 with $\lambda = 0.15$.

STEP 4 is not necessary in this example because the meridional stress, σ_{sm}, is tensile.

Table E4.3.6.1 – Design Loads from VIII-2

Table 4.1.1 – Design Loads	
Design Load Parameter	**Description**
P	Internal of External Specified Design Pressure (see paragraph 4.1.5.2.a)
P_s	Static head from liquid or bulk materials (e.g., catalyst)
D	Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following: • Weight of vessel including internals, supports (e.g., skirts, lugs, saddles, and legs), and appurtenances (e.g., platforms, ladders, etc.) • Weight of vessel contents under design and test conditions • Refractory linings, insulation • Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping • Transportation loads (the static forces obtained as equivalent to the dynamic loads experienced during normal operation of a transport vessel [see paragraph 1.2.1.3(b)]
L	• Appurtenance live loading • Effects of fluid flow, steady state or transient • Loads resulting from wave action
E	Earthquake loads [see paragraph 4.1.5.3(b)]
W	Wind loads [see paragraph 4.1.5.3(b)]
S_s	Snow loads
F	Loads due to Deflagration

Table E4.3.6.2 – Design Load Combinations from VIII-2

Table 4.1.2 – Design Load Combinations	
Design Load Combination [Note (1) and (2)]	**General Primary Membrane Allowable Stress [Note (3)]**
$P + P_s + D$	S
$P + P_s + D + L$	S
$P + P_s + D + S_s$	S
$\Omega P + P_s + D + 0.75L + 0.75S_s$	S
$\Omega P + P_s + D + (0.6W \; or \; 0.7E)$	S
$\Omega P + P_s + D + 0.75(0.6W \; or \; 0.7E) + 0.75L + 0.75S_s$	S
$0.6D + (0.6W \; or \; 0.7E)$ (Note (4))	S
$P_s + D + F$	See Annex 4-D
Other load combinations as defined in the UDS	S

Notes:

1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.
2) See paragraph 4.1.5.3 for additional requirements.
3) S is the allowable stress for the load case combination [see paragraph 4.1.5.3(c)].
4) This load combination addresses an overturning condition for foundation design. It does not apply to design of anchorage (if any) to the foundation. Refer to ASCE/SEI 7, 2.4.1 Exception 2 for an additional reduction to W that may be applicable.

Table E4.3.6.3 – Design Loads (Net-Section Axial Force and Bending Moment) at the Location of Interest

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)	$P = 356.0$
P_s	Static head from liquid or bulk materials (e.g., catalyst)	$P_s = 0.0$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest	$D_F = -66152.5\ lbs$ $D_M = 0.0\ in - lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = 0.0\ lbs$ $L_M = 0.0\ in - lbs$
E	Earthquake loads	$E_F = 0.0\ lbs$ $E_M = 0.0\ in - lbs$
W	Wind Loads	$W_F = 0.0\ lbs$ $W_M = 5.08E + 0.6\ in - lbs$
S_s	Snow Loads	$S_F = 0.0\ lbs$ $S_M = 0.0\ in - lbs$
F	Loads due to Deflagration	$F_F = 0.0\ lbs$ $F_M = 0.0\ in - lbs$

Based on these loads, the shell is required to be designed for the design load combinations shown in Table E4.3.6.4. Note that this table is given in terms of the load combinations shown in VIII-2, Table 4.1.2 (Table E4.3.6.2 of this example).

Table E4.3.6.4 – Design Load Combination at the Location of Interest

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P + P_s + D$	$P + P_s = 356.0\ psi$ $F_1 = -66152.5\ lbs$ $M_1 = 0.0\ in-lbs$	S
2	$P + P_s + D + L$	$P + P_s = 356.0\ psi$ $F_2 = -66152.5\ lbs$ $M_2 = 0.0\ in-lbs$	S
3	$P + P_s + D + S_s$	$P + P_s = 356.0\ psi$ $F_3 = -66152.5\ lbs$ $M_3 = 0.0\ in-lbs$	S
4	$0.9P + P_s + D + 0.75L + 0.75S_s$	$0.9P + P_s = 320.4\ psi$ $F_4 = -66152.5\ lbs$ $M_4 = 0.0\ in-lbs$	S
5	$0.9P + P_s + D + (0.6W\ or\ 0.7E)$	$0.9P + P_s = 320.4\ psi$ $F_5 = -66152.5\ lbs$ $M_5 = 3048000\ in-lbs$	S
6	$(0.9P + P_s + D + 0.75(0.6W\ or\ 0.7E) + 0.75L + 0.75S_s)$	$0.9P + P_s = 320.4\ psi$ $F_6 = -66152.5\ lbs$ $M_6 = 2286000\ in-lbs$	S
7	$0.6D + (0.6W\ or\ 0.7E)$	$F_7 = -39691.5\ lbs$ $M_7 = 3048000\ in-lbs$	S
8	$P_s + D + F$	$P_s = 0.0\ psi$ $F_8 = -66152.5\ lbs$ $M_8 = 0.0\ in-lbs$	See Annex 4-D

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4.3.7 Example E4.3.7 – Conical Transitions Without a Knuckle

Determine if the proposed large and small end cylinder-to-cone transitions are adequately designed considering the following design conditions and applied forces and moments. Evaluate the stresses in the cylinder and cone at both the large and small end junction.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psig @ 300°F$
- Inside Radius (Large End) = $75.0\ in$
- Thickness (Large End) = $1.8125\ in$
- Cylinder Length (Large End) = $60.0\ in$
- Inside Radius (Small End) = $45.0\ in$
- Thickness (Small End) = $1.125\ in$
- Cylinder Length (Small End) = $48.0\ in$
- Thickness (Conical Section) = $1.9375\ in$
- Length of Conical Section = $78.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle (See Figure E4.3.7) = $21.0375\ deg$
- Axial Force (Large End) = $-99167\ lbs$
- Net Section Bending Moment (Large End) = $5.406E+06\ in-lbs$
- Axial Force (Small End) = $-78104\ lbs$
- Net Section Bending Moment (Small End) = $4.301E+06\ in-lbs$

Adjust variables for corrosion.

$$R_L = 75.0 + Corrosion\ \ Allowance = 75.0 + 0.125 = 75.125\ in$$
$$R_S = 45.0 + Corrosion\ \ Allowance = 45.0 + 0.125 = 45.125\ in$$
$$t_L = 1.8125 - Corrosion\ \ Allowance = 1.8125 - 0.125 = 1.6875\ in$$
$$t_S = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$
$$t_C = 1.9375 - Corrosion\ \ Allowance = 1.9375 - 0.125 = 1.8125\ in$$
$$\alpha = 21.0375\ deg$$

Section VIII, Division 1 Solution

Rules for conical reducer sections subject to internal pressure are covered in Appendix 1-5. Rules are provided for the design of reinforcement, if needed, at the cone-to-cylinder junctions for conical reducer sections and conical head where all the elements have a common axis, and the half-apex angle satisfies $\alpha \leq 30\ deg$.

Large End

Cylindrical shell thickness per UG-27(c)(1) and conical shell thickness per UG-32(g).

$$t = \frac{PR}{SE - 0.6P} = \frac{356(75.125)}{20000(1.0) - 0.6(356)} = 1.3517\ in$$

$$t = 1.3517 + Corrosion\ \ Allowance = 1.3517 + 0.125 = 1.4767\ in$$

$$t_r = \frac{PD}{2\cos[\alpha](SE - 0.6P)} = \frac{356(2(75.125))}{2\cos[21.0375](20000(1.0) - 0.6(356))} = 1.4482\ in$$

$$t_r = 1.4482 + Corrosion\ \ Allowance = 1.4482 + 0.125 = 1.5732\ in$$

Appendix 1-5(c)(1), when a cylinder having a minimum length of $2.0\sqrt{R_L t_s}$ is attached to the large end of the cone, determine the ratio $P/S_s E_1$ and then determine Δ per Equation (1). Note: if a cylinder is not present or does not meet the minimum length requirement, Δ is not calculated.

$$\{Cylinder\ \ Length = 60.0\ in\} \geq \left\{2.0\sqrt{R_L t_s} = 2.0\sqrt{(75.125)(1.6875)} = 22.5187\ in\right\} \quad True$$

$$\Delta = 326.6\sqrt{\frac{P}{S_S E_1}} = 326.6\sqrt{\frac{356}{20000(1.0)}} = 43.5\ deg$$

Appendix 1-5(d)(1), for cones attached to a cylinder having a minimum length of $2.0\sqrt{R_L t_s}$, reinforcement shall be provided at the junction of the cone with the large cylinder for conical heads and reducers without knuckles when the value of Δ obtained from Equation (1), using the appropriate ratio $P/S_s E_1$, is less than α.

$$\{\Delta = 43.5\ deg\} \geq \{\alpha = 21.0375\ deg\};\ reinforcement\ is\ not\ required\ at\ the\ large\ end$$

Appendix 1-5(c)(3) revisited, since reinforcement is not required at the large end, $k = 1.0$.

Small End

Cylindrical shell thickness per UG-27(c)(1) and conical shell thickness per UG-32(g).

$$t = \frac{PR}{SE - 0.6P} = \frac{356(45.125)}{20000(1.0) - 0.6(356)} = 0.8119\ in$$

$$t = 0.8119 + Corrosion\ \ Allowance = 0.8119 + 0.125 = 0.9369\ in$$

$$t_r = \frac{PD}{2\cos[\alpha](SE - 0.6P)} = \frac{356(2(45.125))}{2\cos[21.0375](20000(1.0) - 0.6(356))} = 0.8699\ in$$

$$t_r = 0.8699 + Corrosion\ \ Allowance = 0.8699 + 0.125 = 0.9949\ in$$

Appendix 1-5(c)(2), when a cylinder having a minimum length of $1.4\sqrt{R_S t_S}$ is attached to the small end of the cone, determine the ratio $P/S_S E_1$ and the corresponding Δ per Equation (6). Note: if a cylinder is not present or does not meet the minimum length requirement, Δ is not calculated.

$$\{Cylinder\ Length = 48.0\ in\} \geq \left\{1.4\sqrt{R_S t_s} = 1.4\sqrt{(45.125)(1.0)} = 9.4045\ in\right\} \qquad True$$

$$\Delta = 89\sqrt{\frac{P}{S_S E_1}} = 89\sqrt{\frac{356}{20000(1.0)}} = 11.8741\ deg$$

Appendix 1-5(e)(1), for cones attached to a cylinder having a minimum length of $1.4\sqrt{R_S t_S}$ reinforcement shall be provided at the junction of the conical shell of a reducer without a flare and the small cylinder when the value of Δ obtained from Equation (6), using the appropriate ratio $P/S_S E_1$, is less than α.

$$\{\Delta = 11.8741\ deg\} < \{\alpha = 21.0375\ deg\};\ reinforcement\ is\ required\ at\ the\ small\ end$$

Appendix 1-5(c)(3) revisited, since reinforcement is required at the small end, determine the value k. Assuming the reinforcement will be place on the cylinder, if required.

$$k = \frac{y}{S_r E_r} = \frac{20000}{20000(1.0)} = 1.0$$

$$where,$$

$$y = S_s E_s = 20000(1.0) = 20000$$

Appendix 1-5(e)(1), the required area of reinforcement, A_{rs}, shall be at least equal to that indicated by the following equation when Q_s is in tension. At the small end of the cone-to-cylinder juncture, the $PR_s/2$ term is in tension. When f_2 is in compression and the quantity is larger than the $PR_s/2$ term, the design shall be in accordance with U–2(g).

$$A_{rs} = \frac{kQ_s R_s}{S_s E_1}\left(1 - \frac{\Delta}{\alpha}\right)\tan[\alpha]$$

$$A_{rs} = \frac{1.0(8429.1122)(45.125)}{20000(1.0)}\left(1 - \frac{11.8741}{21.0375}\right)\tan[21.0375] = 3.1861\ in^2$$

where,

$$Q_s = \frac{PR_s}{2} + f_2 = \left\{\begin{array}{l} \dfrac{356(45.125)}{2} + 396.8622 = 8429.1122\dfrac{lbs}{in\ of\ cir} \\ \dfrac{356(45.125)}{2} + (-947.8060) = 7084.4440\dfrac{lbs}{in\ of\ cir} \end{array}\right\}$$

$$Q_s = 8429.1122\frac{lbs}{in\ of\ cir} \qquad Use\ the\ maximum\ positive\ value$$

and,

$$f_2 = \frac{F_S}{2\pi R_S} \pm \frac{M_S}{\pi R_S^2} = \begin{Bmatrix} \dfrac{-78104}{2\pi(45.125)} + \dfrac{4.301E+06}{\pi(45.125)^2} = +396.8622 \ \dfrac{lbs}{in \ of \ cir} \\ \dfrac{-78104}{2\pi(45.125)} - \dfrac{4.301E+06}{\pi(45.125)^2} = -947.8060 \ \dfrac{lbs}{in \ of \ cir} \end{Bmatrix}$$

The effective area of reinforcement can be determined in accordance with the following:

$$A_{es} = 0.78\left[\sqrt{R_s t_s}\,(t_s - t) + \sqrt{\left(\frac{R_s t_s}{\cos[\alpha]}\right)}(t_c - t_r)\right]$$

$$A_{es} = 0.78\left[\sqrt{(45.125)(1.0)}\,(1.0 - 0.8119) + \sqrt{\left(\frac{(45.125)(1.8125)}{\cos[21.0375]}\right)}(1.8125 - 0.8699)\right]$$

$$A_{es} = 7.8900 \ in^2$$

The effective area of available reinforcement due to the excess thickness in the cylindrical shell and conical shell, A_{es}, exceeds the required reinforcement, A_{rs}.

$$\left\{A_{es} = 7.8900 \ in^2\right\} \geq \left\{A_{rs} = 3.1861 \ in^2\right\} \qquad True$$

If this was not true, reinforcement would need to be added to the cylindrical or conical shell using a thick insert plate or reinforcing ring. Any additional area of reinforcement which is required shall be situated within a distance of $\sqrt{R_S t_S}$ from the junction, and the centroid of the added area shall be within a distance of $0.25\sqrt{R_S t_S}$ from the junction. In addition, note that in the above solution, the net-section axial force and net-section bending moment are included as an equivalent axial load per unit circumference.

<u>Section VIII, Division 2 Solution with VIII-1 Allowable Stresses</u>

Evaluate per VIII-2, paragraph 4.3.11.

Per VIII-2, paragraph 4.3.11.3, the length of the conical shell, measured parallel to the surface of the cone, shall be equal to or greater than the following:

$$L_C \geq 2.0\sqrt{\frac{R_L t_C}{\cos[\alpha]}} + 1.4\sqrt{\frac{R_S t_C}{\cos[\alpha]}}$$

$$\left\{L_C = 78.0 \ in\right\} \geq \left\{2.0\sqrt{\frac{75.125(1.8125)}{\cos[21.0375]}} + 1.4\sqrt{\frac{45.125(1.8125)}{\cos[21.0375]}} = 37.2624 \ in\right\} \qquad True$$

Evaluate the Large End cylinder-to-cone junction per VIII-2, paragraph 4.3.11.4.

a) STEP 1 – Compute the required thickness of the cylinder at the large end of the cone-to-cylinder junction using VIII-2, paragraph 4.3.3., and select the nominal thickness, t_L (as specified in design conditions).

$$t_L = 1.6875\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, compute the required thickness of the cone at the large end of the cone-to-cylinder junction using VIII-2, paragraph 4.3.4., and select the nominal thickness, t_C (as specified in design conditions).

$$\alpha = 21.0375\ deg$$

$$t_C = 1.8125\ in$$

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with VIII-2, Part 5. In the calculations, if $0\ deg < \alpha \leq 10\ deg$, then use $\alpha = 10\ deg$.

$$20 \leq \left\{ \frac{R_L}{t_L} = \frac{75.125}{1.6875} = 44.5185 \right\} \leq 500 \qquad True$$

$$1 \leq \left\{ \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741 \right\} \leq 2 \qquad True$$

$$\{\alpha = 21.0375\ deg\} \leq \{60\ deg\} \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force, F_L. Calculate the equivalent line load, X_L, using the specified net section axial force, F_L, and bending moment, M_L.

$$X_L = \frac{F_L}{2\pi R_L} \pm \frac{M_L}{\pi R_L^2} = \begin{Bmatrix} \dfrac{-99167}{2\pi(75.125)} + \dfrac{5406000}{\pi(75.125)^2} = 94.8111\ \dfrac{lbs}{in} \\ \dfrac{-99167}{2\pi(75.125)} - \dfrac{5406000}{\pi(75.125)^2} = -514.9886\ \dfrac{lbs}{in} \end{Bmatrix}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per VIII-2, Table 4.3.3, and Table 4.3.4, respectively. For calculated values of n other than those presented in VIII-2, Table 4.3.3 and Table 4.3.4, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741$$

$$H = \sqrt{\frac{R_L}{t_L}} = \sqrt{\frac{75.125}{1.6875}} = 6.6722$$

$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in VIII-2, Table 4.3.3 and Table 4.3.4 is required. The results of the interpolation are summarized with the following values for C_i.

Equation Coefficients C_i	VIII-2, Table 4.3.3		VIII-2, Table 4.3.4	
	Pressure Applied Junction Moment Resultant M_{sN}	Pressure Applied Junction Shear Force Resultant Q_N	Equivalent Line Load Junction Moment Resultant M_{sN}	Equivalent Line Load Junction Shear Force Resultant Q_N
1	-3.079751	-1.962370	-5.706141	-4.878520
2	3.662099	2.375540	0.004705	0.006808
3	0.788301	0.582454	0.474988	-0.018569
4	-0.226515	-0.107299	-0.213112	-0.197037
5	-0.080019	-0.103635	2.241065	2.033876
6	0.049314	0.151522	0.000025	-0.000085
7	0.026266	0.010704	0.002759	-0.000109
8	-0.015486	-0.018356	-0.001786	-0.004071
9	0.001773	0.006551	-0.214046	-0.208830
10	-0.007868	-0.021739	0.000065	-0.000781
11	---	---	-0.106223	0.004724

For the applied pressure case both M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\begin{bmatrix} C_1 + C_2\ln[H] + C_3\ln[B] + C_4(\ln[H])^2 + C_5(\ln[B])^2 + \\ C_6\ln[H]\ln[B] + C_7(\ln[H])^3 + C_8(\ln[B])^3 + \\ C_9\ln[H](\ln[B])^2 + C_{10}(\ln[H])^2\ln[B] \end{bmatrix}$$

This results in:

$$M_{sN} = -\exp\begin{bmatrix} -3.079751 + 3.662099 \cdot \ln[6.6722] + 0.788301 \cdot \ln[0.3846] + \\ (-0.226515)(\ln[6.6722])^2 + (-0.080019)(\ln[0.3846])^2 + \\ 0.049314 \cdot \ln[6.6722] \cdot \ln[0.3846] + \\ 0.026266(\ln[6.6722])^3 + (-0.015486)(\ln[0.3846])^3 + \\ 0.001773 \cdot \ln[6.6722] \cdot (\ln[0.3846])^2 + \\ (-0.007868)(\ln[6.6722])^2 \cdot \ln[0.3846] \end{bmatrix} = -10.6148$$

$$Q_N = -\exp\begin{bmatrix} -1.962370 + 2.375540 \cdot \ln[6.6722] + 0.582454 \cdot \ln[0.3846] + \\ (-0.107299)(\ln[6.6722])^2 + (-0.103635)(\ln[0.3846])^2 + \\ 0.151522 \cdot \ln[6.6722] \cdot \ln[0.3846] + \\ 0.010704(\ln[6.6722])^3 + (-0.018356)(\ln[0.3846])^3 + \\ 0.006551 \cdot \ln[6.6722] \cdot (\ln[0.3846])^2 + \\ (-0.021739)(\ln[6.6722])^2 \cdot \ln[0.3846] \end{bmatrix} = -4.0925$$

For the Equivalent Line Load case, M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\left[\frac{\begin{pmatrix} C_1 + C_3 \ln\left[H^2\right] + C_5 \ln[\alpha] + C_7\left(\ln\left[H^2\right]\right)^2 + \\ C_9(\ln[\alpha])^2 + C_{11} \ln\left[H^2\right] \ln[\alpha] \end{pmatrix}}{\begin{pmatrix} 1 + C_2 \ln\left[H^2\right] + C_4 \ln[\alpha] + C_6\left(\ln\left[H^2\right]\right)^2 + \\ C_8(\ln[\alpha])^2 + C_{10} \ln\left[H^2\right] \ln[\alpha] \end{pmatrix}}\right]$$

This results in:

$$M_{sN} = -\exp\left[\frac{\left(\begin{array}{l} -5.706141 + 0.474988 \cdot \ln\left[6.6722^2\right] + \\ 2.241065 \cdot \ln\left[21.0375\right] + 0.002759\left(\ln\left[6.6722^2\right]\right)^2 + \\ (-0.214046)\left(\ln\left[21.0375\right]\right)^2 + \\ (-0.106223)\ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{array}\right)}{\left(\begin{array}{l} 1 + 0.004705 \cdot \ln\left[6.6722^2\right] + (-0.213112)\ln\left[21.0375\right] + \\ 0.000025\left(\ln\left[6.6722^2\right]\right)^2 + (-0.001786)\left(\ln\left[21.0375\right]\right)^2 + \\ 0.000065 \cdot \ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{array}\right)}\right] = -0.4912$$

$$Q_{N} = -\exp\left[\frac{\left(\begin{array}{l} -4.878520 + (-0.018569)\ln\left[6.6722^2\right] + \\ 2.033876 \cdot \ln\left[21.0375\right] + (-0.000109)\left(\ln\left[6.6722^2\right]\right)^2 + \\ (-0.208830)\left(\ln\left[21.0375\right]\right)^2 + \\ 0.004724 \cdot \ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{array}\right)}{\left(\begin{array}{l} 1 + 0.006808 \cdot \ln\left[6.6722^2\right] + (-0.197037)\ln\left[21.0375\right] + \\ (-0.000085)\left(\ln\left[6.6722^2\right]\right)^2 + (-0.004071)\left(\ln\left[21.0375\right]\right)^2 + \\ (-0.000781)\ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{array}\right)}\right] = -0.1845$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$\begin{array}{lll} \textit{Internal Pressure}: & M_{sN} = -10.6148, & Q_N = -4.0925 \\ \textit{Equivalent Line Load}: & M_{sN} = -0.4912, & Q_N = -0.1845 \end{array}$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in VIII-2, Table 4.3.1 for the Large End Junction.

Evaluate the Cylinder at the Large End:

Stress Resultant Calculations:

$$M_{sP} = Pt_L^2 M_{sN} = 356(1.6875)^2(-10.6148) = -10760.9194\ \frac{in-lbs}{in}$$

$$M_{sX} = X_L t_L M_{sN} = \begin{cases} 94.8111(1.6875)(-0.4912) = -78.5889\ \dfrac{in-lbs}{in} \\ -514.9886(1.6875)(-0.4912) = 426.8741\ \dfrac{in-lbs}{in} \end{cases}$$

$$M_s = M_{sP} + M_{sX} = \begin{cases} -10760.9194 + (-78.5889) = -10839.5083\ \dfrac{in-lbs}{in} \\ -10760.9194 + 426.8741 = -10334.0453\ \dfrac{in-lbs}{in} \end{cases}$$

$$Q_P = Pt_L Q_N = 356(1.6875)(-4.0925) = -2458.5694\ \frac{lbs}{in}$$

$$Q_X = X_L Q_N = \begin{cases} 94.8111(-0.1845) = -17.4926\ \dfrac{lbs}{in} \\ -514.9886(-0.1845) = 95.0154\ \dfrac{lbs}{in} \end{cases}$$

$$Q = Q_P + Q_X = \begin{cases} -2458.5694 + (-17.4926) = -2476.0620\ \dfrac{lbs}{in} \\ -2458.5694 + 95.0154 = -2363.5540\ \dfrac{lbs}{in} \end{cases}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_L^2 t_L^2}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(75.125)^2(1.6875)^2}\right]^{0.25} = 0.1142\ in^{-1}$$

$$N_s = \frac{PR_L}{2} + X_L = \begin{cases} \dfrac{356(75.125)}{2} + 94.8111 = 13467.0611\ \dfrac{lbs}{in} \\ \dfrac{356(75.125)}{2} + (-514.9886) = 12857.2614\ \dfrac{lbs}{in} \end{cases}$$

$$N_{\theta} = PR_L + 2\beta_{cy} R_L \left(-M_s \beta_{cy} + Q\right)$$

$$N_{\theta} = \begin{Bmatrix} 356(75.125) + 2(0.1142)(75.125)\left(-(-10839.5083)(0.1142) + (-2476.0620)\right) \\ 356(75.125) + 2(0.1142)(75.125)\left(-(-10334.0453)(0.1142) + (-2363.553)\right) \end{Bmatrix}$$

$$N_{\theta} = \begin{Bmatrix} 5498.9524 \ \dfrac{lbs}{in} \\ 6438.9685 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{pc} = 1.0$$

Stress Calculations:

Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_s}{t_L} = \begin{Bmatrix} \dfrac{13467.0611}{1.6875} = 7980.4807 \ psi \\ \dfrac{12857.2614}{1.6875} = 7619.1179 \ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_s}{t_L^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(-10839.5083)}{(1.6875)^2(1.0)} = -22838.7994 \ psi \\ \dfrac{6(-10334.0453)}{(1.6875)^2(1.0)} = -21773.7909 \ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_{\theta}}{t_L} = \begin{Bmatrix} \dfrac{5498.9524}{1.6875} = 3258.6385 \ psi \\ \dfrac{6438.9685}{1.6875} = 3815.6850 \ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_L^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(0.3)(-10839.5083)}{(1.6875)^2(1.0)} = -6851.6398 \ psi \\ \dfrac{6(0.3)(-10334.0453)}{(1.6875)^2(1.0)} = -6532.1373 \ psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\left\{\begin{array}{l}\sigma_{sm} = 7980.4807\ psi \\ \sigma_{sm} = 7619.1179\ psi\end{array}\right\} \le \{1.5S = 1.5(20000) = 30000\ psi\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{sm} + \sigma_{sb} = 7980.4807 + (-22838.7994) = |-14858.3\ psi| \\ \sigma_{sm} - \sigma_{sb} = 7980.4807 - (-22838.7994) = 30819.3\ psi \\ \sigma_{sm} + \sigma_{sb} = 7619.1179 + (-21773.7909) = |-14154.7\ psi| \\ \sigma_{sm} - \sigma_{sb} = 7619.1179 - (-21773.7909) = 29392.9\ psi\end{array}\right\} \le \{S_{PS} = 60000\ psi\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m} = 3258.6385\ psi \\ \sigma_{\theta m} = 3815.6850\ psi\end{array}\right\} \le \{1.5S = 1.5(20000) = 30000\ psi\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m} + \sigma_{\theta b} = 3258.6385 + (-6851.6398) = |-3593.0\ psi| \\ \sigma_{\theta m} - \sigma_{\theta b} = 3258.6385 - (-6851.6398) = 10110.3\ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 3815.6850 + (-6532.1373) = |-2716.5\ psi| \\ \sigma_{\theta m} - \sigma_{\theta b} = 3815.6850 - (-6532.1373) = 10347.8\ psi\end{array}\right\} \le \{S_{PS} = 60000\ psi\} \qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cylinder at the cylinder-to-cone junction at the large end is adequately designed.

Evaluate the Cone at the Large End:

Stress Resultant Calculations – as determined above:

$$M_{csP} = M_{sP} = -10760.9194\ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \left\{\begin{array}{l}-78.5889\ \dfrac{in-lbs}{in} \\ 426.8741\ \dfrac{in-lbs}{in}\end{array}\right\}$$

$$M_{cs} = M_{csP} + M_{csX} = \left\{\begin{array}{l}-10760.9194 + (-78.5889) = -10839.5083\ \dfrac{in-lbs}{in} \\ -10760.9194 + 426.8741 = -10334.0453\ \dfrac{in-lbs}{in}\end{array}\right\}$$

$$Q_c = Q\cos[\alpha] + N_s\sin[\alpha]$$

$$Q_c = \left\{\begin{array}{l}((-2476.0620)\cos[21.0375] + (13467.0611)\sin[21.0375]) = 2523.3690\ \dfrac{lbs}{in} \\ ((-2363.5540)\cos[21.0375] + (12857.2614)\sin[21.0375]) = 2409.4726\ \dfrac{lbs}{in}\end{array}\right\}$$

$$R_C = \frac{R_L}{\cos[\alpha]} = \frac{75.125}{\cos[21.0375]} = 80.4900\ in$$

$$\beta_{co} = \left[\frac{3\left(1-\nu^2\right)}{R_C^2 t_C^2}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(80.4900)^2(1.8125)^2}\right]^{0.25} = 0.1064\ in^{-1}$$

$$N_{cs} = N_s \cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{Bmatrix} (13467.0611)\cos[21.0375] - (-2476.0620)\sin[21.0375] = 13458.2772\ \dfrac{lbs}{in} \\ (12857.2614)\cos[21.0375] - (-2363.5540)\sin[21.0375] = 12848.7353\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_{c\theta} = \frac{PR_L}{\cos[\alpha]} + 2\beta_{co}R_C\left(-M_{cs}\beta_{co} - Q_c\right)$$

$$N_{c\theta} = \begin{Bmatrix} \dfrac{356(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(-10839.5083)(0.1064) - 2523.3690\right) \\ \dfrac{356(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(-10334.0453)(0.1064) - 2409.4726\right) \end{Bmatrix}$$

$$N_{c\theta} = \begin{Bmatrix} 5187.9337\ \dfrac{lbs}{in} \\ 6217.6021\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{cpc} = 1.0$$

Stress Calculations:

Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \left\{ \begin{array}{l} \dfrac{13458.2772}{1.8125} = 7425.2564 \;\; psi \\ \dfrac{12848.7353}{1.8125} = 7088.9574 \;\; psi \end{array} \right\}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \left\{ \begin{array}{l} \dfrac{6(-10839.5083)}{(1.8125)^2(1.0)} = -19797.2470 \;\; psi \\ \dfrac{6(-10334.0453)}{(1.8125)^2(1.0)} = -18874.0708 \;\; psi \end{array} \right\}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \left\{ \begin{array}{l} \dfrac{5187.9337}{1.8125} = 2862.3082 \;\; psi \\ \dfrac{6217.6021}{1.8125} = 3430.4012 \;\; psi \end{array} \right\}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \left\{ \begin{array}{l} \dfrac{6(0.3)(-10839.5083)}{(1.8125)^2(1.0)} = -5939.1741 \;\; psi \\ \dfrac{6(0.3)(-10334.0453)}{(1.8125)^2(1.0)} = -5662.2213 \;\; psi \end{array} \right\}$$

Check Acceptance Criteria:

$$\left\{ \begin{array}{l} \sigma_{sm} = 7425.2564 \;\; psi \\ \sigma_{sm} = 7088.9574 \;\; psi \end{array} \right\} \le \left\{ 1.5S = 1.5(20000) = 30000 \;\; psi \right\} \qquad True$$

$$\left\{ \begin{array}{l} \sigma_{sm} + \sigma_{sb} = 7425.2564 + (-19797.2470) = |-12371.9906 \;\; psi| \\ \sigma_{sm} - \sigma_{sb} = 7425.2564 - (-19797.2470) = 27222.5034 \;\; psi \\ \sigma_{sm} + \sigma_{sb} = 7088.9574 + (-18874.0708) = |-11785.1 \;\; psi| \\ \sigma_{sm} - \sigma_{sb} = 7088.9574 - (-18874.0708) = 25963.0 \;\; psi \end{array} \right\} \le \left\{ S_{PS} = 60000 \;\; psi \right\} True$$

$$\left\{ \begin{array}{l} \sigma_{\theta m} = 2862.3082 \;\; psi \\ \sigma_{\theta m} = 3430.4012 \;\; psi \end{array} \right\} \le \left\{ 1.5S = 1.5(20000) = 30000 \;\; psi \right\} \qquad True$$

$$\left\{ \begin{array}{l} \sigma_{\theta m} + \sigma_{\theta b} = 2862.3082 + (-5939.1741) = |-3076.8659 \;\; psi| \\ \sigma_{\theta m} - \sigma_{\theta b} = 2862.3082 - (-5939.1741) = 8801.4823 \; psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 3430.4012 + (-5662.2213) = |-2231.8 \;\; psi| \\ \sigma_{\theta m} - \sigma_{\theta b} = 3430.4012 - (-5662.2213) = 9092.6 \; psi \end{array} \right\} \le \left\{ S_{PS} = 60000 \;\; psi \right\} \quad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cone at the cylinder-to-cone junction at the large end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore, the design is complete.

Evaluate the Small End cylinder-to-cone junction per VIII-2, paragraph 4.3.11.5.

a) STEP 1 – Compute the required thickness of the cylinder at the small end of the cone-to-cylinder junction using VIII-2, paragraph 4.3.3., and select the nominal thickness, t_S, (as specified in design conditions).

b) STEP 2 – Determine the cone half-apex angle, α, compute the required thickness of the cone at the small end of the cone-to-cylinder junction using VIII-2, paragraph 4.3.4., t_C (as specified in design conditions).

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with VIII-2, Part 5. In the calculations, if $0\ deg < \alpha \le 10\ deg$, then use $\alpha = 10\ deg$.

$$20 \le \left(\frac{R_S}{t_S} = \frac{45.125}{1.0} = 45.125 \right) \le 500 \qquad True$$

$$1 \le \left(\frac{t_C}{t_S} = \frac{1.8125}{1.0} = 1.8125 \right) \le 2 \qquad True$$

$$\{\alpha = 21.0375\ deg\} \le \{60\ deg\} \qquad True$$

d) STEP 4 – Calculate the equivalent line load, X_S, given the net section axial force, F_S, and bending moment, M_S, applied at the conical transition.

$$X_S = \frac{F_S}{2\pi R_S} \pm \frac{M_S}{\pi R_S^2} = \begin{Bmatrix} \dfrac{-78104.2}{2\pi(45.125)} + \dfrac{4301000}{\pi(45.125)^2} = 396.8622\ \dfrac{lbs}{in} \\ \dfrac{-78104.2}{2\pi(45.125)} - \dfrac{4301000}{\pi(45.125)^2} = -947.8060\ \dfrac{lbs}{in} \end{Bmatrix}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per VIII-2, Table 4.3.5, and Table 4.3.6, respectively. For calculated values of n other than those presented in VIII-2, Table 4.3.5 and Table 4.3.6, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_S} = \frac{1.8125}{1.000} = 1.8125$$

$$H = \sqrt{\frac{R_S}{t_S}} = \sqrt{\frac{45.125}{1.000}} = 6.7175$$

$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in VIII-2, Table 4.3.5 and Table 4.3.6 is required. The result of the interpolation is summarized with the following values for C_i.

Equation Coefficients C_i	VIII-2, Table 4.3.5		VIII-2, Table 4.3.6	
	Pressure Applied Junction Moment Resultant M_{sN}	Pressure Applied Junction Shear Force Resultant Q_N	Equivalent Line Load Junction Moment Resultant M_{sN}	Equivalent Line Load Junction Shear Force Resultant Q_N
1	-15.144683	0.569891	0.006792	-0.408044
2	3.036812	-0.000027	0.000290	0.021200
3	6.460714	0.008431	-0.000928	-0.325518
4	-0.155909	0.002690	0.121611	-0.003988
5	-1.462862	-0.002884	0.010581	-0.111262
6	-0.369444	0.000000	-0.000011	0.002204
7	0.007742	-0.000005	-0.000008	0.000255
8	0.143191	-0.000117	0.005957	-0.014431
9	0.040944	-0.000087	0.001310	0.000820
10	0.007178	0.000001	0.000186	0.000106
11	---	-0.003778	0.194433	---

For the applied pressure case M_{sN} is calculated using the following equation

$$M_{sN} = \exp\left[\begin{array}{l} C_1 + C_2 \ln\left[H^2\right] + C_3 \ln\left[\alpha\right] + C_4\left(\ln\left[H^2\right]\right)^2 + C_5\left(\ln\left[\alpha\right]\right)^2 + \\ C_6 \ln\left[H^2\right]\ln\left[\alpha\right] + C_7\left(\ln\left[H^2\right]\right)^3 + C_8\left(\ln\left[\alpha\right]\right)^3 + \\ C_9 \ln\left[H^2\right]\left(\ln\left[\alpha\right]\right)^2 + C_{10}\left(\ln\left[H^2\right]\right)^2 \ln\left[\alpha\right] \end{array}\right]$$

This results in:

$$M_{sN} = \exp\left[\begin{array}{l} -15.144683 + 3.036812 \cdot \ln\left[6.7175^2\right] + 6.460714 \cdot \ln\left[21.0375\right] + \\ (-0.155909)\left(\ln\left[6.7175^2\right]\right)^2 + (-1.462862)\left(\ln\left[21.0375\right]\right)^2 + \\ (-0.369444)\ln\left[6.7175^2\right] \cdot \ln\left[21.0375\right] + \\ 0.007742\left(\ln\left[6.7175^2\right]\right)^3 + 0.143191\left(\ln\left[21.0375\right]\right)^3 + \\ 0.040944 \cdot \ln\left[6.7175^2\right] \cdot \left(\ln\left[21.0375\right]\right)^2 + \\ 0.007178\left(\ln\left[6.7175^2\right]\right)^2 \cdot \ln\left[21.0375\right] \end{array}\right] = 9.2135$$

For the applied pressure case Q_N is calculated using the following equation,

$$Q_N = \left(\frac{C_1 + C_3 H^2 + C_5 \alpha + C_7 H^4 + C_9 \alpha^2 + C_{11} H^2 \alpha}{1 + C_2 H^2 + C_4 \alpha + C_6 H^4 + C_8 \alpha^2 + C_{10} H^2 \alpha}\right)$$

This results in:

$$Q_N = \left(\frac{\left(\begin{array}{l} 0.569891 + 0.008431(6.7175)^2 + (-0.002884)(21.0375) + \\ (-0.000005)(6.7175)^4 + (-0.000087)(21.0375)^2 + \\ (-0.003778)(6.7175)^2(21.0375) \end{array}\right)}{\left(\begin{array}{l} 1 + (-0.000027)(6.7175)^2 + 0.002690(21.0375) + \\ 0.000000(6.7175)^4 + (-0.000117)(21.0375)^2 + \\ 0.000001(6.7175)^2(21.0375) \end{array}\right)}\right) = -2.7333$$

For the Equivalent Line Load case, M_{sN} is calculated using the following equation,

$$M_{sN} = \left(\frac{C_1 + C_3 H + C_5 B + C_7 H^2 + C_9 B^2 + C_{11} HB}{1 + C_2 H + C_4 B + C_6 H^2 + C_8 B^2 + C_{10} HB}\right)$$

This results in:

$$M_{sN} = \left(\frac{\left(\begin{array}{l} 0.006792 + (-0.000928)(6.7175) + 0.010581(0.3846) + \\ (-0.000008)(6.7175)^2 + 0.001310(0.3846)^2 + \\ 0.194433(6.7175)(0.3846) \end{array} \right)}{\left(\begin{array}{l} 1 + 0.000290(6.7175) + 0.121611(0.3846) + \\ (-0.000011)(6.7175)^2 + 0.005957(0.3846)^2 + \\ 0.000186(6.7175)(0.3846) \end{array} \right)} \right) = 0.4828$$

For the Equivalent Line Load case, Q_N is calculated using the following equation,

$$Q_N = \left(\begin{array}{l} C_1 + C_2 \ln[H] + C_3 \ln[B] + C_4 (\ln[H])^2 + C_5 (\ln[B])^2 + C_6 \ln[H] \ln[B] + \\ C_7 (\ln[H])^3 + C_8 (\ln[B])^3 + C_9 \ln[H] (\ln[B])^2 + C_{10} (\ln[H])^2 \ln[B] \end{array} \right)$$

This results in:

$$Q_N = \left(\begin{array}{l} -0.408044 + 0.021200 \cdot \ln[6.7175] + (-0.325518) \ln[0.3846] + \\ (-0.003988)(\ln[6.7175])^2 + (-0.111262)(\ln[0.3846])^2 + \\ 0.002204 \cdot \ln[6.7175] \cdot \ln[0.3846] + 0.000255 \cdot (\ln[6.7175])^3 + \\ (-0.014431)(\ln[0.3846])^3 + 0.000820 \cdot \ln[6.7175] \cdot (\ln[0.3846])^2 + \\ 0.000106 (\ln[6.7175])^2 \cdot \ln[0.3846] \end{array} \right) = -0.1613$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$\begin{array}{lll} \textit{Internal Pressure}: & M_{sN} = 9.2135, & Q_N = -2.7333 \\ \textit{Equivalent Line Load}: & M_{sN} = 0.4828, & Q_N = -0.1613 \end{array}$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in VIII-2, Table 4.3.2 for the Small End Junction.

Evaluate the Cylinder at the Small End:

Stress Resultant Calculations:

$$M_{sP} = Pt_S^2 M_{sN} = 356(1.000)^2(9.2135) = 3280.0060\ \frac{in-lbs}{in}$$

$$M_{sX} = X_S t_S M_{sN} = \begin{Bmatrix} 396.8622(1.0000)(0.4828) = -191.6051\ \dfrac{in-lbs}{in} \\ -947.8060(1.0000)(0.4828) = -457.6007\ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$M_s = M_{sP} + M_{sX} = \begin{Bmatrix} 3280.0060 + (191.6051) = 3471.6111\ \dfrac{in-lbs}{in} \\ 3280.0060 + (-457.6007) = 2822.4053\ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$Q_P = Pt_S Q_N = 356(1.0000)(-2.7333) = -973.0548\ \frac{lbs}{in}$$

$$Q_X = X_S Q_N = \begin{Bmatrix} 396.8622(-0.1613) = -64.0139\ \dfrac{lbs}{in} \\ -947.8060(-0.1613) = 152.8811\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$Q = Q_P + Q_X = \begin{Bmatrix} -973.0548 + (-64.0139) = -1037.0687\dfrac{lbs}{in} \\ -973.0548 + 152.8811 = -820.1737\dfrac{lbs}{in} \end{Bmatrix}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_S^2 t_S^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(45.1250)^2(1.000)^2}\right]^{0.25} = 0.1914\ in^{-1}$$

$$N_s = \frac{PR_S}{2} + X_S = \begin{Bmatrix} \dfrac{356(45.125)}{2} + 396.8622 = 8429.1122\ \dfrac{lbs}{in} \\ \dfrac{356(45.125)}{2} + (-947.8060) = 7084.4440\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_\theta = PR_S + 2\beta_{cy}R_S\left(-M_s\beta_{cy} - Q\right)$$

$$N_\theta = \begin{Bmatrix} 356(45.125)+2(0.1914)(45.125)\left(-(3471.6111)(0.1914)-(-1037.0687)\right) \\ 356(45.125)+2(0.1914)(45.125)\left(-(2822.4053)(0.1914)-(-820.1737)\right) \end{Bmatrix}$$

$$N_\theta = \begin{Bmatrix} 22500.7769\ \dfrac{lbs}{in} \\ 20900.5790\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{pc} = 1.0$$

Stress Calculations:

Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_s}{t_S} = \begin{Bmatrix} \dfrac{8429.1122}{1.0000} = 8429.1122\ psi \\ \dfrac{7084.4440}{1.0000} = 7084.4440\ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(3471.6111)}{(1.0000)^2(1.0)} = 20829.6666\ psi \\ \dfrac{6(2822.4053)}{(1.0000)^2(1.0)} = 16934.4318\ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_S} = \begin{Bmatrix} \dfrac{22500.7769}{1.0000} = 22500.7769\ psi \\ \dfrac{20900.5790}{1.0000} = 20900.5790\ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(0.3)(3471.6111)}{(1.0000)^2(1.0)} = 6248.8999\ psi \\ \dfrac{6(0.3)(2822.4053)}{(1.0000)^2(1.0)} = 5080.3295\ psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\left\{\begin{array}{l}\sigma_{sm} = 8429.1122 \ psi \\ \sigma_{sm} = 7084.4440 \ psi\end{array}\right\} \leq \left\{1.5S = 1.5(20000) = 30000 \ psi\right\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{sm} + \sigma_{sb} = 8429.1122 + (20829.6666) = 29258.8 \ psi \\ \sigma_{sm} - \sigma_{sb} = 8429.1122 - (20829.6666) = |-12400.6 \ psi| \\ \sigma_{sm} + \sigma_{sb} = 7084.4440 + (16934.4318) = 24018.9 \ psi \\ \sigma_{sm} - \sigma_{sb} = 7084.4440 - (16934.4318) = |-9850.0 \ psi|\end{array}\right\} \leq \left\{S_{PS} = 60000 \ psi\right\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m} = 22500.7769 \\ \sigma_{\theta m} = 20900.5790\end{array}\right\} \leq \left\{1.5S = 1.5(20000) = 30000 \ psi\right\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m} + \sigma_{\theta b} = 22500.7769 + (6248.8999) = 28749.7 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 22500.7769 - (6248.8999) = 16251.9 \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 20900.5790 + (5080.3295) = 25981.0 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 20900.5790 - (5080.3295) = 15820.2 \ psi\end{array}\right\} \leq \left\{S_{PS} = 60000 \ psi\right\} \qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cylinder at the cylinder-to-cone junction at the small end is adequately designed.

<u>Evaluate the Cone at the Small End:</u>

Stress Resultant Calculations:

$$M_{csP} = M_{sP} = 3280.0060 \ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \left\{\begin{array}{l}191.6051 \ \dfrac{in-lbs}{in} \\ -457.6007 \ \dfrac{in-lbs}{in}\end{array}\right\}$$

$$M_{cs} = M_{csP} + M_{csX} = \left\{\begin{array}{l}3280.0060 + 191.6051 = 3471.6111 \ \dfrac{in-lbs}{in} \\ 3280.0060 + (-457.6007) = 2822.4053 \ \dfrac{in-lbs}{in}\end{array}\right\}$$

$$Q_c = Q\cos[\alpha] + N_s \sin[\alpha]$$

$$Q_c = \left\{\begin{array}{l}-1037.0687\cos[21.0375] + 8429.1122\sin[21.0375] = 2057.9298 \ \dfrac{lbs}{in} \\ -820.1737\cos[21.0375] + 7084.4440\sin[21.0375] = 1777.6603 \ \dfrac{lbs}{in}\end{array}\right\}$$

$$R_c = \frac{R_c}{\cos[\alpha]} = \frac{45.1250}{\cos[21.0375]} = 48.3476\ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_c^{\,2} t_c^{\,2}}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(48.3476)^2(1.8125)^2}\right]^{0.25} = 0.1373\ in^{-1}$$

$$N_{cs} = N_s \cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{cases} \left(8429.1122\cos[21.0375] - (-1037.0687)\sin[21.0375]\right) = 8239.5612\ \dfrac{lbs}{in} \\ \left(7084.4440\cos[21.0375] - (-820.1737)\sin[21.0375]\right) = 6906.6602\ \dfrac{lbs}{in} \end{cases}$$

$$N_{c\theta} = \frac{PR_s}{\cos[\alpha]} + 2\beta_{co} R_c\left(-M_{cs}\beta_{co} + Q_c\right)$$

$$N_{c\theta} = \begin{cases} \dfrac{356(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(3471.6111)(0.1373) + 2057.9298\right) \\ \dfrac{356(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(2822.4053)(0.1373) + 1777.6603\right) \end{cases}$$

$$N_{c\theta} = \begin{cases} 38205.1749\ \dfrac{lbs}{in} \\ 35667.6380\ \dfrac{lbs}{in} \end{cases}$$

$$K_{cpc} = 1.0$$

Stress Calculations:

Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \begin{cases} \dfrac{8239.5612}{1.8125} = 4545.9648\ psi \\ \dfrac{6906.6602}{1.8125} = 3810.5711\ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \begin{cases} \dfrac{6(3471.6111)}{(1.8125)^2(1.0)} = 6340.5406\ psi \\ \dfrac{6(2822.4053)}{(1.8125)^2(1.0)} = 5154.8330\ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \begin{cases} \dfrac{38205.1749}{1.8125} = 21078.7172\ psi \\ \dfrac{35667.6380}{1.8125} = 19678.6968\ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(0.3)(3471.6111)}{(1.8125)^2(1.0)} = 1902.1622 \ psi \\ \dfrac{6(0.3)(2822.4053)}{(1.8125)^2(1.0)} = 1546.4499 \ psi \end{Bmatrix}$$

Check Acceptable Criteria:

$$\begin{Bmatrix} \sigma_{sm} = 4545.9648 \ psi \\ \sigma_{sm} = 3810.5711 \ psi \end{Bmatrix} \le \{1.5S = 1.5(20000) = 30000 \ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = 4545.9648 + (6340.5406) = 10886.5 \ psi \\ \sigma_{sm} - \sigma_{sb} = 4545.9648 - (6340.5406) = |-1794.6 \ psi| \\ \sigma_{sm} + \sigma_{sb} = 3810.5711 + (5154.8330) = 8965.4 \ psi \\ \sigma_{sm} - \sigma_{sb} = 3810.5711 - (5154.8330) = |-1344.3 \ psi| \end{Bmatrix} \le \{S_{PS} = 60000 \ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} = 21078.7172 \\ \sigma_{\theta m} = 19678.6968 \end{Bmatrix} \le \{1.5S = 1.5(20000) = 30000 \ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = 21078.7172 + (1902.1622) = 22980.9 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 21078.7172 - (1902.1622) = 19176.6 \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 19678.6968 + (1546.4499) = 21225.1 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 19678.6968 - (1546.4499) = 18132.2 \ psi \end{Bmatrix} \le \{S_{PS} = 60000 \ psi\} \qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cone at the cylinder-to-cone junction at the small end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore, the design is complete.

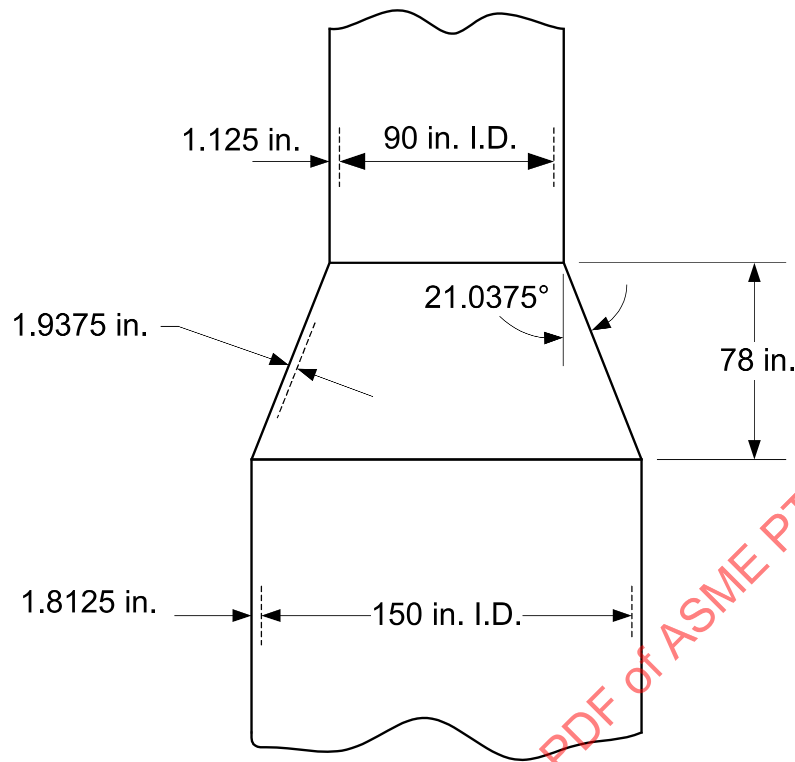

Figure E4.3.7 – Conical Transition

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4.3.8 Example E4.3.8 – Conical Transitions with a Knuckle

Determine if the proposed design for the large end of a cylinder-to-cone junction with a knuckle is adequately designed considering the following design conditions and applied forces and moments, see Figure E4.3.8 for details.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $280\ psig@300°F$
- Inside Diameter (Large End) = $120.0\ in$
- Inside Radius (Large End) = $60.0\ in$
- Knuckle Radius = $10.0\ in$
- Large End Thickness = $1.0\ in$
- Cone Thickness = $1.0\ in$
- Knuckle Thickness = $1.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle = $30.0\ deg$
- Axial Force (Large End) = $-10000\ lbs$
- Net Section Bending Moment (Large End) = $2.0E+06\ in-lbs$

Section VIII, Division 1 Solution

VIII-1 does not provide rules for the required thickness of toriconical heads and section subject to pressure and supplemental loadings. However, UG-22 requires consideration of such loadings and the provisions of U-2(g) apply. This example provides one possible method of satisfying U-2(g); however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

This example accounts for the specified net-section axial force, F_L, and bending moment, M_L, applied to the conical transition at the location of the knuckle by calculating an effective pressure, P_L. The effective pressure from the applied loading and the specified design pressure are summed to determine the equivalent pressure, P_e to be used in the procedure, $P_e = P + P_L$.

Evaluate per UG-32(h). The required thickness of the conical portion of a toriconical head or section, in which the knuckle radius is neither less than 6% of the outside diameter of the head skirt nor less than three times the knuckle thickness, shall be determined by UG-32(g) and substituting D_i for D. The required thickness of the knuckle shall be determined using Mandatory Appendix 1-4(d) with a modified value of L.

$$D_i = D - 2r\left(1-\cos[\alpha]\right) = 120.0 - 2(10.0)\left(1-\cos[30.0]\right) = 117.3205\ in$$

$$L = \frac{D_i}{2\cos[\alpha]} = \frac{117.3205}{2\cos[30.0]} = 67.7350\ in$$

The equivalent design pressure is computed as follows.

$$P_e = P + P_L = \begin{Bmatrix} 280.0 + (5.3828) = 285.3828 \ psi \\ 280.0 + (-7.2329) = 272.7671 \ psi \end{Bmatrix} \quad Use\ the\ maximum\ positive\ value$$

where,

$$P_L = \frac{4f_L}{D_i} = \begin{Bmatrix} \dfrac{4(157.8771)}{117.3205} = 5.3828 \ psi \\ \dfrac{4(-212.1404)}{117.3205} = -7.2329 \ psi \end{Bmatrix}$$

$$f_L = \frac{F_L}{\pi D_i} \pm \frac{4M_L}{\pi D_i^2} = \begin{Bmatrix} \dfrac{-10000}{\pi(117.3205)} + \dfrac{4(2.0E+06)}{\pi(117.3205)^2} = 157.8771 \ \dfrac{lbs}{in} \\ \dfrac{-10000}{\pi(117.3205)} - \dfrac{4(2.0E+06)}{\pi(117.3205)^2} = -212.1404 \ \dfrac{lbs}{in} \end{Bmatrix}$$

Determine the required thickness of the knuckle per UG-32(h) and Mandatory Appendix 1-4(d) using the equivalent design pressure.

$$M = 0.25\left(3 + \sqrt{\frac{L}{r}}\right) = 0.25\left(3 + \sqrt{\frac{67.7350}{10.0}}\right) = 1.4006$$

$$t_k = \frac{P_e LM}{2SE - 0.2P_e} = \frac{285.3828(67.7350)(1.4006)}{2(20000)(1.0) - 0.2(285.3828)} = 0.6778 \ in$$

The required knuckle thickness is less than the design thickness; therefore, the knuckle is adequately designed for the internal pressure and applied forces and moments.

Determine the required thickness of the cone at the knuckle-to-cone intersection at the large end using UG-32(g) using the equivalent design pressure.

$$t_c = \frac{P_e D_i}{2\cos[\alpha](SE - 0.6P_e)} = \frac{285.3828(117.3205)}{2\cos[30.0](20000(1.0) - 0.6(285.3828))} = 0.9749 \ in$$

The required cone thickness is less than the design thickness; therefore, the cone is adequately designed for the internal pressure and applied forces and moments.

<u>Section VIII, Division 2 Solution with VIII-1 Allowable Stresses</u>

Evaluate per VIII-2, paragraph 4.3.12.

a) STEP 1 – Compute the required thickness of the cylinder at the large end of the cone-to-cylinder junction using VIII-2, paragraph 4.3.3., and select the nominal thickness, t_L (as specified in design conditions).

$$t_L = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right] - 1\right) = \frac{120.0}{2}\left(\exp\left[\frac{280.0}{20000.0}\right] - 1\right) = 0.8459 \ in$$

As specified in the design conditions,

$$t_L = 1.0\ in$$

Since the required thickness is less than the design thickness, the cylinder is adequately designed for internal pressure.

b) STEP 2 – Determine the cone half-apex angle, α, compute the required thickness of the cone at the large end of the cone-to-cylinder junction using VIII-2, paragraph 4.3.4., and select the nominal thickness, t_C.

$$\alpha = 30.0\ deg$$

$$t_C = \frac{D}{2\cos[\alpha]}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{120.0}{2\cos[30.0]}\left(\exp\left[\frac{280.0}{20000.0}\right]-1\right) = 0.9768\ in$$

As specified in the design conditions,

$$t_C = 1.0\ in$$

Since the required thickness is less than the design thickness, the cone is adequately designed for internal pressure.

c) STEP 3 – Proportion the transition geometry by assuming a value for the knuckle radius, r_k, and knuckle thickness, t_k, such that the following equations are satisfied. If all these equations cannot be satisfied, the cylinder-to-cone junction shall be designed in accordance with VIII-2, Part 5.

$$\{t_k = 1.0\ in\} \geq \{t_L = 1.0\ in\} \qquad True$$

$$\{r_k = 10.0\ in\} > \{3t_k = 3.0\ in\} \qquad True$$

$$\left\{\frac{r_k}{R_L} = \frac{10.0}{60.0} = 0.1667\right\} > \{0.03\} \qquad True$$

$$\{\alpha = 30\ deg\} \leq \{60\ deg\} \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition at the location of the knuckle. The thrust load due to pressure shall not be included as part of the axial force, F_L.

$$F_L = -10000\ lbs$$

$$M_L = 2.0E+06\ in-lbs$$

e) STEP 5 – Compute the stresses in the knuckle at the junction using the equations in VIII-2, Table 4.3.7.

Determine if the knuckle is considered to be compact or non-compact.

$$\alpha r_k < 2K_m\left(\left\{R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5} + r_k\right\}t_k\right)^{0.5}$$

$$\{0.5236(10.0)\} = \left\{2(0.7)\left(\left\{50.0\left((0.5236)^{-1}\tan[0.5236]\right)^{0.5}+10\right\}1\right)^{0.5}\right\}$$

$$\{5.2360\ in\} < \{11.0683\ in\} \qquad True$$

where,

$$K_m = 0.7$$

$$\alpha = \frac{30.0}{180}\pi = 0.5236\ rad$$

$$R_k = R_L - r_k = 60.0 - 10.0 = 50.0\ in$$

Therefore, analyze the knuckle junction as a compact knuckle.

Stress Calculations:

Determine the hoop and axial membrane stresses at the knuckle:

$$\sigma_{\theta m} = \frac{PK_m\left(R_L\sqrt{R_L t_L}+L_k\sqrt{L_k t_C}\right)+\alpha\left(PL_{1k}r_k-0.5P_e L_{1k}^2\right)}{K_m\left(t_L\sqrt{R_L t_L}+t_C\sqrt{L_k t_C}\right)+\alpha t_k r_k}$$

$$\sigma_{sm} = \frac{P_e L_{1k}}{2t_k}$$

where,

$$L_{1k} = R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5}+r_k = 50.0\left((0.5236)^{-1}\tan[0.5236]\right)^{0.5}+10.0 = 62.5038\ in$$

$$L_k = \frac{R_k}{\cos[\alpha]}+r_k = \frac{50.0}{\cos[0.5236]}+10.0 = 67.7351\ in$$

$$P_e = P + \frac{F_L}{\pi L_{1k}^2\cos^2\left[\frac{\alpha}{2}\right]} \pm \frac{2M_L}{\pi L_{1k}^3\cos^3\left[\frac{\alpha}{2}\right]}$$

$$P_e = \begin{Bmatrix} 280+\dfrac{-10000.0}{\pi(62.5038)^2\cos^2\left[\dfrac{0.5236}{2}\right]}+\dfrac{2(2.0E+06)}{\pi(62.5038)^3\cos^3\left[\dfrac{0.5236}{2}\right]} \\ 280+\dfrac{-10000.0}{\pi(62.5038)^2\cos^2\left[\dfrac{0.5236}{2}\right]}-\dfrac{2(2.0E+06)}{\pi(62.5038)^3\cos^3\left[\dfrac{0.5236}{2}\right]} \end{Bmatrix}$$

$$P_e = \begin{Bmatrix} 284.9125\ psi \\ 273.3410\ psi \end{Bmatrix}$$

therefore,

$$\sigma_{\theta m}=\left\{\begin{array}{l}\dfrac{\left(\begin{array}{l}280(0.7)\left(60.0\sqrt{60.0(1.0)}+67.7351\sqrt{67.7351(1.0)}\right)+\\0.5236\left(280(62.5038)(10.0)-0.5(284.9125)(62.5038)^2\right)\end{array}\right)}{0.7\left(1.0\sqrt{60.0(1.0)}+1.0\sqrt{67.7351(1.0)}\right)+0.5236(1.0)(10.0)}=35.8767\ psi\\\dfrac{\left(\begin{array}{l}280(0.7)\left(60.0\sqrt{60.0(1.0)}+67.7351\sqrt{67.7351(1.0)}\right)+\\0.5236\left(280(62.5038)(10.0)-0.5(273.3410)(62.5038)^2\right)\end{array}\right)}{0.7\left(1.0\sqrt{60.0(1.0)}+1.0\sqrt{67.7351(1.0)}\right)+0.5236(1.0)(10.0)}=756.6825\ psi\end{array}\right\}$$

and,

$$\sigma_{sm}=\left\{\begin{array}{l}\dfrac{P_eL_{1k}}{2t_k}=\dfrac{284.9125(62.5038)}{2(1.0)}=8904.0570\ psi\\\dfrac{P_eL_{1k}}{2t_k}=\dfrac{273.3410(62.5038)}{2(1.0)}=8542.4256\ psi\end{array}\right\}$$

Check Acceptable Criteria:

$$\left\{\begin{array}{l}\sigma_{\theta m}=35.9\ psi\\\sigma_{\theta m}=756.7\ psi\end{array}\right\}\le\left\{S=20000\ psi\right\}\qquad True$$

$$\left\{\begin{array}{l}\sigma_{sm}=8904.1\ psi\\\sigma_{sm}=8542.4\ psi\end{array}\right\}\le\left\{S=20000\ psi\right\}\qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ in the knuckle are both tensile, the condition of local buckling need not be considered. Therefore, the knuckle at the cylinder-to-cone junction at the large end is adequately designed.

f) STEP 6 – The stress acceptance criterion in STEP 5 is satisfied for the knuckle. Therefore, the design is complete.

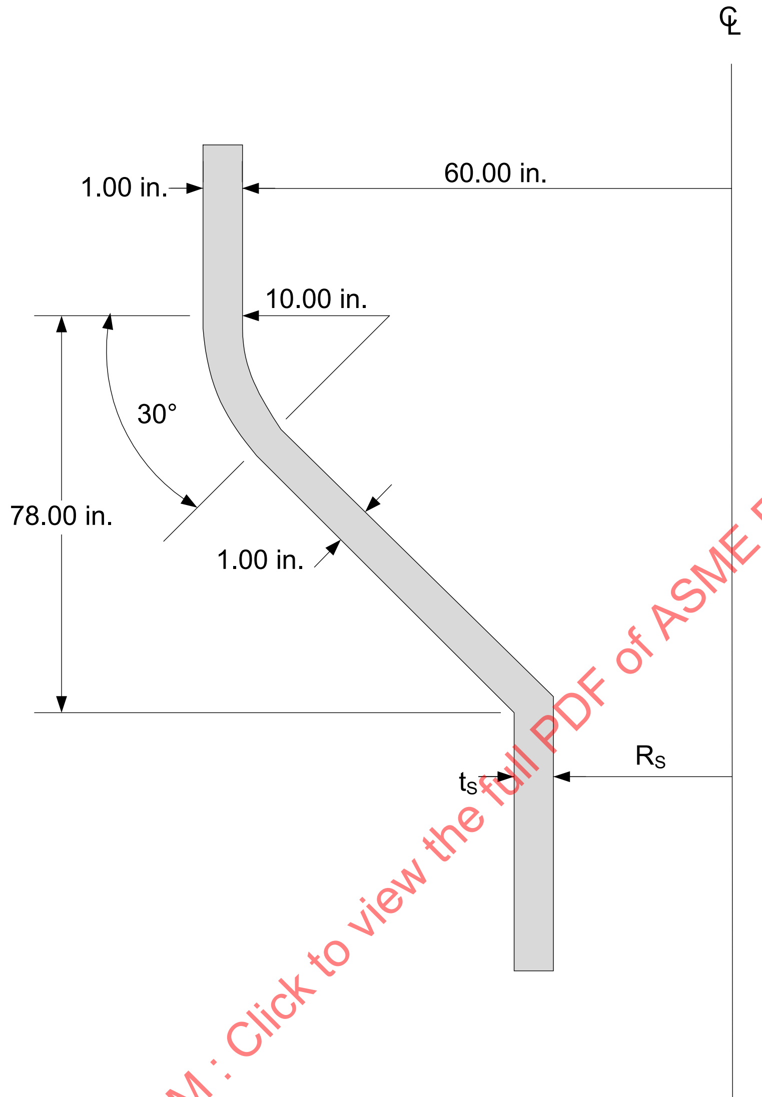

Figure E4.3.8 – Knuckle Detail

4.4 Shells Under External Pressure and Allowable Compressive Stresses

4.4.1 Example E4.4.1 – Cylindrical Shell

Determine the Maximum Allowable External Pressure (MAEP) for a cylindrical shell considering the following design conditions.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Temperature = $300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Unsupported Length = $636.0\ in$
- Mod of Elasticity at Design Temp = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$

Section VIII, Division 1 Solution

Evaluate per paragraph UG-28(c).

a) STEP 1 – UG-28(c)(1), Cylinders having D_o/t.

$$\left\{\frac{D_o}{t}=\frac{92.25}{1}=92.25\right\}\geq 10 \qquad True$$

where,

$$D_o = D+2(Uncorroded\ Thickness)=90.0+2(1.125)=92.25\ in$$

$$t = t-Corrosion\ Allowance=1.125-0.125=1.0\ in$$

$$L = 636.0\ in$$

Assume a value for t and determine the ratios L/D_o and D_o/t.

$$\frac{L}{D_o}=\frac{636.0}{92.25}=6.8943$$

$$\frac{D_o}{t}=92.25$$

b) STEP 2 – Enter Figure G in Subpart 3 of Section II, Part D at the value of L/D_o determined in STEP 1. For values of $L/D_o > 50$, enter the chart at a value of $L/D_o = 50$. For values of $L/D_o < 50$, enter the chart at a value of $L/D_o = 0.05$.

c) STEP 3 – Move horizontally to the line for the value of D_o/t determined in STEP 1. Interpolation may be made for intermediate values of D_o/t; extrapolation is not permitted. From this point of intersection move vertically downward to determine the value of factor A.

$$A = 0.0002$$

d) STEP 4 – Using the value of A calculated in STEP 3, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically to an intersection with the material/temperature line for the design temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 7.

Per Section II Part D, Table 1A, a material specification of $SA-516-70,\ Normalized$ is assigned an External Pressure Chart No. CS-2.

e) STEP 5 – From the intersection obtained in STEP 4, move horizontally to the right and read the value of factor B.

$$B = 2800$$

f) STEP 6 – Using this value of B, calculate the value of the maximum allowable external working pressure P_a using the following formula:

$$P_a = \frac{4B}{3\left(\dfrac{D_o}{t}\right)} = \frac{4(2800)}{3\left(\dfrac{92.25}{1.0}\right)} = 40.7 \ psi$$

g) STEP 7 – For values of A falling to the left of the applicable material/temperature line, the value of P_a can be calculated using the following formula:

$$P_a = \frac{2AE}{3\left(\dfrac{D_o}{t}\right)} \qquad \textit{Not required}$$

h) STEP 8 – Compare the calculated value of P_a obtained in STEPS 6 or 7 with P. If P_a is smaller than P, select a larger value of t and repeat the design procedure until a value of P_a is obtained that is equal to or greater than P.

The allowable external pressure is $P_a = 40.7\ psi$.

Commentary:

As permitted in UG-28, Table G and Table CS-2 may be used to determine the values of A and B in lieu of using Figure G and Figure CS-2. Linear interpolation or any other rational interpolation method may be used. Since Figure G and Figure CS-2 are presented with a log-log scale, it is appropriate to perform log-log linear interpolation of the values of Table G and Table CS-2 to determine the values of A and B. The results of such an exercise produces the following values.

$$A = 1.8838E-04$$

$$B = 2702.6 \ psi$$

$$P_a = 39.1 \ psi$$

Section VIII, Division 2 Solution

Evaluate per VIII-2, paragraph 4.4.5.

a) STEP 1 – Assume an initial thickness, t, and unsupported length, L.

$$t = t - Corrosion\ Allowance = 1.125 - 0.125 = 1.0\ in$$

$$L = 636.0\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{1.6C_h E_y t}{D_o} = \frac{1.6(0.00919142)(28.3\text{E+}06)(1.0)}{92.25} = 4511.5189\ psi$$

where,

$$D_o = D + 2(Uncorroded\ Thickness) = 90.0 + 2(1.125) = 92.25\ in$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{\left(\frac{92.25}{2}\right)1.0}} = 93.6459$$

$$2\left(\frac{D_o}{t}\right)^{0.94} = 2\left(\frac{92.25}{1.0}\right)^{0.94} = 140.6366$$

Since $13 < M_x < 2(D_o/t)^{0.94}$, calculate C_h as follows:

$$C_h = 1.12M_x^{-1.058} = 1.12(93.6459)^{-1.058} = 0.00919142$$

c) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

1) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$F_{he} = 4511.5189\ psi$ (as determined in paragraph 4.4.5, STEP 2)

2) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{4511.5189}{28.3E+06} = 0.00015942$$

3) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

$$\frac{F_{ic}}{E_t} = A_e$$

Commentary:

The model used to develop the stress-strain curve is in paragraph 3-D.3. Though outside the scope of this example problem, the user is encouraged to develop the stress-strain curve for the material of interest at the design temperature. Several of the design parameters and supplemental variables used in the development of the stress-strain curve are also used in the determination of the tangent modulus.

Per paragraph 3-D.3: Stress-Strain Curve Model / Tangent Modulus Parameters

- Engineering Ultimate Tensile Stress at Temperature $\sigma_{uts} = 70000 \ psi$
- Engineering Yield Stress at Temperature $\sigma_{ys} = 33600 \ psi$
- 0.2% Engineering Offset Strain $\varepsilon_{ys} = 0.002$
- Stress-Strain Curve Fitting Parameter, see Table 3-D.1 $\varepsilon_p = 0.00002$
- Modulus of Elasticity at Temperature $E_y = 28.3E+06 \ psi$

Engineering yield to engineering tensile ratio,

$$R = \frac{\sigma_{ys}}{\sigma_{uts}} = \frac{33600}{70000} = 0.4800$$

Curve fitting exponent for the stress-strain curve,

$$m_1 = \frac{\left[\ln\left[R\right] + \left(\varepsilon_p - \varepsilon_{ys}\right)\right]}{\ln\left[\dfrac{\ln\left[1+\varepsilon_p\right]}{\ln\left[1+\varepsilon_{ys}\right]}\right]} = \frac{\left[\ln\left[0.48\right] + \left(0.00002 - 0.002\right)\right]}{\ln\left[\dfrac{\ln\left[1+0.00002\right]}{\ln\left[1+0.002\right]}\right]} = 0.15984367$$

Curve fitting constant for the elastic region of the stress-strain curve,

$$A_1 = \frac{\sigma_{ys}\left(1+\varepsilon_{ys}\right)}{\left(\ln\left[1+\varepsilon_{ys}\right]\right)^{m_1}} = \frac{33600\left(1+0.002\right)}{\left(\ln\left[1+0.002\right]\right)^{0.1598}} = 90926.3215$$

Curve fitting exponent for the stress-strain curve, see Table 3-D.1,

$$m_2 = 0.6\left(1.00 - R\right) = 0.6\left(1.00 - 0.4800\right) = 0.3120$$

Curve fitting constant for the plastic region of the stress-strain curve,

$$A_2 = \frac{\sigma_{uts} \cdot \exp\left[m_2\right]}{m_2^{\ m_2}} = \frac{70000 \cdot \exp\left[0.3120\right]}{\left(0.3120\right)^{0.3120}} = 137537.6771$$

Material parameter for stress-strain curve model,

$$K = 1.5\left(R\right)^{1.5} - 0.5\left(R\right)^{2.5} - \left(R\right)^{3.5}$$

$$K = 1.5\left(0.4800\right)^{1.5} - 0.5\left(0.4800\right)^{2.5} - \left(0.4800\right)^{3.5} = 0.34239735$$

Stress-Strain curve fitting parameter,

$$H = \frac{2\left(\sigma_t - \left(\sigma_{ys} + K\left(\sigma_{uts} - \sigma_{ys}\right)\right)\right)}{K\left(\sigma_{uts} - \sigma_{ys}\right)} = \frac{2\left(70000 - \left(33600 + 0.3424\left(70000 - 33600\right)\right)\right)}{0.3424\left(70000 - 33600\right)}$$

$$H = 3.84116670$$

Per paragraph 3-D.5.1: Tangent Modulus Parameters.

- Tangent Modulus, E_t.
- Coefficient used in Tangent Modulus, D_1, D_2, D_3, D_4.

Tangent Modulus:

$$E_t = \left(\frac{1}{E_y} + D_1 + D_2 + D_3 + D_4\right)^{-1}$$

Coefficients:

$$D_1 = \frac{\sigma_t^{\left(\frac{1}{m_1}-1\right)}}{2m_1 A_1^{\left(\frac{1}{m_1}\right)}}$$

$$D_2 = -\frac{1}{2}\left(\frac{1}{A_1^{\left(\frac{1}{m_1}\right)}}\right)\cdot\left(\sigma_t^{\left(\frac{1}{m_1}\right)}\left(\frac{2}{K\left(\sigma_{uts} - \sigma_{ys}\right)}\right)\left(1 - \tanh^2\left[H\right]\right) + \frac{1}{m_1}\sigma_t^{\left(\frac{1}{m_1}-1\right)}\tanh^2\left[H\right]\right)$$

$$D_3 = \frac{\sigma_t^{\left(\frac{1}{m_2}-1\right)}}{2m_2 A_2^{\left(\frac{1}{m_2}\right)}}$$

$$D_4 = -\frac{1}{2}\left(\frac{1}{A_2^{\left(\frac{1}{m_2}\right)}}\right)\cdot\left(\sigma_t^{\left(\frac{1}{m_2}\right)}\left(\frac{2}{K\left(\sigma_{uts} - \sigma_{ys}\right)}\right)\left(1 - \tanh^2\left[H\right]\right) + \frac{1}{m_2}\sigma_t^{\left(\frac{1}{m_2}-1\right)}\tanh^2\left[H\right]\right)$$

An example of an iterative solution to determine F_{ic} is shown in Table 4.4.2. When using the procedure of paragraph 3-D.5.1, the value of F_{ic} is substituted for σ_t, which is the value of true stress at which true strain will be evaluated.

The suggested algorithm is an interval-halving approach to "guess" at a value of F_{ic}, and determine the corresponding value of E_t. The relationship, $F_{ic}/E_t = A_e$ is checked and based upon the proximity of the ratio F_{ic}/E_t to the value of A_e, an adjustment is made to the next "guess" of F_{ic}. The iteration continues until the relationship $F_{ic}/E_t = A_e$ is satisfied within a specified tolerance, at which point the iteration process is stopped and the final value of F_{ic} is reported.

Table 4.4.2 – Algorithm for Computation of Predicted Inelastic buckling Stress, F_{ic}.

$$Read\left(F_e, E\right)$$

$$A_e = \frac{F_e}{E}$$

$$F_{icup} = MSTS$$

$$F_{iclow} = 0.0$$

$$TOLA_{diff} = 0.00000001$$

$$A_{diff} = 1.0$$

$$Do\ While\ A_{diff} > TOLA_{diff}$$

$$F_{icg} = 0.5\left(F_{icup} + F_{iclow}\right)$$

$$E_{tg} = \left(\frac{1}{E} + D_1 + D_2 + D_3 + D_4\right)^{-1}$$

$$A_i = \frac{F_{icg}}{E_{tg}}$$

$$A_{diff} = A_i - A_e$$

$$IF\left(A_{diff} < 0.0\right)$$

$$F_{iclow} = F_{icg}$$

$$ELSE$$

$$F_{icup} = F_{icg}$$

$$END\ IF$$

$$A_{diff} = \left|A_{diff}\right|$$

$$End\ Do$$

$$F_{ic} = F_{icg}$$

A tabulated summary of the iterative procedure is shown below.

Iteration	***A diff***	***F icup***	***F iclow***	***F icg***	***E t***
1	1.000000000	70000.0000	0.0000	35000.0000	1.6780582E+06
2	0.020698024	35000.0000	0.0000	17500.0000	2.1133306E+07
3	0.000668659	17500.0000	0.0000	8750.0000	2.8052001E+07
4	0.000152503	8750.0000	0.0000	4375.0000	2.8293447E+07
5	0.000004788	8750.0000	4375.0000	6562.5000	2.8244947E+07
6	0.000072925	6562.5000	4375.0000	5468.7500	2.8278853E+07
7	0.000033969	5468.7500	4375.0000	4921.8750	2.8287838E+07
8	0.000014575	4921.8750	4375.0000	4648.4375	2.8290991E+07
9	0.000004890	4648.4375	4375.0000	4511.7188	2.8292298E+07
10	0.000000050	4511.7188	4375.0000	4443.3594	2.8292892E+07
11	0.000002369	4511.7188	4443.3594	4477.5391	2.8292600E+07
12	0.000001159	4511.7188	4477.5391	4494.6289	2.8292450E+07
13	0.000000554	4511.7188	4494.6289	4503.1738	2.8292375E+07
14	0.000000252	4511.7188	4503.1738	4507.4463	2.8292336E+07
15	0.000000101	4511.7188	4507.4463	4509.5825	2.8292317E+07
16	0.000000025	4511.7188	4509.5825	4510.6506	2.8292308E+07
17	0.000000013	4510.6506	4509.5825	**4510.1166**	**2.8292313E+07**
18	**0.000000006**				

$$F_{ic} = 4510.1166 \ psi$$

d) STEP 4 – Calculate the value of design factor, FS, per paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0 \ psi$$

Since $F_{ic} \leq 0.55S_y$, calculate FS as follows:

$$FS = 2.0$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{D_o}\right) = 2(2255.0583)\left(\frac{1.0}{92.25}\right) = 48.9 \ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{4510.1166}{2.0} = 2255.0583 \ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness or reduce the unsupported length of the shell (i.e., by the addition of a stiffening rings) and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The allowable external pressure is $P_a = 48.9 \ psi$.

Combined Loadings – cylindrical shells subject to external pressure and other loadings shall satisfy the requirements of VIII-2, paragraph 4.4.12. In this example problem, the cylindrical shell is only subject to external pressure.

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4.4.2 Example E4.4.2 – Conical Shell

Determine the Maximum Allowable External Pressure (MAEP) for a conical shell considering the following design conditions.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Temperature = $300°F$
- Inside Diameter (Large End) = $150.0\ in$
- Thickness (Large End) = $1.8125\ in$
- Inside Diameter (Small End) = $90.0\ in$
- Thickness (Small End) = $1.125\ in$
- Thickness (Conical Section) = $1.9375\ in$
- Axial Cone Length = $78.0\ in$
- One-Half Apex Angle = $21.0375\ deg$
- Corrosion Allowance = $0.125\ in$
- Mod. of Elasticity at Design Temp. = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$

Section VIII, Division 1 Solution

Evaluate per paragraph UG-33(f): Conical heads and Sections. When the cone-to-cylinder junction is not a line of support, the required thickness of a conical head or section under pressure on the convex side, either seamless or of built-up construction with butt joints shall not be less than the minimum required thickness of the adjacent cylindrical shell and, when a knuckle is not provided, the reinforcement requirement of Appendix 1-8 shall be satisfied. When the cone-to-cylinder junction is a line of support the required thickness shall be determined in accordance with the following procedure.

For this example, it is assumed that the cone-to-cylinder junction is a line of support. However, the supplemental checks on reinforcement and moment of inertia per Appendix 1-8 are not performed.

a) STEP 1 – UG-33(f)(1), when $\alpha \leq 60\ deg$ and cones having $D_L/t_e \geq 10$.

$$\left\{\frac{D_L}{t_e} = \frac{153.875}{1.6917} = 90.9588\right\} \geq 10 \qquad True$$

where,

$$D_L = \left\{\begin{array}{l} Inside\ Diameter + 2\left(Uncorroded\ Cone\ Thickness\right) \\ 150.0 + 2\left(1.9375\right) = 153.875\ in \end{array}\right\}$$

$$t = t - Corrosion\ Allowance = 1.9375 - 0.125 = 1.8125\ in$$

$$t_e = t\cos\left[\alpha\right] = 1.8125 \cdot \cos\left[21.0375\right] = 1.6917\ in$$

and, per UG-33(g) and Figure UG-33.1, sketch (a):

$$D_L = \begin{cases} \text{Inside Diameter} + 2(\text{Uncorroded Large Cylinder Thickness}) \\ 150.0 + 2(1.8125) = 153.625 \text{ in} \end{cases}$$

$$D_S = \begin{cases} \text{Inside Diameter} + 2(\text{Uncorroded Small Cylinder Thickness}) \\ 90.0 + 2(1.125) = 92.25 \text{ in} \end{cases}$$

$$L_e = \frac{L_c}{2}\left(1 + \frac{D_S}{D_L}\right) = \frac{78.0}{2}\left(1 + \frac{92.25}{153.625}\right) = 62.4190 \text{ in}$$

Assume a value for t_e and determine the ratios L_e/D_L and D_L/t_e.

$$\frac{L_e}{D_L} = \frac{62.7926}{153.875} = 0.4056$$

$$\frac{D_L}{t_e} = \frac{153.875}{1.6917} = 90.9588$$

b) STEP 2 – Enter Figure G in Subpart 3 of Section II, Part D at the value of L/D_o equivalent to the value of L_e/D_L determined in STEP 1. For values of $L_e/D_L > 50$ enter the chart at a value of $L_e/D_L => 50$.

c) STEP 3 – Move horizontally to the line for the value of D_o/t equivalent to the value of D_L/t_e determined in STEP 1. Interpolation may be made for intermediate values of D_L/t_e; extrapolation is not permitted. From this point of intersection move vertically downward to determine the value of factor A.

$$A = 0.0045$$

d) STEP 4 – Using the value of A calculated in STEP 3, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically to an intersection with the material/temperature line for the design temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 7.

Per Section II Part D, Table 1A, a material specification of $SA-516,\ Grade\ 70,\ Normalized$ is assigned an External Pressure Chart No. CS-2.

e) STEP 5 – From the intersection obtained in STEP 4, move horizontally to the right and read the value of factor B.

$$B = 17000$$

f) STEP 6 – Using this value of B, calculate the value of the maximum allowable external working pressure P_a using the following formula:

$$P_a = \frac{4B}{3\left(\dfrac{D_L}{t_e}\right)} = \frac{4(17000)}{3\left(\dfrac{153.875}{1.6917}\right)} = 249.2 \text{ psi}$$

g) STEP 7 – For values of A falling to the left of the applicable material/temperature line, the value of P_a can be calculated using the following formula:

$$P_a = \frac{2AE}{3\left(\frac{D_L}{t_e}\right)} \qquad \textit{Not required}$$

h) STEP 8 – Compare the calculated value of P_a obtained in STEPS 6 or 7 with P. If P_a is smaller than P, select a larger value of t and repeat the design procedure until a value of P_a is obtained that is equal to or greater than P.

The allowable external pressure is $P_a = 248.7\ psi$.

Commentary:

As permitted in UG-28, Table G and Table CS-2 may be used to determine the values of A and B in lieu of using Figure G and Figure CS-2. Linear interpolation or any other rational interpolation method may be used. Since Figure G and Figure CS-2 are presented with a log-log scale, it is appropriate to perform log-log linear interpolation of the values of Table G and Table CS-2 to determine the values of A and B. The results of such an exercise produces the following values.

$$A = 4.15546E-03$$

$$B = 16867.7\ psi$$

$$P_a = 247.1\ psi$$

Section VIII, Division 2 Solution

Evaluate per VIII-2, paragraph 4.4.6. and 4.4.5.

The required thickness of a conical shell subjected to external pressure loading shall be determined using the equations for a cylinder by making the following substitutions:

a) The value of t_c is substituted for t in the equations in paragraph 4.4.5.

$$t_c = t = 1.9375 - Corrosion\ Allowance = 1.9375 - 0.125 = 1.8125\ in$$

b) For offset cones, the cone angle, α, shall satisfy the requirements of paragraph 4.3.4.

The conical shell in this example problem is not of the offset type. Therefore, no additional requirements are necessary.

c) The value of $0.5(D_L + D_S)/\cos[\alpha]$ is substituted for D_o in the equations in VIII-2, paragraph 4.4.5, (concentric cone design with common center line per VIII-2, Figure 4.4.7 Sketch (a)).

$$D_o = \frac{0.5(D_L + D_s)}{\cos[\alpha]} = \frac{0.5\left[\left(150.0 + 2(1.9375)\right) + \left(90.0 + 2(1.9375)\right)\right]}{\cos[21.0375]} = 132.7215\ in$$

d) The value of $L_{ce}/\cos[\alpha]$ is substituted for L in the equations in VIII-2, paragraph 4.4.5 where L_{ce} is determined as shown below. For Sketches (a) and (e) in VIII-2, Figure 4.4.7:

$$L_{ce} = L_c$$

$$L = \frac{L_{ce}}{\cos[\alpha]} = \frac{78.0}{\cos[21.0375]} = 83.5703\ in$$

e) Note that the half-apex angle of a conical transition can be computed knowing the shell geometry with the following equations. These equations were developed with the assumption that the conical transition contains a cone section, knuckle, or flare. If the transition does not contain a knuckle or flare, the radii of these components should be set to zero when computing the half-apex angle (see VIII-2, Figure 4.4.7).

$$If\ (R_L - r_k) \geq (R_S + r_f)$$

$$\alpha = \beta + \phi = 0.3672 - 0 = 0.3672\ rad = 21.0375\ deg$$

$$\beta = \arctan\left[\frac{(R_L - r_k) - (R_S + r_f)}{L_c}\right] = \arctan\left[\frac{(75.0 - 0) - (45.0 + 0)}{78.0}\right] = 0.3672\ rad$$

$$\phi = \arcsin\left[\frac{(r_f + r_k)\cos[\beta]}{L_c}\right] = \arcsin\left[\frac{(0.0 + 0.0)\cos[0.3672]}{78.0}\right] = 0.0\ rad$$

Proceed with the design following the steps outlined in VIII-2, paragraph 4.4.5.

a) STEP 1 – Assume an initial thickness, t, and unsupported length, L (see VIII-2, Figures 4.4.1 and 4.4.2).

$$t = 1.8125\ in$$

$$L = 83.5703\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{1.6 C_h E_y t}{D_o} = \frac{1.6(0.1306633)(28.3E+06)(1.8125)}{132.7215} = 80796.7762\ psi$$

where,

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{83.5703}{\sqrt{\left(\frac{132.7215}{2.0}\right)1.8125}} = 7.6200$$

$$2\left(\frac{D_o}{t}\right)^{0.94} = 2\left(\frac{132.7215}{1.8125}\right)^{0.94} = 113.1914$$

Since $1.5 < M_x < 13$, calculate C_h as follows:

$$C_h = \frac{0.92}{(M_x - 0.579)} = \frac{0.92}{(7.6200 - 0.579)} = 0.1306633$$

c) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

1) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$F_{he} = 80796.7762 \ psi$ (as determined in paragraph 4.4.5, STEP 2)

2) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{80796.7762}{28.3E+06} = 0.00285501$$

3) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

A tabulated summary of the iterative procedure is shown below.

Iteration	A_{diff}	F_{icup}	F_{iclow}	F_{icg}	E_t
1	1.000000000	70000.0000	0.0000	35000.0000	1.6780582E+06
2	0.018002431	35000.0000	0.0000	17500.0000	2.1133306E+07
3	0.002026933	35000.0000	17500.0000	26250.0000	7.1896405E+06
4	0.000796077	26250.0000	17500.0000	21875.0000	1.3450426E+07
5	0.001228667	26250.0000	21875.0000	24062.5000	9.9795641E+06
6	0.000443832	26250.0000	24062.5000	25156.2500	8.4945724E+06
7	0.000106440	25156.2500	24062.5000	24609.3750	9.2145856E+06
8	0.000184312	25156.2500	24609.3750	24882.8125	8.8489245E+06
9	0.000043051	25156.2500	24882.8125	25019.5313	8.6703335E+06
10	0.000030638	25019.5313	24882.8125	24951.1719	8.7592753E+06
11	0.000006467	25019.5313	24951.1719	24985.3516	8.7147160E+06
12	0.000012020	24985.3516	24951.1719	24968.2617	8.7369735E+06
13	0.000002760	24968.2617	24951.1719	24959.7168	8.7481189E+06
14	0.000001857	24968.2617	24959.7168	24963.9893	8.7425448E+06
15	0.000000450	24963.9893	24959.7168	24961.8530	8.7453315E+06
16	0.000000704	24963.9893	24961.8530	24962.9211	8.7439381E+06
17	0.000000127	24963.9893	24962.9211	24963.4552	8.7432415E+06
18	0.000000162	24963.4552	24962.9211	24963.1882	8.7435898E+06
19	0.000000018	24963.1882	24962.9211	24963.0547	8.7435898E+06
20	0.000000055	24963.1882	24963.0547	24963.1214	8.7436769E+06
21	0.000000018	24963.1882	24963.1214	**24963.1548**	**8.7436333E+06**
22	**0.000000000**				

$F_{ic} = 24963.1548 \ psi$

d) STEP 4 – Calculate the value of design factor, FS, per VIII-2, paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0 \ psi$$

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{24963.1548}{33600.0}\right) = 1.8565$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{D_o}\right) = 2(13446.3533)\left(\frac{1.8125}{132.7215}\right) = 367.3 \; psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{24963.1548}{1.8565} = 13446.3533 \; psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness or reduce the unsupported length of the shell (i.e., by the addition of a stiffening rings) and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 367.2\, psi$.

Combined Loadings – conical shells subject to external pressure and other loadings shall satisfy the requirements of VIII-2, paragraph 4.4.12. In this example problem, the conical shell is only subject to external pressure.

4.4.3 Example E4.4.3 – Spherical Shell and Hemispherical Head

Determine the Maximum Allowable External Pressure (MAEP) for a hemispherical head considering the following design conditions.

Vessel Data:

- Material = $SA-542,\ Type\ D,\ Class\ 4a$
- Design Temperature = $350°F$
- Inside Diameter = $149.0\ in$
- Thickness = $2.8125\ in$
- Corrosion Allowance = $0.0\ in$
- Modulus of Elasticity at Design Temperature = $29.1E+06\ psi$
- Yield Strength = $58000\ psi$

Section VIII, Division 1 Solution

Evaluate per paragraph UG-28(d). As noted in paragraph UG-33(c), the required thickness of a hemispherical head having pressure on the convex side shall be determined in the same manner as outlined in paragraph UG-28(d) for determining the thickness for a spherical shell.

a) STEP 1 – UG-28(d)(1), Assume a value for t and calculate the value of factor A using the following formula:

$$A=\frac{0.125}{\left(\dfrac{R_o}{t}\right)}=\frac{0.125}{\left(\dfrac{77.3125}{2.8125}\right)}=0.00455$$

where,

$$R_o=\frac{D+2\left(Uncorroded\ Thickness\right)}{2}=\frac{149.0+2\left(2.8125\right)}{2}=77.3125\ in$$

$$t=t-Corrosion\ Allowance=2.8125-0.0=2.8125\ in$$

b) STEP 2 – Using the value of A calculated in STEP 1, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically to an intersection with the material/temperature line for the design temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 5.

Per Section II Part D, Table 1A, a material specification of $SA-542\ Type\ D\ Cl.\ 4a$ is assigned an External Pressure Chart No. CS-2.

c) STEP 3 – From the intersection obtained in Step 2, move horizontally to the right and read the value of factor B.

$$B=15700$$

d) STEP 4 – Using the value of B obtained in STEP 3, calculate the value of the maximum allowable external working pressure P_a using the following formula:

$$P_a = \frac{B}{\left(\frac{R_o}{t}\right)} = \frac{15700}{\left(\frac{77.3125}{2.8125}\right)} = 571.1\ psi$$

e) STEP 5 – For values of A falling to the left of the applicable material/temperature line, the value of P_a can be calculated using the following formula:

$$P_a = \frac{0.0625E}{\left(\frac{R_o}{t}\right)^2} \qquad \textit{Not required}$$

f) STEP 6 – Compare the calculated value of P_a obtained in STEPS 4 or 5 with P. If P_a is smaller than P, select a larger value of t and repeat the design procedure until a value of P_a is obtained that is equal to or greater than P.

The allowable external pressure is $P_a = 571.1\ psi$.

Commentary:

As permitted in UG-28, Table G and Table CS-2 may be used to determine the values of A and B in lieu of using Figure G and Figure CS-2. Linear interpolation or any other rational interpolation method may be used. Since Figure G and Figure CS-2 are presented with a log-log scale, it is appropriate to perform log-log linear interpolation of the values of Table G and Table CS-2 to determine the values of A and B. The results of such an exercise produces the following values. For spheres and hemispherical heads, Figure G/Table G is not necessary as the value of A is calculated directly.

$$A = 4.5472918E-03$$
$$B = 16054.9\ psi$$
$$P_a = 584.1\ psi$$

Section VIII, Division 2 Solution

Evaluate per VIII-2, paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the spherical shell.

$$t = 2.8125\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075E_y\left(\frac{t}{R_o}\right) = 0.075(29.1E+06)\left(\frac{2.8125}{77.3125}\right) = 79395.7154\ psi$$

c) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

1) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$$F_{he} = 79395.7154 \ psi$$ (as determined in paragraph 4.4.7, STEP 2)

2) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{79395.7154}{29.1E+06} = 0.00272838$$

3) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

A tabulated summary of the iterative procedure is shown below.

Iteration	A_{diff}	F_{icup}	F_{iclow}	F_{icg}	E_t
1	1.000000000	85000.0000	0.0000	42500.0000	2.0983586E+07
2	0.000702983	85000.0000	42500.0000	63750.0000	6.1671934E+05
3	0.100641174	63750.0000	42500.0000	53125.0000	5.1280086E+06
4	0.007631397	53125.0000	42500.0000	47812.5000	1.2010241E+07
5	0.001252603	47812.5000	42500.0000	45156.2500	1.6580360E+07
6	0.000004897	47812.5000	45156.2500	46484.3750	1.4259656E+07
7	0.000531477	46484.3750	45156.2500	45820.3125	1.5417952E+07
8	0.000243505	45820.3125	45156.2500	45488.2813	1.5999552E+07
9	0.000114722	45488.2813	45156.2500	45322.2656	1.6290170E+07
10	0.000053810	45322.2656	45156.2500	45239.2578	1.6435333E+07
11	0.000024186	45239.2578	45156.2500	45197.7539	1.6507865E+07
12	0.000009578	45197.7539	45156.2500	45177.0020	1.6544117E+07
13	0.000002324	45177.0020	45156.2500	45166.6260	1.6562240E+00
14	0.000001291	45177.0020	45166.6260	45171.8140	1.6553179E+07
15	0.000000515	45171.8140	45166.6260	45169.2200	1.6557709E+07
16	0.000000388	45171.8140	45169.2200	45170.5170	1.6555444E+07
17	0.000000064	45170.5170	45169.2200	45169.8685	1.6556577E+07
18	0.000000162	45170.5170	45169.8685	45170.1927	1.6556011E+07
19	0.000000049	45170.5170	45170.1927	**45170.3548**	**1.6555727E+07**
20	**0.000000007**				

$$F_{ic} = 45170.3548 \ psi$$

d) STEP 4 – Calculate the value of design margin, FS, per VIII-2, paragraph 4.4.2.

$$0.55S_y = 0.55(58000.0) = 31900.0 \ psi$$

Since $0.55S_y < F_{ic} < S_y$, calculate the FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{45170.3548}{58000.0}\right) = 1.8299$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(24684.6)\left(\frac{2.8125}{77.3125}\right) = 1796.0 \ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{45170.3548}{1.8299} = 24684.6 \ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 1796.0\, psi$.

Combined Loadings – spherical shells and hemispherical heads subject to external pressure and other loadings shall satisfy the requirements of VIII-2, paragraph 4.4.12. In this example problem, the torispherical head is only subject to external pressure.

4.4.4 Example E4.4.4 – Torispherical Head

Determine the Maximum Allowable External Pressure (MAEP) for a torispherical head considering the following design conditions.

Vessel Data:

- Material = $SA-387,\ Grade\ 11,\ Class\ 1$
- Design Temperature = $650°F$
- Inside Diameter = $72.0\ in$
- Crown Radius = $72.0\ in$
- Knuckle Radius = $4.375\ in$
- Thickness = $0.625\ in$
- Corrosion Allowance = $0.125\ in$
- Modulus of Elasticity at Design Temperature = $26.55E+06\ psi$
- Yield Strength at Design Temperature = $26900\ psi$

Section VIII, Division 1 Solution

As noted in paragraph UG-33(e), the required thickness of a torispherical head having pressure on the convex side shall be determined in the same manner as outlined in paragraph UG-28(d) for determining the thickness for a spherical shell, with the appropriate value of R_o.

a) STEP 1 – UG-28(d)(1), Assume a value for t and calculate the value of factor A using the following formula:

$$A = \frac{0.125}{\left(\frac{R_o}{t}\right)} = \frac{0.125}{\left(\frac{72.625}{0.500}\right)} = 0.00086$$

$$R_o = Inside\ Crown\ Radius + Uncorroded\ Thickness = 72.0 + 0.625 = 72.625\ in$$

$$t = t - Corrosion\ Allowance = 0.625 - 0.125 = 0.500\ in$$

b) STEP 2 – Using the value of A calculated in STEP 1, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically to an intersection with the material/temperature line for the design temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 5.

Per Section II Part D, Table 1A, a material specification of $SA-387-11,\ Class\ 1$ is assigned an External Pressure Chart No. CS-2.

c) STEP 3 – From the intersection obtained in Step 2, move horizontally to the right and read the value of factor B.

$$B = 8100$$

d) STEP 4 – Using the value of B obtained in STEP 3, calculate the value of the maximum allowable external working pressure P_a using the following formula:

$$P_a = \frac{B}{\left(\frac{R_o}{t}\right)} = \frac{8100}{\left(\frac{72.625}{0.500}\right)} = 55.8 \ psi$$

e) STEP 5 – For values of A falling to the left of the applicable material/temperature line, the value of P_a can be calculated using the following formula:

$$P_a = \frac{0.0625E}{\left(\frac{R_o}{t}\right)^2} \qquad \textit{Not required}$$

f) STEP 6 – Compare the calculated value of P_a obtained in STEPS 4 or 5 with P. If P_a is smaller than P, select a larger value of t and repeat the design procedure until a value of P_a is obtained that is equal to or greater than P.

The allowable external pressure is $P_a = 55.8\, psi$.

Commentary:

As permitted in UG-28, Table G and Table CS-2 may be used to determine the values of A and B in lieu of using Figure G and Figure CS-2. Linear interpolation or any other rational interpolation method may be used. Since Figure G and Figure CS-2 are presented with a log-log scale, it is appropriate to perform log-log linear interpolation of the values of Table G and Table CS-2 to determine the values of A and B. The results of such an exercise produces the following values. For spheres and torispherical heads, Figure G/Table G is not necessary as the value of A is calculated directly.

$$A = 8.6058519E-03$$
$$B = 8136.2 \ psi$$
$$P_a = 56.0 \ psi$$

Section VIII, Division 2 Solution

Evaluate per VIII-2, paragraph 4.4.8 and 4.4.7.

The required thickness of a torispherical head subjected to external pressure loading shall be determined using the equations for a spherical shell in VIII-2, paragraph 4.4.7 by substituting the outside crown radius for R_o.

$$R_o = 72.0 + 0.625 = 72.625 \ in$$

Restrictions on Torispherical Head Geometry – the restriction of VIII-2 paragraph 4.3.6 shall apply. See VIII-2 paragraph 4.3.6.1.b and STEP 2 of E4.3.4.

Torispherical heads With Different Dome and Knuckle Thickness – heads with this configuration shall be designed in accordance with VIII-2, Part 5. In this example problem, the dome and knuckle thickness are the same.

Proceed with the design following the steps outlined in VIII-2, paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the torispherical head.

$$t = 0.625 - Corrosion\ Allowance = 0.625 - 0.125 = 0.500\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075E_y\left(\frac{t}{R_o}\right) = 0.075(26.55E+06)\left(\frac{0.500}{72.625}\right) = 13709.1222\ psi$$

c) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

1) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$F_{he} = 13709.1222\ psi$ (as determined in paragraph 4.4.7, STEP 2)

2) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{13709.1222}{26.55E+06} = 0.00051635$$

3) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

A tabulated summary of the iterative procedure is shown below.

Iteration	A_{diff}	F_{icup}	F_{iclow}	F_{icg}	E_t
1	1.000000000	60000.0000	0.0000	30000.0000	9.9279405E+05
2	0.029701396	30000.0000	0.0000	15000.0000	1.5506005E+07
3	0.000451016	15000.0000	0.0000	7500.0000	2.5857734E+07
4	0.000226303	15000.0000	7500.0000	11250.0000	2.2459426E+07
5	0.000015448	15000.0000	11250.0000	13125.0000	1.9267796E+07
6	0.000164837	13125.0000	11250.0000	12187.5000	2.0970842E+07
7	0.000064813	12187.5000	11250.0000	11718.7500	2.1745287E+07
8	0.000022559	11718.7500	11250.0000	11484.3750	2.2110214E+07
9	0.000003064	11484.3750	11250.0000	11367.1875	2.2286818E+07
10	0.000006310	11484.3750	11367.1875	11425.7813	2.2199012E+07
11	0.000001653	11484.3750	11425.7813	11455.0781	2.2154736E+07
12	0.000000698	11455.0781	11425.7813	11440.4297	2.2176905E+07
13	0.000000480	11455.0781	11440.4297	11447.7539	2.2165828E+07
14	0.000000108	11447.7539	11440.4297	11444.0918	2.2171369E+07
15	0.000000186	11447.7539	11444.0918	11445.9229	2.2168599E+07
16	0.000000039	11447.7539	11445.9229	11446.8384	2.2167214E+07
17	0.000000035	11446.8384	11445.9229	**11446.3806**	**2.2167906E+07**
18	**0.000000002**				

$F_{ic} = 11446.3806\ psi$

d) STEP 4 – Calculate the value of design margin, FS, per VIII-2, paragraph 4.4.2.

$$0.55S_y = 0.55(26900.0) = 14795.0 \ psi$$

Since $F_{ic} \le 0.55S_y$, calculate the FS as follows:

$$FS = 2.0$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(5723.1903)\left(\frac{0.500}{72.625}\right) = 78.8 \ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{11446.3806}{2.0} = 5723.1903 \ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 78.8\,psi$.

Combined Loadings – torispherical heads subject to external pressure and other loadings shall satisfy the requirements of VIII-2, paragraph 4.4.12. In this example problem, the torispherical head is only subject to external pressure.

4.4.5 Example E4.4.5 – Elliptical Head

Determine the maximum allowable external pressure (MAEP) for a 2:1 elliptical head considering the following design conditions.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Temperature = $300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$

Section VIII, Division 1 Solution

As noted in paragraph UG-33(d), the required thickness of an ellipsoidal head having pressure on the convex side shall be determined in the same manner as outlined in paragraph UG-28(d) for determining the thickness for a spherical shell, with the appropriate value of R_o.

a) STEP 1 – UG-28(d)(1), Assume a value for t and calculate the value of factor A using the following formula:

$$A = \frac{0.125}{\left(\frac{R_o}{t}\right)} = \frac{0.125}{\left(\frac{83.025}{1.0}\right)} = 0.00151$$

$$R_o = K_o D_o = K_o\left(D + 2\left(Uncorroded\ Thickness\right)\right) = 0.9\left(90.0 + 2\left(1.125\right)\right) = 83.025\ in$$

where,

$$K_o\ is\ taken\ from\ Table\ UG-33.1\ for\ a\ 2:1\ ellipse$$
$$t = t - Corrosion\ Allowance = 1.125 - 0.125 = 1.0\ in$$

b) STEP 2 – Using the value of A calculated in STEP 1, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically to an intersection with the material/temperature line for the design temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 5.

Per Section II Part D, Table 1A, a material specification of $SA-516,\ Grade\ 70,\ Normalized$ is assigned an External Pressure Chart No. CS-2.

c) STEP 3 – From the intersection obtained in Step 2, move horizontally to the right and read the value of factor B.

$$B = 13800$$

d) STEP 4 – Using the value of B obtained in STEP 3, calculate the value of the maximum allowable external working pressure P_a using the following formula:

$$P_a = \frac{B}{\left(\frac{R_o}{t}\right)} = \frac{13800}{\left(\frac{83.025}{1.0}\right)} = 166.2 \ psi$$

e) STEP 5 – For values of A falling to the left of the applicable material/temperature line, the value of P_a can be calculated using the following formula:

$$P_a = \frac{0.0625E}{\left(\frac{R_o}{t}\right)^2} \qquad \textit{Not required}$$

f) STEP 6 – Compare the calculated value of P_a obtained in STEPS 4 or 5 with P. If P_a is smaller than P, select a larger value of t and repeat the design procedure until a value of P_a is obtained that is equal to or greater than P.

The allowable external pressure is $P_a = 166.2\,psi$.

Commentary:

As permitted in UG-28, Table G and Table CS-2 may be used to determine the values of A and B in lieu of using Figure G and Figure CS-2. Linear interpolation or any other rational interpolation method may be used. Since Figure G and Figure CS-2 are presented with a log-log scale, it is appropriate to perform log-log linear interpolation of the values of Table G and Table CS-2 to determine the values of A and B. The results of such an exercise produces the following values. For spheres and elliptical heads, Figure G/Table G is not necessary as the value of A is calculated directly.

$$A = 1.50557E-03$$
$$B = 13828.7 \ psi$$
$$P_a = 166.5 \ psi$$

Section VIII, Division 2 Solution

Evaluate per VIII-2, paragraph 4.4.9 and 4.4.7.

The required thickness of an elliptical head subjected to external pressure loading shall be determined using the equations for a spherical shell in VIII-2, paragraph 4.4.7 by substituting $K_o D_o$ for R_o where K_o is given by the following equation.

$$K_o = 0.25346 + 0.13995\left(\frac{D_o}{2h_o}\right) + 0.12238\left(\frac{D_o}{2h_o}\right)^2 - 0.015297\left(\frac{D_o}{2h_o}\right)^3$$

$$K_o = \begin{pmatrix} 0.25346 + 0.13995\left(\dfrac{92.25}{2(23.0625)}\right) + 0.12238\left(\dfrac{92.25}{2(23.0625)}\right)^2 - \\ 0.015297\left(\dfrac{92.25}{2(23.0625)}\right)^3 \end{pmatrix} = 0.900504$$

$$D_o = 90.0 + 2(1.125) = 92.25\ in$$

$$h_o = \left(\frac{D_o}{4}\right) = \frac{92.25}{4} = 23.0625\ in$$

therefore,

$$R_o = K_o D_o = 0.9005040(92.25) = 83.0715\ in$$

Proceed with the design following the steps outlined in VIII-2, paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the elliptical head.

$$t = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075E_y\left(\frac{t}{R_o}\right) = 0.075(28.3E+06)\left(\frac{1.0}{83.0715}\right) = 25550.2790\ \ psi$$

c) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic} see paragraph 4.4.2.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

1) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$F_{he} = 25550.2790\ psi$ (as determined in paragraph 4.4.7, STEP 2)

2) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{25550.2790}{28.3E+06} = 0.00090284$$

3) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

A tabulated summary of the iterative procedure is shown below.

Iteration	A_{diff}	F_{icup}	F_{iclow}	F_{icg}	E_t
1	1.000000000	70000.0000	0.0000	35000.0000	1.6780582E+06
2	0.019954604	35000.0000	0.0000	17500.0000	1.6780582E+07
3	0.000074760	35000.0000	17500.0000	26250.0000	7.1896405E+06
4	0.002748250	26250.0000	17500.0000	21875.0000	1.3450426E+07
5	0.000723506	21875.0000	17500.0000	19687.5000	1.7344726E+07
6	0.000232234	19687.5000	17500.0000	18593.7500	1.9291520E+07
7	0.000060993	18593.7500	17500.0000	18046.8750	2.0230363E+07
8	0.000010768	18593.7500	18046.8750	18320.3125	1.9764850E+07
9	0.000024077	18320.3125	18046.8750	18183.5938	1.9998658E+07
10	0.000006404	18183.5938	18046.8750	18115.2344	2.0114783E+07
11	0.000002244	18183.5938	18115.2344	18149.4141	2.0056787E+07
12	0.000002065	18149.4141	18115.2344	18132.3242	2.0085802E+07
13	0.000000093	18149.4141	18132.3242	18140.8691	2.0071299E+07
14	0.000000985	18140.8691	18132.3242	18136.5967	2.0078551E+07
15	0.000000445	18136.5967	18132.3242	18134.4604	2.0082177E+07
16	0.000000176	18134.4604	18132.3242	18133.3923	2.0083989E+07
17	0.000000041	18133.3923	18132.3242	18132.8583	2.0084896E+07
18	0.000000026	18133.3923	18132.8583	**18133.1253**	**2.0084442E+07**
19	**0.000000008**				

$$F_{ic} = 18133.1253 \ psi$$

d) STEP 4 – Calculate the value of design margin, FS, per VIII-2, paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0 \ psi$$

Since $F_{ic} \le 0.55S_y$, calculate the FS as follows:

$$FS = 2.0$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(9066.5627)\left(\frac{1.0}{83.0715}\right) = 218.3 \ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{18133.1253}{2.0} = 9066.5627 \ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 218.3 \, psi$.

Combined Loadings – ellipsoidal heads subject to external pressure and other loadings shall satisfy the requirements of VIII-2, paragraph 4.4.12. In this example problem, the ellipsoidal head is only subject to external pressure.

4.4.6 Example E4.4.6 – Combined Loadings and Allowable Compressive Stresses

Determine the allowable compressive stresses of the proposed cylindrical shell section considering the following design conditions and specified applied loadings. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $-14.7\ psig\ @\ 300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Unsupported Length = $636.0\ in$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$
- Applied Axial Force = $-66152.5\ lbs$
- Applied Net Section Bending Moment = $5.08E+06\ in-lbs$
- Applied Shear Force = $18762.6\ lbs$

Adjust variables for corrosion and determine outside dimensions.

$$D = 90.0 + 2(Corrosion\ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

$$R = \frac{D}{2} = \frac{90.25}{2} = 45.125\ in$$

$$t = 1.125 - Corosion\ Allowance = 1.125 - 0.125 = 1.0\ in$$

$$D_o = 90.0 + 2(Uncorroded\ Thickness) = 90.0 + 2(1.125) = 92.25\ in$$

$$R_o = \frac{D}{2} = \frac{92.25}{2} = 46.125\ in$$

Section VIII, Division 1 Solution

VIII-1 does not provide rules on the loadings to be considered in the design of a vessel. However, UG-22 requires consideration of such loadings and the provisions of U-2(g) apply. This example provides one possible method of satisfying U-2(g) via Mandatory Appendix 46 as referenced in U-2(g)(1)(a); however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

This example uses VIII-2, paragraph 4.1 which provides specific requirements to account for both loads and load case combinations used in the design of a vessel. These loads and load case combinations (Table 4.1.1 and Table 4.1.2 of VIII-2, respectively) are shown in this example problem in Table E4.4.6.1 and Table E4.4.6.2 for reference. The load factor, Ω_p, shown in Table 4.1.2 of VIII-2 is used to simulate the maximum anticipated operating pressure acting simultaneously with the occasional loads specified. For this example, problem, $\Omega_p = 1.0$.

Additionally, VIII-1 does not provide a procedure for the calculation of combined stresses. VIII-2, paragraph 4.3.10.2 provides a procedure, and this procedure is used in this example problem with modifications to address specific requirements of VIII-1.

In accordance with VIII-2, paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. By inspection of the results shown in Table E4.4.6.3 and Table E4.4.6.4, Design Load Combination 5 is determined to be the governing load combination. The pressure, net section axial force, and bending moment at the location of interest for Design Load Combination are as follows.

$$\Omega P + P_s = (1.0)P + P_s = -14.7 \ psi$$

$$F_5 = -66152.5 \ lbs$$

$$M_5 = 3048000 \ in-lbs$$

$$V_5 = 11257.6 \ lbs \quad \rightarrow \quad \begin{array}{l} Not \ addressed \ in \ VIII-1 \\ and \ is \ not \ included \ in \ example \end{array}$$

a) STEP 1 – Calculate the membrane stress for the cylindrical shell. Note that the circumferential membrane stress, $\sigma_{\theta m}$, is determined based on the equations in UG-27(c)(1) and the longitudinal membrane stress, σ_{sm}, is determined based on the equations provided in UG-27(c)(2). The shear stress is computed based on the known strength of materials solution.

Note: θ is defined as the angle measured around the circumference from the direction of the applied bending moment to the point under consideration. For this example, problem $\theta = 0.0 \ deg$ to maximize the bending stress.

$$\sigma_{\theta m} = \frac{1}{E}\left(\frac{PR}{t} + 0.6P\right) = \frac{1}{1.0}\left(\frac{-14.7(45.125)}{1.0} + 0.6(-14.7)\right) = -672.1575 \ psi$$

$$\sigma_{sm} = \frac{1}{E}\left(\left(\frac{PR}{2t} - 0.2P\right) + \frac{4F}{\pi\left(D_o^2 - D^2\right)} \pm \frac{32MD_o\cos[\theta]}{\pi\left(D_o^4 - D^4\right)}\right)$$

$$\sigma_{sm} = \frac{1}{1.0}\left(\begin{array}{l}\left(\frac{-14.7(45.125)}{2(1.0)} - 0.2(-14.7)\right) + \frac{4(-66152.5)}{\pi\left((92.25)^2 - (90.25)^2\right)} \\ \pm \frac{32(3048000)(92.25)\cos[0.0]}{\pi\left((92.25)^4 - (90.25)^4\right)}\end{array}\right)$$

$$\sigma_{sm} = \begin{Bmatrix} -328.7288 - 230.7616 + 471.1299 = -88.3605 \ psi \\ -328.7288 - 230.7616 - 471.1299 = -1030.6203 \ psi \end{Bmatrix}$$

$$\tau = \frac{16M_tD_o}{\pi\left(D_o^4 - D^4\right)} = \frac{16(0.0)(92.25)}{\pi\left((92.25)^4 - (90.25)^4\right)} = 0.0 \ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\left(\tau\right)^2}\right)$$

$$\sigma_1 = 0.5\left(\left(-672.1575\right) + \left(-1030.6203\right) + \sqrt{\left(\left(-672.1575\right) - \left(-1030.6203\right)\right)^2 + 4\left(0\right)^2}\right)$$

$$\sigma_1 = -672.1575 \ psi$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\left(\tau\right)^2}\right)$$

$$\sigma_2 = 0.5\left(\left(-672.1575\right) + \left(-1030.6203\right) - \sqrt{\left(\left(-672.1575\right) - \left(-1030.6203\right)\right)^2 + 4\left(0\right)^2}\right)$$

$$\sigma_2 = -1030.6203 \ psi$$

$$\sigma_3 = \sigma_r = 0.0 \ psi \qquad For\ stress\ on\ the\ outside\ surface$$

c) STEP 3 – Check the allowable stress acceptance criteria.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]^{0.5}$$

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\begin{array}{l}\left(\left(-672.1575\right) - \left(-1030.6203\right)\right)^2 + \left(\left(-1030.6203\right) - \left(0.0\right)\right)^2 \\ +\left(\left(0.0\right) - \left(-672.1575\right)\right)^2\end{array}\right]^{0.5}$$

$$\sigma_e = 906.2200 \ psi$$

$$\left\{\sigma_e = 906.2 \ psi\right\} \le \left\{S = 20000 \ psi\right\} \qquad True$$

Note that VIII-2 uses an acceptance criterion based on von Mises Stress. Per Mandatory Appendix 46, the acceptance criteria for tensile stress in VIII-1 is in accordance with UG-23. Therefore,

$$\max\left[\sigma_1, \sigma_2, \sigma_3\right] \le S$$

$$\left\{\max\left[\left|-672.2\right|, \left|-1030.6\right|, \left|0.0\right|\right] = 1030.6 \ psi\right\} \le \left\{S = 20000 \ psi\right\} \quad True$$

d) STEP 4 – For cylindrical and conical shells, if the meridional stress, σ_{sm} is compressive, then check the allowable compressive stress using paragraph 4.4.12.2 with $\lambda = 0.15$. Per Mandatory Appendix 46, the maximum allowable compressive stress shall be limited as prescribed in VIII-2, paragraph 4.4.12 in lieu of the rules of UG-23(b).

Since σ_{sm} is compressive, $\{\sigma_{sm} = -1030.6203 \, psi\}$, a compressive stress check is required.

In accordance with VIII-2, paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

1) STEP 4.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$$F_{xe} = \frac{C_x E_y t}{D_o} = \frac{0.8499\left(28.3E+06\right)\left(1.0\right)}{92.25} = 260728.1301 \ psi$$

where,

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{46.125(1.0)}} = 93.6459$$

Since $D_o/t < 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \frac{D_o}{t}\right)},\ 0.9\right] = \min\left[\frac{409(1.0)}{389 + \frac{92.25}{1.0}},\ 0.9\right] = 0.8499$$

Since $M_x \geq 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

2) STEP 4.2 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{xe}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 4.2.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$F_{xe} = 260728.1301\ psi$ (as determined in STEP 2 above)

ii) STEP 4.2.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{xe}}{E} = \frac{260728.1301}{28.3E+06} = 0.00921301$$

iii) STEP 4.2.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 30967.6147\ psi$$

3) STEP 4.3 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{30967.6147}{33600}\right) = 1.7241$$

4) STEP 4.4 – Calculate the allowable axial compressive membrane stress as follows:

$$F_{xa} = \frac{F_{ic}}{FS} = \frac{30967.6147}{1.7241} = 17961.6117\ psi$$

5) STEP 4.5 – Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria.

$$\left\{\sigma_{sm} = 1030.6 \ psi\right\} \le \left\{F_{xa} = 17961.6 \ psi\right\} \qquad True$$

The allowable compressive stress criterion is satisfied.

Section VIII, Division 2 Solution

Evaluate per VIII-2, paragraph 4.4.12.2

The loads transmitted to the cylindrical shell are given in the Table E4.4.6.3. Note that this table is given in terms of the load parameters shown in VIII-2, Table 4.1.1, and Table 4.1.2. (Table E4.4.6.1 and Table E4.4.6.2 of this example). As shown in Table E4.4.6.2, the acceptance criteria are that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition.

In accordance with VIII-2, paragraph 4.4.12.2, the following procedure shall be used to determine the allowable compressive stresses for cylindrical shells that are based on loading conditions. By inspection of the results shown in Table E4.4.6.3 and Table E4.4.6.4, Load Case 5 is determined to be the governing load case. The pressure, net section axial force, bending moment, and radial shear force at the location o $f_q = \dfrac{P\pi D_i^2}{4A} = \dfrac{14.7(\pi)(90.25)^2}{4(286.6703)} = 328.0341 \ psi$ f interest for Load Case 5 are as follows. Note, the load factor, Ω_p, shown in Table 4.1.2 of VIII-2 is used to simulate the maximum anticipated operating pressure acting simultaneously with the occasional loads specified. For this example, problem, $\Omega_p = 1.0$.

$$\Omega P + P_s = (1.0)P + P_s = -14.7 \ psi$$

$$F_5 = -66152.5 \ lbs$$

$$M_5 = 3048000 \ in-lbs$$

$$V_5 = 11257.6 \ lbs$$

Common parameters used in each of the loading conditions are given in VIII-2, paragraph 4.4.12.2.k.

Per VIII-2, paragraph 4.4.12.2.k:

$$A = \frac{\pi\left(D_o^2 - D_i^2\right)}{4} = \frac{\pi\left(92.25^2 - 90.25^2\right)}{4} = 286.6703 \ in^2$$

$$S = \frac{\pi\left(D_o^4 - D_i^4\right)}{32D_o} = \frac{\pi\left(92.25^4 - 90.25^4\right)}{32(92.25)} = 6469.5531 \ in^3$$

$$f_h = \frac{PD_o}{2t} = \frac{14.7(92.25)}{2(1.0)} = 678.0375 \ psi$$

$$f_b = \frac{M}{S} = \frac{3.048E+06}{6469.5531} = 471.1299 \ psi$$

$$f_a = \frac{F}{A} = \frac{66152.5}{286.6703} = 230.7616 \ psi$$

$$f_v = \frac{V\sin[\phi]}{A} = \frac{11257.6\sin[90.0]}{286.6703} = 39.2702\ psi$$

Note: ϕ is defined as the angle measured around the circumference from the direction of the applied shear force to the point under consideration. For this example, problem, $\phi = 90\ deg$ to maximize the shear force.

$$r_g = 0.25\sqrt{D_o^{\ 2} + D_i^{\ 2}} = 0.25\sqrt{\left(92.25^2 + 90.25^2\right)} = 32.2637\ in$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{(46.125)1.0}} = 93.6459$$

The value of the slenderness factor for column buckling, λ_c is calculated in VIII-2, paragraph 4.4.12.2.b.

<u>**Per VIII-2, paragraph 4.4.12.2:**</u>

a) <u>External Pressure Acting Alone, (paragraph 4.4.12.2.a)</u> – the allowable hoop compressive membrane stress of a cylinder subject to external pressure acting alone, F_{ha}, is computed using the equations in VIII-2, paragraph 4.4.5.

From Example E4.4.1,

$$F_{ha} = 2255.0583\ psi$$

b) <u>Axial Compressive Stress Acting Alone, (paragraph 4.4.12.2.b)</u> – the allowable axial compressive membrane stress of a cylinder subject to an axial compressive load acting alone, F_{xa}, is computed using the following equations:

For $\lambda_c \leq 0.15$ (Local Buckling):

1) STEP 1 – Calculate the predicted elastic buckling stress, F_{xe}.

$$F_{xe} = \frac{C_x E_y t}{D_o} = \frac{0.8499(28.3E+06)(1.0)}{92.25} = 260728.1301\ psi$$

where,

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{46.125(1.0)}} = 93.6459$$

Since $D_o/t < 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \frac{D_o}{t}\right)},\ 0.9\right] = \min\left[\frac{409(1.0)}{389 + \frac{92.25}{1.0}},\ 0.9\right] = 0.8499$$

Since $M_x \geq 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

2) STEP 2 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{xe}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic} , is determined using the following procedure.

i) STEP 2.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$F_{xe} = 260728.1301\ psi$ (as determined in STEP 2 above)

ii) STEP 2.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{xe}}{E} = \frac{260728.1301}{28.3E+06} = 0.00921301$$

iii) STEP 2.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 30967.6147\ psi$$

3) STEP 3 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{30967.6147}{33600}\right) = 1.7241$$

4) STEP 4 – Calculate the allowable axial compressive membrane stress as follows:

$$F_{xa} = \frac{F_{ic}}{FS} = \frac{30967.6147}{1.7241} = 17961.6117\ psi$$

For $\lambda_c > 0.15$ and $K_u L_u / r_g < 200$, (Column Buckling)

With F_{xa} calculated, determine the value of λ_c from paragraph 4.4.12.2.k. For a cylinder with end conditions with one end free and the other end fixed, $K_u = 2.1$.

$$\left\{\frac{K_u L_u}{r_g} = \frac{2.1(636.0)}{32.2637} = 41.3964\right\} < \{200\} \qquad True$$

$$\lambda_c = \frac{K_u L_u}{\pi r_g}\left(\frac{F_{xa}\cdot FS}{E_y}\right)^{0.5} = \frac{2.1(636.0)}{\pi(32.2637)}\left(\frac{17961.6117(1.7241)}{28.3E+06}\right)^{0.5} = 0.4359$$

$$F_{ca} = F_{xa}\left[1 - 0.74(\lambda_c - 0.15)\right]^{0.3}$$

$$F_{ca} = 17961.6117\left[1 - 0.74(0.4359 - 0.15)\right]^{0.3} = 16725.3381\ psi$$

c) Compressive Bending Stress, (paragraph 4.4.12.2.c) – the allowable axial compressive membrane stress of a cylindrical shell subject to a bending moment acting across the full circular cross section, F_{ba}, is computed using the procedure in paragraph 4.4.12.2.b. For this example, problem, since $0.15 < \{\lambda_c = 0.4359\} \le 1.2$, $F_{ba} = F_{ca}$.

As shown, $F_{ba} = F_{ca} = 16725.3381\ psi$

d) Shear Stress, (paragraph 4.4.12.2.d) – the allowable shear stress of a cylindrical shell, F_{va}, is computed using the following equations:

1) STEP 1 – Calculate the predicted elastic buckling stress, F_{ve}.

$$F_{ve} = \alpha_v C_v E_y \left(\frac{t}{D_o}\right) = 0.8(0.1542)(28.3E+06)\left(\frac{1.0}{92.25}\right) = 37843.7724\ psi$$

where,

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{46.125(1.0)}} = 93.6459$$

$$4.347\left(\frac{D_o}{t}\right) = 4.347(92.25) = 401.0108$$

Since $26 \le \{M_x = 93.6459\} < \{4.347(D_o/t)\}$, calculate C_v as follows:

$$C_v = \frac{1.492}{M_x^{0.5}} = \frac{1.492}{(93.6459)^{0.5}} = 0.1542$$

Since $\{D_o/t = 92.25\} \le 500$, calculate α_v as follows:

$$\alpha_v = 0.8$$

2) STEP 2 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{ve}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 2.1 – Calculate the predicted elastic buckling stress, F_{ve}.

$F_{ve} = 37843.7724\ psi$ (as determined in STEP 2 above)

ii) STEP 2.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{ve}}{E} = \frac{37843.7724}{28.3E+06} = 0.00133724$$

iii) STEP 2.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 20714.3593 \ psi$$

3) STEP 3 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{20714.3593}{33600}\right) = 1.9502$$

4) STEP 4 – Calculate the allowable compressive shear stress as follows:

$$F_{va} = \frac{F_{ic}}{FS} = \frac{20714.3593}{1.9502} = 10621.7984 \ psi$$

e) Axial Compressive Stress and Hoop Compression, (paragraph 4.4.12.2.e) – the allowable compressive stress for the combination of uniform axial compression and hoop compression, F_{xha}, is computed using the following equations:

1) For $\lambda_c = 0.15$, F_{xha} is computed using the following equation with F_{ha} and F_{xa} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.a and 4.4.12.2.b.1, respectively.

$$F_{xha} = \left[\left(\frac{1}{F_{xa}^2}\right) - \left(\frac{C_1}{C_2 F_{xa} F_{ha}}\right) + \left(\frac{1}{C_2^2 F_{ha}^2}\right)\right]^{-0.5}$$

$$F_{xha} = \left[\begin{array}{l} \left(\dfrac{1}{(17961.6117)^2}\right) - \\ \left(\dfrac{0.0559}{0.8241(17961.6117)(2255.0583)}\right) + \\ \left(\dfrac{1}{(0.8241)^2(2255.0583)^2}\right) \end{array}\right]^{-0.5} = 1853.8375 \ psi$$

where,

$$C_1 = \frac{(F_{xa} \cdot FS + F_{ha} \cdot FS)}{S_y} - 1.0 = \frac{17961.6117(1.7241) + 2255.0583(2.0)}{33600} - 1.0$$

$$C_1 = 0.0559$$

$$C_2 = \frac{f_x}{f_h} = \frac{558.7957}{678.0375} = 0.8241$$

$$f_x = f_a + f_q = 230.7616 + 328.0341 = 558.7957 \ psi$$

Note: this step is not required since $\lambda_c > 0.15$.

2) For $0.15 < \lambda_c \leq 1.2$, F_{xha}, is computed from the following equation with $F_{ah1} = F_{xha}$ evaluated using the equations in VIII-2, paragraph 4.4.12.2.e.1, and F_{ah2} using the following procedure. The value of F_{ca} used in the calculation for F_{ah2} is evaluated using the equation in VIII-2, paragraph 4.4.12.2.b.2 with $F_{xa} = F_{xha}$ as determined in VIII-2, paragraph 4.4.12.2.e.1. As noted, the load on the end of a cylinder due to external pressure does not contribute to column buckling and therefore F_{ah1} is compared with f_a rather than f_x. The stress due to the pressure load does, however, lower the effective yield stress and the quantity in $(1 - f_q/S_y)$ accounts for this reduction.

$$F_{xha} = \min\left[F_{ah1}, \ F_{ah2}\right] = \min\left[1853.8375, 1709.3873\right] = 1709.3873 \ psi$$

$$F_{ah1} = F_{xha} = 1853.8375 \ psi$$

$$F_{ah2} = F_{ca}\left(1 - \frac{f_q}{S_y}\right) = 1726.2404\left(1 - \frac{328.0341}{33600}\right) = 1709.3873 \ psi$$

where,

$$F_{ca} = F_{xa}\left[1 - 0.74\left(\lambda_c - 0.15\right)\right]^{0.3}$$

$$F_{ca} = 1853.8375\left[1 - 0.74\left(0.4359 - 0.15\right)\right]^{0.3} = 1726.2404 \ psi$$

3) For $\lambda_c \leq 0.15$, the allowable hoop compressive membrane stress, F_{hxa}, is given by the following equation.

$$F_{hxa} = \frac{F_{xha}}{C_2} = \frac{1853.8375}{0.8241} = 2249.5298 \ psi$$

Note: this step is not required since $\lambda_c > 0.15$.

4) For $\lambda_c \geq 1.2$, the rules of <u>paragraph 4.4.12.2.e do not apply.</u>

f) <u>Compressive Bending Stress and Hoop Compression, (paragraph 4.4.12.2.f)</u> – the allowable compressive stress for the combination of axial compression due to a bending moment and hoop compression, F_{bha}, is computed using the following equations.

1) An iterative solution procedure is utilized to solve these equations for C_3 with F_{ha} and F_{ba} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.a and 4.4.12.2.c, respectively. For this example, problem, since $0.15 < \{\lambda_c = 0.4359\} \leq 1.2$, $F_{ba} = F_{bc}$.

$$F_{bha} = C_3 C_4 F_{ba} = (0.9926)(0.0937)(16725.3381) = 1555.5672 \ psi$$

where,

$$C_4 = \left(\frac{f_b}{f_h}\right)\left(\frac{F_{ha}}{F_{ba}}\right) = \left(\frac{471.1299}{678.0375}\right)\left(\frac{2255.0583}{16725.3381}\right) = 0.0937$$

$$C_3^2\left(C_4^2+0.6C_4\right)+C_3^{2n}-1=0$$

$$n=5-\frac{4F_{ha}\cdot FS}{S_y}=5-\frac{4(2255.0583)(2.0)}{33600}=4.4631$$

A tabulated summary of the iterative procedure to solve for C_3 is shown below.

Iteration	A_{diff}	$C3_{up}$	$C3_{low}$	$C3_g$	$Equation_g$
1	1.000000000	1.000000000	0.000000000	0.500000000	-9.81697352E-01
2	0.981697352	1.000000000	0.500000000	0.750000000	-8.86747671E-01
3	0.886747671	1.000000000	0.750000000	0.875000000	-6.46606859E-01
4	0.646606859	1.000000000	0.875000000	0.937500000	-3.80785061E-01
5	0.380785061	1.000000000	0.937500000	0.968750000	-1.85787513E-01
6	0.185787513	1.000000000	0.968750000	0.984375000	-6.81664050E-02
7	0.068166405	1.000000000	0.984375000	0.992187500	-3.63864700E-03
8	0.003638647	1.000000000	0.992187500	0.996093750	3.01482310E-02
9	0.030148231	0.996093750	0.992187500	0.994140625	1.31249790E-02
10	0.013124979	0.994140625	0.992187500	0.993164063	4.71093300E-03
11	0.004710933	0.993164063	0.992187500	0.992675781	5.28112000E-04
12	0.000528112	0.992675781	0.992187500	0.992431641	-1.55727200E-03
13	0.001557272	0.992675781	0.992431641	0.992553711	-5.15082000E-04
14	0.000515082	0.992675781	0.992553711	0.992614746	6.38970000E-06
15	0.000006390	0.992614746	0.992553711	0.992584229	-2.54377000E-04
16	0.000254377	0.992614746	0.992584229	0.992599487	-1.24002000E-04
17	0.000124002	0.992614746	0.992599487	0.992607117	-5.88079000E-05
18	0.000058808	0.992614746	0.992607117	0.992610931	-2.62096000E-05
19	0.000026210	0.992614746	0.992610931	0.992612839	-9.91008000E-06
20	0.000009910	0.992614746	0.992612839	0.992613792	-1.76022000E-06
21	0.000001760	0.992614746	0.992613792	0.992614269	2.31473000E-06
22	0.000002315	0.992614269	0.992613792	**0.992614031**	**2.77253000E-07**
23	**0.000000277**	0.992614031	0.992613792		

$$C_3=0.9926$$

2) The allowable hoop compressive membrane stress, F_{hba}, is given by the following equation.

$$F_{hba}=F_{bha}\left(\frac{f_h}{f_b}\right)=1555.5672\left(\frac{678.0375}{471.1299}\right)=2238.7305\ psi$$

g) Shear Stress and Hoop Compression, (paragraph 4.4.12.2.g) – the allowable compressive stress for the combination of shear, F_{vha}, and hoop compression is computed using the following equations.

Note: This load combination is only applicable for shear stress and hoop compression, in the absence of axial compressive stress and compressive bending stress. It is shown in this example problem for informational purposes only. The effect of shear is accounted for in the interaction equations of paragraphs 4.4.12.2.h and 4.4.12.2.i through the variable K_s.

1) The allowable shear stress is given by the following equation with F_{ha} and F_{va} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.and 4.4.12.2.d, respectively.

$$F_{vha}=\left[\left(\frac{F_{va}^2}{2C_5F_{ha}}\right)^2+F_{va}^2\right]^{0.5}-\frac{F_{va}^2}{2C_5F_{ha}}$$

$$F_{vha} = \left[\left(\frac{(10621.7984)^2}{2(0.0579)(2255.0583)} \right)^2 + (10621.7984)^2 \right]^{0.5} - \left[\frac{(10621.7984)^2}{2(0.0579)(2255.0583)} \right]$$

$$F_{vha} = 130.5482 \ psi$$

where,

$$C_5 = \frac{f_v}{f_h} = \frac{39.2702}{678.0375} = 0.0579$$

2) The allowable hoop compressive membrane stress, F_{hva}, is given by the following equation.

$$F_{hva} = \frac{F_{vha}}{C_5} = \frac{130.5482}{0.0579} = 2254.7185 \ psi$$

h) Axial Compressive Stress, Compressive Bending Stress, Shear Stress, and Hoop Compression, (paragraph 4.4.12.2.h) – the allowable compressive stress for the combination of uniform axial compression, axial compression due to a bending moment, and shear in the presence of hoop compression is computed using the following interaction equations.

1) The shear coefficient is determined using the following equation with F_{va} from VIII-2, paragraph 4.4.12.2.d.

$$K_s = 1.0 - \left(\frac{f_v}{F_{va}} \right)^2 = 1.0 - \left(\frac{39.2702}{10621.7984} \right)^2 = 0.9999$$

2) For $\lambda_c \leq 0.15$, the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{xha} and F_{bha} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.e.1 and 4.4.12.2.f.1, respectively.

$$\left(\frac{f_a}{K_s F_{xha}} \right)^{1.7} + \left(\frac{f_b}{K_s F_{bha}} \right) \leq 1.0$$

$$\left\{ \left(\frac{230.7616}{0.9999(1853.8375)} \right)^{1.7} + \left(\frac{471.1299}{0.9999(1555.5672)} \right) = 0.3319 \right\} \leq 1.0 \qquad True$$

Note: this step is not required since $\lambda_c > 0.15$.

3) For $0.15 < \lambda_c \leq 1.2$, the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{xha} and F_{bha} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.e.2 and 4.4.12.2.f.1, respectively.

$$\frac{f_a}{K_s F_{xha}} = \frac{230.7616}{0.9999(1709.3873)} = 0.1350$$

Since $f_a / K_s F_{xha} < 0.2$, the following equation shall be used:

$$\left(\frac{f_a}{2K_s F_{xha}}\right)+\left(\frac{\Delta f_b}{K_s F_{bha}}\right)\le 1.0$$

$$\left\{\left(\frac{230.7616}{2(0.9999)(1709.3873)}\right)+\left(\frac{1.0024(471.1299)}{0.9999(1555.5672)}\right)=0.3711\right\}\le\{1.0\} \qquad True$$

where,

$$\Delta=\frac{C_m}{1-\left(\frac{f_a \cdot FS}{F_e}\right)}=\frac{1.0}{1-\left(\frac{230.7616(1.7241)}{162990.2785}\right)}=1.0024$$

$$F_e=\frac{\pi^2 E_y}{\left(\frac{K_u L_u}{r_g}\right)^2}=\frac{\pi^2(28.3E+06)}{\left(\frac{2.1(636.0)}{32.2637}\right)^2}=162990.2785 \ psi$$

Note: $C_m = 1.0$ for unbraced skirt supported vessels, see paragraph 4.4.15.

i) Axial Compressive Stress, Compressive Bending Stress, and Shear Stress, (paragraph 4.4.12.2.i) – the allowable compressive stress for the combination of uniform axial compression, axial compression due to a bending moment, and shear in the absence of hoop compression is computed using the following interaction equations.

1) The shear coefficient is determined using the equation in VIII-2, paragraph 4.4.12.2.h.1 with F_{va} from paragraph 4.4.12.2.d.

$$K_s=1.0-\left(\frac{f_v}{F_{va}}\right)^2=1.0-\left(\frac{39.2702}{10621.7984}\right)^2=0.9999$$

2) For $\lambda_c \le 0.15$ the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{xa} and F_{ba} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.b.1 and 4.4.12.2.c, respectively.

$$\left(\frac{f_a}{K_s F_{xa}}\right)^{1.7}+\left(\frac{f_b}{K_s F_{ba}}\right)\le 1.0$$

$$\left\{\left(\frac{230.7616}{0.9999(17961.6117)}\right)^{1.7}+\left(\frac{471.1299}{0.9999(16725.3381)}\right)=0.0288\right\}\le 1.0 \qquad True$$

Note: this step is not required since $\lambda_c > 0.15$.

3) For $0.15 < \lambda_c \le 1.2$ the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{ca} and F_{ba} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.b.2 and 4.4.12.2.c, respectively. The coefficient Δ is evaluated using the equations in VIII-2, paragraph 4.4.12.2.h.3.

$$\frac{f_a}{K_s F_{ca}} = \frac{230.7616}{0.9999(16725.3381)} = 0.0138$$

Since $f_a/K_sF_{ca} < 0.2$, the following equation shall be used:

$$\left(\frac{f_a}{2K_s F_{ca}}\right) + \left(\frac{\Delta f_b}{K_s F_{ba}}\right) \le 1.0$$

$$\left\{\left(\frac{230.7616}{2(0.9999)(16725.3381)}\right) + \left(\frac{1.0024(471.1299)}{0.9999(16725.3381)}\right) = 0.0351\right\} \le 1.0 \quad True$$

j) (Paragraph 4.4.12.2.j) – The maximum deviation, e, may exceed the value e_x given in VIII-2, paragraph 4.4.4.2 if the maximum axial stress is less than F_{xa} for shells designed for axial compression only, or less than F_{xha} for shells designed for combinations of axial compression and external pressure. The change in buckling stress, F'_{xe}, is given by VIII-2, Equation (4.4.114). The reduced allowable buckling stress, $F_{xa(reduced)}$, is determined using VIII-2, Equation (4.4.115) where e is the new maximum deviation, F_{xa} is determined using VIII-2, Equation 4.4.61, and FS_{xa} is the value of the stress reduction factor used to determine F_{xa}.

From VIII-2, paragraph 4.4.4.1:

$$e = \min[e_c,\ 2t] = \min\left[1.3007,\ \{2(1.0) = 2.0\}\right] = 1.3007\ in$$

Where, e_c is valid for the following range:

$$\{0.1t = 0.1(1.0) = 0.1\ in\} \le e_c \le \{0.0282R_o = 0.0282(46.125) = 1.3007\ in\}$$

$$e_c = 0.0165t\left(\frac{L}{\sqrt{R_o t}} + 3.25\right)^{1.069} = 0.0165(1.0)\left(\frac{636.0}{\sqrt{46.125(1.0)}} + 3.25\right)^{1.069} = 2.1920\ in$$

From VIII-2, paragraph 4.4.4.2:

$$e_x = 0.002R_m = 0.002(45.625) = 0.0913\ in$$

where,

$$R_m = \frac{(D_o + D_i)}{4} = \frac{(92.25 + 90.25)}{4} = 45.625\ in$$

For axial compression only,

$$\{f_a + f_b\} \le F_{xa}$$

$$\{230.7616 + 471.1299 = 701.8915\ psi\} \le 16725.3381\ psi \qquad True$$

For axial compression and external pressure,

$$\{f_a + f_b + f_q\} \le F_{xha}$$

$$\{230.7616 + 471.1299 + 328.0341 = 1029.9256\ psi\} \le 1709.3873\ psi \qquad True$$

Since, both criteria are satisfied,

$$F_{xe}^{'} = \left(0.944 - \left|0.286\log\left[\frac{0.0005e}{e_x}\right]\right|\right)\left(\frac{E_y t}{R}\right)$$

$$F_{xe}^{'} = \left(0.944 - \left|0.286\log\left[\frac{0.0005(1.3007)}{(0.0913)}\right]\right|\right)\left(\frac{(28.3E+06)(1.0)}{46.125}\right) = 202388.986\ psi$$

Therefore, the reduced allowable bucking stress is determined as follows.

$$F_{xa(reduced)} = \frac{F_{xa} \cdot FS_{xa} - F_{xe}^{'}}{FS_{xa}}$$

$$F_{xa(reduced)} = \frac{17961.6117(1.7241) - (202388.986)}{1.7241} = -99426.5827\ psi$$

A summary of the allowable compressive stresses are as follows:

Paragraph 4.4.12.2.a, External Pressure Acting Alone

$$F_{ha} = 2255.0583\ psi$$

Paragraph 4.4.12.2.b, Axial Compressive Stress Acting Alone

$$F_{xa} = 17961.6117\ psi$$

$$F_{ca} = 16725.3381\ psi$$

Paragraph 4.4.12.2.c, Compressive Bending Stress

$$F_{ba} = 16725.3381\ psi$$

Paragraph 4.4.12.2.d, Shear Stress

$$F_{va} = 10621.7984\ psi$$

Paragraph 4.4.12.2.e, Axial Compressive Stress and Hoop Compression

$$F_{xha} = 1709.3873\ psi$$

Paragraph 4.4.12.2.f, Compressive Bending Stress and Hoop Compression

$$F_{bha} = 1555.5672\ psi$$

$$F_{hba} = 2238.7305\ psi$$

Paragraph 4.4.12.2.g, Shear Stress and Hoop Compression

$$F_{vha} = 130.5482\ psi$$

$$F_{hva} = 2254.7185\ psi$$

Table E4.4.6.1 – Design Loads from VIII-2

Table 4.1.1 – Design Loads	
Design Load Parameter	**Description**
P	Internal of External Specified Design Pressure (see paragraph 4.1.5.2.a)
P_s	Static head from liquid or bulk materials (e.g., catalyst)
D	Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following: • Weight of vessel including internals, supports (e.g., skirts, lugs, saddles, and legs), and appurtenances (e.g., platforms, ladders, etc.) • Weight of vessel contents under design and test conditions • Refractory linings, insulation • Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping • Transportation loads (the static forces obtained as equivalent to the dynamic loads experienced during normal operation of a transport vessel [see paragraph 1.2.1.3(b)]
L	• Appurtenance live loading • Effects of fluid flow, steady state or transient • Loads resulting from wave action
E	Earthquake loads [see paragraph 4.1.5.3(b)]
W	Wind loads [see paragraph 4.1.5.3(b)]
S_s	Snow loads
F	Loads due to Deflagration

Table E4.4.6.2 – Design Load Combinations from VIII-2

Table 4.1.2 – Design Load Combinations	
Design Load Combination [Note (1) and (2)]	**General Primary Membrane Allowable Stress [Note (3)]**
$P+P_s+D$	S
$P+P_s+D+L$	S
$P+P_s+D+S_s$	S
$\Omega P+P_s+D+0.75L+0.75S_s$	S
$\Omega P+P_s+D+\left(0.6W\ or\ 0.7E\right)$	S
$\Omega P+P_s+D+0.75\left(0.6W\ or\ 0.7E\right)+0.75L+0.75S_s$	S
$0.6D+\left(0.6W\ or\ 0.7E\right)$ [Note(4)]	S
P_s+D+F	See Annex 4-D
Other load combinations as defined in the UDS	S

Notes:

1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.
2) See 4.1.5.3 for additional requirements.
3) S is the allowable stress for the load case combination [see paragraph 4.1.5.3(c)].
4) This load combination addresses an overturning condition for foundation design. It does not apply to design of anchorage (if any) to the foundation. Refer to ASCE/SEI 7, 2.4.1 Exception 2 for an additional reduction to W that may be applicable.

Table E4.4.6.3 – Design Loads (Net-Section Axial Force and Bending Moment) at the Location of Interest

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)	$P = -14.7\ psi$
P_s	Static head from liquid or bulk materials (e.g., catalyst)	$P_s = 0.0\ psi$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest	$D_F = -66152.5\ lbs$ $D_M = 0.0\ in-lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = 0.0\ lbs$ $L_M = 0.0\ in-lbs$
E	Earthquake loads	$E_F = 0.0\ lbs$ $E_M = 0.0\ in-lbs$
W	Wind Loads	$W_F = 0.0\ lbs$ $W_M = 5.08E+06\ in-lbs$ $W_V = 18762.6\ lbs$
S_s	Snow Loads	$S_F = 0.0\ lbs$ $S_M = 0.0\ in-lbs$
F	Loads due to Deflagration	$F_F = 0.0\ lbs$ $F_M = 0.0\ in-lbs$

Based on these loads, the shell is required to be designed for the load case combinations shown in Table E4.4.6.4. Note that this table is given in terms of the load combinations shown in VIII-2, Table 4.1.2 (Table E4.4.6.2 of this example).

Table E4.4.6.4 – Load Case Combination at the Location of Interest

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P + P_s + D$	$P + P_s = -14.7\ psi$ $F_1 = -66152.5\ lbs$ $M_1 = 0.0\ in-lbs$	S
2	$P + P_s + D + L$	$P + P_s = -14.7\ psi$ $F_2 = -66152.5\ lbs$ $M_2 = 0.0\ in-lbs$	S
3	$P + P_s + D + S_s$	$P + P_s = -14.7\ psi$ $F_3 = -66152.5\ lbs$ $M_3 = 0.0\ in-lbs$	S
4	$\Omega P + P_s + D + 0.75L + 0.75S_s$	$\Omega P + P_s = -14.7\ psi$ $F_4 = -66152.5\ lbs$ $M_4 = 0.0\ in-lbs$	S
5	$\Omega P + P_s + D + (0.6W\ or\ 0.7E)$	$\Omega P + P_s = -14.7\ psi$ $F_5 = -66152.5\ lbs$ $M_5 = 3048000\ in-lbs$ $V_5 = 11257.6\ lbs$	S
6	$\Omega P + P_s + D + 0.75(0.6W\ or\ 0.7E) +$ $0.75L + 0.75S_s$	$\Omega P + P_s = -14.7\ psi$ $F_6 = -66152.5\ lbs$ $M_6 = 2286000\ in-lbs$ $V_6 = 8443.0\ lbs$	S
7	$0.6D + (0.6W\ or\ 0.7E)$	$F_6 = -39691.5\ lbs$ $M_6 = 3048000\ in-lbs$	S
8	$P_s + D + F$	$P_s = 0.0\ psi$ $F_8 = -66152.5\ lbs$ $M_8 = 0.0\ in-lbs$	See Annex 4-D

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4.4.7 Example E4.4.7 – Conical Transitions without a Knuckle

Determine if the proposed large and small end cylinder-to-cone transitions are adequately designed considering the following design conditions and applied forces and moments.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $-14.7\ psig\ @\ 300°F$
- Inside Radius (Large End) = $75.0\ in$
- Thickness (Large End) = $1.8125\ in$
- Inside Radius (Small End) = $45.0\ in$
- Thickness (Small End) = $1.125\ in$
- Thickness (Conical Section) = $1.9375\ in$
- Length of Conical Section = $78.0\ in$
- Unsupported Length of Large Cylinder = $732.0\ in$
- Unsupported Length of Small Cylinder = $636.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Yield Strength = $33600\ psi$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle (See E4.3.2) = $21.0375\ deg$
- Axial Force (Large End) = $-99167\ lbs$
- Net Section Bending Moment (Large End) = $5.406E+06\ in-lbs$
- Axial Force (Small End) = $-78104\ lbs$
- Net Section Bending Moment (Small End) = $4.301E+06\ in-lbs$

Adjust variables for corrosion and determine outside dimensions.

$$t_L = 1.8125 - Corrosion\ Allowance = 1.8125 - 0.125 = 1.6875\ in$$
$$t_S = 1.125 - Corrosion\ Allowance = 1.125 - 0.125 = 1.0\ in$$
$$t_C = 1.9375 - Corrosion\ Allowance = 1.9375 - 0.125 = 1.8125\ in$$

$$R_L = 75.0 + Uncorroded\ Thickness = 75.0 + 1.9375 = 76.8125\ in$$
$$R_S = 45.0 + Uncorroded\ Thickness = 45.0 + 1.9375 = 46.125\ in$$

$$D_L = 2R_L = 2(76.8125) = 153.625\ in$$
$$D_S = 2R_S = 2(46.125) = 92.25\ in$$

Section VIII, Division 1 Solution

Evaluate per paragraph UG-33(f): Conical heads and Sections. When the cone-to-cylinder junction is not a line of support, the required thickness of a conical head or section under pressure on the convex side, either seamless or of built-up construction with butt joints shall not be less than the required thickness of the adjacent cylindrical shell and, when a knuckle is not provided, the reinforcement requirement of Appendix 1-8 shall be satisfied. When the cone-to-cylinder junction is a line of support the required thickness shall be determined in accordance with the following procedure.

For this example, it is assumed that the cone-to-cylinder junction is a line of support.

Rules for conical reducer sections subject to external pressure are covered in Appendix 1-8. Rules are provided for the design of reinforcement, if needed, and for verification of adequate moment of inertia, when the cone-to-cylinder junction is a line of support, for conical reducer sections and conical heads where all the elements have a common axis, and the half-apex angle satisfies $\alpha \leq 60\ deg$.

Large End

In accordance with Appendix 1-8(b), reinforcement shall be provided at the large end of the cone when required by (b)(1) or (b)(2). When the large end of the cone is considered a line of support, the moment of inertia for a stiffening ring shall be determined in accordance with (b)(3).

Appendix 1-8(b)(1), for cones attached to a cylinder having a minimum length of $2.0\sqrt{R_L t\_s}$, reinforcement shall be provided at the junction of the cone with the large cylinder for conical heads and reducers without knuckles when the value of Δ obtained from Equation (1) using the appropriate ratio $P/S_s E_1$ is less than α.

$$\left\{Cylinder\ \ Length = 732.0\ in\right\} \geq \left\{2.0\sqrt{R_L t_s} = 2.0\sqrt{(76.8125)(1.6875)} = 22.7703\ in\right\} True$$

$$\Delta = 104\sqrt{\frac{P}{S_s E_1}} = 104\sqrt{\frac{14.7}{\left(20000(1.0)\right)}} = 2.8195\ \deg$$

$$\left\{\ \Delta = 2.8195\right\} < \left\{\alpha = 21.0375\right\};\ reinforcement\ is\ required\ at\ the\ large\ end$$

Appendix 1-8(a), since reinforcement is required at the large end, determine the value k. Assuming the reinforcement will be place on the cylinder, if required.

$$k = \frac{y}{S_r E_r} = \frac{20000}{20000(1.0)} = 1.0$$

where,

$$y = S_s E_s = 20000(1.0) = 20000$$

Appendix 1-8(b)(1), the required area of reinforcement, A_{rL}, shall be at least equal to that indicated by the following equation when Q_L is in compression. At the large end of the cone-to-cylinder juncture, the $PR_L/2$ term is in compression. When f_1 is in tension and the quantity is larger than the $PR_L/2$ term, the design shall be in accordance with U–2(g). The localized stress at the discontinuity shall not exceed the stress values specified in Appendix 1-5(g)(1) and (2).

$$A_{rL}=\frac{kQ_L R_L \tan[\alpha]}{S_s E_1}\left(1-\frac{1}{4}\left(\frac{PR_L-Q_L}{Q_L}\right)\cdot\left(\frac{\Delta}{\alpha}\right)\right)$$

$$A_{rL}=\left\{\begin{array}{l}\dfrac{(1.0)(1061.6955)(76.8125)\cdot\tan[21.0375]}{20000(1.0)}\cdot \\ \left(1-\dfrac{1}{4}\left(\dfrac{(14.7)(76.8125)-1061.6955}{1061.6955}\right)\cdot\left(\dfrac{2.8195}{21.0375}\right)\right)\end{array}\right\}=1.5650\ in^2$$

where,

$$Q_L=\frac{P(R_L)}{2}+f_1=\left\{\begin{array}{l}\dfrac{-14.7(76.8125)}{2}+86.1770=-478.3949\dfrac{lbs}{in\ of\ cir} \\ \dfrac{-14.7(76.8125)}{2}+(-497.1236)=-1061.6955\dfrac{lbs}{in\ of\ cir}\end{array}\right\}$$

$$Q_L=|-1061.6955|\frac{lbs}{in\ of\ cir}\quad \textit{Use the absolute value of the maximum negative value}$$

and,

$$f_1=\frac{F_L}{2\pi R_L}\pm\frac{M_L}{\pi R_L^2}=\left\{\begin{array}{l}\dfrac{-99167}{2\pi(76.8125)}+\dfrac{5.406E+06}{\pi(76.8125)^2}=+86.1770\ \dfrac{lbs}{in\ of\ cir} \\ \dfrac{-99167}{2\pi(76.8125)}-\dfrac{5.406E+06}{\pi(76.8125)^2}=-497.1236\ \dfrac{lbs}{in\ of\ cir}\end{array}\right\}$$

The effective area of reinforcement can be determined in accordance with the following:

$$A_{eL}=0.55\left[\sqrt{D_L t_L}\,(t_L-t)+\sqrt{\left(\frac{D_L t_c}{\cos[\alpha]}\right)}(t_c-t_r)\right]$$

$$A_{eL}=0.55\left[\sqrt{(153.625)(1.6875)}\,(1.6875-0.9452)+\sqrt{\left(\frac{(153.625)(1.8125)}{\cos[21.0375]}\right)}(1.8125-0.3590)\right]$$

$$A_{eL}=20.3810\ in^2$$

where,

$$t=0.9452\ in^2\ \ (\textit{see Example E4.4.1 for Methodology})$$

$$t_r=0.3590\ in^2\ (\textit{see Example E4.4.2 for Methodology})$$

The effective area of available reinforcement due to the excess thickness in the cylindrical shell and conical shell, A_{eL}, exceeds the required reinforcement, A_{rL}.

$$\left\{A_{eL}=20.3810\ in^2\right\}\geq\left\{A_{rL}=1.5650\ in^2\right\}\qquad \textit{True}$$

If this was not true, reinforcement would need to be added to the cylindrical or conical shell using a thick insert plate or reinforcing ring. Any additional area of reinforcement which is required shall be situated within a distance of $\sqrt{R_L t_s}$ from the junction, and the centroid of the added area shall be within a distance of $0.25\sqrt{R_L t_s}$ from the junction.

Appendix 1-8(b)(3), when the cone-to-cylinder or knuckle-to-cylinder juncture is a line of support, the moment of inertia for a stiffening ring at the large end shall be determined by the following procedure.

a) STEP 1 – Assuming that the shell has been designed and D_L, L_L, and t are known, select a member to be used for the stiffening ring and determine the cross-sectional area A_{TL}.

$$A_{TL} = \frac{L_L t_s}{2} + \frac{L_c t_c}{2} + A_s = \frac{732.0(1.6875)}{2} + \frac{83.8196(1.8125)}{2} + 0.0 = 693.5865\ in^2$$

where,

$$L_L = 732.0\ in$$

$$L_c = \sqrt{L^2 + (R_L - R_s)^2} = \sqrt{78.0^2 + (76.8125 - 46.125)^2} = 83.8196\ in$$

$$A_s = 0.0\ in^2 \qquad Assume\ no\ stiffening\ ring\ area$$

Calculate factor B using the following formula. If F_L is a negative number, the design shall be in accordance with U-2(g).

$$B = \frac{3}{4}\left(\frac{F_L D_L}{A_{TL}}\right) = \frac{3}{4}\left(\frac{5979.9834(153.625)}{693.5865}\right) = 993.3962\ psi$$

where,

$$F_L = PM + f_1 \tan[\alpha] = 14.7(393.7947) + (497.1236)\cdot\tan[21.0375] = 5979.9834\ \frac{lbs}{in}$$

$$f_1 = |-497.1236|\ \frac{lbs}{in\ of\ circ}$$

and,

$$M = \frac{-R_L \tan[\alpha]}{2} + \frac{L_L}{2} + \frac{R_L^2 - R_s^2}{3R_L \tan[\alpha]}$$

$$M = \frac{-(76.8125)\cdot\tan[21.0375]}{2} + \frac{732.0}{2} + \frac{(76.8125)^2 - (46.125)^2}{3(76.8125)\cdot\tan[21.0375]} = 393.7947\ in$$

b) STEP 2 – Enter the right-hand side of the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration at the value of B determined by STEP 1. If different materials are used for the shell and stiffening ring, use the material chart resulting in the larger value of A in STEP 4.

Per Section II Part D, Table 1A, a material specification of $SA-516,\ Grade\ 70,\ Normalized$ is assigned an External Pressure Chart No. CS-2.

c) STEP 3 – Move horizontally to the left to the material/temperature line for the design metal temperature. For values of B falling below the left end of the material/temperature line, see STEP 5.

d) STEP 4 – Move vertically to the bottom of the chart and read the value of A.

This step is not required as the value of B falls below the left end of the material/temperature line.

e) STEP 5 – For values of B falling below the left end of the material/temperature line for the design temperature, the value of A can be calculated using the following:

$$A = \frac{2B}{E_x} = \frac{2(993.3962)}{28.3E+06} = 0.00007$$

where,

$$E_x = \min\left[E_c, E_s, E_r\right], \; (min\ of\ \ the\ cone,\ shell,\ or\ stiffening\ \ ring)$$

f) STEP 6 – Compute the value of the required moment of inertia from the formulas for I_s or I'_s. For the circumferential stiffening ring only,

$$I_s = \frac{AD_L^2 A_{TL}}{14.0} = \frac{0.00007(153.625)^2(693.5865)}{14.0} = 81.8454\ in^4$$

For the shell-cone or ring-shell-cone section,

$$I_s^{'} = \frac{AD_L^2 A_{TL}}{10.9} = \frac{0.00007(153.625)^2(693.5865)}{10.9} = 105.1226\ in^4$$

g) STEP 7 – Determine the available moment of inertia of the ring only, I, or the shell-cone or ring-shell-cone, I'.

h) STEP 8 – When the ring only is used,

$$I \geq I_s$$

And when the shell-cone- or ring-shell-cone is used,

$$I^{'} \geq I_s^{'}$$

If the equation is not satisfied, a new section with a larger moment of inertia must be selected, and the calculation shall be done again until the equation is met. The requirements of UG-29(b), (c), (d), (e), and (f) and UG-30 are to be met in attaching stiffening rings to the shell.

VIII-1 does not provide a procedure to calculate the available moment of inertia of the shell-cone or ring-shell-cone junction. The designer must consider the following options.

a) Size a structural member to satisfy the requirement of $I \geq I_S$.

b) Size a structural member to be used in conjunction with the available moment of inertia of the cone and cylinder to satisfy the requirement of $I' \geq I'_S$.

c) The cost of material, fabrication, welding, inspection, and engineering.

Small End

In accordance with Appendix 1-8(c), reinforcement shall be provided at the small end of the cone when required by (c)(1) or (c)(2). When the small end of the cone is considered a line of support, the moment of inertia for a stiffening ring shall be determined in accordance with (c)(3).

Appendix 1-8(c)(1), for cones attached to a cylinder having a minimum length of $1.4\sqrt{R_S t_S}$, reinforcement shall be provided at the junction of the conical shell of a reducer without a flare and the small cylinder.

$$\{Cylinder\ Length = 636.0\ in\} \geq \left\{2.0\sqrt{R_S t_s} = 2.0\sqrt{(46.125)(1.0)} = 9.5081\ in\right\} \qquad True$$

Appendix 1-8(a), since reinforcement is required at the small end, determine the value k. Assuming the reinforcement will be place on the cylinder, if required.

$$k = \frac{y}{S_r E_r} = \frac{20000}{20000(1.0)} = 1.0$$

where,

$$y = S_s E_s = 20000(1.0) = 20000$$

Appendix 1-8(c)(1), the required area of reinforcement, A_{rs}, shall be at least equal to that indicated by the following equation when Q_s is in compression. At the small end of the cone-to-cylinder juncture, the $PR_s/2$ term is in compression. When f_2 is in tension and the quantity is larger than the $PR_s/2$ term, the design shall be in accordance with U–2(g). The localized stress at the discontinuity shall not exceed the stress values specified in Appendix 1-5(g)(1) and (2).

$$A_{rs} = \frac{kQ_s R_s \tan[\alpha]}{S_s E_1} = \frac{(1.0)(1252.0151)(46.125)\cdot\tan[21.0375]}{20000(1.0)} = 1.1106\ in^2$$

where,

$$Q_s = \frac{P(R_s)}{2} + f_2 = \begin{Bmatrix} \dfrac{-14.7(46.125)}{2} + 373.9985 = +34.9798\dfrac{lbs}{in\ of\ cir} \\ \dfrac{-14.7(46.125)}{2} + (-912.9963) = -1252.0151\dfrac{lbs}{in\ of\ cir} \end{Bmatrix}$$

$$Q_s = |-1252.0151|\frac{lbs}{in\ of\ cir} \qquad Use\ the\ absolute\ value\ of\ the\ maximum\ negative\ value$$

and

$$f_2 = \frac{F_s}{2\pi R_s} \pm \frac{M_s}{\pi R_s^2} = \begin{Bmatrix} \dfrac{-78104}{2\pi(46.125)} + \dfrac{4.301E+06}{\pi(46.125)^2} = +373.9985\ \dfrac{lbs}{in\ of\ cir} \\ \dfrac{-78104}{2\pi(46.125)} - \dfrac{4.301E+06}{\pi(46.125)^2} = -912.9963\ \dfrac{lbs}{in\ of\ cir} \end{Bmatrix}$$

The effective area of reinforcement can be determined in accordance with the following:

$$A_{es}=0.55\left[\sqrt{D_s t_s}\left(t_s-t\right)+\sqrt{\left(\frac{D_s t_c}{\cos[\alpha]}\right)}\left(t_c-t_r\right)\right]$$

$$A_{es}=0.55\left[\sqrt{(92.25)(1.0)}\left(1.0-0.6699\right)+\sqrt{\left(\frac{(92.25)(1.8125)}{\cos[21.0375]}\right)}\left(1.8125-0.2669\right)\right]=13.1216\ in^2$$

where,

$$t=0.6699\ in^2\ \left(see\ Example\ E4.4.1\ for\ Methodology\right)$$

$$t_r=0.2669\ in^2\left(see\ Example\ E4.4.2\ for\ Methodology\right)$$

Commentary:

The required thickness for the cone at the small end junction, t_r, was calculated using the methodology as shown in Example E.4.4.2. The equivalent cone length, L_e, and the equivalent cone thickness, t_e, were maintained; however, the small end diameter, D_s, was used in the ratios L_e/D_s and D_s/t_e.

The effective area of available reinforcement due to the excess thickness in the cylindrical shell and conical shell, A_{es}, exceeds the required reinforcement, A_{rs}.

$$\left\{A_{es}=13.1216\ in^2\right\}\geq\left\{A_{rs}=1.1106\ in^2\right\}\qquad True$$

If this was not true, reinforcement would need to be added to the cylindrical or conical shell using a thick insert plate or reinforcing ring. Any additional area of reinforcement which is required shall be situated within a distance of $\sqrt{R_s t_s}$ from the junction, and the centroid of the added area shall be within a distance of $0.25\sqrt{R_s t_s}$ from the junction.

Appendix 1-8(c)(3), when the cone-to-cylinder of knuckle-to-cylinder juncture is a line of support, the moment of inertia for a stiffening ring at the small end shall be determined by the following procedure.

a) STEP 1 – Assuming that the shell has been designed and D_s, L_s, and t are known, select a member to be used for the stiffening ring and determine the cross-sectional area A_{TS}.

$$A_{TS}=\frac{L_s t_s}{2}+\frac{L_c t_c}{2}+A_s=\frac{636.0(1.0)}{2}+\frac{83.8196(1.8125)}{2}+0.0=393.9615\ in^2$$

where,

$$L_s=636.0\ in$$

$$L_c=\sqrt{L^2+\left(R_L-R_s\right)^2}=\sqrt{78.0^2+\left(76.8125-46.125\right)^2}=83.8196\ in$$

$$A_s=0.0\ in^2\qquad Assume\ no\ stiffening\ ring\ area$$

Calculate factor B using the following formula. If F_s is a negative number, the design shall be in accordance with U-2(g).

$$B = \frac{3}{4}\left(\frac{F_s D_s}{A_{TS}}\right) = \frac{3}{4}\left(\frac{5677.1577(92.25)}{393.9615}\right) = 997.0222 \ psi$$

where,

$$F_s = PN + f_2 \tan[\alpha] = 14.7(362.3133) + (912.9963)\cdot\tan[21.0375] = 5677.1577 \ \frac{lbs}{in}$$

$$f_2 = |-912.9963| \ \frac{lbs}{in \ of \ circ}$$

and,

$$N = \frac{R_s \tan[\alpha]}{2} + \frac{L_s}{2} + \frac{R_L^2 - R_s^2}{6R_s \tan[\alpha]}$$

$$N = \frac{(46.125)\cdot\tan[21.0375]}{2} + \frac{636.0}{2} + \frac{(76.8125)^2 - (46.125)^2}{6(46.125)\cdot\tan[21.0375]} = 362.3133 \ in$$

b) STEP 2 – Enter the right-hand side of the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration at the value of B determined by STEP 1. If different materials are used for the shell and stiffening ring, use the material chart resulting in the larger value of A in STEP 4.

Per Section II Part D, Table 1A, a material specification of $SA-516,\ Grade\ 70,\ Normalized$ is assigned an External Pressure Chart No. CS-2.

c) STEP 3 – Move horizontally to the left to the material/temperature line for the design metal temperature. For values of B falling below the left end of the material/temperature line, see STEP 5.

d) STEP 4 – Move vertically to the bottom of the chart and read the value of A.

This step is not required as the value of B falls below the left end of the material/temperature line.

e) STEP 5 – For values of B falling below the left end of the material/temperature line for the design temperature, the value of A can be calculated using the following:

$$A = \frac{2B}{E_x} = \frac{2(997.0222)}{28.3E+06} = 0.00007$$

where,

$$E_x = \min[E_c, E_s, E_r], \ that \ is \ the \ min \ of \ the \ cone, \ shell, \ or \ stiffening \ ring$$

f) STEP 6 – Compute the value of the required moment of inertia from the formulas for I_s or I'_s. For the circumferential stiffening ring only,

$$I_s = \frac{AD_s^2 A_{TS}}{14.0} = \frac{0.00007(92.25)^2(393.9615)}{14.0} = 16.7632 \ in^4$$

For the shell-cone or ring-shell-cone section,

$$I_s' = \frac{AD_s^2 A_{TS}}{10.9} = \frac{0.00007(92.25)^2(393.9615)}{10.9} = 21.5307\ in^4$$

g) STEP 7 – Determine the available moment of inertia of the ring only, I, or the shell-cone or ring-shell-cone, I'.

h) STEP 8 – When the ring only is used,

$$I \geq I_s$$

And when the shell-cone- or ring-shell-cone is used,

$$I' \geq I_s'$$

If the equation is not satisfied, a new section with a larger moment of inertia must be selected, and the calculation shall be done again until the equation is met. The requirements of UG-29(b), (c), (d), (e), and (f) and UG-30 are to be met in attaching stiffening rings to the shell.

VIII-1 does not provide a procedure to calculate the available moment of inertia of the shell-cone or ring-shell-cone junction. The designer must consider the following options.

a) Size a structural member to satisfy the requirement of $I \geq I_s$.

b) Size a structural member to be used in conjunction with the available moment of inertia of the cone and cylinder to satisfy the requirement of $I' \geq I'_s$.

c) The cost of material, fabrication, welding, inspection, and engineering.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraphs 4.4.13 and 4.3.11.

The design rules in VIII-2, paragraph 4.3.11 shall be satisfied. In these calculations, a negative value of pressure shall be used in all applicable equations.

Per VIII-2, paragraph 4.3.11.3, the length of the conical shell, measured parallel to the surface of the cone, shall be equal to or greater than the following:

$$\{L_C = 78.0\} \geq \left\{ \begin{array}{l} 2.0\sqrt{\dfrac{R_L t_C}{\cos[\alpha]}} + 1.4\sqrt{\dfrac{R_S t_C}{\cos[\alpha]}} \\ 2.0\sqrt{\dfrac{75.125(1.8125)}{\cos[21.0375]}} + 1.4\sqrt{\dfrac{45.125(1.8125)}{\cos[21.0375]}} = 37.2624\ in \end{array} \right\} True$$

Evaluate the Large End cylinder-to-cone junction per VIII-2, paragraph 4.3.11.4.

a) STEP 1 – Compute the large end cylinder thickness, t_L, using VIII-2, paragraph 4.3.3., (as specified in design conditions)

$$t_L = 1.6875\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the large end using VIII-2, paragraph 4.3.4., (as specified in design conditions)

$$\alpha = 21.0375\ deg$$

$$t_C = 1.8125\ in$$

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with VIII-2, Part 5. In the calculations if $0\ deg < \alpha \leq 10\ deg$, then use $\alpha = 10\ deg$.

$$20 \leq \left\{ \frac{R_L}{t_L} = \frac{75.125}{1.6875} = 44.5185 \right\} \leq 500 \qquad True$$

$$1 \leq \left\{ \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741 \right\} \leq 2 \qquad True$$

$$\left\{ \alpha = 21.0375\ deg \right\} \leq 60\ deg \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force, F_L. Calculate the equivalent line load, X_L, using the specified net section axial force, F_L, and bending moment, M_L.

$$X_L = \frac{F_L}{2\pi R_L} \pm \frac{M_L}{\pi R_L^2} = \begin{Bmatrix} \dfrac{-99167}{2\pi(75.125)} + \dfrac{5.406E+06}{\pi(75.125)^2} = 94.8111\ \dfrac{lbs}{in} \\ \dfrac{-99167}{2\pi(75.125)} - \dfrac{5.406E+06}{\pi(75.125)^2} = -514.9886\ \dfrac{lbs}{in} \end{Bmatrix}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per VIII-2, Table 4.3.3 and VIII-2, Table 4.3.4, respectively. For calculated values of n other than those presented in VIII-2, Table 4.3.3 and Table 4.3.4, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741$$

$$H = \sqrt{\frac{R_L}{t_L}} = \sqrt{\frac{75.125}{1.6875}} = 6.6722$$

$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in VIII-2, Table 4.3.3 and Table 4.3.4 is required. The results of the interpolation are summarized with the following values for C_i (see VIII-2, paragraph 4.3.11.4 and STEP 5 of E4.3.7).

For the applied pressure case both M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\begin{bmatrix} C_1 + C_2\ln[H] + C_3\ln[B] + C_4\left(\ln[H]\right)^2 + C_5\left(\ln[B]\right)^2 + \\ C_6\ln[H]\ln[B] + C_7\left(\ln[H]\right)^3 + C_8\left(\ln[B]\right)^3 + \\ C_9\ln[H]\left(\ln[B]\right)^2 + C_{10}\left(\ln[H]\right)^2\ln[B] \end{bmatrix}$$

This results in the following (see VIII-2, paragraph 4.3.11.4 and STEP 5 of E4.3.7):

$$M_{sN} = -10.6148$$
$$Q_N = -4.0925$$

For the Equivalent Line Load case, M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\left[\frac{\begin{pmatrix} C_1 + C_3\ln\left[H^2\right] + C_5\ln[\alpha] + C_7\left(\ln\left[H^2\right]\right)^2 + \\ C_9\left(\ln[\alpha]\right)^2 + C_{11}\ln\left[H^2\right]\ln[\alpha] \end{pmatrix}}{\begin{pmatrix} 1 + C_2\ln\left[H^2\right] + C_4\ln[\alpha] + C_6\left(\ln\left[H^2\right]\right)^2 + \\ C_8\left(\ln[\alpha]\right)^2 + C_{10}\ln\left[H^2\right]\ln[\alpha] \end{pmatrix}}\right]$$

This results in the following (see VIII-2, paragraph 4.3.11.4 and STEP 5 of E4.3.7):

$$M_{sN} = -0.4912$$
$$Q_N = -0.1845$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$\begin{array}{lll} Internal\ Pressure: & M_{sN} = -10.6148, & Q_N = -4.0925 \\ Equivalent\ Line\ Load: & M_{sN} = -0.4912, & Q_N = -0.1845 \end{array}$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in VIII-2, Table 4.3.1 for the Large End Junction.

Evaluate the Cylinder at the Large End:

Stress Resultant Calculations:

$$M_{sP} = Pt_L^2 M_{sN} = -14.7(1.6875)^2(-10.6148) = 444.3413\ \frac{in-lbs}{in}$$

$$M_{sX} = X_L t_L M_{sN} = \begin{Bmatrix} 94.8111(1.6875)(-0.4912) = -78.5889\ \dfrac{in-lbs}{in} \\ -514.9886(1.6875)(-0.4912) = 426.8741\ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$M_s = M_{sP} + M_{sX} = \begin{Bmatrix} 444.3413 + (-78.5889) = 365.7524 \ \dfrac{in-lbs}{in} \\ 444.3413 + 426.8741 = 871.2154 \ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$Q_P = Pt_L Q_N = -14.7(1.6875)(-4.0925) = 101.5196 \ \frac{lbs}{in}$$

$$Q_X = X_L Q_N = \begin{Bmatrix} 94.8111(-0.1845) = -17.4926 \ \dfrac{lbs}{in} \\ -514.9886(-0.1845) = 95.0154 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$Q = Q_P + Q_X = \begin{Bmatrix} 101.5196 + (-17.4926) = 84.0270 \ \dfrac{lbs}{in} \\ 101.5196 + 95.0154 = 196.5350 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_L^2 t_L^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(75.125)^2(1.6875)^2}\right]^{0.25} = 0.1142 \ in^{-1}$$

$$N_s = \frac{PR_L}{2} + X_L = \begin{Bmatrix} \dfrac{-14.7(75.125)}{2} + 94.8111 = -457.3577 \ \dfrac{lbs}{in} \\ \dfrac{-14.7(75.125)}{2} + (-514.9886) = -1067.1574 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_\theta = PR_L + 2\beta_{cy} R_L \left(-M_s \beta_{cy} + Q\right)$$

$$N_\theta = \begin{Bmatrix} -14.7(75.125) + 2(0.1142)(75.125)\left(-(365.7524)(0.1142) + 84.0270\right) \\ -14.7(75.125) + 2(0.1142)(75.125)\left(-(871.2154)(0.1142) + 196.5350\right) \end{Bmatrix}$$

$$N_\theta = \begin{Bmatrix} -379.2502 \ \dfrac{lbs}{in} \\ 560.7660 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{pc} = 1.0$$

Stress Calculations:

Determine the axial and hoop membrane and bending stresses.

$$\sigma_{sm} = \frac{N_s}{t_L} = \begin{Bmatrix} \dfrac{-457.3577}{1.6875} = -271.0268 \ psi \\ \dfrac{-1067.1574}{1.6875} = -632.3895 \ psi \end{Bmatrix}$$

$$\sigma_{sb}=\frac{6M_s}{t_L^2K_{pc}}=\begin{Bmatrix}\dfrac{6(365.7524)}{(1.6875)^2(1.0)}=770.6388\ psi\\ \dfrac{6(871.2154)}{(1.6875)^2(1.0)}=1835.6472\ psi\end{Bmatrix}$$

$$\sigma_{\theta m}=\frac{N_\theta}{t_L}=\begin{Bmatrix}\dfrac{-379.2502}{1.6875}=-224.7409\ psi\\ \dfrac{560.7660}{1.6875}=332.3058\ psi\end{Bmatrix}$$

$$\sigma_{\theta b}=\frac{6\nu M_s}{t_L^2K_{pc}}=\begin{Bmatrix}\dfrac{6(0.3)(365.7524)}{(1.6875)^2(1.0)}=231.1916\ psi\\ \dfrac{6(0.3)(871.2154)}{(1.6875)^2(1.0)}=550.6942\ psi\end{Bmatrix}$$

Check Acceptance Criteria:

$$\begin{Bmatrix}\sigma_{sm}=-271.0268\ psi\\ \sigma_{sm}=-632.3895\ psi\end{Bmatrix}\le\{1.5S,\ not\ applicable\ due\ to\ compressive\ stress\}$$

$$\begin{Bmatrix}\sigma_{sm}+\sigma_{sb}=-271.0268+770.6388=499.6\ psi\\ \sigma_{sm}-\sigma_{sb}=-271.0268-770.6388=|-1041.7|\ psi\\ \sigma_{sm}+\sigma_{sb}=-632.3895+1835.6472=1203.3\ psi\\ \sigma_{sm}-\sigma_{sb}=-632.3895-1835.6472=|-2468.0|\ psi\end{Bmatrix}\le\{S_{PS}=60000\ psi\}\qquad True$$

$$\begin{Bmatrix}\sigma_{\theta m}=-224.7409\\ \sigma_{\theta m}=332.3058\end{Bmatrix}\le\begin{Bmatrix}1.5S,\ not\ applicable\ due\ to\ compressive\ stress\\ 1.5S=1.5(20000)=30000\ psi\end{Bmatrix}\qquad True$$

$$\begin{Bmatrix}\sigma_{\theta m}+\sigma_{\theta b}=-224.7409+231.1916=6.5\ psi\\ \sigma_{\theta m}-\sigma_{\theta b}=-224.7409-231.1916=|-455.9|\ psi\\ \sigma_{\theta m}+\sigma_{\theta b}=332.3058+550.6942=883.0\ psi\\ \sigma_{\theta m}-\sigma_{\theta b}=332.3058-550.6942=|-218.4|\ psi\end{Bmatrix}\le\{S_{PS}=60000\ psi\}\qquad True$$

Since the hoop membrane stress, $\sigma_{\theta m}$ and the axial membrane stress, σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m}\le F_{ha}$$

$$\sigma_{sm}\le F_{xa}$$

F_{ha} is evaluated using VIII-2, paragraph 4.4.5.1, but substituting F_{he} with the following equation.

$$F_{he}=\frac{0.4E_yt}{D_o}$$

F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2.b with $\lambda=0.15$.

In accordance with VIII-2, paragraph 4.4.5.1, the value of F_{ha} is calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

$$t = 1.6875\ in$$

$$L \rightarrow Not\ required,\ as\ the\ equation\ for\ F_{he}\ is\ independent\ of\ L$$

2) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{0.4E_y t}{D_o} = \frac{0.4(28.3E+06)(1.6875)}{153.625} = 124344.9959\ psi$$

3) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$F_{he} = 124344.9959\ psi$ (as determined in STEP 2 above)

ii) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{124344.9959}{28.3E+06} = 0.00439382$$

iii) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 27206.0299\ psi$$

4) STEP 4 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{27206.0299}{33600}\right) = 1.8070$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{27206.0299}{1.8070} = 15055.9103\ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ to the allowable hoop compressive membrane stress, F_{ha} per following criteria.

$$\{\sigma_{\theta m} = 224.7\ psi\} \le \{F_{ha} = 15055.9\ psi\} \qquad True$$

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

In accordance with VIII-2, paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

1) STEP 1 – Calculate the predicted elastic buckling stress, F_{xe}.

$$F_{xe} = \frac{C_x E_y t}{D_o} = \frac{0.8520(28.3E+06)(1.6875)}{153.625} = 264854.8413 \ psi$$

where,

$$\frac{D_o}{t} = \frac{153.625}{1.6875} = 91.0370$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{732.0}{\sqrt{76.8125(1.6875)}} = 64.2944$$

Since $\frac{D_o}{t} < 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \frac{D_o}{t}\right)},\ 0.9\right] = \min\left[\frac{409(1.0)}{389 + \frac{153.625}{1.6875}},\ 0.9\right] = 0.8520$$

Since $M_x \geq 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

2) STEP 2 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{xe}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 2.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$F_{xe} = 264854.8413 \ psi$ (as determined in STEP 2 above)

ii) STEP 2.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{xe}}{E} = \frac{264854.8413}{28.3E+06} = 0.00935883$$

iii) STEP 2.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 31046.7970 \ psi$$

3) STEP 3 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{31046.7970}{33600}\right) = 1.7223$$

4) STEP 4 – Calculate the allowable axial compressive membrane stress as follows:

$$F_{xa} = \frac{F_{ic}}{FS} = \frac{31046.7970}{1.7223} = 18026.3584 \ psi$$

5) STEP 5 – Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria.

$$\{\sigma_{sm} = 632.4 \ psi\} \le \{F_{xa} = 18026.4 \ psi\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

Evaluate the Cone at the Large End:

Stress Resultant Calculations, as determined above.

$$M_{csP} = M_{sP} = 444.3413 \ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \begin{Bmatrix} -78.5889 \ \frac{in-lbs}{in} \\ 426.8741 \ \frac{in-lbs}{in} \end{Bmatrix}$$

$$M_{cs} = M_{csP} + M_{csX} = \begin{Bmatrix} 444.3413 + (-78.5889) = 365.7524 \ \frac{in-lbs}{in} \\ 444.3413 + 426.8741 = 871.2154 \ \frac{in-lbs}{in} \end{Bmatrix}$$

$$Q_c = Q\cos[\alpha] + N_s \sin[\alpha]$$

$$Q_c = \begin{Bmatrix} 84.0270(\cos[21.0375]) + (-457.3577)\sin[21.0375] = -85.7555 \ \frac{lbs}{in} \\ 196.5350(\cos[21.0375]) + (-1067.1574)\sin[21.0375] = -199.6519 \ \frac{lbs}{in} \end{Bmatrix}$$

$$R_C = \frac{R_L}{\cos[\alpha]} = \frac{75.125}{\cos[21.0375]} = 80.4900 \ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_C^2 t_C^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(80.4900)^2 (1.8125)^2}\right]^{0.25} = 0.1064 \ in^{-1}$$

$$N_{cs} = N_s \cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{Bmatrix} -457.3577(\cos[21.0375]) - 84.0270\sin[21.0375] = -457.0368 \dfrac{lbs}{in} \\ -1067.1574(\cos[21.0375]) - 196.5350\sin[21.0375] = -1066.5786 \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_{c\theta} = \frac{PR_L}{\cos[\alpha]} + 2\beta_{co}R_C(-M_{cs}\beta_{co} - Q_c)$$

$$N_{c\theta} = \begin{Bmatrix} \dfrac{-14.7(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)(-(365.7524)(0.1064) - (-85.7555)) \\ \dfrac{-14.7(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)(-(871.2154)(0.1064) - (-199.6519)) \end{Bmatrix}$$

$$N_{c\theta} = \begin{Bmatrix} -380.9244 \dfrac{lbs}{in} \\ 648.7441 \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{cpc} = 1.0$$

Stress Calculations:

Determine the axial and hoop membrane and bending stresses.

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \begin{Bmatrix} \dfrac{-457.0368}{1.8125} = -252.1582 \ psi \\ \dfrac{-1066.5786}{1.8125} = -588.4572 \ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(365.7524)}{(1.8125)^2(1.0)} = 668.0091 \ psi \\ \dfrac{6(871.2154)}{(1.8125)^2(1.0)} = 1591.1853 \ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \begin{Bmatrix} \dfrac{-380.9244}{1.8125} = -210.1652 \ psi \\ \dfrac{648.7441}{1.8125} = 357.9278 \ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(0.3)(365.7524)}{(1.8125)^2(1.0)} = 200.4027 \ psi \\ \dfrac{6(0.3)(871.2154)}{(1.8125)^2(1.0)} = 477.3556 \ psi \end{Bmatrix}$$

f) STEP 6 – Check Acceptance Criteria:

$$\begin{Bmatrix} \sigma_{sm} = -252.1582 \ psi \\ \sigma_{sm} = -588.4572 \ psi \end{Bmatrix} \le \{1.5S, \ not \ applicable \ due \ to \ compressive \ stress\}$$

$$\left\{\begin{array}{l} \sigma_{sm} + \sigma_{sb} = -252.1582 + 668.0091 = 415.6 \ psi \\ \sigma_{sm} - \sigma_{sb} = -252.1582 - 668.0091 = |-920.2| \ psi \\ \sigma_{sm} + \sigma_{sb} = -588.4572 + 1591.1853 = 1002.7 \ psi \\ \sigma_{sm} - \sigma_{sb} = -588.4572 - 1591.1853 = |-2179.6| \ psi \end{array}\right\} \le \{S_{PS} = 60000 \ psi\} \quad True$$

$$\begin{Bmatrix} \sigma_{\theta m} = -210.1652 \\ \sigma_{\theta m} = 357.9278 \end{Bmatrix} \le \begin{Bmatrix} 1.5S, \ not \ applicable \ due \ to \ compressive \ stress \\ 1.5S = 1.5(20000) = 30000 \ psi \end{Bmatrix} \quad True$$

$$\left\{\begin{array}{l} \sigma_{\theta m} + \sigma_{\theta b} = -210.1652 + 200.4027 = |-9.7| \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -210.1652 - 200.4027 = -410.6 \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 357.9278 + 477.3556 = 835.3 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 357.9278 - 477.3556 = |-119.4| \ psi \end{array}\right\} \le \{S_{PS} = 60000 \ psi\} \quad True$$

Since the hoop membrane stress, $\sigma_{\theta m}$ and the axial membrane stress, σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m} \le F_{ha}$$
$$\sigma_{sm} \le F_{xa}$$

F_{ha} is evaluated using VIII-2, paragraph 4.4.5.1, but substituting F_{he} with the following equation.

$$F_{he} = \frac{0.4 E_y t}{D_o}$$

where,

$$t = t_c = 1.8125 \ in$$

$$D_o = \frac{0.5(D_{cL} + D_{cs})}{\cos[\alpha]} = \frac{0.5(153.875 + 93.875)}{\cos[21.0375]} = 132.7215 \ in$$

F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2.b with $\lambda = 0.15$ and the following substitutions.

$$t = t_c = 1.8125 \ in$$

$$D_o = 132.7215 \ in$$

$$L = \frac{L_{ce}}{\cos[\alpha]} = \frac{L_c}{\cos[\alpha]} = \frac{78.0}{\cos[21.0375]} = 83.5703 \ in$$

Using the procedure shown above for the large end cylindrical shell and the above noted substitutions, the allowable compressive hoop membrane and axial membrane stresses, F_{ha} and F_{xa}, respectively, are as follows.

$$F_{ha} = 15888.9\ psi$$
$$F_{xa} = 19150.9\ psi$$

Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ and axial compressive membrane stress, σ_{sm}, to the allowable hoop compressive membrane stress, F_{ha} and axial compressive membrane stress, F_{xa} per following criteria.

$$\{\sigma_{\theta m} = 210.2\ psi\} \le \{F_{ha} = 15888.9\ psi\} \qquad True$$
$$\{\sigma_{sm} = 588.5\ psi\} \le \{F_{xa} = 19150.9\ psi\} \qquad True$$

Therefore, local buckling due to hoop and axial compressive membrane stress is not a concern.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore, the design is complete.

<u>Evaluate the Small End cylinder-to-cone junction per VIII-2, paragraph 4.3.11.5.</u>

a) STEP 1 – Compute the small end cylinder thickness, t_S, using VIII-2, paragraph 4.3.3., (as specified in design conditions).

$$t_S = 1.0\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the small end using VIII-2, paragraph 4.3.4., (as specified in design conditions)

$$\alpha = 21.0375\ deg$$
$$t_C = 1.8125\ in$$

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with VIII-2, Part 5. In the calculations, if $0\ deg < \alpha \le 10\ deg$, then use $\alpha = 10\ deg$.

$$20 \le \left\{\frac{R_S}{t_S} = \frac{45.125}{1.0} = 45.125\right\} \le 500 \qquad True$$

$$1 \le \left(\frac{t_C}{t_S} = \frac{1.8125}{1.0} = 1.8125\right) \le 2 \qquad True$$

$$\{\alpha = 21.0375\ deg\} \le 60\ deg \qquad True$$

d) STEP 4 – Determine the net section axial force, F_S, and bending moment, M_S, applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force, F_S. Calculate the equivalent line load, X_S, using the specified net section axial force, F_S, and bending moment, M_S.

$$X_S = \frac{F_S}{2\pi R_S} \pm \frac{M_S}{\pi R_S^2} = \begin{Bmatrix} \dfrac{-78104}{2\pi(45.125)} + \dfrac{4.301E+06}{\pi(45.125)^2} = 396.8629\ \dfrac{lbs}{in} \\ \dfrac{-78104}{2\pi(45.125)} - \dfrac{4.301E+06}{\pi(45.125)^2} = -947.8053\ \dfrac{lbs}{in} \end{Bmatrix}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per VIII-2, Table 4.3.5 and VIII-2, Table 4.3.6, respectively. For calculated values of n other than those presented in VIII-2, Table 4.3.5 and Table 4.3.6, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_S} = \frac{1.8125}{1.0} = 1.8125$$

$$H = \sqrt{\frac{R_S}{t_S}} = \sqrt{\frac{45.125}{1.0}} = 6.7175$$

$$B = \tan\left[\alpha\right] = \tan\left[21.0375\right] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in VIII-2, Table 4.3.5 and Table 4.3.6 is required. The results of the interpolation are summarized with the following values for C_i (see VIII-2, paragraph 4.3.11.5 and STEP 5 of E4.3.7).

For the applied pressure case M_{sN} is calculated using the following equation

$$M_{sN} = \exp\begin{bmatrix} C_1 + C_2\ln\left[H^2\right] + C_3\ln\left[\alpha\right] + C_4\left(\ln\left[H^2\right]\right)^2 + C_5\left(\ln\left[\alpha\right]\right)^2 + \\ C_6\ln\left[H^2\right]\ln\left[\alpha\right] + C_7\left(\ln\left[H^2\right]\right)^3 + C_8\left(\ln\left[\alpha\right]\right)^3 + \\ C_9\ln\left[H^2\right]\left(\ln\left[\alpha\right]\right)^2 + C_{10}\left(\ln\left[H^2\right]\right)^2\ln\left[\alpha\right] \end{bmatrix}$$

This results in the following (see VIII-2, paragraph 4.3.11.5 and STEP 5 of E4.3.7).

$$M_{sN} = 9.2135$$

For the applied pressure case Q_N is calculated using the following equation:

$$Q_N = \left(\frac{C_1 + C_3H^2 + C_5\alpha + C_7H^4 + C_9\alpha^2 + C_{11}H^2\alpha}{1 + C_2H^2 + C_4\alpha + C_6H^4 + C_8\alpha^2 + C_{10}H^2\alpha}\right)$$

This results in the following (see VIII-2, paragraph 4.3.11.5 and STEP 5 of E4.3.7).

$$Q_N = -2.7333$$

For the Equivalent Line Load case, M_{sN} is calculated using the following equation:

$$M_{sN} = \left(\frac{C_1 + C_3H + C_5B + C_7H^2 + C_9B^2 + C_{11}HB}{1 + C_2H + C_4B + C_6H^2 + C_8B^2 + C_{10}HB}\right)$$

This results in the following (see VIII-2, paragraph 4.3.11.5 and STEP 5 of E4.3.7).

$$M_{sN} = 0.4828$$

For the Equivalent Line Load case, Q_N is calculated using the following equation:

$$Q_N = \begin{pmatrix} C_1 + C_2\ln\left[H\right] + C_3\ln\left[B\right] + C_4\left(\ln\left[H\right]\right)^2 + C_5\left(\ln\left[B\right]\right)^2 + C_6\ln\left[H\right]\ln\left[B\right] + \\ C_7\left(\ln\left[H\right]\right)^3 + C_8\left(\ln\left[B\right]\right)^3 + C_9\ln\left[H\right]\left(\ln\left[B\right]\right)^2 + C_{10}\left(\ln\left[H\right]\right)^2\ln\left[B\right] \end{pmatrix}$$

This results in the following (see VIII-2, paragraph 4.3.11.5 and STEP 5 of E4.3.7).

$$Q_N = -0.1613$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$\begin{array}{lll} Internal\ Pressure: & M_{sN} = 9.2135, & Q_N = -2.7333 \\ Equivalent\ Line\ Load: & M_{sN} = 0.4828, & Q_N = -0.1613 \end{array}$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in VIII-2, Table 4.3.2. for the Small End Junction.

Evaluate the Cylinder at the Small End.

Stress Resultant Calculations.

$$M_{sP} = Pt_S^2 M_{sN} = -14.7(1.0)^2(9.2135) = -135.4385\ \frac{in-lbs}{in}$$

$$M_{sX} = X_S t_S M_{sN} = \begin{Bmatrix} 396.8629(1.0)(0.4828) = 191.6054\ \dfrac{in-lbs}{in} \\ -947.8053(1.0)(0.4828) = -457.6004\ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$M_s = M_{sP} + M_{sX} = \begin{Bmatrix} -135.4385 + (191.6054) = 56.1669\ \dfrac{in-lbs}{in} \\ -135.4385 + (-457.6004) = -593.0389\ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$Q_P = Pt_S Q_N = -14.7(1.0)(-2.7333) = 40.1795\ \frac{lbs}{in}$$

$$Q_X = X_S Q_N = \begin{Bmatrix} 396.8629(-0.1613) = -64.0140\ \dfrac{lbs}{in} \\ -947.8053(-0.1613) = 152.8810\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$Q = Q_P + Q_X = \begin{Bmatrix} 40.1795 + (-64.0140) = -23.8345\ \dfrac{lbs}{in} \\ 40.1795 + 152.8810 = 193.0605\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_S^2 t_S^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(45.1250)^2(1.000)^2}\right]^{0.25} = 0.1914\ in^{-1}$$

$$N_s = \frac{PR_S}{2} + X_S = \begin{Bmatrix} \dfrac{-14.7(45.125)}{2} + 396.8629 = 65.1942 \dfrac{lbs}{in} \\ \dfrac{-14.7(45.125)}{2} + (-947.8053) = -1279.4741 \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_\theta = PR_S + 2\beta_{cy} R_S \left(-M_s \beta_{cy} - Q\right)$$

$$N_\theta = \begin{Bmatrix} -14.7(45.125) + 2(0.1914)(45.125)(-(56.1669)(0.1914) - (-23.8345)) \\ -14.7(45.125) + 2(0.1914)(45.125)(-(-593.0389)(0.1914) - 193.0605) \end{Bmatrix}$$

$$N_\theta = \begin{Bmatrix} -437.3238 \dfrac{lbs}{in} \\ -2037.5216 \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{pc} = 1.0$$

Stress Calculations:

Determine the axial and hoop membrane and bending stresses:

$$\sigma_{sm} = \frac{N_s}{t_S} = \begin{Bmatrix} \dfrac{65.1942}{1.0} = 65.1942 \ psi \\ \dfrac{-1279.4741}{1.0} = -1279.4741 \ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(56.1669)}{(1.0)^2(1.0)} = 337.0014 \ psi \\ \dfrac{6(-593.0389)}{(1.0)^2(1.0)} = -3558.2334 \ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_S} = \begin{Bmatrix} \dfrac{-437.3238}{1.0} = -437.3238 \ psi \\ \dfrac{-2037.5216}{1.0} = -2037.5216 \ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(0.3)(56.1669)}{(1.0)^2(1.0)} = 101.1004 \ psi \\ \dfrac{6(0.3)(-593.0389)}{(1.0)^2(1.0)} = -1067.4700 \ psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\left\{\begin{matrix} \sigma_{sm} = 65.1942 \ psi \\ \sigma_{sm} = -1279.4741 \ psi \end{matrix}\right\} \leq \left\{\begin{matrix} 1.5S = 1.5(20000) = 30000 \ psi \\ 1.5S, \ not \ applicable \ due \ to \ compressive \ stress \end{matrix}\right\} \qquad True$$

$$\left\{\begin{matrix} \sigma_{sm} + \sigma_{sb} = 65.1942 + 337.0014 = 402.2 \ psi \\ \sigma_{sm} - \sigma_{sb} = 65.1942 - 337.0014 = |-271.8| \ psi \\ \sigma_{sm} + \sigma_{sb} = -1279.4741 + (-3558.2334) = |-4837.7| \ psi \\ \sigma_{sm} - \sigma_{sb} = -1279.4741 - (-3558.2334) = 2278.8 \ psi \end{matrix}\right\} \leq \{S_{PS} = 60000 \ psi\} \qquad True$$

$$\left\{\begin{matrix} \sigma_{\theta m} = -437.3238 \\ \sigma_{\theta m} = -2037.5216 \end{matrix}\right\} \leq \{1.5S, \ not \ applicable \ due \ to \ compressive \ stress\}$$

$$\left\{\begin{matrix} \sigma_{\theta m} + \sigma_{\theta b} = -437.3238 + 101.1004 = |-336.2| \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -437.3238 - 101.1004 = |-538.4| \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = -2037.5216 + (-1067.4700) = |-3105.0| \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -2037.5216 - (-1067.4700) = |-970.1| \ psi \end{matrix}\right\} \leq \{S_{PS} = 60000 \ psi\} \qquad True$$

Since the hoop membrane stress $\sigma_{\theta m}$ and axial membrane stress σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m} \leq F_{ha}$$

$$\sigma_{sm} \leq F_{xa}$$

F_{ha} is evaluated using VIII-2, paragraph 4.4.5.1, but substituting F_{he} with the following equation.

$$F_{he} = \frac{0.4 E_y t}{D_o}$$

F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2.b with $\lambda = 0.15$.

In accordance with VIII-2, paragraph 4.4.5.1, the value of F_{ha} is calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

$$t = 1.0 \ in$$

$$L \rightarrow Not \ required, \ as \ the \ equation \ for \ F_{he} \ is \ independent \ of \ L$$

2) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{0.4 E_y t}{D_o} = \frac{0.4(28.3E+06)(1.0)}{92.25} = 122710.0271 \ psi$$

3) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$$F_{he} = 122710.0271\ psi$$ (as determined in STEP 2 above)

ii) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{122710.0271}{28.3E+06} = 0.00433604$$

iii) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 27137.9709\ psi$$

4) STEP 4 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{27137.9709}{33600}\right) = 1.8085$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{27137.9709}{1.8085} = 15005.8\ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ to the allowable hoop compressive membrane stress, F_{ha} per following criteria.

$$\{\sigma_{\theta m} = 2037.5\ psi\} \leq \{F_{ha} = 15005.8\ psi\} \qquad True$$

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

In accordance with VIII-2, paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

i) STEP 6.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$$F_{xe} = \frac{C_x E_y t}{D_o} = \frac{0.8499(28.3E+06)(1.0)}{92.25} = 260728.1301\ psi$$

where,

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{46.125(1.0)}} = 93.6459$$

Since $D_o/t < 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\overline{c}}{\left(389+\frac{D_o}{t}\right)},\ 0.9\right] = \min\left[\frac{409(1.0)}{389+\frac{92.25}{1.0}},\ 0.9\right] = 0.8499$$

Since $M_x \geq 15$, calculate $\overline{c}$ as follows:

$$\overline{c} = 1.0$$

ii) STEP 6.2 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{xe}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

1. STEP 6.2.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$$F_{xe} = 260728.1301\ psi$$ (as determined in STEP 2 above)

2. STEP 6.2.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{xe}}{E} = \frac{260728.1301}{28.3E+06} = 0.00921301$$

3. STEP 6.2.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 30967.6147\ psi$$

iii) STEP 6.3 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{30967.6147}{33600}\right) = 1.7241$$

iv) STEP 6.4 – Calculate the allowable axial compressive membrane stress as follows:

$$F_{xa} = \frac{F_{ic}}{FS} = \frac{30967.6147}{1.7241} = 17961.6117\ psi$$

v) STEP 6.5 – Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria.

$$\{\sigma_{sm} = 632.4\ psi\} \leq \{F_{xa} = 17961.6\ psi\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

Evaluate the Cone at the Small End.

Stress Resultant Calculations as determined above.

$$M_{csP} = M_{sP} = -135.4385 \ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \begin{cases} 191.6054 \ \dfrac{in-lbs}{in} \\ -457.6004 \ \dfrac{in-lbs}{in} \end{cases}$$

$$M_{cs} = M_{csP} + M_{csX} = \begin{cases} -135.4385 + 191.6054 = 56.1669 \ \dfrac{in-lbs}{in} \\ -135.4385 + (-457.6004) = -593.0389 \ \dfrac{in-lbs}{in} \end{cases}$$

$$Q_c = Q\cos[\alpha] + N_s \sin[\alpha]$$

$$Q_c = \begin{cases} (-23.8345)\cos[21.0375] + 65.1942\sin[21.0375] = 1.1575 \ \dfrac{lbs}{in} \\ 193.0605\cos[21.0375] + (-1279.4741)\sin[21.0375] = -279.1120 \ \dfrac{lbs}{in} \end{cases}$$

$$R_c = \frac{R_s}{\cos[\alpha]} = \frac{45.1250}{\cos[21.0375]} = 48.3476 \ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_c^{\,2} t_c^{\,2}}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(48.3476)^2(1.8125)^2}\right]^{0.25} = 0.1373 \ in^{-1}$$

$$N_{cs} = N_s\cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{cases} 65.1942\cos[21.0375] - (-23.8345)\sin[21.0375] = 69.4048 \ \dfrac{lbs}{in} \\ (-1279.4741)\cos[21.0375] - 193.0605\sin[21.0375] = -1263.4963 \ \dfrac{lbs}{in} \end{cases}$$

$$N_{c\theta} = \frac{PR_s}{\cos[\alpha]} + 2\beta_{co}R_c\left(-M_{cs}\beta_{co} + Q_c\right)$$

$$N_{c\theta} = \begin{cases} \dfrac{-14.7(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(56.1669)(0.1373) + 1.1575\right) \\ \dfrac{-14.7(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(-593.0389)(0.1373) + (-279.1120)\right) \end{cases}$$

$$N_{c\theta} = \begin{cases} -797.7248 \dfrac{lbs}{in} \\ -3335.2619 \dfrac{lbs}{in} \end{cases}$$

$$K_{cpc} = 1.0$$

Stress Calculations:

Determine the axial and hoop membrane and bending stresses:

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \left\{ \begin{array}{l} \dfrac{69.4048}{1.8125} = 38.2923 \;\; psi \\ \dfrac{-1263.4963}{1.8125} = -697.1014 \;\; psi \end{array} \right\}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \left\{ \begin{array}{l} \dfrac{6(56.1669)}{(1.8125)^2(1.0)} = 102.5831 \;\; psi \\ \dfrac{6(-593.0389)}{(1.8125)^2(1.0)} = -1083.1246 \;\; psi \end{array} \right\}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \left\{ \begin{array}{l} \dfrac{-797.7248}{1.8125} = -440.1240 \;\; psi \\ \dfrac{-3335.2619}{1.8125} = -1840.1445 \;\; psi \end{array} \right\}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \left\{ \begin{array}{l} \dfrac{6(0.3)(56.1669)}{(1.8125)^2(1.0)} = 30.7749 \;\; psi \\ \dfrac{6(0.3)(-593.0389)}{(1.8125)^2(1.0)} = -324.9374 \;\; psi \end{array} \right\}$$

Check Acceptable Criteria:

$$\left\{\begin{array}{l}\sigma_{sm}=38.2923\ psi\\ \sigma_{sm}=-697.1014\ psi\end{array}\right\}\le\left\{\begin{array}{l}1.5S=1.5(20000)=30000\ psi\\ 1.5S,\ not\ applicable\ due\ to\ compressive\ stress\end{array}\right\}\qquad True$$

$$\left\{\begin{array}{l}\sigma_{sm}+\sigma_{sb}=38.2923+102.5831=140.9\ psi\\ \sigma_{sm}-\sigma_{sb}=38.2923-102.5831=|-64.3|\ psi\\ \sigma_{sm}+\sigma_{sb}=-697.1014+(-1083.1246)=|-1780.2|\ psi\\ \sigma_{sm}-\sigma_{sb}=-697.1014-(-1083.1246)=386.0\ psi\end{array}\right\}\le\{S_{PS}=60000\ psi\}\qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m}=-440.1240\\ \sigma_{\theta m}=-1840.1445\end{array}\right\}\le\{1.5S,\ not\ applicable\ due\ to\ compressive\ stress\}$$

$$\left\{\begin{array}{l}\sigma_{\theta m}+\sigma_{\theta b}=-440.1240+30.7749=|-409.3|\ psi\\ \sigma_{\theta m}-\sigma_{\theta b}=-440.1240-30.7749=|-470.9|\ psi\\ \sigma_{\theta m}+\sigma_{\theta b}=-1840.1445+(-324.9374)=|-2164.9|\ psi\\ \sigma_{\theta m}-\sigma_{\theta b}=-1840.1445-(-324.9374)=|-1515.1|\ psi\end{array}\right\}\le\{S_{PS}=60000\ psi\}\qquad True$$

Since the hoop membrane stress, $\sigma_{\theta m}$ and the axial membrane stress, σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m}\le F_{ha}$$
$$\sigma_{sm}\le F_{xa}$$

F_{ha} is evaluated using VIII-2, paragraph 4.4.5.1, but substituting F_{he} with the following equation.

$$F_{he}=\frac{0.4E_y t}{D_o}$$

where,

$$t=t_c=1.8125\ in$$

$$D_o=\frac{0.5(D_{cL}+D_{cs})}{\cos[\alpha]}=\frac{0.5(153.875+93.875)}{\cos[21.0375]}=132.7215\ in$$

F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2.b with $\lambda=0.15$ and the following substitutions.

$$t=t_c=1.8125\ in$$

$$D_o=132.7215\ in$$

$$L=\frac{L_{ce}}{\cos[\alpha]}=\frac{L_c}{\cos[\alpha]}=\frac{78.0}{\cos[21.0375]}=83.5703\ in$$

Using the procedure shown above for the large end cylindrical shell and the above noted substitutions, the allowable compressive hoop membrane and axial membrane stresses, F_{ha} and F_{xa}, respectively, are as follows.

$$F_{ha} = 15888.9 \ psi$$
$$F_{xa} = 19150.9 \ psi$$

Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ and axial compressive membrane stress, σ_{sm}, to the allowable hoop compressive membrane stress, F_{ha} and axial compressive membrane stress, F_{xa} per following criteria.

$$\left\{\sigma_{\theta m} = 697.1 \ psi\right\} \le \left\{F_{ha} = 15888.9 \ psi\right\} \qquad True$$
$$\left\{\sigma_{sm} = 1840.1 \ psi\right\} \le \left\{F_{xa} = 19150.9 \ psi\right\} \qquad True$$

Therefore, local buckling due to hoop and axial compressive membrane stress is not a concern.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore, the design is complete.

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4.4.8 Example E4.4.8 – Conical Transitions with a Knuckle

Determine if the proposed design for the large end of a cylinder-to-cone junction with a knuckle is adequately designed considering the following design conditions and applied forces and moments.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $-14.7\ psig @ 300°F$
- Inside Diameter (Large End) = $120.0\ in$
- Large End Thickness = $1.0\ in$
- Inside Diameter (Small End) = $33.0\ in$
- Small End Thickness = $1.0\ in$
- Knuckle Radius = $10.0\ in$
- Cone Thickness = $1.0\ in$
- Knuckle Thickness = $1.0\ in$
- Length of Conical Section = $73.0\ in$
- Unsupported Length of Large Cylinder = $240.0\ in$
- Unsupported Length of Small Cylinder = $360.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $20000\ psi$
- Yield Strength = $33600\ psi$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle = $30.0\ deg$
- Axial Force (Large End) = $-10000\ lbs$
- Net Section Bending Moment (Large End) = $2.0E+06\ in-lbs$

Section VIII, Division 1 Solution

VIII-1 does not provide design rules for knuckles or flares under pressure on the convex side. However, in accordance with paragraph UG-33(f) for a cone-to-cylinder junction with a knuckle that is a line of support, the moment of inertia calculation of Appendix 1-8 must be performed. The reinforcement calculation, however, is not required. The moment of inertia calculation can be performed either by considering the presence of the knuckle or by assuming the knuckle is not present whereby the cone is assumed to intersect the adjacent cylinder.

For this example, it is assumed that the cone-to-cylinder junction with a knuckle is a line of support, and the knuckle is not present, and the cone is assumed to intersect the adjacent cylinder.

Determine outside dimensions.

$$D_L = 120.0 + 2(Uncorroded\ Thickness) = 122.0\ in$$

$$R_L = 60.0 + Uncorroded\ Thickness = 60.0 + 1.0 = 61.0\ in$$

$$D_s = 33.0 + 2(Uncorroded\ Thickness) = 35.0\ in$$

$$R_s = 16.5 + Uncorroded\ Thickness = 16.5 + 1.0 = 17.5\ in$$

Evaluate per Mandatory Appendix 1-8. The moment of inertia for a stiffening ring at the large end shall be determined by the following procedure.

a) STEP 1 – Assuming that the shell has been designed and D_L, L_L, and t are known, select a member to be used for the stiffening ring and determine the cross-sectional area A_{TL}.

$$A_{TL} = \frac{L_L t_s}{2} + \frac{L_c t_c}{2} + A_s = \frac{240.0(1.0)}{2} + \frac{84.9779(1.0)}{2} + 0.0 = 162.4890\ in^2$$

where,

$$L_L = 240.0\ in$$

$$L_c = \sqrt{L^2 + (R_L - R_s)^2} = \sqrt{73.0^2 + (61.0 - 17.5)^2} = 84.9779\ in$$

$$A_s = 0.0\ in^2 \qquad \textit{Assume no stiffening ring area}$$

Calculate factor B using the following formula. If F_L is a negative number, the design shall be in accordance with U-2(g).

$$B = \frac{3}{4}\left(\frac{F_L D_L}{A_{TL}}\right) = \left\{\begin{array}{l} \dfrac{3}{4}\left(\dfrac{2063.9601(122.0)}{162.4890}\right) = 1162.2470\ psi \\ \dfrac{3}{4}\left(\dfrac{1866.4043(122.0)}{162.4890}\right) = 1051.0003\ psi \end{array}\right\} \textit{Use maximum value}$$

where,

$$F_L = PM + f_1 \tan[\alpha] = \left\{\begin{array}{l} 14.7(134.7106) + (144.9974)\cdot\tan[30] = 2063.9601\ \dfrac{lbs}{in} \\ 14.7(134.7106) + (-197.1793)\cdot\tan[30] = 1866.4043\ \dfrac{lbs}{in} \end{array}\right\}$$

and,

$$f_1 = \frac{F_L}{2\pi R_L} \pm \frac{M_t}{\pi R_L^2} = \left\{\begin{array}{l} \dfrac{-10000}{2\pi(61.0)} + \dfrac{2.0E+06}{\pi(61.0)^2} = 144.9974\ \dfrac{lbs}{in\ of\ cir} \\ \dfrac{-10000}{2\pi(61.0)} - \dfrac{2.0E+06}{\pi(61.0)^2} = -197.1793\ \dfrac{lbs}{in\ of\ cir} \end{array}\right\}$$

and,

$$M = \frac{-R_L \tan[\alpha]}{2} + \frac{L_L}{2} + \frac{R_L^2 - R_s^2}{3R_L \tan[\alpha]}$$

$$M = \frac{-(61.0)\cdot\tan[30]}{2} + \frac{240.0}{2} + \frac{(61.0)^2 - (17.5)^2}{3(61.0)\cdot\tan[30]} = 134.7106\ in$$

b) STEP 2 – Enter the right-hand side of the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration at the value of B determined by STEP 1. If different materials are used for the shell and stiffening ring, use the material chart resulting in the larger value of A in STEP 4.

Per Section II Part D, Table 1A, a material specification of $SA-516-70N$ is assigned an External Pressure Chart No. CS-2.

c) STEP 3 – Move horizontally to the left to the material/temperature line for the design metal temperature. For values of B falling below the left end of the material/temperature line, see STEP 5.

d) STEP 4 – Move vertically to the bottom of the chart and read the value of A.

This step is not required as the value of B falls below the left end of the material/temperature line.

e) STEP 5 – For values of B falling below the left end of the material/temperature line for the design temperature, the value of A can be calculated using the following:

$$A = \frac{2B}{E_x} = \frac{2(1162.2470)}{28.3E+06} = 0.00008$$

where,

$$E_x = \min\left[E_c, E_s, E_r\right], \; \left(min\ of\ the\ cone,\ shell,\ or\ stiffening\ ring\right)$$

f) STEP 6 – Compute the value of the required moment of inertia from the formulas for I_s or I'_s. For the circumferential stiffening ring only,

$$I_s = \frac{AD_L^2 A_{TL}}{14.0} = \frac{0.00008(122.0)^2(162.4890)}{14.0} = 13.8199\ in^4$$

For the shell-cone or ring-shell-cone section,

$$I_s^{'} = \frac{AD_L^2 A_{TL}}{10.9} = \frac{0.00008(122.0)^2(162.4890)}{10.9} = 17.7504\ in^4$$

g) STEP 7 – Determine the available moment of inertia of the ring only, I, or the shell-cone or ring-shell-cone, I'.

h) STEP 8 – When the ring only is used,

$$I \geq I_s$$

and when the shell-cone- or ring-shell-cone is used,

$$I^{'} \geq I_s^{'}$$

If the equation is not satisfied, a new section with a larger moment of inertia must be selected, and the calculation shall be done again until the equation is met. The requirements of UG-29(b), (c), (d), (e), and (f) and UG-30 are to be met in attaching stiffening rings to the shell.

VIII-1 does not provide a procedure to calculate the available moment of inertia of the shell-cone or ring-shell-cone junction. The designer must consider the following options.

a) Size a structural member to satisfy the requirement of $I \geq I_s$.

b) Size a structural member to be used in conjunction with the available moment of inertia of the cone and cylinder to satisfy the requirement of $I' \geq I'_s$.

c) The cost of material, fabrication, welding, inspection, and engineering.

<u>Section VIII, Division 2 Solution with VIII-1 Allowable Stresses</u>

Evaluate per VIII-2, paragraphs 4.4.14 and 4.3.12.

The design rules in VIII-2, paragraph 4.3.12 shall be satisfied. In these calculations, a negative value of pressure shall be used in all applicable equations.

a) STEP 1 – Compute the large end cylinder thickness, t_L, using VIII-2, paragraph 4.4.5, (as specified in design conditions)

$$t_L = 1.0\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the large end using VIII-2, paragraph 4.4.5, (as specified in design conditions).

$$\alpha = 30\ deg$$

$$t_C = 1.0\ in$$

c) STEP 3 – Proportion the transition geometry by assuming a value for the knuckle radius, r_k, and knuckle thickness, t_k, such that the following equations are satisfied. If all these equations cannot be satisfied, the cylinder-to-cone junction shall be designed in accordance with VIII-2, Part 5.

$$\{t_k = 1.0\ in\} \geq \{t_L = 1.0\ in\} \qquad True$$

$$\{r_k = 10.0\ in\} > \{3t_k = 3.0\ in\} \qquad True$$

$$\left\{\frac{r_k}{R_L} = \frac{10.0}{60.0} = 0.1667\right\} > \{0.03\} \qquad True$$

$$\{\alpha = 30\ deg\} \leq \{60\ deg\} \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition at the location of the knuckle. The thrust load due to pressure shall not be included as part of the axial force, F_L.

$$F_L = -10000\ lbs$$

$$M_L = 2.0E+06\ in-lbs$$

e) STEP 5 – Compute the stresses in the knuckle at the junction using the equations in VIII-2, Table 4.3.7.

Determine if the knuckle is considered to be compact or non-compact.

$$\alpha r_k < 2K_m\left(\left\{R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5} + r_k\right\}t_k\right)^{0.5}$$

$$\{0.5236(10.0)\} < \left\{2(0.7)\left(\left\{50.0\left((0.5236)^{-1}\cdot\tan[0.5236]\right)^{0.5} + 10\right\}1\right)^{0.5}\right\}$$

$$\{5.2360\ in\} < \{11.0683\ in\} \qquad True$$

where,

$$K_m = 0.7$$

$$\alpha = \frac{30.0}{180}\pi = 0.5236\ rad$$

$$R_k = R_L - r_k = 60.0 - 10.0 = 50.0\ in$$

Therefore, analyze the knuckle junction as a compact knuckle.

Stress Calculations:

Determine the hoop and axial membrane stresses at the knuckle:

$$\sigma_{\theta m} = \frac{PK_m\left(R_L\sqrt{R_L t_L} + L_k\sqrt{L_k t_C}\right) + \alpha\left(PL_{1k}r_k - 0.5P_e L_{1k}^2\right)}{K_m\left(t_L\sqrt{R_L t_L} + t_C\sqrt{L_k t_C}\right)\alpha t_k r_k}$$

$$\sigma_{sm} = \frac{P_e L_{1k}}{2t_k}$$

where,

$$L_{1k} = R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5} + r_k = 50.0\left((0.5236)^{-1}\tan[0.5236]\right)^{0.5} + 10.0 = 62.5038\ in$$

$$L_k = \frac{R_k}{\cos[\alpha]} + r_k = \frac{50.0}{\cos[0.5236]} + 10.0 = 67.7351\ in$$

$$P_e = P + \frac{F_L}{\pi L_{1k}^2\cos^2\left[\dfrac{\alpha}{2}\right]} \pm \frac{2M_L}{\pi L_{1k}^3\cos^3\left[\dfrac{\alpha}{2}\right]}$$

$$P_e = \begin{Bmatrix} -14.7 + \dfrac{-10000.0}{\pi(62.5038)^2\cdot\cos^2\left[\dfrac{0.5236}{2}\right]} + \dfrac{2(2.0E+06)}{\pi(62.5038)^3\cdot\cos^3\left[\dfrac{0.5236}{2}\right]} \\ -14.7 + \dfrac{-10000.0}{\pi(62.5038)^2\cdot\cos^2\left[\dfrac{0.5236}{2}\right]} - \dfrac{2(2.0E+06)}{\pi(62.5038)^3\cdot\cos^3\left[\dfrac{0.5236}{2}\right]} \end{Bmatrix}$$

$$P_e = \begin{Bmatrix} -9.7875\ psi \\ -21.3590\ psi \end{Bmatrix}$$

therefore,

$$\sigma_{\theta m}=\left\{\begin{array}{l}\dfrac{\left(\begin{array}{l}(-14.7)(0.7)\left(60.0\sqrt{60.0(1.0)}+67.7351\sqrt{67.7351(1.0)}\right)+\\0.5236\left((-14.7)(62.5038)(10.0)-0.5(-9.7875)(62.5038)^2\right)\end{array}\right)}{0.7\left(1.0\sqrt{60.0(1.0)}+1.0\sqrt{67.7351(1.0)}\right)+0.5236(1.0)(10.0)}=-323.9558\ psi\\ \dfrac{\left(\begin{array}{l}(-14.7)(0.7)\left(60.0\sqrt{60.0(1.0)}+67.7351\sqrt{67.7351(1.0)}\right)+\\0.5236\left((-14.7)(62.5038)(10.0)-0.5(-21.3590)(62.5038)^2\right)\end{array}\right)}{0.7\left(1.0\sqrt{60.0(1.0)}+1.0\sqrt{67.7351(1.0)}\right)+0.5236(1.0)(10.0)}=396.8501\ psi\end{array}\right\}$$

and,

$$\sigma_{sm}=\left\{\begin{array}{l}\dfrac{P_eL_{1k}}{2t_k}=\dfrac{-9.7875(62.5038)}{2(1.0)}=-305.8780\ psi\\ \dfrac{P_eL_{1k}}{2t_k}=\dfrac{-21.3590(62.5038)}{2(1.0)}=-667.5093\ psi\end{array}\right\}$$

<u>**Check Acceptable Criteria:**</u>

$$\left\{\begin{array}{l}\sigma_{\theta m}=-324.0\ psi\\ \sigma_{\theta m}=396.9\ psi\end{array}\right\}\leq\left\{\begin{array}{ll}S,\ not\ applicable\ due\ to\ compressive\ stress & \\ 20000 & True\end{array}\right\}$$

$$\left\{\begin{array}{l}\sigma_{sm}=-305.9\ psi\\ \sigma_{sm}=-667.5\ psi\end{array}\right\}\leq\left\{S,\ not\ applicable\ due\ to\ compressive\ stress\right\}$$

Since the hoop membrane stress $\sigma_{\theta m}$ and axial membrane stress σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m}\leq F_{ha}$$

$$\sigma_{sm}\leq F_{xa}$$

F_{ha} is evaluated using VIII-2, paragraph 4.4.5.1, but substituting F_{he} with the following equation.

$$F_{he}=\frac{0.4E_yt}{D_o}$$

F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2.b with $\lambda=0.15$.

In accordance with VIII-2, paragraph 4.4.5.1, the value of F_{ha} is calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

$$t=1.0\ in$$

$$L\rightarrow Not\ required,\ as\ the\ equation\ for\ F_{he}\ is\ independent\ of\ L$$

2) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{0.4E_y t}{D_o} = \frac{0.4(28.3E+06)(1.0)}{122.0} = 92786.8853 \ psi$$

3) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$F_{he} = 92786.8853 \ psi$ (as determined in STEP 2 above)

ii) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{92786.8853}{28.3E+06} = 0.00327869$$

iii) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.2 FOR THE ITERATIVE PROCEDURE.

$F_{ic} = 25689.9738 \ psi$

4) STEP 4 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{25689.9738}{33600}\right) = 1.8404$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{25689.9738}{1.8404} = 13958.9077 \ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ to the allowable hoop compressive membrane stress, F_{ha} per following criteria.

$$\{\sigma_{\theta m} = 324.0 \ psi\} \le \{F_{ha} = 13958.9 \ psi\} \qquad True$$

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

In accordance with VIII-2, paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

1) STEP 1 – Calculate the predicted elastic buckling stress, F_{xe}.

$$F_{xe} = \frac{C_x E_y t}{D_o} = \frac{0.8004(28.3E+06)(1.0)}{122.0} = 185666.5574 \ psi$$

where,

$$\frac{D_o}{t} = \frac{122.0}{1.0} = 122.0$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{240.0}{\sqrt{61.0(1.0)}} = 30.7289$$

Since $D_o/t < 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \dfrac{D_o}{t}\right)},\ 0.9\right] = \min\left[\frac{409(1.0)}{\left(389 + \dfrac{122.0}{1.0}\right)},\ 0.9\right] = 0.8004$$

Since $M_x \geq 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

2) STEP 2 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{xe}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 2.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$F_{xe} = 185666.5574 \ psi$ (as determined in STEP 2 above)

ii) STEP 2.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{xe}}{E} = \frac{185666.5574}{28.3E+06} = 0.00656066$$

iii) STEP 2.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 29252.6889 \ psi$$

3) STEP 3 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{29252.6889}{33600}\right) = 1.7619$$

4) STEP 4 – Calculate the allowable axial compressive membrane stress as follows:

$$F_{xa} = \frac{F_{ic}}{FS} = \frac{29252.6889}{1.7619} = 16602.9224 \ \ psi$$

5) STEP 5 – Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria.

$$\left\{\sigma_{sm} = 667.5 \ \ psi\right\} \leq \left\{F_{xa} = 16602.9 \ \ psi\right\} \qquad\qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

f) STEP 6 – The stress acceptance criterion in STEP 5 is satisfied. Therefore, the design is complete.

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4.5 Shells Openings in Shells and Heads

4.5.1 Example E4.5.1 – Radial Nozzle in Cylindrical Shell

Design an integral nozzle in a cylindrical shell based on the vessel and nozzle data below. The parameters used in this design procedure are shown in Figure E4.5.1.

Vessel and Nozzle Data:

- Design Conditions = $356\ psig@300°F$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0
- Shell Material = $SA-516,\ Grade\ 70,\ Normalized$
- Shell Allowable Stress = $20000\ psi$
- Yield Strength = $33600\ psi$
- Nozzle Material = $SA-105$
- Nozzle Allowable Stress = $20000\ psi$
- Shell Inside Diameter = $150.0\ in$
- Shell Thickness = $1.8125\ in$
- Nozzle Outside Diameter = $19.0\ in$
- Nozzle Hub Outside Diameter = $25.5\ in$
- Nozzle Thickness = $1.5\ in$
- Nozzle Hub Thickness = $4.75\ in$
- External Nozzle Projection = $14.1875\ in$
- Internal Nozzle Projection = $0.0\,in$

The nozzle is inserted through the shell, i.e., set–in type nozzle, see Figure UW-16.1(g).

Establish the corroded dimensions.

Shell:

$$D = 150.0 + 2(Corrosion\ Allowance) = 150.0 + 2(0.125) = 150.25\ in$$

$$R = \frac{D}{2} = \frac{150.25}{2} = 75.125\ in$$

$$t = 1.8125 - Corrosion\ Allowance = 1.8125 - 0.125 = 1.6875\ in$$

Nozzle:

$$t_n = 1.5 - Corrosion\ Allowance = 1.5 - 0.125 = 1.375\ in$$

$$t_x = 4.75 - Corrosion\ Allowance = 4.75 - 0.125 = 4.625\ in$$

$$R_n = \frac{D_n - 2(t_n)}{2} = \frac{19.0 - 2(1.375)}{2} = 8.125\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-37.

The required thickness of the shell based on circumferential stress is given by UG-27(c)(1).

$$t_r = \frac{PR}{SE-0.6P} = \frac{356(75.125)}{20000(1.0)-0.6(356)} = 1.3517\ in$$

The required thickness of the nozzle based on circumferential stress is given by UG-27(c)(1).

$$t_{rn} = \frac{PR_n}{SE-0.6P} = \frac{356(8.125)}{20000(1.0)-0.6(356)} = 0.1462\ in$$

a) STEP 1 – Calculate the Limits of Reinforcement per UG-40.

1) Reinforcing dimensions for an integrally reinforced nozzle per Figures UG-40(e), UG-40(e-1), UG-40(e-2): See Figure E4.5.1 of this example.

$$t_x = 4.625\ in$$
$$L = 7.1875\ in$$
$$\{L = 7.1875\ in\} < \{2.5t_x = 2.5(4.625) = 11.5625\ in\}$$
$$Therefore\ use\ UG-40(e-1)\quad \left\{\begin{array}{l} t_n = 1.375\ in \\ t_e = \dfrac{4.625-1.375}{\tan[30]} = 5.6292\ in \\ D_p = 25.5\ in \end{array}\right\}$$

2) Finished opening chord length.

$$d = 2R_n = 2(8.125) = 16.25\ in$$

3) The limits of reinforcement measured parallel to the vessel wall in the corroded condition.

$$\max[d,\ R_n + t_n + t] = \max[16.25,\ \{8.125+1.375+1.6875\}] = 16.25\ in$$

4) The limits of reinforcement measured normal to the vessel wall in the corroded condition.

$$\min[2.5t,\ 2.5t_n + t_e] = \min[2.5(1.6875),\ \{2.5(1.375)+5.6292\}] = 4.2188\ in$$

b) STEP 2 – Calculate reinforcement strength parameters per UG-37.

1) Strength Reduction Factors:

$$f_{r1} = S_n/S_v = 20000/20000 = 1.0$$
$$f_{r2} = S_n/S_v = 20000/20000 = 1.0$$
$$f_{r3} = \min[S_n, S_p]/S_v = 20000/20000 = 1.0$$
$$f_{r4} = S_p/S_v = 20000/20000 = 1.0$$

2) Joint Efficiency Parameter: For a nozzle located in a solid plate, $E_1 = 1.0$.

3) Correction Factor for variation of internal pressure stresses on different planes with respect to the axis

of the vessel: For a radial nozzle in a cylindrical shell, $F = 1.0$.

c) STEP 3 – Calculate the Areas of Reinforcement, see Figure UG–37.1 (with Reinforcing Element, per Figure UG-40(e-1)).

1) Area Required, A:

$$A = dt_r F + 2t_n t_r F(1 - f_{r1})$$
$$A = 16.25(1.3517)(1.0) + 2(1.375)(1.3517)(1.0)(1 - 1.0) = 21.9651\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11} = d(E_1 t - F t_r) - 2t_n(E_1 t - F t_r)(1 - f_{r1})$$
$$A_{11} = \begin{Bmatrix} 16.25(1.0(1.6875) - 1.0(1.3517)) - \\ 2(1.375)\{(1.0(1.6875) - 1.0(1.3517))(1 - 1.0)\} \end{Bmatrix} = 5.4568\ in^2$$
$$A_{12} = 2(t + t_n)(E_1 t - F t_r) - 2t_n(E_1 t - F t_r)(1 - f_{r1})$$
$$A_{12} = \begin{Bmatrix} 2(1.6875 + 1.375)(1.0(1.6875) - 1.0(1.3517)) - \\ 2(1.375)\{(1.0(1.6875) - 1.0(1.3517))(1 - 1.0)\} \end{Bmatrix} = 2.0568\ in^2$$
$$A_1 = \max[5.4568,\ 2.0568] = 5.4568\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use smaller value:

$$A_{21} = 5(t_n - t_{rn}) f_{r2} t$$
$$A_{21} = 5(1.375 - 0.1462)(1.0)(1.6875) = 10.3680\ in^2$$
$$A_{22} = 2(t_n - t_{rn})(2.5t_n + t_e) f_{r2}$$
$$A_{22} = 2(1.375 - 0.1462)(2.5(1.375) + 5.6292)(1.0) = 22.2823\ in^2$$
$$A_2 = \min[10.3680,\ 22.2823] = 10.3680\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3. Use smaller value:

$$A_3 = \min[5tt_i f_{r2}, 5t_i t_i f_{r2}, 2ht_i f_{r2}]$$
$$A_3 = 0.0 \qquad since\ t_i = 0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.1 of this example:

$$\begin{aligned} &Outer\ Nozzle\ Fillet\ Weld\ Leg: && 0.375\ in \\ &Outer\ Element\ Fillet\ Weld\ Leg: && 0.0\ in \\ &Inner\ Nozzle\ Fillet\ Weld\ Leg: && 0.0\ in \end{aligned}$$

$$A_{41} = leg^2 f_{r3} = (0.375)^2(1.0) = 0.1406\ in^2$$
$$A_{42} = 0.0\ in^2$$
$$A_{43} = 0.0\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = \left(D_p - d - 2t_n\right)t_e f_{r4} = \left(25.5 - 16.25 - 2\left(1.375\right)\right)\left(4.2188\right)\left(1.0\right) = 27.4222\ in^2$$

Note: The thickness of the reinforcing pad, t_e, exceeds the outside vertical reinforcement zone limit. Therefore, the reinforcement area in the pad is limited to within the zone.

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + \left(A_{41} + A_{42} + A_{43}\right) + A_5$$

$$A_{avail} = 5.4568 + 10.3680 + 0.0 + \left(0.1406 + 0.0 + 0.0\right) + 27.4222 = 43.3876\ in^2$$

d) STEP 4 – Nozzle reinforcement acceptance criterion:

$$\left\{A_{avail} = 43.3876\ in^2\right\} \ge \left\{A = 21.9651\ in^2\right\} \qquad True$$

Therefore, the nozzle is adequately reinforced.

Commentary:

The procedure shown above considered the hub thickened section of the nozzle as a separate reinforcement pad, although it is integral with the nozzle. The VIII-1 code does not provide explicit rules for determination of the available area in the nozzle, A_2 for thickened integrally reinforced nozzles. However, the various nozzle sketches in Fig UW-16.1 consistently indicate the nozzle thickness, t_n is that of the upper nozzle neck section. Therefore, it is believed appropriate to remain consistent with the provided nomenclature and evaluate the hub thickened portion of the nozzle as separate reinforcement, paying close attention to the proper use of the strength reduction factor, f_r and the limit of reinforcement measured normal to the vessel.

For comparison purposes, if the value of t_n was re–defined to that of the hub thickened section, where $t_n = 4.625\ in$, the integrally reinforced nozzle would follow the nomenclature of UG-40(e-2). The designer must pay close attention to the following:

1) Limit of reinforcement measured parallel to the vessel wall considering the increased nozzle thickness, and

2) Limits of reinforcement measured normal to the vessel wall considering the length of the reinforced section of the nozzle, denoted as L.

a) STEP 1 – Calculate the Limits of Reinforcement per UG-40.

1) Reinforcing dimensions for an integrally reinforced nozzle per Figure UG-40(e-2).

$$UG-40(e-2) \quad \begin{Bmatrix} t_n = 4.625\ in \\ t_e = 0.0\ in \\ D_p = 0.0\ in \end{Bmatrix}$$

2) Finished opening chord length.

$$d = 2R_n = 2\left(8.125\right) = 16.25\ in$$

3) The limits of reinforcement measured parallel to the vessel wall in the corroded condition.

$$\max\left[d,\ R_n + t_n + t\right] = \max\left[16.25,\ \left\{8.125 + 4.625 + 1.6875\right\}\right] = 16.25\ in$$

4) The limits of reinforcement measured normal to the vessel wall in the corroded condition.

$$\min\left[2.5t,\ 2.5t_n+t_e\right]=\min\left[2.5(1.6875),\ \{2.5(4.625)+0.0\}\right]=4.2188\ in$$

b) STEP 2 – Calculate reinforcement strength parameters per UG-37.

Same as provided previously.

c) STEP 3 – Calculate the Areas of Reinforcement, see Figure UG–37.1 (Without Reinforcing Element, per Figure UG-40(e-2)).

1) Area Required, A:

$$A=dt_rF+2t_nt_rF(1-f_{r1})$$

$$A=16.25(1.3517)(1.0)+2(4.625)(1.3517)(1.0)(1-1.0)=21.9651\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11}=d\left(E_1t-Ft_r\right)-2t_n\left(E_1t-Ft_r\right)(1-f_{r1})$$

$$A_{11}=\begin{Bmatrix}16.25(1.0(1.6875)-1.0(1.3517))-\\2(4.625)\{(1.0(1.6875)-1.0(1.3517))(1-1.0)\}\end{Bmatrix}=5.4568\ in^2$$

$$A_{12}=2(t+t_n)\left(E_1t-Ft_r\right)-2t_n\left(E_1t-Ft_r\right)(1-f_{r1})$$

$$A_{12}=\begin{Bmatrix}2(1.6875+4.625)(1.0(1.6875)-1.0(1.3517))-\\2(4.625)\{(1.0(1.6875)-1.0(1.3517))(1-1.0)\}\end{Bmatrix}=4.2395\ in^2$$

$$A_1=\max\left[5.4568,\ 4.2395\right]=5.4568\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use smaller value:

$$A_{21}=5\left(t_n-t_{rn}\right)f_{r2}t$$

$$A_{21}=5(4.625-0.1462)(1.0)(1.6875)=37.7899\ in^2$$

$$A_{22}=2\left(t_n-t_{rn}\right)\left(2.5t_n+t_e\right)f_{r2}$$

$$A_{22}=2(4.625-0.1462)(2.5(4.625)+0.0)(1.0)=103.5723\ in^2$$

$$A_2=\min\left[37.7899,\ 103.5723\right]=37.7899\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3. Use smaller value:

$$A_3=\min\left[5tt_if_{r2},\ 5t_it_if_{r2},\ 2ht_if_{r2}\right]$$

$$A_3=0.0 \qquad since\ t_i=0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.1 of this example:

Outer Nozzle Fillet Weld Leg:	$0.375\ in$
Outer Element Fillet Weld Leg:	$0.0\ in$
Inner Nozzle Fillet Weld Leg:	$0.0\ in$

$$A_{41} = leg^2 f_{r3} = (0.375)^2 (1.0) = 0.1406\ in^2$$
$$A_{42} = 0.0\ in^2$$
$$A_{43} = 0.0\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = 0.0\ in^2$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + (A_{41} + A_{42} + A_{43}) + A_5$$
$$A_{avail} = 5.4568 + 37.7899 + 0.0 + (0.1406 + 0.0 + 0.0) + 0.0 = 43.3873\ in^2$$

d) STEP 4 – Nozzle reinforcement acceptance criterion:

$$\{A_{avail} = 43.3873\ in^2\} \geq \{A = 21.9651\ in^2\} \qquad True$$

Therefore, it is shown that either procedure will provide equivalent total available area, A_{avail}; both satisfying the required area acceptance criterion.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

The procedure, per VIII-2, paragraph 4.5.5, to design a radial nozzle in a cylindrical shell subject to pressure loading is shown below.

a) STEP 1 – Determine the effective radius of the shell as follows.

$$R_{eff} = 0.5 D_i = 0.5(150.25) = 75.125\ in$$

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall.

For set–in integrally reinforced nozzles,

$$L_R = \min\left[\sqrt{R_{eff} t},\ 2R_n\right]$$
$$L_R = \min\left[\sqrt{(75.125)(1.6875)},\ \ 2(8.125)\right] = \min[11.2594, 16.25] = 11.2594\ in$$

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in VIII-2, paragraph 4.5.13 would need to be checked.

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface.

For set–in nozzles,

$$L_{H1} = \min[1.5t,\ t_e] + \sqrt{R_n t_n} = \min[1.5(1.6875), 0.0] + \sqrt{8.125(4.625)} = 6.1301\ in$$

$$L_{H2} = L_{pr1} = 14.1875\ in$$

$$L_{H3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5\ in$$

$$L_H = \min[L_{H1},\ L_{H2},\ L_{H3}] + t = \min[6.1301,\ 15.875,\ 13.5] + 1.6875 = 7.8176\ in$$

d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable.

$$L_{I1} = \sqrt{R_n t_n} = \sqrt{8.125(4.625)} = 6.1301\ in$$

$$L_{I2} = L_{pr2} = 0.0$$

$$L_{I3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5\ in$$

$$L_I = \min[L_{I1},\ L_{I2},\ L_{I3}] = \min[6.1301,\ 0.0,\ 13.5] = 0.0$$

e) STEP 5 – Determine the total available area near the nozzle opening (see VIII-2, Figure 4.5.1). Do not include any area that falls outside of the limits defined by L_H, L_R, and L_I.

For set–in nozzles,

$$A_T = A_1 + f_{rn}(A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp} A_5$$

$$A_1 = (tL_R)\cdot \max\left[\left(\frac{\lambda}{5}\right)^{0.85},\ 1.0\right] = 1.6875(11.2594)\cdot \max\left[\left(\frac{1.3037}{5}\right)^{0.85},\ 1.0\right]$$

$$A_1 = 19.0002\ in^2$$

$$\lambda = \min\left[\left\{\frac{(2R_n + t_n)}{\sqrt{(D_i + t_{eff})t_{eff}}}\right\},\ 12.0\right] = \min\left[\frac{2(8.125) + 4.625}{\sqrt{150.25 + 1.6875}(1.6875)},\ 12.0\right] = 1.3037$$

$$t_{eff} = t + \left(\frac{A_5 f_{rp}}{L_R}\right) = 1.6875 + \left(\frac{0.0(0.0)}{11.2594}\right) = 1.6875\ in$$

$$f_{rp} = \min\left[\frac{S_p}{S}, 1\right] = 0.0$$

$$f_{rn} = \min\left[\frac{S_n}{S}, 1\right] = \frac{20000}{20000} = 1.0$$

Since $\{L_H = 7.8176\ in\} \le \{L_{x3} = L_{pr3} + t = 7.1875 + 1.6875 = 8.875\ in\}$, calculate A_2 as follows, see VIII-2, Figure 4.5.13,

$$A_2 = t_n L_H = 4.625(7.8176) = 36.1564\ in^2$$

$$A_3 = t_n L_I = 0.0$$

$$A_{41} = 0.5L_{41}^2 = 0.5(0.375)^2 = 0.0703\ in^2$$

$$A_{42} = 0.5L_{42}^2 = 0.0$$

$$A_{43} = 0.5L_{43}^2 = 0.0$$

$$A_5 = \min\left[A_{5a},\, A_{5b}\right]$$
$$A_{5a} = Wt_e = 0.0$$
$$A_{5b} = L_R t_e = 0.0$$
$$A_5 = 0.0$$

$$A_T = 19.0002 + 1.0\left(36.1564 + 0.0\right) + 0.0703 + 0.0 + 0.0 + 0.0 = 55.2269\ in^2$$

f) STEP 6 – Determine the applicable forces.

For set–in nozzles,

$$f_N = PR_{xn}L_H = 356\left(10.2644\right)\left(7.8176\right) = 28566.4985\ lbs$$

$$R_{xn} = \frac{t_n}{\ln\left[\dfrac{R_n + t_n}{R_n}\right]} = \frac{4.625}{\ln\left[\dfrac{8.125 + 4.625}{8.125}\right]} = 10.2644\ in$$

$$f_S = PR_{xs}\left(L_R + t_n\right) = 356\left(75.9656\right)\left(11.2594 + 4.625\right) = 429573.7997\ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[\dfrac{R_{eff} + t_{eff}}{R_{eff}}\right]} = \frac{1.6875}{\ln\left[\dfrac{75.125 + 1.6875}{75.125}\right]} = 75.9656\ in$$

$$f_Y = PR_{xs}R_{nc} = 356\left(75.9656\right)\left(8.125\right) = 219730.4980\ lbs$$

Note: For radial nozzles, $R_{nc} = R_n$.

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection.

$$\sigma_{avg} = \frac{\left(f_N + f_S + f_Y\right)}{A_T} = \frac{28566.4985 + 429573.7997 + 219730.4980}{55.2269} = 12274.2866\ psi$$

$$\sigma_{circ} = \frac{PR_{xs}}{t_{eff}} = \frac{356\left(75.9656\right)}{1.6875} = 16025.9281\ psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection.

$$P_L = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\},\, \sigma_{circ}\right]$$
$$P_L = \max\left[\left\{2\left(12274.2866\right) - 16025.9281\right\},\, 16025.9281\right] = 16025.9281\ psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy VIII-2, Equation 4.5.56. If the nozzle is subjected to internal pressure, then the allowable stress, S_{allow}, is given by VIII-2, Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by VIII-2, Equation 4.5.58 where F_{ha} is evaluated in VIII-2, paragraph 4.4 for the shell geometry being evaluated (e.g., cylinder, spherical shell, or formed head). The allowable stress shall be the minimum of the shell or nozzle material evaluated at the design temperature.

$$\left\{P_L = 16025.9281\ psi\right\} \le \left\{S_{allow} = 1.5SE = 1.5\left(20000\right)\left(1.0\right) = 30000\ psi\right\}$$

j) STEP 10 – Determine the maximum allowable working pressure at the nozzle intersection.

$$P_{\max 1} = \frac{S_{allow}}{\frac{2A_p}{A_T} - \frac{R_{xs}}{t_{eff}}} = \frac{1.5(20000)(1.0)}{\left(\frac{2(1904.1315)}{55.2269}\right) - \left(\frac{75.9656}{1.6875}\right)} = 1253.1320 \; psi$$

$$A_p = \frac{f_N + f_S + f_Y}{P} = \frac{28566.4985 + 429573.7997 + 219730.4980}{356.0} = 1904.1315 \; in^2$$

$$P_{\max 2} = S\left(\frac{t}{R_{xs}}\right) = 20000\left(\frac{1.6875}{75.9656}\right) = 444.28 \; psi$$

$$P_{\max} = \min\left[P_{\max 1}, \; P_{\max 2}\right] = \min\left[1253.1320, \; 444.28\right] = 444.28 \; psi$$

The nozzle is acceptable because $P_{\max} = 444.28 \; psi$ is greater than the specified design pressure of $356 \; psig$.

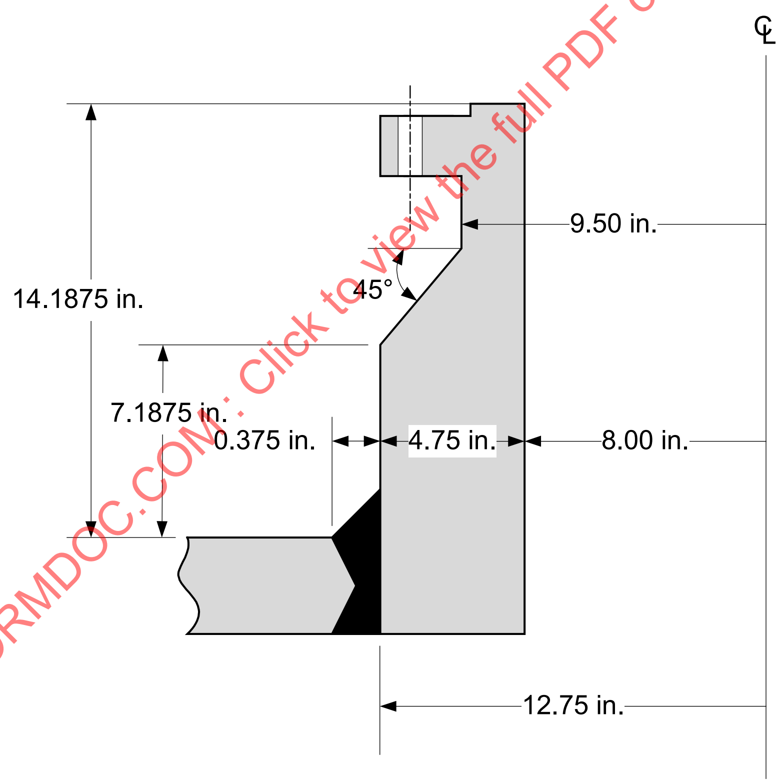

Figure E4.5.1 – Nozzle Detail

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4.5.2 Example E4.5.2 – Hillside Nozzle in Cylindrical Shell

Design an integral hillside nozzle in a cylindrical shell based on the vessel and nozzle data below. The parameters used in this design procedure are shown in Figure E4.5.2.

Vessel and Nozzle Data:

- Design Conditions = $356\ psig@300°F$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0
- Shell Material = $SA-516,\ Grade\ 70,\ Normalized$
- Shell Allowable Stress = $20000\ psi$
- Shell Yield Strength = $33600\ psi$
- Nozzle Material = $SA-105$
- Nozzle Allowable Stress = $20000\ psi$
- Shell Inside Diameter = $150.0\ in$
- Shell Thickness = $1.8125\ in$
- Nozzle Outside Diameter = $11.56\ in$
- Nozzle Thickness = $1.97\ in$
- External Nozzle Projection = $19.0610\ in$
- Internal Nozzle Projection = $0.0\ in$
- Nozzle Offset = $34.875\ in$

The nozzle is inserted through the shell, i.e., set–in type nozzle, see Figure UW-16.1(d).

Establish the corroded dimensions.

Shell:

$$D = 150.0 + 2(Corrosion\ Allowance) = 150.0 + 2(0.125) = 150.25\ in$$

$$R = \frac{D}{2} = \frac{150.25}{2} = 75.125\ in$$

$$t = 1.8125 - Corrosion\ Allowance = 1.8125 - 0.125 = 1.6875\ in$$

Nozzle:

$$t_n = 1.97 - Corrosion\ Allowance = 1.97 - 0.125 = 1.845\ in$$

$$R_n = \frac{D_{n,OD} - 2(t_n)}{2} = \frac{11.56 - 2(1.845)}{2} = 3.935\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-37.

The required thickness of the shell based on circumferential stress is given by UG-27(c)(1).

$$t_r = \frac{PR}{SE-0.6P} = \frac{356(75.125)}{20000(1.0)-0.6(356)} = 1.3517\ in$$

The required thickness of the nozzle based on circumferential stress is given by UG-27(c)(1).

$$t_{rn} = \frac{PR_n}{SE-0.6P} = \frac{356(3.935)}{20000(1.0)-0.6(356)} = 0.0708\ in$$

a) STEP 1 – Calculate the Limits of Reinforcement per UG-40.

1) Reinforcing dimensions for an integrally reinforced nozzle per Figures UG-40(e), UG-40(e-1), UG-40(e-2): See Figure E4.5.2 of this example:

$$t_x = 1.845\ in$$

$$L \approx 16.0\ in$$

$$\{L = 16.0\ in\} \geq \{2.5t_x = 2.5(1.845) = 4.6125\ in\}$$

$$Therefore\ use\ UG-40(e-2)\quad \begin{Bmatrix} t_n = 1.845\ in \\ t_e = 0.0 \end{Bmatrix}$$

Note: Figure UG-40 does not provide a sketch for an integral uniform thickness nozzle with full penetration weld inserted through the shell without a reinforcing pad. Therefore, sketch (e-1) was used with $t_e = 0.0$. Additionally, the value of L is approximate and is determined by subtracting the flange thickness from the external nozzle projection, see Figure E4.5.2.

2) Finished opening chord length.

i) Perpendicular to longitudinal axis, see Figure E4.5.2.

$$R_m = Mean\ Cylinder\ Radius = R + \frac{t_r}{2} = 75.125 + \frac{1.3517}{2} = 75.8009\ in$$

$$L_{off} = Offset\ Length = 34.875\ in$$

$$x_1 = L_{off} + R_n = 34.875 + 3.935 = 38.81\ in$$

$$x_2 = L_{off} - R_n = 34.875 + 3.935 = 30.94\ in$$

$$y_1 = \sqrt{R_m^2 - x_1^2} = \sqrt{75.8009^2 - 38.81^2} = 65.1119\ in$$

$$y_2 = \sqrt{R_m^2 - x_2^2} = \sqrt{75.8009^2 - 30.94^2} = 69.1989\ in$$

$$d = \sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2} = \sqrt{(38.81-30.94)^2 + (69.1989-65.1119)^2} = 8.8679\ in$$

ii) Parallel to longitudinal axis.

$$d = 2R_n = 2(3.935) = 7.870\ in$$

3) The limits of reinforcement measured parallel to the vessel wall in the corroded condition.

i) Perpendicular to longitudinal axis.

$$\max\left[d,\ R_n + t_n + t\right] = \max\left[8.8679,\ \{3.935 + 1.8450 + 1.6875\}\right] = 8.8679\ in$$

ii) Parallel to longitudinal axis.

$$\max\left[d,\ R_n + t_n + t\right] = \max\left[7.870,\ \{3.935 + 1.8450 + 1.6875\}\right] = 7.870\ in$$

4) The limits of reinforcement measured normal to the vessel wall in the corroded condition.

$$\min\left[2.5t,\ 2.5t_n + t_e\right] = \min\left[2.5(1.6875),\ \{2.5(1.845) + 0.0\}\right] = 4.2188\ in$$

b) STEP 2 – Calculate reinforcement strength parameters per UG-37.

1) Strength Reduction Factors:

$$f_{r1} = S_n / S_v = 20000/20000 = 1.0$$
$$f_{r2} = S_n / S_v = 20000/20000 = 1.0$$
$$f_{r3} = \min\left[S_n, S_p\right] / S_v = 0$$
$$f_{r4} = S_p / S_v = 0$$

2) Joint Efficiency Parameter: For a nozzle located in a solid plate, $E_1 = 1.0$.

3) Correction Factor for variation of internal pressure stresses on different planes with respect to the axis of the vessel. Figure UG-37 may be used for integrally reinforced openings in cylindrical shells and cones. See UW-16(c)(1).

i) For an opening perpendicular to the longitudinal axis, $d = 8.8679\ in\ \rightarrow F = 0.5$.

ii) For an opening parallel to the longitudinal axis, $d = 7.870\ in\ \rightarrow F = 1.0$.

c) STEP 3 – Calculate the Areas of Reinforcement perpendicular to the longitudinal axis, $F = 0.5$. See Figure UG-37.1.

1) Area Required, A:

$$A = dt_rF + 2t_nt_rF(1 - f_{r1})$$
$$A = 8.8679(1.3517)(0.5) + 2(1.845)(1.3517)(0.5)(1 - 1.0) = 5.9934\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11} = d\left(E_1t - Ft_r\right) - 2t_n\left(E_1t - Ft_r\right)(1 - f_{r1})$$
$$A_{11} = \left\{\begin{array}{l} 8.8679\left(1.0(1.6875) - 0.5(1.3517)\right) - \\ 2(1.845)\left\{\left(1.0(1.6875) - 0.5(1.3517)\right)(1 - 1.0)\right\} \end{array}\right\} = 8.9712\ in^2$$
$$A_{12} = 2(t + t_n)\left(E_1t - Ft_r\right) - 2t_n\left(E_1t - Ft_r\right)(1 - f_{r1})$$
$$A_{12} = \left\{\begin{array}{l} 2(1.6875 + 1.845)\left(1.0(1.6875) - 0.5(1.3517)\right) - \\ 2(1.845)\left\{\left(1.0(1.6875) - 0.5(1.3517)\right)(1 - 1.0)\right\} \end{array}\right\} = 7.1473\ in^2$$
$$A_1 = \max\left[8.9712,\ 7.1473\right] = 8.9712\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use smaller value:

$$A_{21} = 5(t_n - t_{rn}) f_{r2} t = 5(1.845 - 0.0708)(1.0)(1.6875) = 14.9698\ in^2$$

$$A_{22} = 5(t_n - t_{rn}) f_{r2} t_n = 5(1.845 - 0.0708)(1.0)(1.845) = 16.3670\ in^2$$

$$A_2 = \min[14.9698,\ 16.3670] = 14.9698\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3. Use smaller value:

$$A_3 = \min[5tt_i f_{r2}, 5t_i t_i f_{r2}, 2ht_i f_{r2}]$$

$$A_3 = 0.0 \qquad since\ t_i = 0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.2 of this example,

$$Outer\ Nozzle\ Fillet\ Weld\ Leg: \quad 0.375\ in$$

$$Outer\ Element\ Fillet\ Weld\ Leg: \quad 0.0\ in$$

$$Inner\ Nozzle\ Fillet\ Weld\ Leg: \quad 0.0\ in$$

$$A_{41} = leg^2 f_{r3} = (0.375)^2 (1.0) = 0.1406\ in^2$$

$$A_{42} = 0.0\ in^2$$

$$A_{43} = 0.0\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = (D_p - d - 2t_n) t_e f_{r4} = 0.0\ in^2$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + (A_{41} + A_{42} + A_{43}) + A_5$$

$$A_{avail} = 8.9712 + 14.9698 + 0.0 + (0.1406 + 0.0 + 0.0) + 0.0 = 24.0816\ in^2$$

d) STEP 4 – Nozzle reinforcement acceptance criterion:

$$\{A_{avail} = 24.0816\ in^2\} \geq \{A = 5.9934\ in^2\} \qquad True$$

Therefore, the nozzle is adequately reinforced in the plane perpendicular to the longitudinal axis.

e) STEP 5 – Calculate the Areas of Reinforcement, parallel to the longitudinal axis, $F = 1.0$. See Figure UG-37.1.

1) Area Required, A:

$$A = dt_r F + 2t_n t_r F(1 - f_{r1})$$

$$A = 7.870(1.3517)(1.0) + 2(1.845)(1.3517)(1.0)(1 - 1.0) = 10.6379\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11} = d\left(E_1 t - F t_r\right) - 2t_n\left(E_1 t - F t_r\right)\left(1 - f_{r1}\right)$$

$$A_{11} = \left\{\begin{array}{l} 7.870\left(1.0\left(1.6875\right) - 1.0\left(1.3517\right)\right) - \\ 2\left(1.845\right)\left\{\left(1.0\left(1.6875\right) - 1.0\left(1.3517\right)\right)\left(1 - 1.0\right)\right\} \end{array}\right\} = 2.6427\ in^2$$

$$A_{12} = 2\left(t + t_n\right)\left(E_1 t - F t_r\right) - 2t_n\left(E_1 t - F t_r\right)\left(1 - f_{r1}\right)$$

$$A_{12} = \left\{\begin{array}{l} 2\left(1.6875 + 1.845\right)\left(1.0\left(1.6875\right) - 1.0\left(1.3517\right)\right) - \\ 2\left(1.845\right)\left\{\left(1.0\left(1.6875\right) - 1.0\left(1.3517\right)\right)\left(1 - 1.0\right)\right\} \end{array}\right\} = 2.3724\ in^2$$

$$A_1 = \max\left[2.6427,\ 2.3724\right] = 2.6427\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use smaller value:

$$A_{21} = 5\left(t_n - t_{rn}\right) f_{r2} t$$

$$A_{21} = 5\left(1.845 - 0.0708\right)\left(1.0\right)\left(1.6875\right) = 14.9698\ in^2$$

$$A_{22} = 5\left(t_n - t_{rn}\right) f_{r2} t_n$$

$$A_{22} = 5\left(1.845 - 0.0708\right)\left(1.0\right)\left(1.845\right) = 16.3670\ in^2$$

$$A_2 = \min\left[14.9698,\ 16.3670\right] = 14.9698\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3. Use smaller value:

$$A_3 = \min\left[5 t t_i f_{r2},\ 5 t_i t_i f_{r2},\ 2 h t_i f_{r2}\right]$$

$$A_3 = 0.0 \qquad since\ t_i = 0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.2 of this example:

$$\begin{array}{ll} Outer\ Nozzle\ Fillet\ Weld\ \ Leg: & 0.375\ inches \\ Outer\ Element\ Fillet\ Weld\ \ Leg: & 0.0\ inches \\ Inner\ Nozzle\ Fillet\ Weld\ \ Leg: & 0.0\ inches \end{array}$$

$$A_{41} = leg^2 f_{r3} = \left(0.375\right)^2\left(1.0\right) = 0.1406\ in^2$$

$$A_{42} = 0.0\ in^2$$

$$A_{43} = 0.0\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = \left(D_p - d - 2t_n\right) t_e f_{r4} = 0.0\ in^2$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + \left(A_{41} + A_{42} + A_{43}\right) + A_5$$

$$A_{avail} = 2.6427 + 14.9698 + 0.0 + \left(0.1406 + 0.0 + 0.0\right) + 0.0 = 17.7531\ in^2$$

f) STEP 6 – Nozzle reinforcement acceptance criterion:

$$\left\{A_{avail} = 17.7531\ in^2\right\} \geq \left\{A = 10.6379\ in^2\right\} \qquad True$$

Therefore, the nozzle is adequately reinforced in the plane parallel to the longitudinal axis.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

For a hillside nozzle in a cylindrical shell (see VIII-2, Figure 4.5.4), the design procedure in VIII-2, paragraph 4.5.5 shall be used with the following substitutions from VIII-2, paragraph 4.5.6.

$$R_{nc} = \max\left[\left(\frac{R_{ncl}}{2}\right),\ R_n\right]$$

where,

$$R_{ncl} = R_{eff}\left(\theta_1 - \theta_2\right)$$

$$\theta_1 = \cos^{-1}\left[\frac{D_X}{R_{eff}}\right] = \cos^{-1}\left[\frac{34.875}{75.125}\right] = 62.3398\ deg = 1.0880\ rad$$

$$\theta_2 = \cos^{-1}\left[\frac{D_X + R_n}{R_{eff}}\right] = \cos^{-1}\left[\frac{34.875 + 3.935}{75.125}\right] = 58.8952\ deg = 1.0279\ rad$$

$$R_{ncl} = 75.125\left(1.0880 - 1.0279\right) = 4.5150\ in$$

$$R_{nc} = \max\left[\left(\frac{4.5150}{2}\right),\ 3.935\right] = 3.935\ in$$

The procedure in VIII-2, paragraph 4.5.5 is shown below.

a) STEP 1 – Determine the effective radius of the shell as follows:

$$R_{eff} = 0.5D_i = 0.5\left(150.25\right) = 75.125\ in$$

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall:

For integrally reinforced nozzles:

$$L_R = \min\left[\sqrt{R_{eff}t},\ 2R_n\right] = \min\left[\sqrt{75.125\left(1.6875\right)},\ 2\left(3.935\right)\right] = 7.87\ in$$

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in VIII-2, paragraph 4.5.13 would need to be checked.

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface.

For set–in nozzles,

$$L_{H1} = \min\left[1.5t, t_e\right] + \sqrt{R_n t_n} = \min\left[1.5 \times 1.6875, 0\right] + \sqrt{3.935\left(1.845\right)} = 2.6945\ in$$

$$L_{H2} = L_{pr1} = 19.0610\ in$$

$$L_{H3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5\ in$$

$$L_H = \min[L_{H1},\ L_{H2},\ L_{H3}] + t = 4.3820\ in$$

d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable:

$$L_{I1} = \sqrt{R_n t_n} = \sqrt{3.935(1.845)} = 2.6945$$

$$L_{I2} = L_{pr2} = 0.0$$

$$L_{I3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5\ in$$

$$L_I = \min[L_{I1},\ L_{I2},\ L_{I3}] = 0.0$$

e) STEP 5 – Determine the total available area near the nozzle opening (see VIII-2, Figures 4.5.1 and 4.5.2). Do not include any area that falls outside of the limits defined by L_H, L_R, and L_I.

For set–in nozzles,

$$A_T = A_1 + f_{rn}(A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp}A_5$$

$$A_1 = (tL_R)\cdot\max\left[\left(\frac{\lambda}{5}\right)^{0.85},\ 1.0\right] = 1.6875(7.87)\cdot\max\left[\left(\frac{0.6067}{5}\right)^{0.85},\ 1.0\right] = 13.2806$$

$$\lambda = \min\left[\left\{\frac{(2R_n + t_n)}{\sqrt{(D_i + t_{eff})t_{eff}}}\right\},\ 12.0\right] = \min\left[\left\{\frac{2(3.935) + 1.845}{\sqrt{(150.25 + 1.6875)(1.6875)}}\right\},\ 12.0\right]$$

$$\lambda = 0.6067$$

$$t_{eff} = t + \left(\frac{A_5 f_{rp}}{L_R}\right) = 1.6875 + \left(\frac{0.0(0.0)}{7.87}\right) = 1.6875\ in$$

$$f_{rn} = \frac{S_n}{S} = \frac{20000}{20000} = 1.0$$

$$f_{rp} = \frac{S_p}{S} = 0.0$$

Since $\{t_n = 1.845\ in\} = \{t_{n2} = 1.845\ in\}$, calculate A_2 as follows:

$$A_2 = t_n L_H = 1.845(4.3820) = 8.0848\ in^2$$

$$A_3 = t_n L_I = 1.845(0.0) = 0.0$$

$$A_{41} = 0.5L_{41}^2 = 0.5(0.375) = 0.0703\ in^2$$

$$A_{42} = 0.5L_{42}^2 = 0.0$$

$$A_{43} = 0.5L_{43}^2 = 0.0$$

$$A_5 = \min\left[A_{5a},\ A_{5b}\right]$$
$$A_{5a} = Wt_e = 0.0$$
$$A_{5b} = L_R t_e = 0.0$$
$$A_5 = 0.0$$

$$A_T = 13.2806 + 1.0(8.0848 + 0.0) + 0.0 + 0.0703 + 0.0 + 0.0 = 21.4357\ in^2$$

f) STEP 6 – Determine the applicable forces:

For set–in nozzles,

$$f_N = PR_{xn}L_H = 356\left(4.7985\right)\left(4.3820\right) = 7485.6216\ lbs$$

$$R_{xn} = \frac{t_n}{\ln\left[\dfrac{R_n + t_n}{R_n}\right]} = \frac{1.845}{\ln\left[\dfrac{3.935 + 1.845}{3.935}\right]} = 4.7985\ in$$

$$f_S = PR_{xs}\left(L_R + t_n\right) = 356\left(75.9656\right)\left(7.87 + 1.845\right) = 262730.0662\ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[\dfrac{R_{eff} + t_{eff}}{R_{eff}}\right]} = \frac{1.6875}{\ln\left[\dfrac{75.125 + 1.6875}{75.125}\right]} = 75.9656\ in$$

$$f_Y = PR_{xs}R_{nc} = 356\left(75.9656\right)\left(3.935\right) = 106417.1704\ lbs$$

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection:

$$\sigma_{avg} = \frac{\left(f_N + f_S + f_Y\right)}{A_T} = \frac{\left(7485.6216 + 262730.0662 + 106417.1704\right)}{21.4357} = 17570.3550\ psi$$

$$\sigma_{circ} = \frac{PR_{xs}}{t_{eff}} = \frac{356\left(75.9656\right)}{1.6875} = 16025.9281\ psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection:

$$P_L = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\},\ \sigma_{circ}\right]$$
$$P_L = \max\left[\left\{2\left(17570.3550\right) - 16025.9281\right\},\ 16025.9281\right] = 19114.7819\ psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy VIII-2, Equation 4.5.56. If the nozzle is subjected to internal pressure, then the allowable stress, S_{allow}, is given by VIII-2, Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by VIII-2, Equation 4.5.58 where F_{ha} is evaluated in VIII-2, paragraph 4.4 for the shell geometry being evaluated (e.g., cylinder, spherical shell, or formed head). The allowable stress shall be the minimum of the shell or nozzle material evaluated at the design temperature.

$$\left\{P_L = 19114.7819\ psi\right\} \le \left\{S_{allow} = 1.5SE = 1.5\left(20000\right)\left(1.0\right) = 30000\ psi\right\}$$

j) STEP 10 – Determine the maximum allowable working pressure of the nozzle:

$$P_{\max 1} = \frac{S_{allow}}{\dfrac{2A_p}{A_T} - \dfrac{R_{xs}}{t_{eff}}} = \frac{1.5(20000)(1.0)}{\dfrac{2(1057.9575)}{21.4357} - \dfrac{75.9656}{1.6875}} = 558.7300\ psi$$

$$A_p = \frac{(f_N + f_S + f_Y)}{A_T} = \frac{(7485.6216 + 262730.0662 + 106417.1704)}{356} = 1057.9575\ in^2$$

$$P_{\max 2} = S\left(\frac{t}{R_{xs}}\right) = 20000\left(\frac{1.6875}{75.9656}\right) = 444.28\ psi$$

$$P_{\max} = \min\left[P_{\max 1}, P_{\max 2}\right] = \min\left[558.73,\ 444.28\right] = 444.28\ psi$$

The nozzle is acceptable because $P_{\max} = 444.28\ psi$ is greater than the specified design pressure of $356\ psig$.

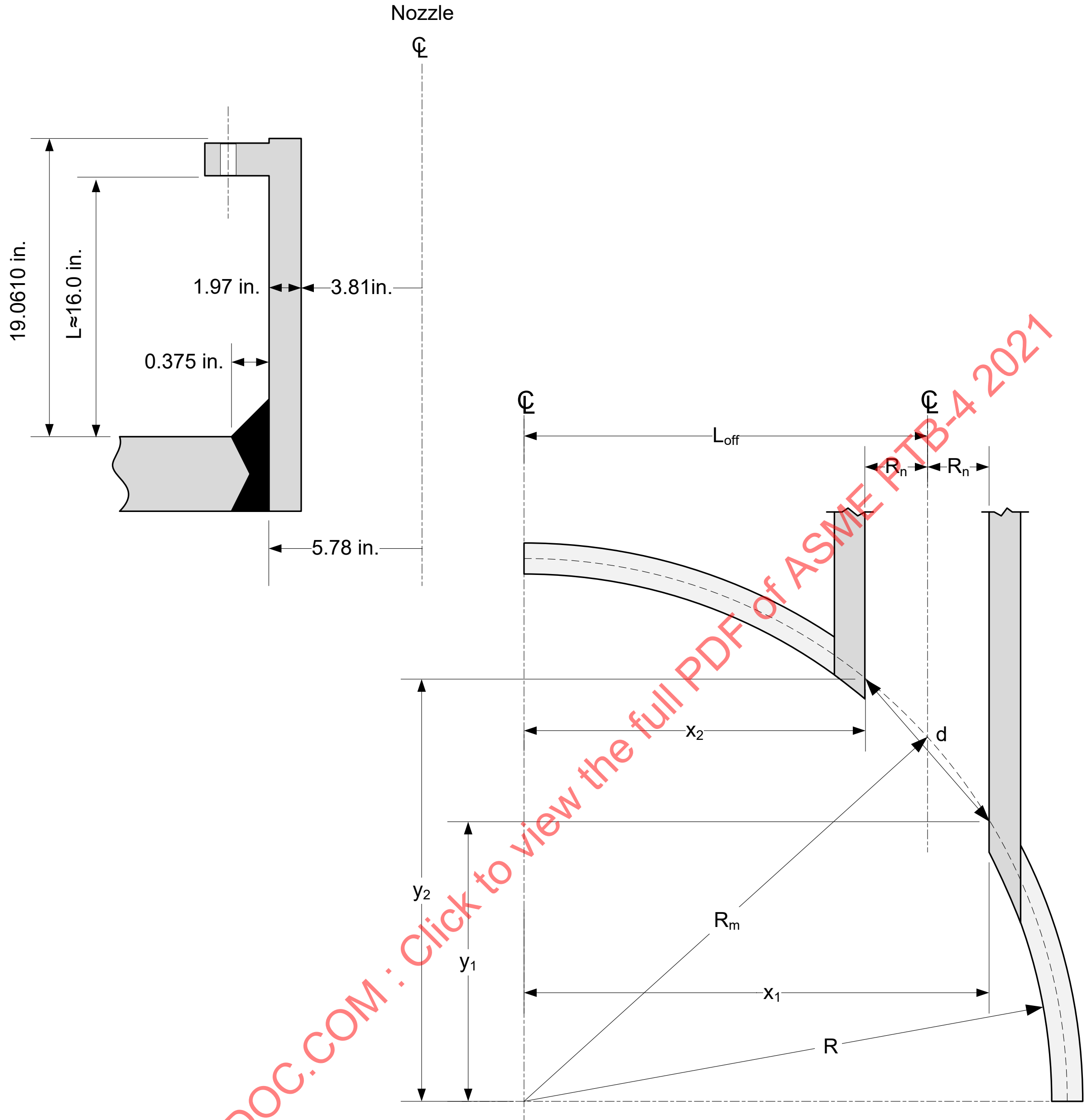

Figure E4.5.2 – Nozzle Detail

4.5.3 Example E4.5.3 – Radial Nozzle in Ellipsoidal Head

Design an integral radial nozzle centrally located in a 2:1 ellipsoidal head based on the vessel and nozzle data below. The parameters used in this design procedure are shown in Figure E4.5.3.

Vessel and Nozzle Data:

- Design Conditions = $356\ psig @ 300°F$
- Vessel and Nozzle Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0
- Head Material = $SA-516,\ Grade\ 70,\ Norm.$
- Head Allowable Stress = $20000\ psi$
- Head Yield Strength = $33600\ psi$
- Nozzle Material = $SA-105$
- Nozzle Allowable Stress = $20000\ psi$
- Head Inside Diameter = $90.0\ in$
- Height of the Elliptical Head, (2:1) = $22.5\ in$
- Head Thickness = $1.0\ in$
- Nozzle Outside Diameter = $15.94\ in$
- Nozzle Thickness = $2.28\ in$
- External Nozzle Projection = $13.5\ in$
- Nozzle Internal Projection = $0.0\ in$

The nozzle is inserted centrally through the head, i.e., set–in type nozzle, see Figure UW-16.1(d).

Establish the corroded dimensions.

Ellipsoidal Head:

$$D = 90.0 + 2(Corrosion\ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

$$R = \frac{D}{2} = \frac{90.25}{2} = 45.125\ in$$

$$t = 1.0 - Corrosion\ Allowance = 1.0 - 0.125 = 0.875\ in$$

Nozzle:

$$t_n = 2.28 - Corrosion\ Allowance = 2.28 - 0.125 = 2.155\ in$$

$$R_n = \frac{D_n - 2(t_n)}{2} = \frac{15.94 - 2(2.155)}{2} = 5.815\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-37.

The required thickness of the 2:1 ellipsoidal head based on circumferential stress is given by UG-32(d). However, per UG-37(a), when an opening and its reinforcement are in an ellipsoidal head and located entirely within a circle the center which coincides with the center of the head and the diameter of which is equal to 80% of the shell diameter, t_r is the thickness required for a seamless sphere of radius $K_1 D$, where K_1 is given in Table UG-37.

Per Table UG-37, for a 2:1 ellipsoidal head where, $D/2h = 90.0/2(22.5) = 2 \rightarrow K_1 = 0.9$.

The required thickness, t_r, per the UG-37 definition for nozzle reinforcement calculations.

$$t_r = \frac{PDK}{2SE - 0.2P} = \frac{356(90.25)(0.9)}{2(20000)(1.0) - 0.2(356)} = 0.7242\ in$$

The required thickness of the nozzle based on circumferential stress is given by UG-27(c)(1).

$$t_{rn} = \frac{PR_n}{SE - 0.6P} = \frac{356(5.815)}{20000(1.0) - 0.6(356)} = 0.1046\ in$$

a) STEP 1 – Calculate the Limits of Reinforcement per UG-40.

1) Reinforcing dimensions for an integrally reinforced nozzle per Figure UG-40(e), UG-40(e-1), UG-40(e-2): See Figure E4.5.3 of this example.

$$t_x = 2.155\ in$$

$$L \approx 12\ in$$

$$\{L = 12\ in\} < \{2.5t_x = 2(2.155) = 4.31\ in\}$$

$$Therefore\ use\ UG-40(e-2)\begin{Bmatrix} t_n = 2.155\ in \\ t_e = 0.0 \end{Bmatrix}$$

Note: Figure UG-40 does not provide a sketch for an integral uniform thickness nozzle with full penetration weld inserted through the shell without a reinforcing pad. Therefore, sketch (e-2) was used with $t_e = 0.0$. Additionally, the value of L is approximate and is determined by subtracting the flange thickness from the external nozzle projection, see Figure E4.5.3.

2) Finished opening chord length.

$$d = 2R_n = 2(5.815) = 11.63\ in$$

3) The limits of reinforcement measured parallel to the vessel wall in the corroded condition.

$$\max[d,\ R_n + t_n + t] = \max[11.63,\ \{5.815 + 2.155 + 1.0\}] = 11.63\ in$$

4) The limits of reinforcement measured normal to the vessel wall in the corroded condition.

$$\min[2.5t,\ 2.5t_n + t_e] = \min[2.5(0.875),\ \{2.5(2.155) + 0.0\}] = 2.1875\ in$$

b) STEP 2 – Calculate the reinforcement strength parameters per UG-37.

1) Strength Reduction Factors:

$$f_{r1} = S_n / S_v = 20000/20000 = 1.0$$
$$f_{r2} = S_n / S_v = 20000/20000 = 1.0$$
$$f_{r3} = \min\left[S_n, S_p\right] / S_v = 0.0$$
$$f_{r4} = S_p / S_v = 0.0$$

2) Joint Efficiency Parameter: For a nozzle located in a solid plate, $E_1 = 1.0$.

3) Correction Factor for variation of internal pressure stresses on different planes with respect to the axis of the vessel: For a nozzle in an ellipsoidal head, $F = 1.0$.

c) STEP 3 – Calculate the Areas of Reinforcement, see Figure UG–37.1

1) Area Required, A:

$$A = dt_r F + 2t_n t_r F\left(1 - f_{r1}\right)$$
$$A = 11.63(0.7242)(1.0) + 2(2.155)(0.7242)(1.0)(1-1.0) = 8.4224\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11} = d\left(E_1 t - F t_r\right) - 2t_n\left(E_1 t - F t_r\right)\left(1 - f_{r1}\right)$$
$$A_{11} = \left\{\begin{array}{l} 11.63\left(1.0(0.875) - 1.0(0.7242)\right) - \\ 2(2.155)\left(1.0(0.875) - 1.0(0.7242)\right)(1-1.0) \end{array}\right\} = 1.7538\ in^2$$
$$A_{12} = 2\left(t + t_n\right)\left(E_1 t - F t_r\right) - 2t_n\left(E_1 t - F t_r\right)\left(1 - f_{r1}\right)$$
$$A_{12} = \left\{\begin{array}{l} 2(0.875 + 2.155)\left(1.0(0.875) - 1.0(0.7242)\right) - \\ 2(2.155)\left(1.0(0.875) - 1.0(0.7242)\right)(1-1.0) \end{array}\right\} = 0.9138\ in^2$$
$$A_1 = \max\left[1.7538,\ 0.9138\right] = 1.7538\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use the smaller value:

$$A_{21} = 5\left(t_n - t_{rn}\right) f_{r2} t$$
$$A_{21} = 5(2.155 - 0.1046)(1.0)(0.875) = 8.9705\ in^2$$
$$A_{22} = 5\left(t_n - t_{rn}\right) f_{r2} t_n$$
$$A_{22} = 5(2.155 - 0.1046)(1.0)(2.155) = 22.0931\ in^2$$
$$A_2 = \min\left[8.9705,\ 22.0931\right] = 8.9705\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3:

$$A_3 = \min\left[5 t t_i f_{r2}, 5 t_i t_i f_{r2}, 2 h t_i f_{r2}\right]$$
$$A_3 = 0.0 \qquad since\ t_i = 0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.3 of this example,

$$\begin{array}{ll} Outer\ Nozzle\ Fillet\ Weld\ Leg: & 0.375\ in \\ Outer\ Element\ Fillet\ Weld\ Leg: & 0.0\ in \\ Inner\ Nozzle\ Fillet\ Weld\ Leg: & 0.0\ in \end{array}$$

$$A_{41} = leg^2 f_{r3} = (0.375)^2 (1.0) = 0.1406\ in^2$$

$$A_{42} = 0.0\ in^2$$

$$A_{43} = 0.0\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = \left(D_p - d - 2t_n\right) t_e f_{r4} = 0.0\ in^2$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + \left(A_{41} + A_{42} + A_{43}\right) + A_5$$

$$A_{avail} = 1.7538 + 8.9705 + 0.0 + \left(0.1406 + 0.0 + 0.0\right) + 0.0 = 10.8649\ in^2$$

d) STEP 4 – Nozzle reinforcement acceptance criterion:

$$\left\{A_{avail} = 10.8649\ in^2\right\} \geq \left\{A = 8.4224\ \ in^2\right\} \qquad True$$

Therefore, the nozzle is adequately reinforced.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

The procedure, per VIII-2, paragraph 4.5.10, to design a radial nozzle in an ellipsoidal head subject to pressure loading is shown below.

a) STEP 1 – Determine the effective radius of the ellipsoidal head as follows.

$$R_{eff} = \frac{0.9D_i}{6}\left[2 + \left(\frac{D_i}{2h}\right)^2\right] = \frac{0.9(90.25)}{6}\left[2 + \left(\frac{90.25}{2(22.625)}\right)^2\right] = 80.9262\ in$$

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall.

For integrally reinforced set–in nozzles in ellipsoidal heads,

$$L_R = \min\left[\sqrt{R_{eff}t},\ 2R_n\right] = \min\left[\sqrt{80.9262(0.875)},\ 2(5.8150)\right] = 8.4149\ in$$

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in VIII-2, paragraph 4.5.13 would need to be checked.

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface. See VIII-2, Figures 4.5.9 and 4.5.10.

For set–in nozzles in ellipsoidal heads,

$$L_H = \min\left[t + t_e + F_p\sqrt{R_n t_n},\ L_{pr1} + t\right]$$

$$X_o = \min\left[D_R + (R_n + t_n)\cdot\cos[\theta],\ \frac{D_i}{2}\right]$$

$$X_o = \min\left[0.0 + (5.8150 + 2.1550)\cdot\cos[0.0],\ \frac{90.25}{2}\right] = 7.97\ in$$

where,

$$\theta = \arctan\left[\left(\frac{h}{R}\right)\cdot\left(\frac{D_R}{\sqrt{R^2 - D_R^2}}\right)\right] = \arctan\left[\left(\frac{22.625}{45.125}\right)\cdot\left(\frac{0.0}{\sqrt{45.125^2 - 0.0^2}}\right)\right] = 0.0\ rad$$

Since $\{X_o = 7.97\ in\} \le \{0.35D_i = 0.35(90.25) = 31.5875\ in\}$, calculate F_p as follows:

$$F_p = C_n = \min\left[\left(\frac{t + t_e}{t_n}\right)^{0.35},\ 1.0\right] = \min\left[\left(\frac{0.875 + 0.0}{2.1550}\right)^{0.35},\ 1.0\right] = 0.7295$$

therefore,

$$L_H = \min\left[0.875 + 0.0 + (0.7295)\sqrt{5.8150(2.1550)},\ 13.5 + 0.875\right] = 3.4574\ in$$

d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable.

$$L_{pr2} = 0.0$$

$$L_I = \min\left[F_p\sqrt{R_n t_n},\ L_{pr2}\right] = 0.0$$

e) STEP 5 – Determine the total available area near the nozzle opening (see VIII-2, Figures 4.5.1 and 4.5.2) where f_{rn} and f_{rp} are given by VIII-2, Equations (4.5.21) and (4.5.22) respectively. Do not include any area that falls outside of the limits defined by L_H, L_R, and L_I.

For set–in nozzles:

$$A_T = A_1 + f_{rn}(A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp}A_5$$

$$A_1 = tL_R = 0.875(8.4149) = 7.3630\ in^2$$

Since $\{t_n = 2.1550\ in\} = \{t_{n2} = 2.1550\ in\}$, calculate A_2 as follows:

$$A_2 = t_n L_H = 2.1550(3.4574) = 7.4507\ in^2$$

$$A_3 = t_n L_I = 0.0$$

$$A_{41} = 0.5L_{41}^2 = 0.5(0.375)^2 = 0.0703\ in^2$$

$$A_{42} = 0.5L_{42}^2 = 0.0$$

$$A_{43} = 0.5L_{43}^2 = 0.0$$

$$t_e = 0.0\ in$$

$$A_{5a} = Wt_e = 0.0$$

$$A_{5b} = \left(L_R - t_n\right)t_e = 0.0$$

$$A_5 = \min\left[A_{5a},\ A_{5b}\right] = 0.0$$

$$f_{rn} = \frac{S_n}{S} = \frac{20000}{20000} = 1.0$$

$$f_{rp} = \frac{S_p}{S} = 0.0$$

$$A_T = 7.363 + 1.0\left(7.4507 + 0.0\right) + 0.0703 + 0.0 + 0.0 + 0.0\left(0.0\right) = 14.8840\ in^2$$

f) STEP 6 – Determine the applicable forces.

For set–in nozzles,

$$f_N = PR_{xn}L_H = 356\left(6.8360\right)\left(3.4572\right) = 8413.4972\ lbs$$

$$R_{xn} = \frac{t_n}{\ln\left[\dfrac{R_n + t_n}{R_n}\right]} = \frac{2.1550}{\ln\left[\dfrac{5.8150 + 2.1550}{5.8150}\right]} = 6.8360\ in$$

$$f_S = \frac{PR_{xs}\left(L_R + t_n\right)}{2} = \frac{356\left(81.3629\right)\left(8.4149 + 2.1550\right)}{2} = 153079.5936\ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[\dfrac{R_{eff} + t_{eff}}{R_{eff}}\right]} = \frac{0.875}{\ln\left[\dfrac{80.9262 + 0.875}{80.9262}\right]} = 81.3629\ in$$

$$t_{eff} = t + \left(\frac{A_5 f_{rp}}{L_R}\right) = 0.875 + \left(\frac{0.0}{8.4149}\right) = 0.875\ in$$

$$f_Y = \frac{PR_{xs}R_{nc}}{2} = \frac{356\left(81.3629\right)\left(5.8150\right)}{2} = 84216.2969\ lbs$$

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection.

$$\sigma_{avg} = \frac{\left(f_N + f_S + f_Y\right)}{A_T} = \frac{8413.4972 + 153079.5936 + 84216.2969}{14.884} = 16508.2900\ psi$$

$$\sigma_{circ} = \frac{PR_{xs}}{2t_{eff}} = \frac{356\left(81.3629\right)}{2\left(0.875\right)} = 16551.5385\ psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection.

$$P_L = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\}, \sigma_{circ}\right]$$

$$P_L = \max\left[\left\{2(16508.29) - 16551.5385\right\}, 16551.5385\right] = 16551.5385 \ psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy VIII-2, Equation 4.5.146. If the nozzle is subjected to internal pressure, then the allowable stress, S_{allow}, is given by VIII-2, Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by VIII-2, Equation 4.5.58.

$$\left\{P_L = 16551.5385\right\} \leq \left\{S_{allow} = 1.5SE = 1.5(20000)(1.0) = 30000 \ psi\right\}$$

j) STEP 10 – Determine the maximum allowable working pressure of the nozzle.

$$P_{\max 1} = \frac{S_{allow}}{\left(\dfrac{2A_p}{A_T}\right) - \left(\dfrac{R_{xs}}{2t_{eff}}\right)} = \frac{1.5(20000)(1.0)}{\left(\dfrac{2(690.1949)}{14.884}\right) - \left(\dfrac{81.3629}{2(0.875)}\right)} = 648.6470 \ psi$$

$$A_p = \frac{(f_N + f_S + f_Y)}{P}$$

$$A_p = \frac{8413.4972 + 153079.5936 + 84216.2969}{356} = 690.1949 \ in^2$$

$$P_{\max 2} = 2S\left(\frac{t}{R_{xs}}\right) = 2(20000)\left(\frac{0.875}{81.3629}\right) = 430.1715 \ psi$$

$$P_{\max} = \min\left[P_{\max 1}, P_{\max 2}\right] = \min\left[648.647, \ 430.1715\right] = 430.1715 \ psi$$

The nozzle is acceptable because $P_{max} = 430.1715\, psi$ is greater than the specified design pressure of $356\, psig$.

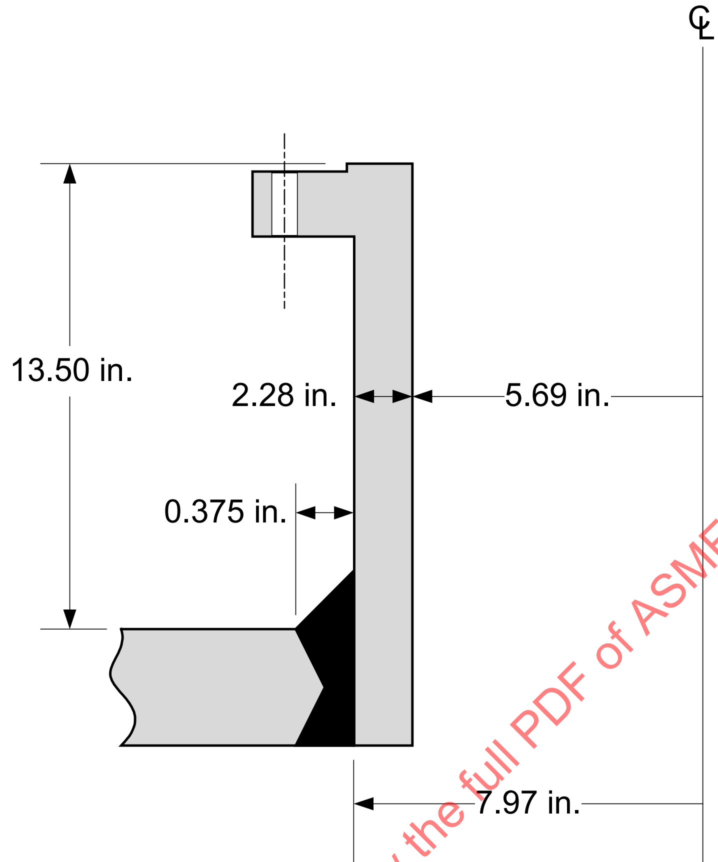

Figure E4.5.3 – Nozzle Details

4.5.4 Example E4.5.4 – Radial Nozzle in Cylindrical Shell

Check the design of an integral radial nozzle in a cylindrical shell based on the vessel and nozzle data below. Verify the adequacy of the attachment welds. The parameters used in this design procedure are shown in Figure E4.5.4.

Vessel and Nozzle Data:

- Design Conditions = $425\ psig @ 800°F$
- Vessel and Nozzle Corrosion Allowance = $0.0625\ in$
- Weld Joint Efficiency = 1.0
- Shell Allowable Stress = $11400\ psi$
- Nozzle Allowable Stress = $12000\ psi$
- Shell Inside Diameter = $96.0\ in$
- Shell Thickness = $2.0\ in$
- Nozzle Inside Diameter = $16.0\ in$
- Nozzle Thickness (seamless) = $1.75\ in$

The nozzle has a set–on type configuration and the opening does not pass through a vessel Category A joint, see Figure UW-16.1(n). All category A joints are to be fully radiographed (see UW-3).

Establish the corroded dimensions.

Shell:

$$D = 96.0 + 2(Corrosion\ Allowance) = 96.0 + 2(0.0625) = 96.125\ in$$

$$R = \frac{D}{2} = \frac{96.125}{2} = 48.0625\ in$$

$$t = 2.0 - Corrosion\ Allowance = 2.0 - 0.0625 = 1.9375\ in$$

Nozzle:

$$t_n = 1.75 - Corrosion\ Allowance = 1.75 - 0.0625 = 1.6875\ in$$

$$R_n = \frac{D_n + 2(Corrosion\ Allowance)}{2} = \frac{16.0 + 2(0.0625)}{2} = 8.0625\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-37.

The required thickness of the shell based on circumferential stress is given by UG-27(c)(1).

$$t_r = \frac{PR}{SE - 0.6P} = \frac{425(48.0625)}{11400(1.0) - 0.6(425)} = 1.8328\ in$$

The required thickness of the nozzle based on circumferential stress is given by UG-27(c)(1).

$$t_{rn} = \frac{PR_n}{SE - 0.6P} = \frac{425(8.0625)}{12000(1.0) - 0.6(425)} = 0.2917\ in$$

a) STEP 1 – Calculate the required weld sizes per UW-16(d) and Figure UW-16.1 Sketch (n).

1) Inner perimeter weld:

$$t_{wr} = 0.7t_{\min} = 0.7(0.75) = 0.525\ in$$

$$t_{wact} = 0.875 - 0.0625 = 0.8125\ in$$

$$\{t_{wact} = 0.8125\ in\} > \{t_{wr} = 0.525\ in\} \qquad True$$

2) Outer perimeter weld.

$$Throat_r = 0.5t_{\min} = 0.5(0.75) = 0.375\ in$$

$$Throat_{act} = 0.7(weld\ \ size) = 0.7(0.75) = 0.525\ in$$

$$\{Throat_{act} = 0.525\ in\} > \{Throat_r = 0.375\ in\} \qquad True$$

b) STEP 2 – Calculate the Limits of Reinforcement per UG-40.

1) Reinforcing dimensions for an integrally reinforced nozzle per Figure UG-40(d). See Figure E4.5.4 of this example.

$$\theta = \arctan\left[\frac{\left(\frac{26.0 - 19.5}{2}\right)}{3.5}\right] = 42.9\ deg$$

Since $\{\theta = 42.9\ deg\} > \{\theta = 30\ deg\}$, Figure UG-40 sketch (d) applies and $t_e = 3.5\ in$.

2) Finished opening chord length.

$$d = 2R_n = 2(8.0625) = 16.125\ in$$

3) The limits of reinforcement measured parallel to the vessel wall in the corroded condition.

$$\max[d,\ R_n + t_n + t] = \max[16.125,\ \{8.0625 + 1.6875 + 1.9375\}] = 16.125\ in$$

4) The limits of reinforcement measured normal to the vessel wall in the corroded condition.

$$\min[2.5t,\ 2.5t_n + t_e] = \min[2.5(1.9375),\ \{2.5(1.6875) + 3.5\}] = 4.8438\ in$$

c) STEP 3 – Calculate the reinforcement strength parameters per UG-37.

1) Strength Reduction Factors:

$$f_{r1} = 1.0 \qquad for\ set-on\ type\ nozzle$$

$$f_{r2} = S_n / S_v = 12000/11400 = 1.0526 \rightarrow\ set\ f_{r2} = 1.0$$

$$f_{r3} = \min[S_n, S_p]/S_v = \min[12000,\ 11400]/11400 = 1.0$$

$$f_{r4} = S_p / S_v = 12000/11400 = 1.0526 \rightarrow\ set\ f_{r4} = 1.0$$

2) Joint Efficiency Parameter: For a nozzle located in a solid plate, $E_1 = 1.0$.

3) Correction Factor for variation of internal pressure stresses on different planes with respect to the axis of the vessel: For a radial nozzle in a cylindrical shell, $F = 1.0$.

d) STEP 4 – Calculate the Areas of Reinforcement, see Figure UG–37.1

1) Area Required, A:

$$A = dt_r F + 2t_n t_r F(1 - f_{r1})$$
$$A = 16.125(1.8328)(1.0) + 2(1.6875)(1.8328)(1.0)(1 - 1.0) = 29.5539\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11} = d(E_1 t - Ft_r) - 2t_n(E_1 t - Ft_r)(1 - f_{r1})$$
$$A_{11} = \begin{Bmatrix} 16.125(1.0(1.9375) - 1.0(1.8328)) - \\ 2(1.6875)(1.0(1.9375) - 1.0(1.8328))(1 - 1.0) \end{Bmatrix} = 1.6883\ in^2$$
$$A_{12} = 2(t + t_n)(E_1 t - Ft_r) - 2t_n(E_1 t - Ft_r)(1 - f_{r1})$$
$$A_{12} = \begin{Bmatrix} 2(1.9375 + 1.6875)(1.0(1.9375) - 1.0(1.8328)) - \\ 2(1.6875)(1.0(1.9375) - 1.0(1.8328))(1 - 1.0) \end{Bmatrix} = 0.7591\ in^2$$
$$A_1 = \max[1.6883,\ 0.7591] = 1.6883\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use the smaller value:

$$A_{21} = 5(t_n - t_{rn}) f_{r2} t$$
$$A_{21} = 5(1.6875 - 0.2917)(1.0)(1.9375) = 13.5218\ in^2$$
$$A_{22} = 2(t_n - t_{rn})(2.5t_n + t_e) f_{r2}$$
$$A_{22} = 2(1.6875 - 0.2917)(2.5(1.6875) + 3.5)(1.0) = 21.5477\ in^2$$
$$A_2 = \min[13.5218,\ 21.5477] = 13.5218\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3:

$$A_3 = \min\left[5tt_i f_{r2},\ 5t_i t_i f_{r2},\ 2ht_i f_{r2}\right]$$
$$A_3 = 0.0 \qquad since\ t_i = 0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.4 of this example:

$$Outer\ Nozzle\ Fillet\ Weld\ Leg: \qquad 0.75\ in$$
$$Outer\ Element\ Fillet\ Weld\ Leg: \qquad 0.0\ in$$
$$Inner\ Nozzle\ Fillet\ Weld\ Leg: \qquad 0.0\ in$$

$$A_{41} = leg^2 f_{r3} = (0.75)^2(1.0) = 0.5625\ in^2$$
$$A_{42} = 0.0\ in^2$$
$$A_{43} = 0.0\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = \left(D_p - d - 2t_n\right)t_e f_{r4} = \left(26.0 - 16.125 - 2\left(1.6875\right)\right)\left(2.75\right)\left(1.0\right) = 17.875\ in^2$$

Where the value of t_e is calculated as the average thickness of the reinforcing element.

$$t_e = \frac{3.5+2}{2} = 2.75\ in$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + \left(A_{41} + A_{42} + A_{43}\right) + A_5$$
$$A_{avail} = 1.6883 + 13.5218 + 0.0 + \left(0.5625 + 0.0 + 0.0\right) + 17.875 = 33.6476\ in^2$$

e) STEP 5 – Nozzle reinforcement acceptance criterion:

$$\left\{A_{avail} = 33.6476\ in^2\right\} \geq \left\{A = 29.5539\ \ in^2\right\} \qquad\qquad True$$

Therefore, the nozzle is adequately reinforced.

The load to be carried by the welds is calculated in accordance with UG-41.

a) STEP 1 – Per Figure UG-41.1, sketch (b) Nozzle Attachment Weld Loads and Weld Strength Paths to be Considered; typical nozzle detail with nozzle neck abutting (set–on) the vessel wall.

Per UG-41(b)(1): Weld Load for Strength Path 1-1, W_{1-1}:

$$W_{1-1} = \left(A_2 + A_5 + A_{41} + A_{42}\right)S_v = \left(13.5218 + 17.875 + 0.5625 + 0.0\right)\left(11400\right) = 364336.0\ lbs$$

Per UG-41(b)(2): Total Weld Load, W.

$$W = \left(A - A_1\right)S_v = \left(29.5539 - 1.6883\right)\left(11400\right) = 317667.8\ lbs$$

Since W is smaller than W_{1-1}, W may be used in place of W_{1-1} for comparing weld capacity to weld load.

b) STEP 2 – Determine the allowable stresses of the attachment welds for weld strength path check. The allowable stress of the welds should be considered equal to the lesser of the two allowable stresses joined. Per UW-15(c) and UG-45(c), the allowable stresses for groove/fillet welds in percentages of stress value for the vessel material, used with UG-41 calculations are as follows:

$$Groove\ Weld\ Tension: 74\%$$
$$Groove\ Weld\ \ Shear: 60\%$$
$$Fillet\ Weld\ \ Shear: 49\%$$
$$Nozzle\ \ Neck\ \ Shear: 70\%$$

1) Groove Weld Shear:

$$S_{gws} = 0.6\left(11400\right) = 6840\ \ psi$$

2) Fillet Weld Shear:

$$S_{fws} = 0.49\left(11400\right) = 5586\ \ psi$$

c) STEP 3 – Determine the Strength of Connection Elements

1) Groove Weld Shear:

$$GWS = \frac{\pi}{2}\left(Mean\ Diameter\ of\ Weld\right)\left(Weld\ Leg\right)\left(S_{gws}\right)$$

$$GWS = \frac{\pi}{2}\left(16.875\right)\left(0.8125\right)\left(6840\right) = 147313.7\ lbs$$

2) Fillet Weld Shear:

$$FWS = \frac{\pi}{2}\left(Nozzle\ OD\right)\left(Weld\ Leg\right)\left(S_{fws}\right)$$

$$FWS = \frac{\pi}{2}\left(26.0\right)\left(0.75\right)\left(5590\right) = 171224.7\ lbs$$

d) STEP 4 – Check Weld Strength Paths

$$Path_{1-1} = GWS + FWS = 147313.7 + 171224.7 = 318538.4\ lbs$$

e) STEP 5 – Weld Path Acceptance Criteria:

Per UG-41(b)(1):

$$Not\ required,\ see\ STEP\ 1$$

Per UG-42(b)(2):

$$\min\left[Path_{1-1},\ Path_{2-2},\ Path_{3-3}\right] \geq W$$

$$\left\{Path_{1-1} = 318538.4\right\} \geq \left\{W = 317667.8\right\} \qquad True$$

Section VIII, Division 2 Solution

There is no comparable weld detail for this nozzle attachment in VIII-2, Part 4.2. Therefore, no calculation is performed.

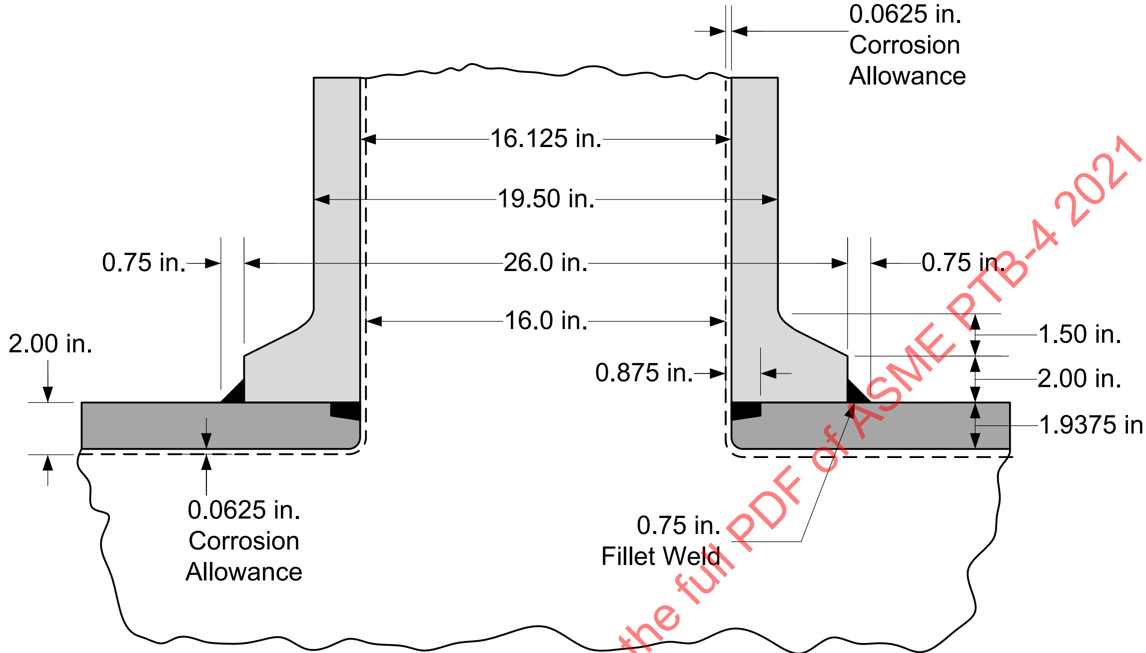

Figure E4.5.4 – Nozzle Details

4.5.5 Example E4.5.5 – Pad Reinforced Radial Nozzle in Cylindrical Shell

Check the design of a radial nozzle in a cylindrical shell based on the vessel and nozzle data below. Verify the adequacy of the attachment welds. Calculate the shear stresses from the applied nozzle loads and compare to the acceptance criteria of UG-45. The parameters used in this design procedure are shown in Figure E4.5.5.

Vessel and Nozzle Data:

- Design Conditions = $500\ psig @ 400°F$
- Vessel and Nozzle Corrosion Allowance = $0.25\ in$
- Weld Joint Efficiency = 1.0
- Shell Allowable Stress = $13700\ psi$
- Nozzle Allowable Stress = $13700\ psi$
- Reinforcement Pad Allowable Stress = $13700\ psi$
- Shell Inside Diameter = $83.0\ in$
- Shell Thickness = $2.0\ in$
- Nozzle Outside Diameter = $16.0\ in$
- Nozzle Thickness (fabricated from plate) = $0.75\ in$
- Reinforcement Pad Diameter = $28.25\ in$
- Reinforcement Pad Thickness = $1.5\ in$
- Applied Shear Load = $25000\ lbs$
- Applied Torsional Moment = $250000\ in-lbs$

The nozzle has a set–in type configuration and the opening does not pass through a vessel Category A joint, see Figure UW-16.1(q). All category A joints are to be fully radiographed (see UW-3).

Establish the corroded dimensions.

Shell:

$$D = 83.0 + 2(Corrosion\ \ Allowance) = 83.0 + 2(0.25) = 83.5\ in$$

$$R = \frac{D}{2} = \frac{83.5}{2} = 41.75\ in$$

$$t = 2.0 - Corrosion\ \ Allowance = 2.0 - 0.25 = 1.75\ in$$

Nozzle:

$$t_n = 0.75 - Corrosion\ \ Allowance = 0.75 - 0.25 = 0.5\ in$$

$$R_n = \frac{D_n - 2(Corroded\ \ Nozzle\ Thickness)}{2} = \frac{16.0 - 2(0.5)}{2} = 7.5\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-37.

The required thickness of the shell based on circumferential stress is given by UG-27(c)(1).

$$t_r = \frac{PR}{SE - 0.6P} = \frac{500(41.75)}{13700(1.0) - 0.6(500)} = 1.5578 \ in$$

The required thickness of the nozzle based on circumferential stress is given by UG-27(c)(1).

$$t_{rn} = \frac{PR_n}{SE - 0.6P} = \frac{500(7.5)}{13700(1.0) - 0.6(500)} = 0.2799 \ in$$

a) STEP 1 – Determine the Minimum Nozzle Thickness per UG-45.

1) For access openings and openings used only for inspection:

$$t_{UG-45} = t_a$$

$$\textit{Not applicable}$$

2) For other nozzles:

$$t_{UG-45} = \max\left[t_a, \ t_b\right]$$

where,

$$t_b = \min\left[t_{b3}, \ \max\left[t_{b1}, \ t_{b2}\right]\right]$$

t_a, the minimum neck thickness required for internal or external pressure using UG-27 and UG-28 (plus corrosion allowance), as applicable. The effects of external forces and moments from supplemental loads (see UG-22) shall be considered. Shear stresses caused by UG-22 loadings shall not exceed 70% of the allowable tensile stress for the nozzle material.

$$t_a = t_{rn} + Corrosion \ Allowance = 0.2799 + 0.25 = 0.5299 \ in$$

t_{b1}, for vessels under internal pressure, the thickness (plus corrosion allowance) required for pressure (assuming $E = 1.0$) for the shell or head at the location where the nozzle neck or other connection attaches to the vessel but in no case less than the minimum thickness specified for the material in UG-16(b).

$$t_{b1} = \max\left[t_{rE=1.0} + Corrosion \ Allowance, \ t_{UG-16b}\right]$$

$$t_{b1} = \max\left[1.5578 + 0.25, \ 0.0625\right] = 1.8078 \ in$$

t_{b2}, for vessels under external pressure, the thickness (plus corrosion allowance) obtained by using the external design pressure as an equivalent internal design pressure (assuming $E = 1.0$) in the formula for the shell or head at the location where the nozzle neck or other connection attaches to the vessel but in no case less than the minimum thickness specified for the material in UG-16(b).

$$t_{b2} = \max\left[t_{rE=1.0} + Corrosion \ Allowance, \ t_{UG-16b}\right]$$

$$\textit{Not applicable}$$

t_{b3}, the thickness given in Table UG-45 plus the thickness added for corrosion allowance.

$$t_{b3} = t_{TABLE UG-45} + Corrosion\ \ Allowance = 0.328 + 0.25 = 0.578\ in$$

therefore,

$$t_b = \min\left[t_{b3},\ \max\left[t_{b1},\ t_{b2}\right]\right] = \min\left[0.578,\ \max\left[1.8078,\ 0.0\right]\right] = 0.578\ in$$

and,

$$t_{UG-45} = \max\left[t_a,\ t_b\right] = \max\left[0.5299,\ 0.578\right] = 0.578\ in$$

Since $\{t_n = 0.75\ in\} \geq \{t_{UG-45} = 0.578\ in\}$ the nozzle thickness satisfies UG-45 criteria.

b) STEP 2 – Calculate the maximum membrane shear stress due to the superimposed shear and torsion loads and compare to the allowable shear stress.

As specified in the definition of t_a in UG-45:

$$S_s = 0.70S = 0.7(13700) = 9590\ \ psi$$

Membrane shear stress from shear load:

$$S_{sl} = \frac{Shear\ \ Load}{\pi r t_n} = \frac{25000}{\pi(7.5)(0.75)} = 1415\ \ psi$$

Membrane shear stress from torsional moment:

$$S_{tl} = \frac{Torsion\ \ Load}{2\pi R_n^2 t} = \frac{250000}{2\pi(7.5)^2(0.5)} = 1415\ \ psi$$

Total membrane shear stress:

$$S_{st} = S_{sl} + S_{tl} = 2122 + 1415 = 3537\ \ psi$$

Since $\{S_{st} = 3537\ psi\} \leq \{S_s = 9590\ psi\}$ the nozzle is adequately designed for the applied shear loads.

c) STEP 3 – Calculate the required weld sizes per UW-16(d) and Figure UW-16.1 Sketch (q). See Figure E4.5.5 of this example.

1) Outer nozzle fillet weld, based on throat dimensions:

$$t_c = \min\left[0.25\ in,\ 0.7t_{min}\right]$$

$$t_c = \min\left[0.25\ in,\ 0.7\left(\min\left[0.75\ in,\ thickness\ \ of\ \ thinner\ \ parts\ \ joined\right]\right)\right]$$

$$t_c = \min\left[0.25,\ 0.7\left(\min\left[0.75,\ 0.5\right]\right)\right] = 0.25\ in$$

$$t_{cact} = 0.7(weld\ \ leg\ \ size) = 0.7(0.375) = 0.2625\ in$$

$$\{t_{cact} = 0.2625\ in\} > \{t_c = 0.25\ in\} \qquad\qquad True$$

2) Outer reinforcing element fillet weld, based on throat dimensions:

$$Throat_r = 0.5t_{\min} = 0.5\left(\min\left[0.75\ in,\ thickness\ of\ thinner\ parts\ joined\right]\right)$$
$$Throat_r = 0.5\left(\min\left[0.75,\ 1.5\right]\right) = 0.375\ in$$
$$Throat_{act} = 0.7\left(weld\ leg\ size\right) = 0.7\left(0.875\right) = 0.6125\ in$$
$$\left\{Throat_{act} = 0.6125\ in\right\} > \left\{Throat_r = 0.375\ in\right\} \qquad True$$

3) Reinforcing element groove weld:

$$t_w = 0.7t_{min} = 0.7\left(\min\left[0.75\ in,\ thickness\ of\ thinner\ parts\ joined\right]\right)$$
$$t_w = 0.7\left(\min\left[0.75,\ 0.5\right]\right) = 0.35\ in$$
$$t_{wact} = 0.375\ in$$
$$\left\{t_{wact} = 0.375\ in\right\} > \left\{t_c = 0.35\ in\right\} \qquad True$$

4) Shell groove weld:

$$t_w = 0.7t_{min} = 0.7\left(\min\left[0.75\ in,\ thickness\ of\ thinner\ parts\ joined\right]\right)$$
$$t_w = 0.7\left(\min\left[0.75,\ 0.5\right]\right) = 0.35\ in$$
$$t_{wact} = 0.375\ in$$
$$\left\{t_{wact} = 0.375\ in\right\} > \left\{t_c = 0.35\ in\right\} \qquad True$$

d) STEP 4 – Calculate the Limits of Reinforcement per UG-40.

1) Reinforcing dimensions for a reinforced nozzle per Figure UG-40 sketch (b-1). See Figure E4.5.5 of this example.

2) Finished opening chord length.

$$d = 2R_n = 2\left(7.5\right) = 15.0\ in$$

3) The limits of reinforcement measured parallel to the vessel wall in the corroded condition.

$$\max\left[d,\ R_n + t_n + t\right] = \max\left[15.0,\ \left\{7.5 + 0.5 + 1.75\right\}\right] = 15.0\ in$$

4) The limits of reinforcement measured normal to the vessel wall in the corroded condition.

$$\min\left[2.5t,\ 2.5t_n + t_e\right] = \min\left[2.5\left(1.75\right),\ \left\{2.5\left(0.5\right) + 1.5\right\}\right] = 2.75\ in$$

e) STEP 5 – Calculate the reinforcement strength parameters per UG-37.

1) Strength Reduction Factors:

$$f_{r1} = S_n / S_v = 13700/13700 = 1.0$$
$$f_{r2} = S_n / S_v = 13700/13700 = 1.0$$
$$f_{r3} = \min\left[S_n, S_p\right] / S_v = \min\left[13700,\ 13700\right]/13700 = 1.0$$
$$f_{r4} = S_p / S_v = 13700/13700 = 1.0$$

2) Joint Efficiency Parameter: For a nozzle located in a solid plate, $E_1 = 1.0$.

3) Correction Factor for variation of internal pressure stresses on different planes with respect to the axis of the vessel: For a pad reinforced radial nozzle in a cylindrical shell, $F = 1.0$.

f) STEP 6 – Calculate the Areas of Reinforcement, see Figure UG–37.1

1) Area Required, A:

$$A = dt_r F + 2t_n t_r F\left(1 - f_{r1}\right)$$

$$A = 15.0(1.5578)(1.0) + 2(0.5)(1.5578)(1.0)(1-1.0) = 23.367\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11} = d\left(E_1 t - Ft_r\right) - 2t_n\left(E_1 t - Ft_r\right)\left(1 - f_{r1}\right)$$

$$A_{11} = \left\{ \begin{array}{l} 15.0\left(1.0(1.75) - 1.0(1.5578)\right) - \\ 2(0.5)\left(1.0(1.75) - 1.0(1.5578)\right)(1-1.0) \end{array} \right\} = 2.883\ in^2$$

$$A_{12} = 2\left(t + t_n\right)\left(E_1 t - Ft_r\right) - 2t_n\left(E_1 t - Ft_r\right)\left(1 - f_{r1}\right)$$

$$A_{12} = \left\{ \begin{array}{l} 2(1.75+0.5)\left(1.0(1.75) - 1.0(1.5578)\right) - \\ 2(0.5)\left(1.0(1.75) - 1.0(1.5578)\right)(1-1.0) \end{array} \right\} = 0.8649\ in^2$$

$$A_1 = \max\left[2.883,\ 0.8649\right] = 2.883\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use the smaller value:

$$A_{21} = 5\left(t_n - t_{rn}\right) f_{r2} t$$

$$A_{21} = 5(0.5 - 0.2799)(1.0)(1.75) = 1.9259\ in^2$$

$$A_{22} = 2\left(t_n - t_{rn}\right)\left(2.5t_n + t_e\right) f_{r2}$$

$$A_{22} = 2(0.5 - 0.2799)\left(2.5(0.5) + 1.5\right)(1.0) = 1.2106\ in^2$$

$$A_2 = \min\left[1.9259,\ 1.2106\right] = 1.2106\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3:

$$A_3 = \min\left[5tt_i f_{r2},\ 5t_i t_i f_{r2},\ 2ht_i f_{r2}\right]$$

$$A_3 = 0.0 \qquad since\ t_i = 0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.4 of this example:

$$\begin{array}{ll} Outer\ Nozzle\ Fillet\ Weld\ Leg: & 0.375\ in \\ Outer\ Element\ Fillet\ Weld\ Leg: & 0.875\ in \\ Inner\ Nozzle\ Fillet\ Weld\ Leg: & 0.0\ in \end{array}$$

$$A_{41} = leg^2 f_{r3} = (0.375)^2(1.0) = 0.1406\ in^2$$

$$A_{42} = leg^2 f_{r4} = (0.875)^2(1.0) = 0.7656\ in^2$$

$$A_{43} = 0.0\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = \left(D_p - d - 2t_n\right)t_e f_{r4} = \left(28.25 - 15.0 - 2\left(0.5\right)\right)\left(1.5\right)\left(1.0\right) = 18.375\ in^2$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + \left(A_{41} + A_{42} + A_{43}\right) + A_5$$
$$A_{avail} = 2.883 + 1.2106 + 0.0 + \left(0.1406 + 0.7656 + 0.0\right) + 18.375 = 23.3748\ in^2$$

g) STEP 7 – Nozzle reinforcement acceptance criterion:

$$\left\{A_{avail} = 23.3748\ in^2\right\} \geq \left\{A = 23.367\ \ in^2\right\} \qquad True$$

Therefore, the nozzle is adequately reinforced.

The load to be carried by the welds is calculated in accordance with UG-41.

a) STEP 1 – Per Figure UG-41.1, sketch (a) Nozzle Attachment Weld Loads and Weld Strength Paths to be Considered; typical nozzle detail with nozzle neck inserted through (set–in) the vessel wall.

Per UG-41(b)(1):

1) Weld Load for Strength Path 1-1, W_{1-1}.

$$W_{1-1} = \left(A_2 + A_5 + A_{41} + A_{42}\right)S_v = \left(1.2106 + 18.375 + 0.1406 + 0.7656\right)\left(13700\right) = 279504.7\ lbs$$

2) Weld Load for Strength Path 2-2, W_{2-2}.

$$W_{2-2} = \left(A_2 + A_3 + A_{41} + A_{43} + 2t_n t f_{r1}\right)S_v$$
$$W_{2-2} = \left(1.2106 + 0.0 + 0.1406 + 0.0 + 2\left(0.5\right)\left(1.75\right)\left(1.0\right)\right)\left(13700\right) = 41993.2\ lbs$$

3) Weld Load for Strength Path 3-3, W_{3-3}.

$$W_{3-3} = \begin{pmatrix} A_2 + A_3 + A_5 + A_{41} + \\ A_{42} + A_{43} + 2 \cdot t_n \cdot t \cdot f_{r1} \end{pmatrix} S_v$$
$$W_{3-3} = \begin{pmatrix} 1.2106 + 0.0 + 18.375 + \\ 0.1406 + 0.7656 + 0 + 2\left(0.5\right)\left(1.75\right)\left(1.0\right) \end{pmatrix}\left(13700\right) = 304712.7\ lbs$$

Per UG-41(b)(2): Total Weld Load, W.

$$W = \left(A - A_1 + 2t_n f_{r1}\left(E_1 t - F t_r\right)\right)S_v$$
$$W = \left(23.367 - 2.883 + 2\left(0.5\right)\left(1.0\right)\left(1.0\left(1.75\right) - 1.0\left(1.5578\right)\right)\right)\left(13700\right) = 283263.9\ lbs$$

Since W is smaller than W_{3-3}, W may be used in place of W_{3-3} for comparing weld capacity to weld load.

b) STEP 2 – Determine the allowable stresses of the attachment welds for weld strength path check. The allowable stress of the welds should be considered equal to the lesser of the two allowable stresses joined. Per UW-15(c) and UG-45(c), the allowable stresses for groove/fillet welds in percentages of stress value for the vessel material, used with UG-41 calculations are as follows:

$$Groove\ Weld\ Tension : 74\%$$
$$Groove\ Weld\ \ Shear : 60\%$$

$$\text{Fillet Weld Shear} : 49\%$$
$$\text{Nozzle Neck Shear} : 70\%$$

1) Fillet Weld Shear – Outer Nozzle Fillet and Outer Element Fillet:

$$S_{nfws} = S_{efws} = 0.49(13700) = 6713 \ psi$$

2) Groove Weld Tension – Nozzle Groove Weld and Element Groove Weld:

$$S_{ngwt} = S_{egwt} = 0.74(13700) = 10138 \ psi$$

3) Groove Weld Shear:

$$S_{gws} = 0.60(13700) = 8220 \ psi$$

4) Nozzle Wall Shear:

$$S_{nws} = 0.70(13700) = 9590 \ psi$$

c) STEP 3 – Determine the Strength of Connection Elements

1) Outer Nozzle Fillet Weld Shear:

$$ONWS = \frac{\pi}{2}(Nozzle \ OD)(Weld \ Leg)(S_{nfws})$$

$$ONWS = \frac{\pi}{2}(16.0)(0.375)(6713) = 63268.5 \ lbs$$

2) Outer Element Fillet Weld Shear:

$$OEWS = \frac{\pi}{2}(Reinforcing \ Element \ OD)(Weld \ Leg)(S_{efws})$$

$$OEWS = \frac{\pi}{2}(28.25)(0.875)(6713) = 260653.2 \ lbs$$

3) Nozzle Groove Weld Tension:

$$NGWT = \frac{\pi}{2}(Nozzle \ OD)(Weld \ Leg)(S_{ngwt})$$

$$NGWT = \frac{\pi}{2}(16.0)(0.375)(10138) = 95548.4 \ lbs$$

4) Element Groove Weld Tension:

$$EGWT = \frac{\pi}{2}(Nozzle \ OD)(Weld \ Leg)(S_{egwt})$$

$$EGWT = \frac{\pi}{2}(16.0)(0.375)(10138) = 95548.4 \ lbs$$

5) Nozzle Wall Shear:

$$NWS = \frac{\pi}{2}(Mean\ Nozzle\ Diameter)(t_n)(S_{nws})$$

$$NWS = \frac{\pi}{2}(15+0.5)(0.5)(9590) = 116745.5\ lbs$$

d) STEP 4 – Check Weld Strength Paths.

1) $Path_{1-1} = OEWS + NWS = 260653.2 + 116745.5 = 377398.7\ lbs$

2) $Path_{2-2} = ONWS + EGWT + NGWT = 63268.5 + 95548.4 + 95548.4 = 254365.3\ lbs$

3) $Path_{3-3} = OEWS + NGWT = 260653.2 + 95548.4 = 356201.6\ lbs$

e) STEP 5 – Weld Path Acceptance Criteria:

Per UG-41(b)(1):

$$\{Path_{1-1} = 377398.7\ lbs\} \geq \{W_{1-1} = 279504.7\ lbs\} \qquad True$$
$$\{Path_{2-2} = 254365.3\ lbs\} \geq \{W_{2-2} = 41993.2\ lbs\} \qquad True$$
$$\{Path_{3-3} = 356201.6\ lbs\} \geq \{W_{3-3} = 304712.7\ lbs\} \qquad True$$

Per UG-42(b)(2):

$$\min[Path_{1-1},\ Path_{2-2},\ Path_{3-3}] \geq W$$
$$\min[377398.7,\ 254365.3,\ 356201.6\ lbs] \geq \{W = 283263.9\ lbs\} \qquad False$$

$Path_{2-2}$ does not have sufficient strength to resist load W but the weld is acceptable by UG-41(b)(1).

Section VIII, Division 2 Solution

There is no comparable weld detail for this nozzle attachment in VIII-2, Part 4.2. Therefore, no calculation is performed.

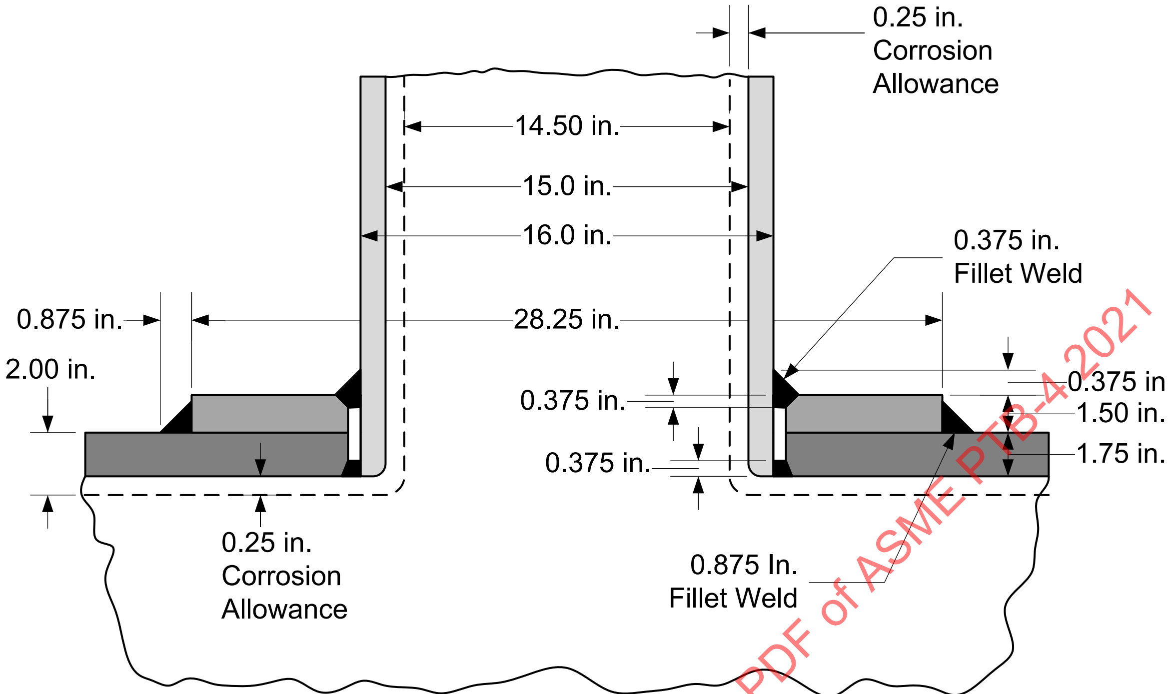

Figure E4.5.5 – Nozzle Details

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4.5.6 Example E4.5.6 – Radial Nozzle in an Ellipsoidal Head with Inside Projection

Check the design of a radial nozzle centrally located in a 2:1 ellipsoidal head based on the vessel and nozzle data below. Verify the adequacy of the attachment welds. The parameters used in this design procedure are shown in Figure E4.5.6.

Vessel and Nozzle Data:

- Design Conditions = $150\ psig@400°F$
- Vessel and Nozzle Corrosion Allowance = $0.0\ in$
- Radiography = $Not\ Performed$
- Shell Allowable Stress = $17500\ psi$
- Nozzle Allowable Stress = $12000\ psi$
- Head Inside Diameter = $23.625\ in$
- Head Thickness = $0.1875\ in$
- Nozzle Outside Diameter = $NPS\ 8 \rightarrow 8.625\ in$
- Nozzle Thickness = $SCH\ 20 \rightarrow 0.25\ in$
- Nozzle Internal Projection = $0.500\ in$

The nozzle has a set–in type configuration with an internal projection. The opening does not pass through a vessel Category A joint, see Figure UW-16.1(i). There is no radiography performed for this vessel.

Establish the dimensions.

Ellipsoidal Head:

$$D = 23.625\ in$$

$$R = \frac{D}{2} = \frac{23.625}{2} = 11.8125\ in$$

$$t = 0.1875\ in$$

Nozzle:

$$t_n = 0.25\ in$$

$$R_n = \frac{D_n - 2(Nozzle\ Thickness)}{2} = \frac{8.625 - 2(0.25)}{2} = 4.0625\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-37.

The required thickness of the 2:1 elliptical head based on circumferential stress is given by UG-32(d). However, per UG-37(a), when an opening and its reinforcement are in an ellipsoidal head and located entirely within a circle the center which coincides with the center of the head and the diameter of which is equal to 80% of the shell diameter, t_r is the thickness required for a seamless sphere of radius $K_1 D$, where K_1 is given in Table UG-37.

Per Table UG-37, for a 2:1 elliptical head where, $D/2h = 2 \rightarrow K_1 = 0.9$.

Since no radiography was specified for this vessel, the requirements of UW-11(a)(5)(b) were not satisfied and a joint efficiency of 0.85 is applied to the Category B weld attaching the cylinder to the seamless 2:1 ellipsoidal head. See UW-12(d).

The required thickness, t_r, for the head per UG-37 definition for nozzle reinforcement calculations.

$$t_r = \frac{PDK}{2SE - 0.2P} = \frac{150(23.625)(0.9)}{2(17500)(1.0) - 0.2(150)} = 0.0912 \ in$$

The required thickness of the nozzle based on circumferential stress is given by UG-27(c)(1).

$$t_{rn} = \frac{PR_n}{SE - 0.6P} = \frac{150(4.0625)}{12000(1.0) - 0.6(150)} = 0.0512 \ in$$

a) STEP 1 – Calculate the required weld sizes per UW-16(d) and Figure UW-16.1 Sketch (i). See Figure E4.5.6 of this example.

Outer/Inner nozzle fillet weld, based on throat dimensions,

$$t_1 \ or \ t_2 \geq \min\left[0.25 \ in, \ 0.7t_{min}\right]$$

$$t_1 \ or \ t_2 \geq \min\left[0.25 \ in, \ 0.7\left(\min\left[0.75 \ in, \ thickness \ of \ thinner \ parts \ joined\right]\right)\right]$$

$$t_1 \ or \ t_2 \geq \min\left[0.25, \ 0.7\left(\min\left[0.75, \ 0.1875\right]\right)\right] = 0.1313 \ in$$

$$t_{1act} = t_{2act} = 0.7\left(weld \ leg \ size\right) = 0.7\left(0.25\right) = 0.175 \ in$$

$$\left\{t_{1act} = t_{2act} = 0.175 \ in\right\} > \left\{t_1 = t_2 = 0.1313 \ in\right\} \qquad True$$

and,

$$t_1 + t_2 \geq 1.25t_{\min}$$

$$t_1 + t_2 \geq 1.25\left(\min\left[0.75 \ in, \ thickness \ of \ thinner \ parts \ joined\right]\right)$$

$$\left\{t_1 + t_2 = 0.175 + 0.175 = 0.350\right\} \geq \left\{1.25\left(\min\left[0.75, \ 0.1875\right]\right) = 0.2344\right\} \qquad True$$

b) STEP 2 – Calculate the Limits of Reinforcement per UG-40.

1) Reinforcing dimensions for a reinforced nozzle per Figure UG-40 sketch (l). See Figure E4.5.6 of this example.

2) Finished opening chord length.

$$d = 2R_n = 2\left(4.0625\right) = 8.125 \ in$$

3) The limits of reinforcement measured parallel to the vessel wall in the corroded condition.

$$\max\left[d, \ R_n + t_n + t\right] = \max\left[8.125, \ \left\{4.0625 + 0.25 + 0.1875\right\}\right] = 8.125 \ in$$

4) The limits of reinforcement measured normal to the vessel wall in the corroded condition.

i) Outside of vessel:

$$\min\left[2.5t, \ 2.5t_n + t_e\right] = \min\left[2.5\left(0.1875\right), \ \left\{2.5\left(0.25\right) + 0.0\right\}\right] = 0.4688 \ in$$

ii) Inside of vessel:

$$\min\left[h,\ 2.5t,\ 2.5t_i\right] = \min\left[0.5,\ 2.5(0.1875),\ 2.5(0.5)\right] = 0.4688\ in$$

c) STEP 3 – Calculate the reinforcement strength parameters per UG-37.

1) Strength Reduction Factors:

$$f_{r1} = S_n / S_v = 12000/17500 = 0.6857$$
$$f_{r2} = S_n / S_v = 12000/17500 = 0.6857$$
$$f_{r3} = \min\left[S_n, S_p\right]/S_v = \min\left[12000,\ 0.0\right]/17500 = 0.0$$
$$f_{r4} = S_p / S_v = 0.0/17500 = 0.0$$

2) Joint Efficiency Parameter: For a nozzle located in a solid plate, $E_1 = 1.0$.

3) Correction Factor for variation of internal pressure stresses on different planes with respect to the axis of the vessel: For a radial nozzle in an ellipsoidal head, $F = 1.0$.

d) STEP 4 – Calculate the Areas of Reinforcement, see Figure UG–37.1

1) Area Required, A:

$$A = dt_rF + 2t_nt_rF\left(1 - f_{r1}\right)$$
$$A = 8.125(0.0912)(1.0) + 2(0.25)(0.0912)(1.0)(1 - 0.6857) = 0.7553\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11} = d\left(E_1t - Ft_r\right) - 2t_n\left(E_1t - Ft_r\right)\left(1 - f_{r1}\right)$$
$$A_{11} = \left\{\begin{array}{l} 8.125(1.0(0.1875) - 1.0(0.0912)) - \\ 2(0.25)(1.0(0.1875) - 1.0(0.0912))(1 - 0.6857) \end{array}\right\} = 0.7673\ in^2$$
$$A_{12} = 2\left(t + t_n\right)\left(E_1t - Ft_r\right) - 2t_n\left(E_1t - Ft_r\right)\left(1 - f_{r1}\right)$$
$$A_{12} = \left\{\begin{array}{l} 2(0.1875 + 0.25)(1.0(0.1875) - 1.0(0.0912)) - \\ 2(0.25)(1.0(0.1875) - 1.0(0.0912))(1 - 0.6857) \end{array}\right\} = 0.0691\ in^2$$
$$A_1 = \max\left[0.7673,\ 0.0691\right] = 0.7673\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use the smaller value:

$$A_{21} = 5\left(t_n - t_{rn}\right)f_{r2}t$$
$$A_{21} = 5(0.25 - 0.0512)(0.6857)(0.1875) = 0.1278\ in^2$$

$$A_{22} = 5\left(t_n - t_{rn}\right)f_{r2}t_n$$
$$A_{22} = 5(0.25 - 0.0512)(0.6857)(0.25) = 0.1704\ in^2$$
$$A_2 = \min\left[0.1278,\ 0.1704\right] = 0.1278\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3:

$$A_3 = \min\left[5tt_i f_{r2}, 5t_i t_i f_{r2}, 2ht_i f_{r2}\right]$$

$$A_3 = \min\left[5(0.1875)(0.25)(0.6857), 5(0.25)^2(0.6857), 2(0.5)(0.25)(0.6857)\right]$$

$$A_3 = \min\left[0.1607, 0.2143, 0.1714\right] = 0.1607\ in^2$$

5) Area Available in Welds, $A_{41}, A_4 2, A_{43}$, use the following minimum specified weld leg dimensions, see Figure E4.5.4 of this example:

$$\begin{array}{ll} Outer\ Nozzle\ Fillet\ Weld\ Leg: & 0.25\ in \\ Outer\ Element\ Fillet\ Weld\ Leg: & 0.0\ in \\ Inner\ Nozzle\ Fillet\ Weld\ Leg: & 0.25\ in \end{array}$$

$$A_{41} = leg^2 f_{r2} = (0.25)^2 (0.6857) = 0.0429\ in^2$$

$$A_{42} = 0.0\ in^2$$

$$A_{43} = leg^2 f_{r2} = (0.25)^2 (0.6857) = 0.0429\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = \left(D_p - d - 2t_n\right) t_e f_{r4} = 0.0\ in^2$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + \left(A_{41} + A_{42} + A_{43}\right) + A_5$$

$$A_{avail} = 0.7673 + 0.1278 + 0.1607 + (0.0429 + 0.0 + 0.0429) + 0.0 = 1.1416\ in^2$$

e) STEP 5 – Nozzle reinforcement acceptance criterion:

$$\left\{A_{avail} = 1.1416\ in^2\right\} \geq \left\{A = 0.7553\ in^2\right\} \qquad True$$

Therefore, the nozzle is adequately reinforced.

The load to be carried by the welds is calculated in accordance with UG-41.

a) STEP 1 – Per Figure UG-41.1, sketch (a) Nozzle Attachment Weld Loads and Weld Strength Paths to be Considered; typical nozzle detail with nozzle neck inserted through (set–in) the vessel wall.

Per UG-41(b)(1):

1) Weld Load for Strength Path 1-1, W_{1-1}.

$$W_{1-1} = \left(A_2 + A_5 + A_{41} + A_{42}\right) S_v = (0.1278 + 0.0 + 0.0429 + 0.0)(17500) = 2987.3\ lbs$$

2) Weld Load for Strength Path 2-2, W_{2-2}.

$$W_{2-2} = \begin{pmatrix} A_2 + A_3 + A_{41} + \\ A_{43} + 2 \cdot t_n \cdot t \cdot f_{r1} \end{pmatrix} S_v$$

$$W_{2-2} = \begin{pmatrix} 0.1278 + 0.1607 + 0.0429 + \\ 0.0429 + 2(0.25)(0.1875)(0.6857) \end{pmatrix} (17500) = 7675.2\ lbs$$

Per UG-41(b)(2):

Total Weld Load, W.

$$W = \left(A - A_1 + 2 \cdot t_n \cdot f_{r1}\left(E_1 t - F t_r\right)\right) S_v$$

$$W = \begin{pmatrix} 0.7553 - 0.7673 + 2(0.25) \cdot (0.6857) \cdot \\ (1.0(0.1875) - 1.0(0.0912)) \end{pmatrix} (17500) = 367.8 \; lbs$$

Since W is smaller than W_{1-1} and W_{2-2}, W may be used in place of W_{1-1} and W_{2-2} for comparing weld capacity to weld load.

b) STEP 2 – Determine the allowable stresses of the attachment welds for weld strength path check. The allowable stress of the welds should be considered equal to the lesser of the two allowable stresses joined. Per UW-15(c) and UG-45(c), the allowable stresses for groove/fillet welds in percentages of stress value for the vessel material, used with UG-41 calculations are as follows:

$$Groove\ Weld\ Tension : 74\%$$

$$Groove\ Weld\ Shear : 60\%$$

$$Fillet\ Weld\ Shear : 49\%$$

$$Nozzle\ Neck\ Shear : 70\%$$

1) Fillet Weld Shear – Outer Nozzle Fillet and Inner Nozzle Fillet:

$$S_{ofws} = S_{ifws} = 0.49(12000) = 5880 \; psi$$

2) Nozzle Wall Shear:

$$S_{nws} = 0.70(12000) = 8400 \; psi$$

c) STEP 3 – Determine the Strength of Connection Elements

1) Outer Nozzle Fillet Weld Shear:

$$ONWS = \frac{\pi}{2}(Nozzle\ OD)(Weld\ Leg)\left(S_{ofws}\right)$$

$$ONWS = \frac{\pi}{2}(8.625)(0.25)(5880) = 19915.7 \; lbs$$

2) Inner Nozzle Fillet Weld Shear:

$$INWS = \frac{\pi}{2}(Nozzle\ OD)(Weld\ Leg)\left(S_{ifws}\right)$$

$$INWS = \frac{\pi}{2}(8.625)(0.25)(5880) = 19915.7 \; lbs$$

3) Nozzle Wall Shear:

$$NWS = \frac{\pi}{2}\left(Mean\ \ Nozzle\ \ Diameter\right)\left(t_n\right)\left(S_{nws}\right)$$

$$NWS = \frac{\pi}{2}\left(8.125 + 0.25\right)\left(0.25\right)\left(8400\right) = 27626.4\ lbs$$

d) STEP 4 – Check Weld Strength Paths

1) $Path_{1-1} = ONWS + NWS = 19915.7 + 27626.4 = 47542.1\ lbs$

2) $Path_{2-2} = ONWS + INWS = 19915.7 + 19915.7 = 39831.4\ lbs$

e) STEP 5 – Weld Path Acceptance Criteria:

Per UG-41(b)(1):

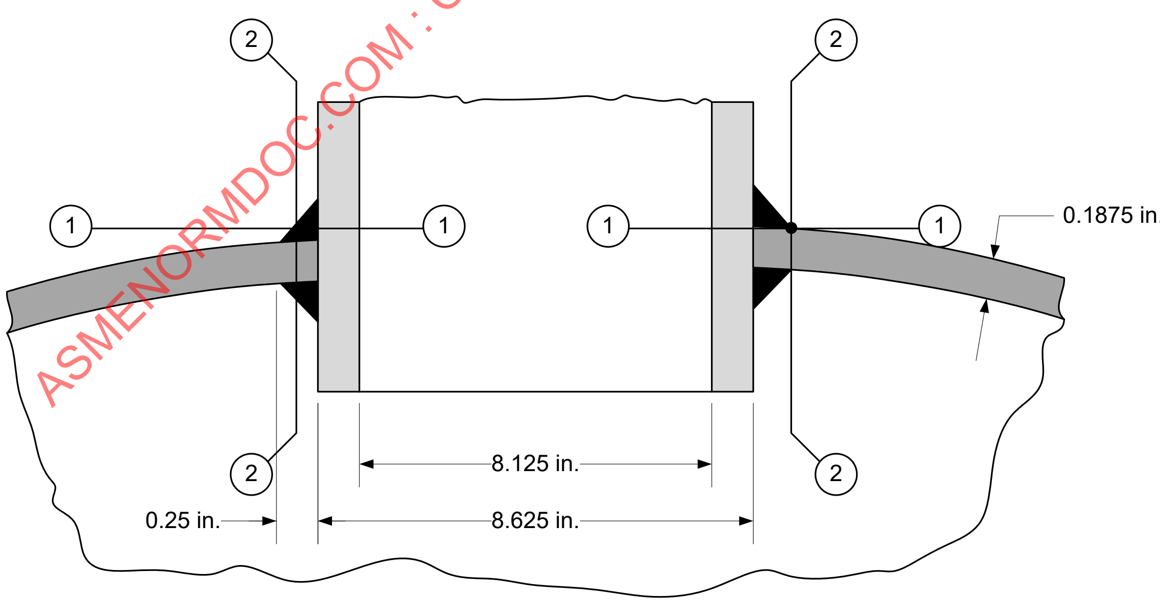
$$\left\{Path_{1-1} = 47542.1\ lbs\right\} \geq \left\{W_{1-1} = 2987.3\ lbs\right\} \qquad True$$
$$\left\{Path_{2-2} = 39831.4\ lbs\right\} \geq \left\{W_{2-2} = 7675.2\ lbs\right\} \qquad True$$

Per UG-42(b)(2):

$$\min\left[Path_{1-1},\ Path_{2-2}\right] \geq W$$
$$\min\left[47542.1,\ 39831.4\ lbs\right] \geq \left\{W = 367.8\ lbs\right\} \qquad True$$

<u>Section VIII, Division 2 Solution</u>

There is no comparable weld detail for this nozzle attachment in VIII-2, Part 4.2. Therefore, no calculation is performed.



4.5.7 Example E4.5.7 – Piping Load Evaluation of ASME B16.5 Nozzle Flange

Evaluate the following ASME B16.5 Class 150 Weld Neck Flange attached to a vessel nozzle considering the anticipated applied axial force and bending moment determined from a pipe flexibility analysis.

Nozzle/Flange and Loading Data:

- Design Conditions = $200\ psi\ @\ 300°F$
- Flange Material = $SA-105$
- Bolt Material = $SA-193,\ Grade\ B7$
- Nominal Pipe Size = $NPS\ 10$
- ASME B16.5 Weld Neck Flange = $Class\ 150$
- ASME B16.5 Pressure/Temperature Rating = $230\ psig\ @\ 300°F$
- Gasket Type = $Class\ 150\ SWG$
- Gasket Outside/Inside Diameter = $12.5\ in\ /\ 11.31\ in$
- External Tensile Axial Force = $5000\ lbs$
- External Moment = $120000.0\ in-lbs$

Reference Standards:

ASME B16.5 *Pipe Flanges and Flanged Fittings NPS ½ Through NPS 24*

Table II-2-1.1 Pressure-Temperature Ratings for Group 1.1 Materials

ASME B16.20 *Metallic Gaskets for Pipe Flanges – Ring Joint, Spiral-Wound, and Jacketed*

Table I-4 Dimensions for Spiral-Wound Gaskets Used with ASME B16.5 Flanges

Section VIII, Division 1 Solution

Evaluate per UG-44(b).

External loads (forces and moments) may be evaluated for flanged joints with welding neck flanges chosen in accordance with ASME B16.5 and ASME B16.47 using the following requirements.

1) The vessel MAWP (corrected for the static pressure acting on the flange) at the design temperature cannot exceed the pressure-temperature rating of the flange.
2) The actual assembly bolt load (see Nonmandatory Appendix S) shall comply with ASME PCC-1, Nonmandatory Appendix O.
3) The bolt material shall have an allowable stress equal to or greater than $SA-193, Grade\ B7, Class\ 2$ at the specified bolt size and temperature.
4) The combination of vessel MAWP (corrected for the static pressure acting on the flange) with external moment and external axial force shall satisfy the following equation (the units of the variables in this equation shall be consistent with the pressure rating).

$$\left\{\begin{array}{l} 16M_E + 4F_E G \le \pi G^3\left[\left(P_R - P_D\right) + F_M P_R\right] \\ 16\left(120000\right) + 4\left(5000\right)\left(11.905\right) \le \pi\left(11.905\right)^3\left[\left(230 - 200\right) + 1.2\left(230\right)\right] \\ 2158100\ in-lbs \le 1622032.3\ in-lbs \end{array}\right\} \qquad False$$

where,

$$F_E = 5000\ lbs$$

$$F_M = 1.2 \qquad \left(per\ Table\ UG-44-1,\ See\ Figure\ E4.5.7\right)$$

$$G = \frac{Gasket\ OD - Gasket\ ID}{2} = \frac{12.5 - 11.31}{2} = 11.905\ in$$

$$M_E = 120000\ in-lbs$$

$$P_D = 230\ psig$$

$$P_R = 200\ psig$$

Since the above expression is not satisfied, the Class 150 flange is not adequate for the proposed combination of applied loads and design pressure. The designer may consider the following options.

1) Reduce the applied loads on the nozzle flange via modifications to the support layout for the piping system,

2) Use a Class 300 flange, pending satisfaction of the above expression.

Table UG-44-1
Moment Factor, F_M

Standard	Size Range	Flange Pressure Rating Class					
		150	300	600	900	1500	2500
ASME B16.5	≤NPS 12	1.2	0.5	0.5	0.5	0.5	0.5
	>NPS 12 and ≤NPS 24	1.2	0.5	0.5	0.3	0.3	...
ASME B16.47							
Series A	All	0.6	0.1	0.1	0.1	...	...
Series B	<NPS 48	[Note (1)]	[Note (1)]	0.13	0.13	...	...
	≥NPS 48	0.1	[Note (2)]	...	...	...	...

GENERAL NOTES:
(a) The combinations of size ranges and flange pressure classes for which this Table gives no moment factor value are outside the scope of this Table.
(b) The designer should consider reducing the moment factor if the loading is primarily sustained in nature and the bolted flange joint operates at a temperature where gasket creep/relaxation will be significant.

NOTES:
(1) F_M = [0.1 + (48 − NPS)]/56.
(2) F_M = 0.1, except for NPS 60, Class 300, in which case F_M = 0.03.

4.6 Flat Heads

4.6.1 Example E4.6.1 – Flat Unstayed Circular Heads Attached by Bolts

Determine the required thickness for a heat exchanger blind flange.

Blind Flange Data:

- Material = $SA-105$
- Design Conditions = $135\ psig@650°F$
- Flange Bolt-Up Temperature = $100°F$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $17800\ psi$
- Allowable Stress at Flange Bolt-Up Temp. = $20000\ psi$
- Weld Joint Efficiency = 1.0
- Mating flange information and gasket details are provided in Example Problem E4.16.1.

Design rules for unstayed flat heads and covers are provided in UG-34. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.6.

Evaluate the blind flange in accordance with VIII-1, UG-34 and Appendix 2.

The minimum required thickness of a flat unstayed circular head, cover, or blind flange that is attached with bolting that results in an edge moment, see VIII-1, Figure UG-34, Sketch (j), shall be calculated by the equations shown below. The operating and gasket seating bolt loads, $W = W_{m1}$ and W, and the moment arm of this load, h_G, in these equations shall be computed based on the flange geometry and gasket material as described in VIII-1, paragraph 2-5 and Table 2-6.

a) STEP 1 – Calculate the gasket moment arm, h_G, and the diameter of the gasket load reaction, d in accordance with VIII-1, Table 2-6, and paragraph 2-3, respectively, as demonstrated in Example Problem E4.16.1.

   See Flange Design Procedure, STEP 6: $h_G = 0.875\ in$

   See Gasket Reaction Diameter, STEP 3: $d = G = 29.5\ in$

b) STEP 2 – Calculate the operating and gasket seating bolt loads, $W = W_{m1}$ and W, in accordance with VIII-1, paragraph 2-5, as demonstrated in Example Problem E4.16.1.

   Design Bolt Loads, STEP 1: $W = W_{m1} = 111329.5\ lbs$

   Design Bolt Loads, STEP 4: $W = 237626.3\ lbs$

c) STEP 3 – Identify the appropriate attachment factor, C, from VIII-1, Figure UG-34 Sketch (j).

$$C = 0.3$$

d) STEP 4 – The required thickness of the blind flange is the maximum of the thickness required for the operating and gasket seating conditions.

$$t = \max\left[t_o,\ t_g\right]$$

   1) The required thickness in the operating condition is in accordance with VIII-1, UG-34, Equation (2).

$$t_o = d\sqrt{\left(\frac{CP}{SE}\right) + \left(\frac{1.9Wh_G}{SEd^3}\right)} + CA$$

$$t_o = (29.5)\sqrt{\left(\frac{0.3(135)}{17800(1.0)}\right) + \left(\frac{1.9(111329.5)(0.875)}{17800(1.0)(29.5)^3}\right)} + 0.125 = 1.6523\ in$$

2) The required thickness in the gasket seating condition is in accordance with VIII-1, UG-34, Equation (2) when $P = 0.0$.

$$t_g = d\sqrt{\frac{1.9Wh_G}{SEd^3}} + CA$$

$$t_g = (29.5)\sqrt{\frac{1.9(237626.3)(0.875)}{20000(1.0)(29.5)^3}} + 0.125 = 0.9943\ in$$

$$t = \max\left[1.6523,\ 0.9943\right] = 1.6523\ in$$

The required thickness is $1.6523\ in$.

4.6.2 Example E4.6.2 – Flat Un-stayed Non-Circular Heads Attached by Welding

Determine the required thickness for an air-cooled heat exchanger end plate. The end plate is welded to the air-cooled heat exchanger box with a full penetration Category C, Type 7 corner joint.

End Plate Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $400\ psig\ @\ 500°F$
- Short Span Length = $7.125\ in$
- Long Span Length = $9.25\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0

Design rules for unstayed flat heads and covers are provided in UG-34. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.6.

Evaluate the welded end plate in accordance with VIII-1, UG-34, and Appendix 13.

The minimum required thickness of a flat unstayed non-circular head or cover that is not attached with bolting that results in an edge moment shall be calculated by the following equations.

a) STEP 1 – Determine the short and long span dimensions of the non-circular plate, d and D, respectively (in the corroded state) as demonstrated in Example Problem E4.12.1.

$$d = H = 7.375\ in$$

$$D = h = 9.500\ in$$

Note, the variables d and D used in VIII-1, UG-34 are denoted as H and h, respectively, in VIII-1, Appendix 13.

b) STEP 2 – Calculate the Z factor in accordance with VIII-1, UG-34, Equation (4).

$$Z = \min\left[2.5, \left(3.4 - \left(\frac{2.4d}{D}\right)\right)\right] = \min\left[2.5, \left(3.4 - \left(\frac{2.4(7.375)}{9.5}\right)\right)\right] = 1.5368\ in$$

c) STEP 3 – The appropriate attachment factor, C, is taken from VIII-1, paragraph 13-4(f). For end closures of non-circular vessels constructed of flat plate, the design rules of VIII-1, UG-34 shall be used except that 0.20 shall be used for the value of C in all the calculations.

$$C = 0.20$$

d) STEP 4 – Calculate the required thickness using VIII-1, UG-34, Equation (3).

$$t = d\sqrt{\frac{ZCP}{SE}} + CA = 7.375\sqrt{\frac{1.5368(0.20)(400)}{20000(1.0)}} + 0.125 = 0.7032\ in$$

The required thickness is $0.7032\ in$.

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4.6.3 Example E4.6.3 – Integral Flat Head with a Centrally Located Opening

Determine if the stresses in the integral flat head with a centrally located opening are within acceptable limits, considering the following design conditions. The head, shell and opening detail is shown in Figure E4.6.3. The vessel is fabricated from Type 304 stainless steel with an allowable stress of 18.8 ksi.

End Plate Data:

- Design Conditions = $100\ psig@100°F$
- Outside diameter of flat head and shell, A = $72\ in$
- Inside diameter of shell, B_s = $70\ in$
- Diameter of central opening, B_n = $40\ in$
- Thickness of the flat head, t = $3.0\ in$
- Thickness of nozzle above the transition, g_{0n} = $0.5625\ in$
- Thickness of nozzle at the flat head, g_{1n} = $1.125\ in$
- Length of nozzle transition, h_n = $2.0\ in$
- Thickness of shell below transition, g_{0s} = $1.0\ in$
- Thickness of shell at head, g_{1s} = $2.0\ in$
- Length of shell transition, h_s = $3.0\ in$
- Allowable stress = $18800\ psi$

Design rules for Integral Flat Head with a Centrally Located Opening are provided in Mandatory Appendix 14. The rules in this appendix are the same as those provided in VIII-2, paragraph 4.6. The design procedure in VIII-2, paragraph 4.6 is used in this example problem with substitute references made to VIII-1, Mandatory Appendix 14, and Appendix 2 paragraphs.

Evaluate the integral flat head with a single, circular, centrally located opening in accordance with VIII-1, Appendix 14.

a) STEP 1 – Determine the design pressure and temperature of the flat head opening.

See the specified data above.

b) STEP 2 – Determine the geometry of the flat head opening.

See Figure E4.6.3 and the specified data above.

c) STEP 3 – Calculate the operating moment, M_o, using the following equation in accordance with VIII-1, paragraph 14-3(a)(1) with reference to paragraphs 2-3, 2-6 and Table 2-6.

$$M_o = M_D + M_T = H_D h_D + M_T h_T$$
$$M_o = 125600(14.44) + 259050(7.5) = 3756225\ in-lbs$$

where, the flange forces, H_D and H_T, are calculated as follows.

$$H_D = 0.785 B_n^2 P = 0.785(40)^2(100) = 125600\ lbs$$
$$H = 0.785 B_s^2 P = 0.785(70)^2(100) = 384650\ lbs$$
$$H_T = H - H_D = 384650 - 125600 = 259050\ lbs$$

And the moment arms, h_D and h_T, are calculated as follows:

$$h_D = R + \frac{g_{1n}}{2} = 13.88 + \frac{1.125}{2} = 14.44 \ in$$

$$h_T = \frac{R + g_{1n}}{2} = \frac{13.88 + 1.125}{2} = 7.5 \ in$$

where,

$$R = \frac{B_s - B_n}{2} - g_{1n} = \frac{70 - 40}{2} - 1.125 = 13.88 \ in$$

d) STEP 4 – Calculate F, V, and f based on B_n, g_{1n}, g_{0n}, and h_n using the equations/direct interpretation from VIII-1 Table 2-7.1 and Figure 2-7.2, Figure 2-7.3 and Figure 2-7.6 and designate the resulting values as F_n, V_n, and f_n.

Figure 2-7.2:

$$g_{rn} = \frac{g_{1n}}{g_{0n}} = \frac{1.125}{0.5625} = 2.0$$

$$h_{0n} = \sqrt{B_n g_{0n}} = \sqrt{(40)(0.5625)} = 4.75 \ in$$

$$h_{rn} = \frac{h_n}{h_{0n}} = \frac{2.0}{4.75} = 0.421$$

Interpretation of Figure 2-7.2, $F_n \approx 0.84$. From the equations of Table 2-7.1, $F_n = 0.843$.

Figure 2-7.3:

With $g_{rn} = 2.0$ and $h_{rn} = 0.421$:

Interpretation of Figure 2-7.3, $V_n \approx 0.25$. From the equations of Table 2-7.1, $V_n = 0.252$.

Figure 2-7.6:

With $g_{rn} = 2.0$ and $h_{rn} = 0.421$:

Interpretation of Figure 2-7.6, $f_n \approx 1.5$. From the equations of Table 2-7.1, $f_n = 1.518$.

e) STEP 5 – Calculate F, V, and f based on B_s, g_{1s}, g_{0s}, and h_s using the equations/direct interpretation from VIII-1 Table 2-7.1 and Figure 2-7.2, Figure2-7.3 and Figure 2-7.6, and designate the resulting values as F_s, V_s, and f_s.

Figure 2-7.2:

$$g_{rs} = \frac{g_{1s}}{g_{0s}} = \frac{2.0}{1} = 2.0$$

$$h_{0s} = \sqrt{B_s g_{0s}} = \sqrt{(70)(1)} = 8.37 \ in$$

$$h_{rs} = \frac{h_s}{h_{0s}} = \frac{3.0}{8.37} = 0.359$$

Interpretation of Figure 2-7.2, $F_s \approx 0.86$. From the equations of Table 2-7.1, $F_s = 0.857$.

Figure 2-7.3:

With $g_{rs} = 2.0$ and $h_{rs} = 0.359$:

Interpretation of Figure 2-7.3, $V_s \approx 0.28$. From the equations of Table 2-7.1, $V_s = 0.276$.

Figure 2-7.6:

With $g_{rs} = 2.0$ and $h_{rs} = 0.359$:

Interpretation of Figure 2-7.6, $f_s \approx 1.8$. From the equations of Table 2-7.1, $f_s = 1.79$.

f) STEP 6 – Calculate Y, T, U, Z, L, e, and d based on $K = A/B_n$ using the equations/direct interpretation from VIII-1 Figure 2-7.1.

$$K = \frac{A}{B_n} = \frac{72}{40} = 1.8$$

$$Y = \frac{1}{K-1}\left[0.66845 + 5.71690\left(\frac{K^2 \log_{10} K}{K^2 - 1}\right)\right]$$

$$Y = \frac{1}{(1.8)-1}\left[0.66845 + 5.71690\left(\frac{(1.8)^2 \log_{10}[1.8]}{(1.8)^2 - 1}\right)\right] = 3.47$$

$$T = \frac{K^2\left(1 + 8.55246 \log_{10} K\right) - 1}{\left(1.04720 + 1.9448K^2\right)(K-1)} = \frac{(1.8)^2\left(1 + 8.55246 \log_{10}[1.8]\right) - 1}{\left(1.04720 + 1.9448(1.8)^2\right)(1.8-1)} = 1.58$$

$$U = \frac{K^2\left(1 + 8.55246 \log_{10} K\right) - 1}{1.36136\left(K^2 - 1\right)(K-1)} = \frac{(1.8)^2\left(1 + 8.55246 \log_{10}[1.8]\right) - 1}{1.36136\left((1.8)^2 - 1\right)\left((1.8)-1\right)} = 3.82$$

$$Z = \frac{\left(K^2 + 1\right)}{\left(K^2 - 1\right)} = \frac{\left((1.8)^2 + 1\right)}{\left((1.8)^2 - 1\right)} = 1.89$$

$$d = \frac{U g_{on}^2 h_{on}}{V_n} = \frac{(3.82)(0.5625)^2(4.75)}{0.252} = 23\ in^3$$

$$e = \frac{F_n}{h_{on}} = \frac{0.843}{4.75} = 0.18\ in^{-1}$$

$$L = \frac{te+1}{T} + \frac{t^3}{d} = \frac{(3)(0.18)+1}{1.58} + \frac{(3)^3}{23} = 2.15\ in$$

g) STEP 7 – Calculate the quantity $(E\theta)^*$ for an opening with an integrally attached nozzle using the following equation, VIII-1, paragraph 14-3(b)(1).

$$(E\theta)^* = \frac{0.91\left(\frac{g_{1n}}{g_{on}}\right)^2 (B_{1n}) V_n}{f_n h_{0n}} \cdot S_H$$

$$(E\theta)^* = \frac{0.91\left(\frac{1.125}{0.5625}\right)^2 (40.5625)(0.252)}{(1.518)(4.75)} \cdot (52287) = 269584 \ psi$$

where, B_{1n} is evaluated from paragraph 2-3 and S_H is evaluated from VIII-1, paragraph 2-7.

$$B_{1n} = B_n + g_{0n} = 40 + 0.5625 = 40.5625 \ in$$

$$S_H = \frac{f_n M_o}{L g_{1n}^2 B_n} = \frac{1.518(3756225)}{2.15(1.125)^2 (40)} = 52287 \ psi$$

h) STEP 8 – Calculate the quantity M_H using the following equation, VIII-1, paragraph 14-3(c).

$$M_H = \frac{(E\theta)^*}{\frac{1.74 h_{0s} V_s}{g_{0s}^3 B_{1s}} + \frac{(E\theta)^*}{M_o}\left(1 + \frac{F_s t}{h_{0s}}\right)}$$

$$M_H = \frac{269584}{\frac{1.74(8.37)(0.276)}{(1)^3 (71.0)} + \frac{269584}{3756225}\left(1 + \frac{(0.857)(3)}{8.37}\right)} = 1792262 \ in-lb$$

where, B_{1s} is evaluated from paragraph 2-3.

$$B_{1s} = B_s + g_{0s} = 70 + 1.0 = 71.0 \ in$$

i) STEP 9 – Calculate the quantity X_1 using the following equation, VIII-1, paragraph 14-3(d).

$$X_1 = \frac{M_o - M_H\left(1 + \frac{F_s t}{h_{0s}}\right)}{M_o} = \frac{3756225 - 1792262\left(1 + \frac{(0.857)(3)}{8.37}\right)}{3756225} = 0.376$$

j) STEP 10 – Calculate the stresses at the shell-to-flat head junction in accordance with VIII-1, paragraph 14-3(e)(1) and the opening-to-flat-head junction in accordance with VIII-1, paragraph 14-3(e)(2).

Longitudinal hub stress in shell:

$$S_{HS} = \frac{1.10 f_s X_1 (E\theta)^* (h_{0s})}{\left(\frac{g_{1s}}{g_{0s}}\right)^2 B_s V_s} = \frac{1.10(1.79)(0.376)(269584)(8.37)}{\left(\frac{2}{1}\right)^2 (70)(0.276)} = 21621 \ psi$$

Radial stress at outside diameter:

$$S_{RS} = \frac{1.91 M_H \left(1 + \frac{F_s t}{h_{0s}}\right)}{B_s t^2} + \frac{0.64 F_s M_H}{B_s h_{0s} t} = \left(\frac{1.91(1792262)\left(1 + \frac{(0.857)(3)}{8.37}\right)}{(70)(3)^2} + \frac{0.64(0.857)(1792262)}{(70)(8.37)(3)} \right) = 7663 \ psi$$

Tangential stress at outside diameter:

$$S_{TS} = \frac{X_1 (E\theta)^* t}{B_s} - \frac{0.57 M_H \left(1 + \frac{F_s t}{h_{0s}}\right)}{B_s t^2} + \frac{0.64 Z F_s M_H}{B_s h_{0s} t}$$

$$S_{TS} = \left(\frac{(0.376)(269584)(3)}{70} - \frac{0.57(1792262)\left(1 + \frac{(0.857)(3)}{8.37}\right)}{(70)(3)^2} + \frac{0.64(1.89)(0.857)(1792262)}{(70)(8.37)(3)} \right)$$

$$S_{TS} = 3286 \ psi$$

Longitudinal hub stress in central opening:

$$S_{HO} = X_1 S_H = (0.376)(52287) = 19672 \ psi$$

Radial stress at central opening:

$$S_{RO} = X_1 S_R = (0.376)(8277) = 3114 \ psi$$

Where, S_R is evaluated from VIII-1, paragraph 2-7.

$$S_R = \frac{(1.33te + 1) M_o}{L t^2 B_n} = \frac{(1.33(3)(0.18) + 1)(3756225)}{(2.15)(3)^2 (40)} = 8277 \ psi$$

Tangential stress at diameter of central opening:

$$S_{TO} = X_1 S_T + \frac{0.64 Z_1 F_s M_H}{B_s h_{os} t} = \left((0.376)(20582) + \frac{0.64(2.89)(0.857)(1792262)}{(70)(8.37)(3)} \right) = 9362 \ psi$$

Where, Z_1 is calculated as follows and S_T is evaluated from VIII-1, paragraph 2-7.

$$Z_1 = \frac{2K^2}{K^2 - 1} = \frac{2(1.8)^2}{\left((1.8)^2 - 1\right)} = 2.89$$

$$S_T = \frac{YM_o}{t^2 B_n} - ZS_R = \frac{(3.47)(3756225)}{(3)^2(40)} - (1.89)(8277) = 20582\ psi$$

k) STEP 11 – Check the flange stress acceptance criteria in VIII-1, paragraph 14-3(f) with reference to paragraph 2-8. If the stress criteria are satisfied, then the design is complete. If the stress criteria are not satisfied, then re-proportion the flat head and/or opening dimensions and go to STEP 3.

Shell-to-flat-head junction:

$$\left\{S_{HS} = 21621\ psi\right\} \le \left\{1.5S_f = 1.5(18800) = 28200\ psi\right\} \qquad True$$

$$\left\{S_{RS} = 7663\ psi\right\} \le \left\{S_f = 18800\ psi\right\} \qquad True$$

$$\left\{S_{TS} = 3286\ psi\right\} \le \left\{S_f = 18800\ psi\right\} \qquad True$$

$$\left\{\frac{(S_{HS} + S_{RS})}{2} = \frac{(21621 + 7663)}{2} = 14642\ psi \le S_f = 18800\ psi\right\} \qquad True$$

$$\left\{\frac{(S_{HS} + S_{TS})}{2} = \frac{(21621 + 3286)}{2} = 12454\ psi \le S_f = 18800\ psi\right\} \qquad True$$

Opening-to-flat-head junction:

$$\left\{S_{HO} = 19672\ psi \le 1.5S_f = 1.5(18800) = 28200\ psi\right\} \qquad True$$

$$\left\{S_{RO} = 3114\ psi\right\} \le \left\{S_f = 18800\ psi\right\} \qquad True$$

$$\left\{S_{TO} = 9362\ psi\right\} \le \left\{S_f = 18800\ psi\right\} \qquad True$$

$$\left\{\frac{(S_{HO} + S_{RO})}{2} = \frac{(19672 + 3114)}{2} = 11393\ psi \le S_f = 18800\ psi\right\} \qquad True$$

$$\left\{\frac{(S_{HO} + S_{TO})}{2} = \frac{(19672 + 9362)}{2} = 14517\ psi \le S_f = 18800\ psi\right\} \qquad True$$

Stress acceptance criteria are satisfied, the design is complete.

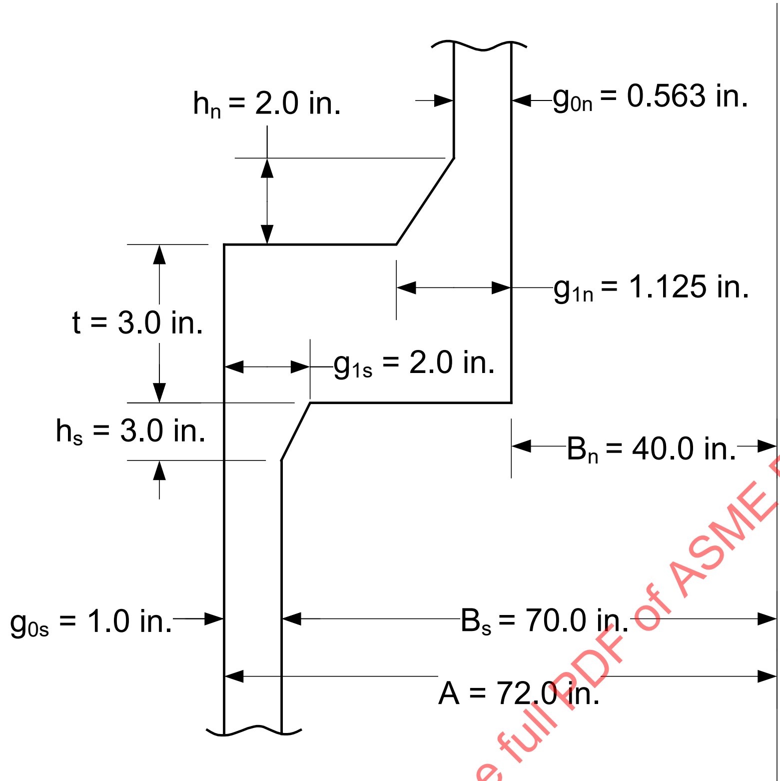

Figure E4.6.3 – Head, Shell, and Nozzle Geometry

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4.7 Spherically Dished Bolted Covers

4.7.1 Example E4.7.1 – Thickness Calculation for a Type D Head

Determine if the proposed Type D spherically dished bolted cover, used in a heat exchanger application, is adequately designed considering the following design conditions. The spherically dished head is seamless. See Figure E4.7.1 for details.

Tubeside Data:

- Design Conditions = $213\ psig\ @\ 400°F$
- Corrosion Allowance (CAT) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Shellside Data:

- Design Conditions = $305\ psig\ @\ 250°F$
- Corrosion Allowance (CAS) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Flange Data:

- Material = $SA-105$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Allowable Stress at Tubeside Design Temperature = $20000\ psi$
- Allowable Stress at Shellside Design Temperature = $20000\ psi$

Head Data:

- Material = $SA-515,\ Grade\ 60$
- Allowable Stress at Ambient Temperature = $17100\ psi$
- Allowable Stress at Tubeside Design Temperature = $17100\ psi$
- Allowable Stress at Shellside Design Temperature = $17100\ psi$
- Yield Stress at Shellside Design Temperature = $28800\ psi$
- Modulus of Elasticity at Shellside Design Temp. = $28.55E+06\ psi$

Bolt Data

- Material = $SA-193,\ Grade\ B7$
- Diameter = $0.75\ in^2$
- Cross-Sectional Root Area = $0.302\ in^2$
- Number of Bolts = 20
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Allowable Stress at Tubeside Design Temperature = $25000\ psi$
- Allowable Stress at Shellside Design Temperature = $25000\ psi$

<u>**Gasket Data**</u>

- Material = Solid Flat Metal (Iron/Soft Steel)
- Gasket Factor = 5.5
- Gasket Seating Factor = $18000\ psi$
- Inside Diameter = $16.1875\ in$
- Outside Diameter = $17.0625\ in$

Design rules for spherically dished bolted covers with ring type gaskets are provided in Mandatory Appendix 1-6 with reference to Mandatory Appendix 2. The rules in the paragraphs of Appendix 1-6 are the same as those provided in VIII-2, paragraph 4.7. The rules in the paragraphs of Appendix 2 are the same as those provided in VIII-2, paragraph 4.16 with noted differences as outlined in Example Problems E4.16.1 and E4.16.2.

The calculations are performed using dimensions in the corroded condition and the uncorroded condition, and the more severe case shall control. This example only evaluates the spherically dished bolted cover in the corroded condition.

Per VIII-1 Appendix 1-6(g), the thickness of the head for a Type D Head Configuration Figure 1-6 Sketch (d) shall be determined by the following equations.

a) Internal pressure (pressure on the concave side) – the head thickness shall be determined using Appendix 1-6, Equation (9).

$$t = \left(\frac{5PL}{6S}\right) = \frac{5(213)(16.125)}{6(17100)} = 0.1674\ in$$

where,

$$L = L + CAS = 16.0 + 0.125 = 16.125\ in$$

This calculated thickness is increased for corrosion allowance on both the shell and tube side.

$$t = t + CAS + CAT$$
$$t = 0.1674 + 0.125 + 0.125 = 0.4174\ in$$

b) External pressure (pressure on the convex side) – the head thickness shall be determined in accordance with the rules of paragraph UG-33(c). As noted in paragraph UG-33(c), the required thickness of a hemispherical head having pressure on the convex side shall be determined in the same manner as outlined in paragraph UG-28(d) for determining the thickness for a spherical shell.

1) STEP 1 – Assume an initial thickness, t, for the spherical shell and calculate the value of factor A using the following equation.

The specified head thickness shall consider corrosion from the tubeside and shellside, therefore, when calculating the outside radius of the head, R_o, the shellside corrosion allowance is deducted from the head thickness.

$$A = \frac{0.125}{\left(\dfrac{R_o}{t}\right)} = \frac{0.125}{\left(\dfrac{16.75}{0.625}\right)} = 0.00466$$

where,

$$R_o = L + t - CAS = 16.0 + 0.875 - 0.125 = 16.75\ in$$
$$t = t - CAS - CAT = 0.875 - 0.125 - 0.125 = 0.625\ in$$

2) STEP 2 – Using the value of A calculated in STEP 1, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically to an intersection with the material/temperature line for the tubeside temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 5.

   Per Section II Part D, Table 1A, a material specification of $SA-515, Grade\ 60$ is assigned an External Pressure Chart No. CS-2.

3) STEP 3 – From the intersection obtained in Step 2, move horizontally to the right and read the value of factor B.

$$B = 15127$$

4) STEP 4 – Using the value of B obtained in STEP 3, calculate the value of the maximum allowable external working pressure P_a using the following equation.

$$P_a = \frac{B}{\left(\dfrac{R_o}{t}\right)} = \frac{15127}{\left(\dfrac{16.75}{0.625}\right)} = 564.4\ psi$$

5) STEP 5 – For values of A falling to the left of the applicable material/temperature line, the value of P_a can be calculated using the following equation.

$$P_a = \frac{0.0625E}{\left(\dfrac{R_o}{t}\right)^2} \qquad Not\ required$$

6) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2.

   Since $\{P_a = 564.4\ psi\} > \{P = 305\ psi\}$, the specified head thickness is acceptable for external pressure.

The flange thickness of the head for a Type D Head Configuration is determined per Appendix 1-6(g), Equation (10). To compute the required flange thickness, the flange operating, and gasket seating moments are determined using VIII-1, Appendix 2.

Establish the design conditions and gasket reaction diameter.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

$$Tubeside\ Conditions: P = 213\ psig\ at\ 400°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 2-5.1.

$$m = 5.5$$

$$y = 18000\ psi$$

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5(GOD - GID) = 0.5(17.0625 - 16.1875) = 0.4375\ in$$

From Table 2-5.2, Facing Sketch Detail 2, Column I,

$$b_o = \frac{w+N}{4} = \frac{(0.125+0.4375)}{4} = 0.1406\ in$$

where,

$$w = raised\ nubbin\ width = 0.125\ in$$

for $b_o \le 0.25\ in$,

$$b = b_o = 0.1406\ in$$

Therefore, from paragraph 2-3, the location of the gasket reaction is calculated as follows.

$$G = mean\ diameter\ of\ the\ gasket\ contact\ face$$

$$G = 0.5(17.0625 + 16.1875) = 16.625\ in$$

Paragraph 2-5 – Calculate the design bolts load for the operating and gasket seating conditions.

a) STEP 1 – Paragraph 2-5(c)(1), determine the design bolt load for the operating condition.

$$W_{m1} = H + H_p = 0.785G^2P + 2b(3.14)GmP \quad for\ non-self-energized\ gaskets$$

$$W_{m1} = 0.785(16.625)^2(213) + 2(0.1406)(3.14)(16.625)(5.5)(213) = 63410.7\ lbs$$

b) STEP 2 – Paragraph 2-5(c)(2), determine the design bolt load for the gasket seating condition.

$$W_{m2} = 3.14bGy$$

$$W_{m2} = (3.14)(0.1406)(16.625)(18000) = 132114.1\ lbs$$

c) STEP 3 – Paragraph 2-5(d), determine the total required and actual bolt areas.

The total cross-sectional area of bolts A_m required for both the operating conditions and gasket seating is determined as follows.

$$A_m = \max\left[A_{m1},\ A_{m2}\right] = \max\left[2.5364,\ 5.2846\right] = 5.2846\ in^2$$

where,

$$A_{m1} = \frac{W_{m1}}{S_b} = \frac{63410.7}{25000} = 2.5364\ in^2$$

$$A_{m2} = \frac{W_{m2}}{S_a} = \frac{132114.1}{25000} = 5.2846\ in^2$$

The actual bolt area A_b is calculated as follows.

$$A_b = (Number\ of\ bolts)(Root\ area\ of\ one\ bolt) = 20(0.302) = 6.04\ in^2$$

Verify that the actual bolt area is equal to or greater than the total required area.

$$\left\{A_b = 6.04\ in^2\right\} \geq \left\{A_m = 5.2846\ in^2\right\} \qquad True$$

d) STEP 4 – Paragraph 2-5(e), determine the flange design bolt load.

For operating conditions,

$$W = W_{m1} = 63410.7\ lbs$$

For gasket seating,

$$W = \frac{(A_m + A_b)S_a}{2} = \frac{(5.2846 + 6.04)25000}{2} = 141557.5\ lbs$$

Commentary:

VIII-1, Appendix 2 does not include an overall step-by-step procedure to design a flange. However, an organized procedure of the steps taken when designing a flange is presented in VIII-2, paragraph 4.16.7. The procedure is applicable to VIII-1, Appendix 2 and is presented in this example problem to assist the designer.

The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

$$Tubeside\ Conditions: P = 213\ psig\ at\ 400°F$$
$$Shellside\ Conditions: P = 305\ psig\ at\ 250°F$$

b) STEP 2 – Determine the design bolt loads for operating condition W, and the gasket seating condition W, and the corresponding actual bolt load area A_b, paragraph 2-5.

$$W = 63410.7\ lbs \qquad Operating\ Condition$$

$$W = 141557.5\ lbs \qquad Gasket\ Seating$$

$$A_b = 6.04\ in^2$$

c) STEP 3 – Determine an initial flange geometry (see Figure E4.7.1), in addition to the information required to determine the bolt load, the following geometric parameters are required.

1) Flange bore

$$B = B + 2(CAT) = 16.25 + 2(0.125) = 16.50 \; in$$

2) Bolt circle diameter

$$C = 18.125 \; in$$

3) Outside diameter of the flange

$$A = A - 2(CAS) = 19.625 - 2(0.125) = 19.375 \; in$$

4) Flange thickness, (see Figure E4.7.1)

$$T = T - 2(CAT) = 2.3125 - 2(0.125) = 2.0625 \; in$$

5) Thickness of the hub at the large end

$$Not \;\; Applicable$$

6) Thickness of the hub at the small end

$$Not \;\; Applicable$$

7) Hub length

$$Not \;\; Applicable$$

d) STEP 4 – Determine the flange stress factors using the equations/direct interpretation from Table 2-7.1 and Figure 2-7.1 – Figure 2-7.6.

$$Not \;\; Applicable$$

e) STEP 5 – Determine the flange forces, paragraph 2-3.

<u>Tubeside Conditions:</u>

$$H_D = 0.785B^2P = (0.785)(16.5)^2(213) = 45521.6 \; lbs$$

$$H = 0.785G^2P = (0.785)(16.625)^2(213) = 46213.9 \; lbs$$

$$H_T = H - H_D = 46213.9 - 45521.6 = 692.3 \; lbs$$

$$H_G = W - H = 63410.7 - 46213.9 = 17196.8 \; lbs \qquad Operating$$

<u>Shellside Conditions:</u>

$$H_D = 0.785B^2P = (0.785)(16.5)^2(305) = 65183.5 \; lbs$$

$$H = 0.785G^2P = (0.785)(16.625)^2(305) = 66174.8 \; lbs$$

$$H_T = H - H_D = 66174.8 - 65183.5 = 991.3 \; lbs$$

$$H_G = Not\ \ Applicable$$

f) STEP 6 – Determine the flange moment for the operating condition using paragraph 2-6 for internal pressure and paragraph 2-11 for external pressure. In these equations, h_D is determined from paragraph 1-6(b), and h_T and h_G are determined from Table 2-6.

For internal pressure (Tubeside Conditions):

$$M_o = H_D h_D + H_T h_T + H_G h_G$$
$$M_o = 45521.6(0.8125) + 692.3(0.7813) + 17196.8(0.75)$$
$$M_o = 50424.8\ in - lbs$$

For external pressure (Shellside Conditions):

$$M_o = H_D(h_D - h_G) + H_T(h_T - h_G)$$
$$M_o = 65183.5(0.8125 - 0.75) + 991.3(0.7813 - 0.75)$$
$$M_o = 4105.0\ in - lbs$$

From paragraph 1-6(b),

$$h_D = \frac{C - B}{2} = \frac{18.125 - 16.50}{2} = 0.8125\ in$$

From Table 2-6 for loose type flanges,

$$h_G = \frac{C - G}{2} = \frac{18.125 - 16.625}{2} = 0.75\ in$$

$$h_T = \frac{h_D + h_G}{2} = \frac{0.8125 + 0.75}{2} = 0.7813\ in$$

g) STEP 7 – Determine the flange moment for the gasket seating condition using paragraph 2-6 for internal pressure and paragraph 2-11 for external pressure.

For internal pressure (Tubeside Conditions):

$$M_o = W\frac{(C - G)}{2} = (141557.5)\left(\frac{(18.125 - 16.625)}{2}\right) = 106168.1\ in - lbs$$

For external pressure (Shellside Conditions):

$$M_o = Wh_G = (141557.5)(0.75) = 106168.1\ in - lbs$$

where,

$$W = \frac{A_{m2} + A_b}{2} S_a = \frac{5.2846 + 6.04}{2}(25000) = 141557.5\ lbs$$

Paragraph 1-6(g)(2) – the flange thickness of the head for a Type D Head Configuration shall be determined by the following equations. When determining the flange design moment for the design condition, M_o, using paragraph 2-6, the following modifications must be made. An additional moment term, M_r, computed as shown in paragraph (1-6(b)) shall be added to M_o as defined in paragraph 2-6. Note that this term may be positive or negative depending on the location of the head–to–flange ring intersection with relation to the flange ring centroid. Since the head–to–flange ring intersection is above the flange centroid, the sign of the M_r value is negative.

$$T = \max\left[T_g,\ T_o\right] = \max\left[T_g,\ \max\left[T_{o(tubeside)},\ T_{o(shellside)}\right]\right]$$

where,

$$T_g = F + \sqrt{F^2 + J} + CAS + CAT$$

$$F = \frac{PB\sqrt{4L^2 - B^2}}{8S(A-B)}$$

$$J = \left(\frac{M_o}{SB}\right)\left(\frac{A+B}{A-B}\right)$$

and,

$$T_o = F + \sqrt{F^2 + J} + CAS + CAT$$

$$F = \frac{PB\sqrt{4L^2 - B^2}}{8S(A-B)}$$

$$J = \left(\frac{M_o}{SB}\right)\left(\frac{A+B}{A-B}\right)$$

a) STEP 1 – Calculate the additional moment, M_r, using paragraph 1-6(b) as follows:

$$M_r = H_r h_r$$

where,

$$H_r = 0.785B^2 P\cot\left[\beta_1\right]$$

$$\beta_1 = \arcsin\left[\frac{B}{2L+t}\right] = \arcsin\left[\frac{(16.5)}{2(16.125)+(0.625)}\right] = \begin{Bmatrix} 0.5258\ rad \\ 30.1259\ deg \end{Bmatrix}$$

and,

$$L = 16.0 + CAT = 16.0 + 0.125 = 16.125\ in$$

$$t = t - CAT - CAS = 0.875 - 0.125 - 0.125 = 0.625\ in$$

Commentary:

VIII-1, Appendix 1-6(b) does not include guidance as to a method to calculate the lever arm, h_r of force H_r about the centroid of the flange ring. The procedure shown in Annex E4.7.1 provides one method to calculate h_r; however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

Referencing Annex E4.7.1,

$$h_r = 0.2654\ in$$

For internal pressure (Tubeside Conditions),

$$H_r = H_D \cot[\beta_1] = 0.785B^2 P \cot[\beta_1]$$

$$H_r = (0.785)(16.5)^2 (213)\cot[30.1259] = 78447.1 \; lbs$$

$$M_r = H_r h_r = 78447.1(0.2654) = 20819.9 \; in-lbs$$

For external pressure (Shellside Conditions),

$$H_r = H_D \cot[\beta_1] = 0.785B^2 P \cot[\beta_1]$$

$$H_r = (0.785)(16.5)^2 (305)\cot[30.1259] = 112330.3 \; lbs$$

$$M_r = H_r h_r = 112330.3(0.2654) = 29812.5 \; in-lbs$$

b) STEP 2 – Calculate the modified flange moment for the design condition, M_o, using paragraph 2-6 including the additional moment, M_r.

For internal pressure (Tubeside Conditions),

$$M_{o(tubeside)} = M_o - M_r = 50424.8 - 20819.9 = 29604.9 \; in-lbs$$

For external pressure (Shellside Conditions),

$$M_{o(shellside)} = M_o - M_r = 4105.0 - 29812.5 = -25707.5 \; in-lbs$$

c) STEP 3 – Calculate the flange thickness for the gasket seating condition, T_g.

$$T_g = F + \sqrt{F^2 + J} + CAS + CAS$$

$$T_g = 0.0 + \sqrt{(0.0)^2 + 4.0145} + 0.125 + 0.125 = 2.2536 \; in$$

where,

$$F = \frac{PB\sqrt{4L^2 - B^2}}{8S(A-B)} = \frac{0.0(16.5)\sqrt{4(16.125)^2 - (16.5)^2}}{8(20000)(19.375-16.5)} = 0.0$$

$$J = \left(\frac{M_o}{SB}\right)\left(\frac{A+B}{A-B}\right) = \left(\frac{106168.1}{(20000)(16.5)}\right) \cdot \left(\frac{(19.375+16.5)}{(19.375-16.5)}\right) = 4.0145$$

d) STEP 4 – Calculate the flange thickness for the operating conditions, $T_{o(tubeside)}$ and $T_{o(shellside)}$.

For internal pressure (Tubeside Conditions),

$$T_o = F + \sqrt{F^2 + J} + CAS + CAS$$

$$T_{o(tubeside)} = 0.2117 + \sqrt{(0.2117)^2 + 1.1194} + 0.125 + 0.125 = 1.5407 \; in$$

where,

$$F = \frac{PB\sqrt{4L^2 - B^2}}{8S(A-B)} = \frac{213(16.5)\sqrt{4(16.125)^2 - (16.5)^2}}{8(20000)(19.375-16.5)} = 0.2117$$

$$J = \left(\frac{M_o}{SB}\right)\left(\frac{A+B}{A-B}\right) = \left(\frac{29604.9}{(20000)(16.5)}\right)\cdot\left(\frac{(19.375+16.5)}{(19.375-16.5)}\right) = 1.1194$$

For external pressure (Shellside Conditions),

$$T_o = F + \sqrt{F^2 + J} + CAS + CAS$$

$$T_{o(shellside)} = 0.3031 + \sqrt{(0.3031)^2 + 0.9721} + 0.125 + 0.125 = 1.5846\ in$$

where,

$$F = \frac{PB\sqrt{4L^2 - B^2}}{8S(A-B)} = \frac{|305|(16.5)\sqrt{4(16.125)^2 - (16.5)^2}}{8(20000)(19.375-16.5)} = 0.3031$$

$$J = \left(\frac{M_o}{SB}\right)\left(\frac{A+B}{A-B}\right) = \left(\frac{|-25707.5|}{(20000)(16.5)}\right)\cdot\left(\frac{(19.375+16.5)}{(19.375-16.5)}\right) = 0.9721$$

e) STEP 5 – Determine the required flange thickness using the thicknesses determined in STEP 3 and STEP 4.

$$T = \max\left[T_g,\ T_o\right] = \max\left[T_g,\ \max\left[T_{o(tubeside)},\ T_{o(shellside)}\right]\right]$$

$$T = \max\left[2.2536,\ \max\left[1.5407,\ 1.5846\right]\right] = 2.2536\ in$$

The specified head thickness, $\{t = 0.875\ in\} > \{t_{req} = 0.4174\ in\}$ for internal pressure (tubeside conditions) and the external pressure (shellside conditions) calculations verified the maximum allowable external pressure, $\{MAEP = 560.9\ psi\} > \{P_{shellside} = 305\ psi\}$.

The specified flange thickness, $\{T = 2.3125\ in\} > \{T_{req} = 2.2536\ in\}$ for design internal pressure (tubeside conditions), external pressure (shellside conditions), and gasket seating conditions. Therefore, the proposed type D spherically dished bolted cover is adequately designed.

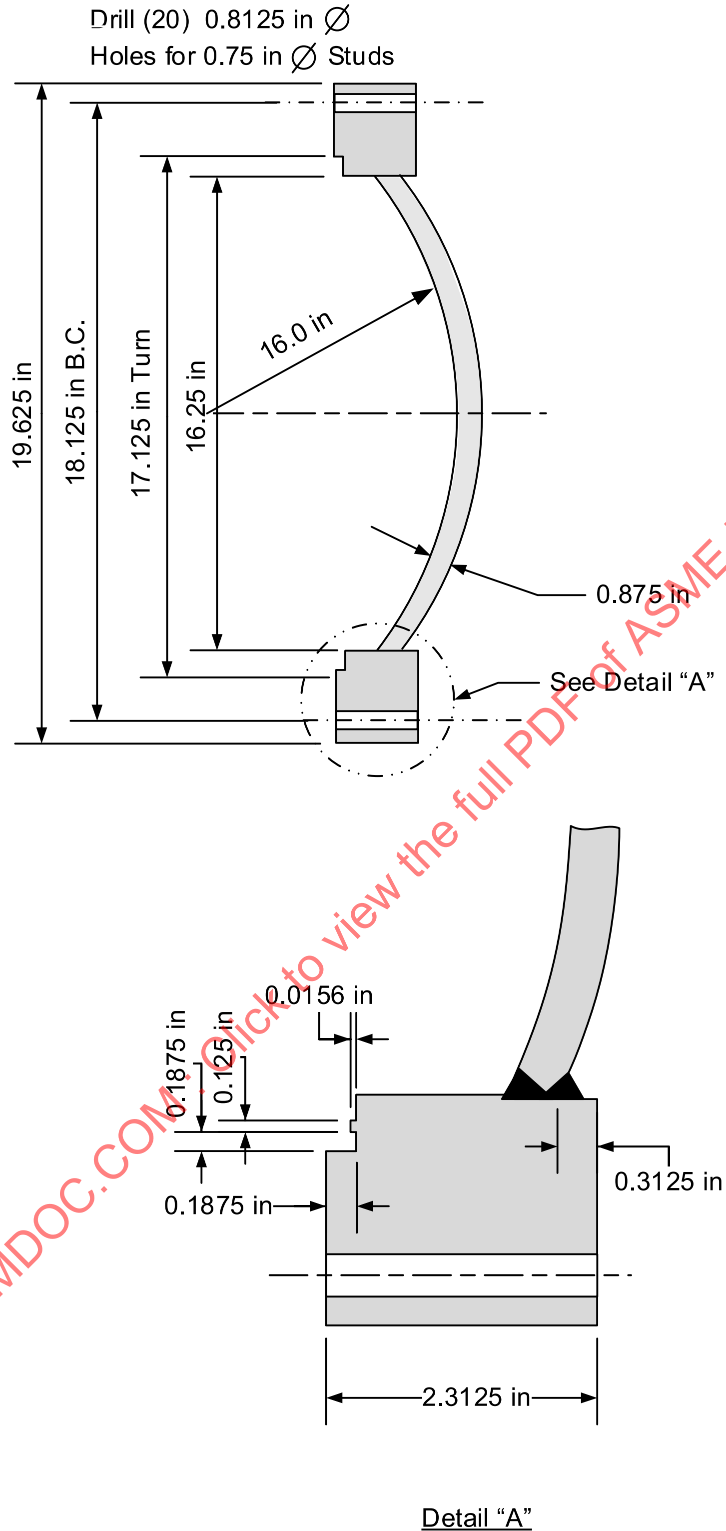

Figure E4.7.1 – Floating Head Geometry

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Annex E4.7.1

ASME VIII-1 does not provide explicit guidance for computing the lever arm, h_r. This Annex provides one possible method of satisfying U-2(g); however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

Assumptions used in the development of the procedure include the following.

1) Shellside and tubeside corrosion allowance is applied to the dished head.
2) Tubeside corrosion allowance is only applied to the flange ring inside radius.
3) Shellside corrosion allowance is applied to the outer surfaces of the flange ring.
4) The geometry of the dished head to flange ring is assumed to be rectilinear.
5) The projected thickness of the dished head and associated dimensions are based off of the angle β_1.
6) The location of the dished head to flange ring attachment point is established from the measured value from the outside depth of the flange ring to the outside base of the head. This variable is referenced as $DFHEAD$, (distance from head).
7) The ring flange is rectangular in shape, i.e., the portion of the flange that is removed by machining for the gasket surface is not considered.

The lever arm h_r measured from the centerline of the projected dished head thickness on the flange ring to the flange ring centroid is determined geometrically considering the above established assumptions. The variable $DFHEAD$ along with the inside radius and thickness of the dished head, L and t, respectively, and the flange ring radius, R set the initial location of the head centerline with the flange ring. The angle β_1 formed by the tangent to the centerline of the dished head at its point of intersection with the flange ring and a line perpendicular to the axis of the dished head is then established. From this point, the dished head and flange ring are subject to the applicable tubeside and shellside corrosion allowances resulting in the final corroded geometry from which the corroded lever arm is determined.

Refer to Figure AE4.7.1

a) STEP 1 – Establish the variables used in the calculations.

Flange (uncorroded):

$$Thickness: \quad T = 2.3125\ in$$

$$Inside\ Diameter: \quad B = 16.25\ in$$

$$Inside\ Radius: \quad R = \frac{B}{2} = \frac{16.25}{2} = 8.125\ in$$

$$Head\ \ Location: \quad DFHEAD = 0.3125\ in$$

Spherically Dished Head (uncorroded):

$$Thickness: \quad t = 0.875\ in$$

$$Inside\ Radius: \quad L = 16.0\ in$$

Shellside and Tubeside Corrosion Allowance:

$$Shellside: \qquad CAS = 0.125\ in$$
$$Tubeside: \qquad CAT = 0.125\ in$$

b) STEP 2 – Establish the corroded dimensions of the input variables.

$$t_c = t - CAS - CAT = 0.875 - 0.125 - 0.125 = 0.625\ \ in$$
$$L_{mc} = L + CAT + \frac{t_c}{2} = 16.0 + 0.125 + \frac{0.625}{2} = 16.4375\ in$$
$$T = T - 2(CAS) = 2.3125 - 2(0.125) = 2.0625\ in$$
$$R_c = R + CAT = 8.125 + 0.125 = 8.25\ in$$
$$DFHEAD_c = DFHEAD - CAS = 0.3125 - 0.125 = 0.1875\ in$$

c) STEP 3 – Calculate the angle β_1 formed by the corroded mean radius of the dished head at the intersection with the flange ring and the corresponding corroded radius of the flange ring, measured to the axis of the dished head assembly.

$$\beta_1 = \arcsin\left[\frac{R_c}{L_{mc}}\right] = \arcsin\left[\frac{8.25}{16.4375}\right] = \begin{Bmatrix} 0.5258\ rad \\ 30.1259\ deg \end{Bmatrix}$$

d) STEP 4 – Calculate the axial adjustment of $DFHEAD$ due to the applied tubeside corrosion allowance on the flange inside radius. See Figure AE4.7.1.

$$X_{DFHEAD} = CAT \cdot \tan[\beta_1] = 0.125 \cdot \tan[30.1259] = 0.0725\ in$$

e) STEP 5 – Calculate the projected shellside corrosion allowance of the dished head on the flange ring, X_{cas}.

$$X_{cas} = \frac{CAS}{\cos[\beta_1]} = \frac{0.125}{\cos[30.1259]} = 0.1445\ in$$

f) STEP 6 – Calculate the projected corroded dished head thickness on the flange ring, X_c.

$$X_c = \frac{t_c}{\cos[\beta_1]} = \frac{0.625}{\cos[30.1259]} = 0.7226\ in$$

g) STEP 7 – Calculate the moment arm based on the corroded dimensions, h_r.

$$h_r = 0.5T_c - [DFHEAD_c + X_{DFHEAD} + X_{cas}] - 0.5X_c$$
$$h_r = 0.5(2.0625) - [0.1875 + 0.0725 + 0.1445] - 0.5(0.7227) = 0.2654\ in$$

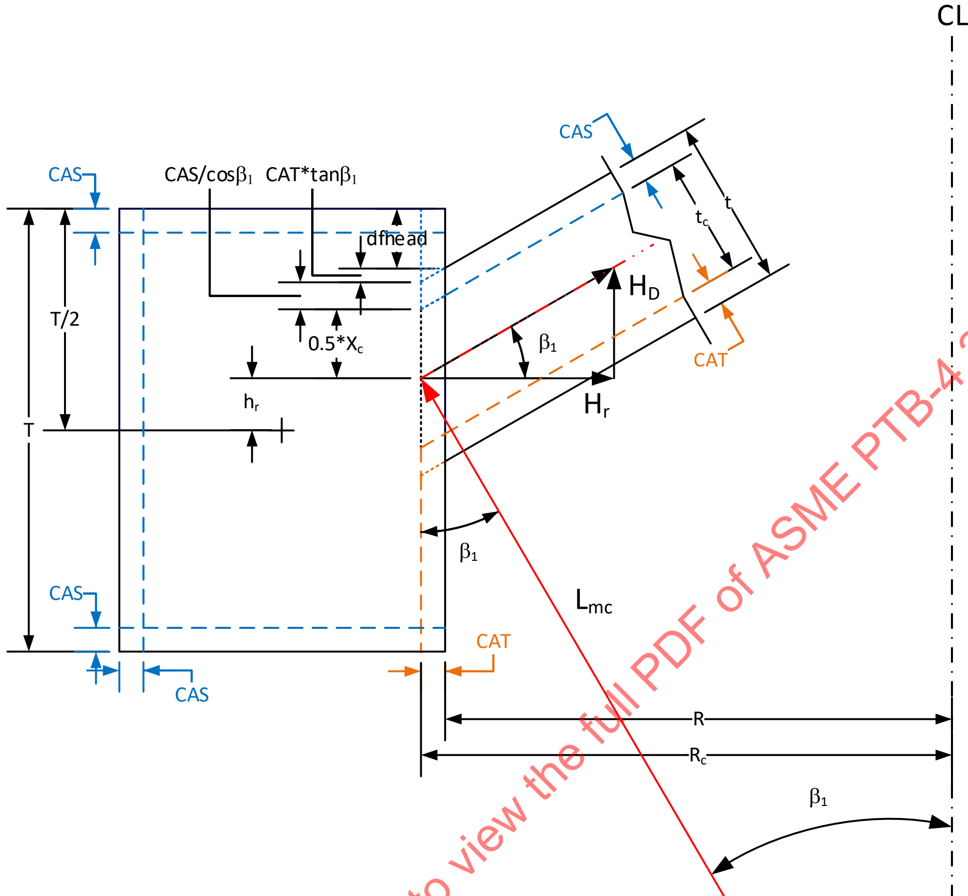

Figure AE4.7.1 – Lever Arm of Floating Head Geometry

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4.7.2 Example E4.7.2 – Thickness Calculation for a Type D Head Using the Alternative Rule in VIII-2, Paragraph 4.7.5.3

Mandatory Appendix 1-6(h) indicates that the equations for the bolted heads with a dished cover are approximate in that they do not consider continuity between the flange ring and the dished head. A more exact method of analysis which takes the continuity of the flange and head into account may be used if it meets the requirements of U-2(g). The alternate design method provided in VIII-2; paragraph 4.7.5.3 satisfies this requirement.

Determine if the proposed Type D spherically dished bolted cover is adequately designed, considering the following design conditions. The spherically dished head is seamless. Evaluate using the alternative procedure in VIII-2, paragraph 4.7.5.3.

Tubeside Data:

- Design Conditions = $213\ psig@400°F$
- Corrosion Allowance (CAT) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Shellside Data:

- Design Conditions = $305\ psig@250°F$
- Corrosion Allowance (CAS) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Flange Data:

- Material = $SA-105$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Allowable Stress at Tubeside Design Temperature = $20000\ psi$
- Allowable Stress at Shellside Design Temperature = $20000\ psi$

Head Data:

- Material = $SA-515,\ Grade\ 60$
- Allowable Stress at Ambient Temperature = $17100\ psi$
- Allowable Stress at Tubeside Design Temperature = $17100\ psi$
- Allowable Stress at Shellside Design Temperature = $17100\ psi$
- Yield Stress at Shellside Design Temperature = $28800\ psi$
- Modulus of Elasticity at Shellside Design Temp. = $28.55E+06\ psi$

Bolt Data

- Material = $SA-193,\ Grade\ B7$
- Diameter = $0.75\ in$
- Cross-Sectional Root Area = $0.302\ in^2$
- Number of Bolts = 20

- Allowable Stress at Ambient Temperature = $25000\ psi$
- Allowable Stress at Tubeside Design Temperature = $25000\ psi$
- Allowable Stress at Shellside Design Temperature = $25000\ psi$

Gasket Data

- Material = Solid Flat Metal (Iron/Soft Steel)
- Gasket Factor = 5.5
- Gasket Seating Factor = $18000\ psi$
- Inside Diameter = $16.1875\ in$
- Outside Diameter = $17.0625\ in$

Per VIII-2, paragraph 4.7.5.3, the following procedure can be used to determine the required head and flange thickness of a Type D head. This procedure accounts for the continuity between the flange ring and the head and represents a more accurate method of analysis.

a) STEP 1 – Determine the design pressure and temperature of the flange joint. When evaluating external pressure, a negative value of the pressure is used in all equations of this procedure.

$$Tubeside\ Conditions: P = 213\ psig\ at\ 400°F$$

$$Shellside\ Conditions: P = 305\ psig\ at\ 250°F$$

b) STEP 2 – Determine an initial Type D head configuration geometry (see Figure E4.7.1). The following geometry parameters are required.

1) Flange bore

$$B = B_{nom} + 2(CAT) = 16.25 + 2(0.125) = 16.50\ in$$

2) Bolt circle diameter

$$C = 18.125\ in$$

3) Outside diameter of the flange

$$A = A - 2(CAS) = 19.625 - 2(0.125) = 19.375\ in$$

4) Flange thickness, (see Figure E4.7.1)

$$T = T - 2(CAS) = 2.3125 - 2(0.125) = 2.0625\ in$$

5) Mean head radius, (see VIII-2, Figure 4.7.5)

$$R = \frac{(L + t_{nom} - CAS) + (L + CAT)}{2}$$

$$R = \frac{(16.0 + 0.875 - 0.125) + (16.0 + 0.125)}{2} = 16.4375\ in$$

6) Head thickness

$$t = t_{nom} - CAT - CAS = 0.875 - 0.125 - 0.125 = 0.625\ in$$

7) Initial inside depth of flange to the base of the head, (see Figure E4.7.2).

$$q_o = q_{nom} - CAS = 1.0 - 0.125 = 0.875\ in$$

Commentary:

Although the procedure shown in VIII-2, paragraph 4.7.5.3 provides an equation to calculate the lever arm e of shell discontinuity force V about the centroid of the flange ring, the original development of the equation did not lend itself well to the consideration of corrosion allowance for the calculation of the variable q. The procedure shown in Annex E4.7.2 provides one method of determining these adjustments to q; however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

Referencing Annex E4.7.2,

$$q = 0.9470\ in$$

c) STEP 3 – Select a gasket configuration and determine the location of the gasket reaction, G, and the design bolt loads for the gasket seating, W_g, and operating conditions, W_o, using the rules of VIII-2, paragraph 4.16. Computations for the following parameters are shown in Example Problem E4.7.1.

$$G = 16.625\ in$$

$$W_g = 141557.5\ lbs$$

$$W_o = 63410.7\ lbs$$

$$W_{oe} = -90799.4\ lbs$$

d) STEP 4 – Determine the geometry parameters.

$$h_1 = \frac{(C-G)}{2} = \frac{(18.125-16.625)}{2} = 0.75\ in$$

$$h_2 = \frac{(G-B)}{2} = \frac{(16.625-16.5)}{2} = 0.0625\ in$$

$$d = \frac{(A-B)}{2} = \frac{(19.375-16.5)}{2} = 1.4375\ in$$

$$n = \frac{T}{t} = \frac{2.0625}{0.625} = 3.3$$

$$K = \frac{A}{B} = \frac{19.375}{16.5} = 1.1742$$

$$\phi = \arcsin\left[\frac{B}{2R}\right] = \arcsin\left[\frac{16.5}{2(16.4375)}\right] = 30.1259\ deg$$

$$e = q - \frac{1}{2}\left[T - \frac{t}{\cos[\phi]}\right] = 0.9470 - \frac{1}{2}\left[2.0625 - \frac{0.625}{\cos[30.1259]}\right] = 0.2770\ in$$

$$k_1 = 1 - \left(\frac{1-2\nu}{2\lambda}\right)\cot[\phi] = 1 - \left[\frac{1-2(0.3)}{2(6.5920)}\right]\cot[30.1259] = 0.9477$$

$$k_2 = 1 - \left(\frac{1+2\nu}{2\lambda}\right)\cot[\phi] = 1 - \left[\frac{1+2(0.3)}{2(6.5920)}\right]\cot[30.1259] = 0.7909$$

where,

$$\nu = 0.3$$

$$\lambda = \left[3\left(1-\nu^2\right)\left(\frac{R}{t}\right)^2\right]^{0.25} = \left\{3\left(1-0.3^2\right)\left(\frac{16.4375}{0.625}\right)^2\right\}^{0.25} = 6.5920$$

e) STEP 5 – Determine the shell discontinuity geometry factors.

$$C_1 = \frac{0.275n^3t\cdot\ln[K]}{k_1} - e = \left(\frac{0.275(3.3)^3(0.625)\cdot\ln[1.1742]}{0.9477}\right) - (0.2770) = 0.7696$$

$$C_2 = \frac{1.1\lambda n^3 t\ln[K]}{Bk_1} + 1 = \left(\frac{1.1(6.5920)(3.3)^3(0.625)\cdot\ln[1.1742]}{(16.5)(0.9477)}\right) + 1 = 2.6726$$

$$C_4 = \frac{\lambda\sin[\phi]}{2}\left(k_2 + \frac{1}{k_1}\right) + \frac{B}{4nd} + \frac{1.65e}{tk_1}$$

$$C_4 = \left[\begin{array}{l}\dfrac{(6.5920)\sin[30.1259]}{2}\left(0.7907 + \dfrac{1}{0.9477}\right) + \\ \dfrac{16.5}{4(3.3)(1.4375)} + \dfrac{1.65(0.2770)}{(0.625)(0.9477)}\end{array}\right] = 4.6951$$

$$C_5 = \frac{1.65}{tk_1}\left(1 + \frac{4\lambda e}{B}\right) = \left(\frac{1.65}{(0.625)(0.9477)}\right)\left(1 + \frac{4(6.5920)(0.2770)}{(16.5)}\right) = 4.0188$$

f) STEP 6 – Determine the shell discontinuity load factors for the operating and gasket seating conditions.

Operating Condition – Tubeside:

$$C_{3o} = \frac{\pi B^2 P}{4}\left[e\cot[\phi] + \frac{2q(T-q)}{B} - h_2\right] - W_o h_1$$

$$C_{3o} = \left(\frac{\pi(16.5)^2(213)}{4}\right)\left[\begin{array}{l}(0.2770)\cot[30.1259] + \\ \dfrac{2(0.9470)(2.0625-0.9470)}{16.5} - 0.0625\end{array}\right] - 63410.7(0.75)$$

$$C_{3o} = -22831.9\ in-lbs$$

$$C_{6o} = \frac{\pi B^2 P}{4}\left(\frac{4q - B\cot[\phi]}{4nd} - \frac{0.35}{\sin[\phi]}\right)$$

$$C_{6o} = \frac{\pi(16.5)^2(213)}{4}\left(\frac{4(0.9470)-(16.5)\cot[30.1259]}{4(3.3)(1.4375)} - \frac{0.35}{\sin[30.1259]}\right)$$

$$C_{6o} = -90917.8\, lbs$$

Operating Condition – Shellside:

$$C_{3o} = \frac{\pi B^2 P}{4}\left[e\cot[\phi] + \frac{2q(T-q)}{B} - h_2\right] - W_o h_1$$

$$C_{3o} = \left(\frac{\pi(16.5)^2(-305)}{4}\right)\left[\begin{array}{l}(0.2770)\cot[30.1259]+ \\ \dfrac{2(0.9470)(2.0625-0.9470)}{16.5} - 0.0625\end{array}\right] - (-90799.4)(0.75)$$

$$C_{3o} = 32693.6\ in-lbs$$

$$C_{6o} = \frac{\pi B^2 P}{4}\left(\frac{4q - B\cot[\phi]}{4nd} - \frac{0.35}{\sin[\phi]}\right)$$

$$C_{6o} = \frac{\pi(16.5)^2(-305)}{4}\left(\frac{4(0.9470)-(16.5)\cot[30.1259]}{4(3.3)(1.4375)} - \frac{0.35}{\sin[30.1259]}\right)$$

$$C_{6o} = 130187.4\ lbs$$

Gasket Seating Condition:

$$C_{3g} = -W_g h_1 = -(141577.5)(0.75) = -106168.1\ in-lbs$$

$$C_{6g} = 0.0$$

g) STEP 7 – Determine the shell discontinuity force and moment for the operating and gasket condition.

Operating Condition – Tubeside:

$$V_{do} = \frac{C_2 C_{6o} - C_{3o} C_5}{C_2 C_4 - C_1 C_5}$$

$$V_{do} = \frac{(2.6726(-90917.8)) - (-22831.9(4.0188))}{(2.6726(4.6951)) - (0.7696(4.0188))} = -15994.3\ lbs$$

$$M_{do} = \frac{C_1 C_{6o} - C_{3o} C_4}{C_2 C_4 - C_1 C_5}$$

$$M_{do} = \frac{(0.7696(-90917.8)) - (-22831.9(4.6951))}{(2.6726(4.6951)) - (0.7696(4.0188))} = 3937.3\ lbs$$

Operating Condition – Shellside:

$$V_{do} = \frac{C_2 C_{6o} - C_{3o} C_5}{C_2 C_4 - C_1 C_5}$$

$$V_{do} = \frac{\left(2.6726\left(130187.4\right)\right)-\left(32693.6\left(4.0188\right)\right)}{\left(2.6726\left(4.6951\right)\right)-\left(0.7696\left(4.0188\right)\right)} = 22902.6\ lbs$$

$$M_{do} = \frac{C_1C_{6o} - C_{3o}C_4}{C_2C_4 - C_1C_5}$$

$$M_{do} = \frac{\left(0.7696\left(130187.4\right)\right)-\left(32693.6\left(4.6951\right)\right)}{\left(2.6726\left(4.6951\right)\right)-\left(0.7696\left(4.0188\right)\right)} = -5637.9\ lbs$$

Gasket Seating Condition:

$$V_{dg} = \frac{C_2C_{6g} - C_{3g}C_5}{C_2C_4 - C_1C_5} = \frac{\left(2.6726\left(0.0\right)\right)-\left(-106168.1\left(4.0188\right)\right)}{\left(2.6726\left(4.6951\right)\right)-\left(0.7696\left(4.0188\right)\right)} = 45125.0\ lbs$$

$$M_{dg} = \frac{C_1C_{6g} - C_{3g}C_4}{C_2C_4 - C_1C_5} = \frac{\left(0.7696\left(0.0\right)\right)-\left(-106168.1\left(4.6951\right)\right)}{\left(2.6726\left(4.6951\right)\right)-\left(0.7696\left(4.0188\right)\right)} = 52718.8\ lbs$$

h) STEP 8 – Calculate the stresses in the head and at the head to flange junction using VIII-2, Table 4.7.1 and check the stress criteria for both the operating and gasket conditions.

Calculated Stresses – Operating Conditions – Tubeside (omitting shellside pressure):

$$S_{hm} = \frac{PR}{2t} + P_e = \frac{213\left(16.4375\right)}{2\left(0.625\right)} + 0.0 = 2801.0\ psi$$

$$S_{hl} = \frac{PR}{2t} + \frac{V_{do}\cos\left[\phi\right]}{\pi Bt} + P_e$$

$$S_{hl} = \frac{213\left(16.4375\right)}{2\left(0.625\right)} + \frac{\left(-15994.3\right)\cos\left[30.1259\right]}{\pi\left(16.5\right)\left(0.625\right)} + 0.0 = 2373.9\ psi$$

$$S_{hb} = \frac{6M_{do}}{\pi Bt^2} = \frac{6\left(3937.3\right)}{\pi\left(16.5\right)\left(0.625\right)^2} = 1166.7\ psi$$

$$S_{hlbi} = S_{hl} - S_{hb} = 2373.9 - 1166.7 = 1207.2\ psi$$

$$S_{hlbo} = S_{hl} + S_{hb} = 2373.9 + 1166.7 = 3540.6\ psi$$

$$S_{fm}=\frac{1}{\pi BT}\left(\frac{\pi B^2 P}{4}\left(\frac{4q}{B}-\cot[\phi]\right)-V_{do}\right)\left(\frac{K^2+1}{K^2-1}\right)+P_e$$

$$S_{fm}=\left(\frac{1}{\pi(16.5)(2.0625)}\right)\left(\begin{array}{l}\left(\dfrac{\pi(16.5)^2(213)}{4}\right)\\ \left(\dfrac{4(0.9470)}{(16.5)}-\cot[30.1259]\right)-\\ (-15994.3)\end{array}\right)\left(\frac{(1.1742)^2+1}{(1.1742)^2-1}\right)+0.0$$

$$S_{fm}=-3056.3\ psi$$

$$S_{fb}=\frac{0.525n}{Btk_1}\left(V_{do}-\frac{4M_{do}\lambda}{B}\right)$$

$$S_{fb}=\frac{0.525(3.3)}{(16.5)(0.625)(0.9477)}\left((-15994.3)-\frac{4(3937.3)(6.5920)}{(16.5)}\right)=-3950.7\ psi$$

$$S_{fmbi}=S_{fm}+S_{fb}=-3056.3+(-3950.7)=-7007.0\ psi$$

$$S_{fmbo}=S_{fm}-S_{fb}=-3056.3-(-3950.7)=894.4\ psi$$

Acceptance Criteria – Operating Conditions – Tubeside:

$\{S_{hm}=2801.0\ psi\}\le\{S_{ho}=17100\ psi\}$ *True*

$\{S_{hl}=2373.9\ psi\}\le\{1.5S_{ho}=1.5(17100)=25650\ psi\}$ *True*

$\{S_{hlbi}=1207.2\ psi\}\le\{1.5S_{ho}=1.5(17100)=25650\ psi\}$ *True*

$\{S_{hlbo}=3540.6\ psi\}\le\{1.5S_{ho}=1.5(17100)=25650\ psi\}$ *True*

$\{S_{fm}=-3056.3\ psi\}\le\{S_{fo}=20000\ psi\}$ *True*

$\{S_{fmbi}=-7007.0\ psi\}\le\{1.5S_{fo}=1.5(20000)=30000\ psi\}$ *True*

$\{S_{fmbo}=894.4\ psi\}\le\{1.5S_{fo}=1.5(20000)=30000\ psi\}$ *True*

Calculated Stresses – Operating Conditions – Shellside (omitting tubeside pressure):

$$S_{hm}=\frac{PR}{2t}+P_e=\frac{(-305)(16.4375)}{2(0.625)}+(-305)=-4315.8\ psi$$

$$S_{hl}=\frac{PR}{2t}+\frac{V_{do}\cos[\phi]}{\pi Bt}+P_e$$

$$S_{hl}=\frac{(-305)(16.4375)}{2(0.625)}+\frac{(22902.6)\cos[30.1259]}{\pi(16.5)(0.625)}+(-305)=-3704.3\ psi$$

$$S_{hb} = \frac{6M_{do}}{\pi B t^2} = \frac{6(-5637.9)}{\pi(16.5)(0.625)^2} = -1670.6 \ psi$$

$$S_{hlbi} = S_{hl} - S_{hb} = -3704.3 - (-1670.6) = -2033.7 \ psi$$

$$S_{hlbo} = S_{hl} + S_{hb} = -3704.3 + (-1670.6) = -5374.9 \ psi$$

$$S_{fm} = \frac{1}{\pi B T}\left(\frac{\pi B^2 P}{4}\left(\frac{4q}{B} - \cot[\phi]\right) - V_{do}\right)\left(\frac{K^2+1}{K^2-1}\right) + P_e$$

$$S_{fm} = \left(\frac{1}{\pi(16.5)(1.875)}\right)\left(\begin{array}{l}\left(\dfrac{\pi(16.5)^2(-305)}{4}\right)\\ \left(\dfrac{4(0.8262)}{(16.5)} - \cot[30.1259]\right) - \\ (22902.6)\end{array}\right)\left(\frac{(1.1742)^2+1}{(1.1742)^2-1}\right) + (-305)$$

$$S_{fm} = 4071.3 \ psi$$

$$S_{fb} = \frac{0.525n}{Btk_1}\left(V_{do} - \frac{4M_{do}\lambda}{B}\right)$$

$$S_{fb} = \frac{0.525(3.3)}{(16.5)(0.625)(0.9477)}\left(22902.6 - \frac{4(-5637.9)(6.5920)}{(16.5)}\right) = 5657.1 \ psi$$

$$S_{fmbi} = S_{fm} + S_{fb} = 4071.3 + 5657.1 = 9728.4 \ psi$$

$$S_{fmbo} = S_{fm} + S_{fb} = 4071.3 - 5657.1 = -1585.8 \ psi$$

Acceptance Criteria – Operating Conditions – Shellside:

$$\{S_{hm} = -4315.8 \ psi\} \le \{S_{ho} = 17100 \ psi\} \qquad True$$

$$\{S_{hl} = -3704.3 \ psi\} \le \{1.5S_{ho} = 1.5(17100) = 25650 \ psi\} \qquad True$$

$$\{S_{hlbi} = -2033.7 \ psi\} \le \{1.5S_{ho} = 1.5(17100) = 25650 \ psi\} \qquad True$$

$$\{S_{hlbo} = -5374.9 \ psi\} \le \{1.5S_{ho} = 1.5(17100) = 25650 \ psi\} \qquad True$$

$$\{S_{fm} = 4071.3 \ psi\} \le \{S_{fo} = 20000 \ psi\} \qquad True$$

$$\{S_{fmbi} = 9728.4 \ psi\} \le \{1.5S_{fo} = 1.5(20000) = 30000 \ psi\} \qquad True$$

$$\{S_{fmbo} = -1585.8 \ psi\} \le \{1.5S_{fo} = 1.5(20000) = 30000 \ psi\} \qquad True$$

Calculated Stresses – Gasket Seating Conditions:

$$S_{hm} = 0.0$$

$$S_{hl} = \frac{V_{dg}\cos[\phi]}{\pi Bt} = \frac{(45125.0)\cos[30.1259]}{\pi(16.5)(0.625)} = 1204.7\ psi$$

$$S_{hb} = \frac{6M_{dg}}{\pi Bt^2} = \frac{6(52718.8)}{\pi(16.5)(0.625)^2} = 15621.5\ psi$$

$$S_{hlbi} = S_{hl} - S_{hb} = 1204.7 - 15621.5 = -14416.8\ psi$$

$$S_{hlbo} = S_{hl} + S_{hb} = 1204.7 + 15621.5 = 16826.2\ psi$$

$$S_{fm} = \frac{1}{\pi BT}(-V_{dg})\left(\frac{K^2+1}{K^2-1}\right)$$

$$S_{fm} = \left(\frac{1}{\pi(16.5)(2.0625)}\right)(-(45125.0))\left(\frac{(1.1742)^2+1}{(1.1742)^2-1}\right) = -2650.9\ psi$$

$$S_{fb} = \frac{0.525n}{Btk_1}\left(V_{dg} - \frac{4M_{dg}\lambda}{B}\right)$$

$$S_{fb} = \frac{0.525(3.3)}{(16.5)(0.625)(0.9477)}\left(45125.0 - \frac{4(52718.8)(6.5920)}{(16.5)}\right) = -6935.4\ psi$$

$$S_{fmbi} = S_{fm} + S_{fb} = -2650.9 + (-6935.4) = -9586.3\ psi$$

$$S_{fmbo} = S_{fm} - S_{fb} = -2650.9 - (-6935.4) = 4284.5\ psi$$

Acceptance Criteria – Gasket Seating Conditions:

$\{S_{hm} = 0.0\ psi\} \le \{S_{hg} = 17100\ psi\}$ *True*

$\{S_{hl} = 1204.7\ psi\} \le \{1.5S_{hg} = 1.5(17100) = 25650\ psi\}$ *True*

$\{S_{hlbi} = -14416.8\ psi\} \le \{1.5S_{hg} = 1.5(17100) = 25650\ psi\}$ *True*

$\{S_{hlbo} = 16826.2\ psi\} \le \{1.5S_{hg} = 1.5(17100) = 25650\ psi\}$ *True*

$\{S_{fm} = -2650.9\ psi\} \le \{S_{fg} = 20000\ psi\}$ *True*

$\{S_{fmbi} = -9586.3\ psi\} \le \{1.5S_{fg} = 1.5(20000) = 30000\ psi\}$ *True*

$\{S_{fmbo} = 4284.5\ psi\} \le \{1.5S_{fg} = 1.5(20000) = 30000\ psi\}$ *True*

Since the calculated stresses in both the head and flange ring are shown to be within the acceptance criteria, for both internal pressure (tubeside conditions), external pressure (shellside conditions), and gasket seating conditions, the proposed Type D spherically dished bolted cover is adequately designed.

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Annex E4.7.2

Although the procedure shown in VIII-2, paragraph 4.7.5.3 provides an equation to calculate the lever arm e of shell discontinuity force V about the centroid of the flange ring, the original development of the equation did not lend itself well to the consideration of corrosion allowance. The variable q is defined as the inside depth of the flange to the base of the head, see VIII-2, Figure 4.7.5, to establish the location of the dished head to flange ring attachment point. However, when the tubeside corrosion allowance is applied to the inside diameter of the dished head and flange ring, the value of q must change accordingly. Additionally, the value of q must be adjusted to account for the shellside corrosion allowance acting on the flange ring. This Annex provides one method of determining these adjustments to q; however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

Assumptions used in the development of the procedure include the following.

1) Shellside and tubeside corrosion allowance is applied to the dished head.
2) Tubeside corrosion allowance is only applied to the flange ring inside radius.
3) Shellside corrosion allowance is applied to the outer surfaces of the flange ring.
4) The geometry of the dished head to flange ring is assumed to be rectilinear.
5) The projected thickness of the dished head and associated dimensions are based off of the angle ϕ.
6) The location of the dished head to flange ring attachment point is established from the measured value from the inside depth of the flange ring to the inside base of the head. This variable is referenced as q.
7) The ring flange is rectangular in shape, i.e., the portion of the flange that is removed by machining for the gasket surface is not considered.

The lever arm e measured from the centerline of the projected dished head thickness on the flange ring to the flange ring centroid is determined geometrically considering the above established assumptions. The variable q along with the inside radius and thickness of the dished head, R_{nom} and t_{nom}, respectively, and the flange ring diameter, B set the initial location of the head centerline with the flange ring. The angle ϕ formed by the tangent to the centerline of the dished head at its point of intersection with the flange ring and a line perpendicular to the axis of the dished head is then established. From this point, the dished head and flange ring are subject to the applicable tubeside and shellside corrosion allowances resulting in the final corroded geometry from which the corroded lever arm is determined.

Refer to Figure AE4.7.2

a) STEP 1 – Establish the variables used in the calculations.

Flange (uncorroded):

$$Thickness: \quad T_{nom} = 2.3125\ in$$

$$Inside\ Diameter: \quad B_{nom} = 16.25\ in$$

$$Head\ Location: \quad q_o = 1.0\ in$$

Spherically Dished Head (uncorroded):

$$Thickness: \quad t_{nom} = 0.875\ in$$
$$Inside\ Radius: \quad L = 16.0\ in$$

Shellside and Tubeside Corrosion Allowance:

$$Shellside: \quad CAS = 0.125\ in$$
$$Tubeside: \quad CAT = 0.125\ in$$

b) STEP 2 – Establish the corroded dimensions of the input variables, as previously defined.

$$t = 0.625\ \ in$$
$$R = 16.4375\ in$$
$$T = 2.0625\ in$$
$$B = 16.5\ in$$
$$q_o = 0.875\ in$$

c) STEP 3 – Calculate the angle ϕ formed by the corroded mean radius of the dished head at the intersection with the flange ring and the corresponding corroded radius of the flange ring, measured to the axis of the dished head assembly.

$$\phi = \arcsin\left[\frac{B}{2R}\right] = \arcsin\left[\frac{16.5}{2(16.4375)}\right] = \begin{Bmatrix} 0.5258\ rad \\ 30.1259\ deg \end{Bmatrix}$$

d) STEP 4 – Calculate the projected tubeside corrosion allowance of the dished head on the flange ring, X_{cat}.

$$X_{cat} = \frac{CAT}{\cos[\phi]} = \frac{0.125}{\cos[30.1259]} = 0.1445\ in$$

e) STEP 5 – Calculate the axial adjustment of X_{adj} due to the applied tubeside corrosion allowance on the flange inside radius.

$$X_{adj} = CAT \cdot \tan[\phi] = 0.125 \cdot \tan[30.1259] = 0.0725\ in$$

f) STEP 6 – Calculate the adjusted value of q based on the corroded dimensions.

$$q = q_o + X_{cat} - X_{adj} = 0.875 + 0.1445 - 0.0725 = 0.9470\ in$$

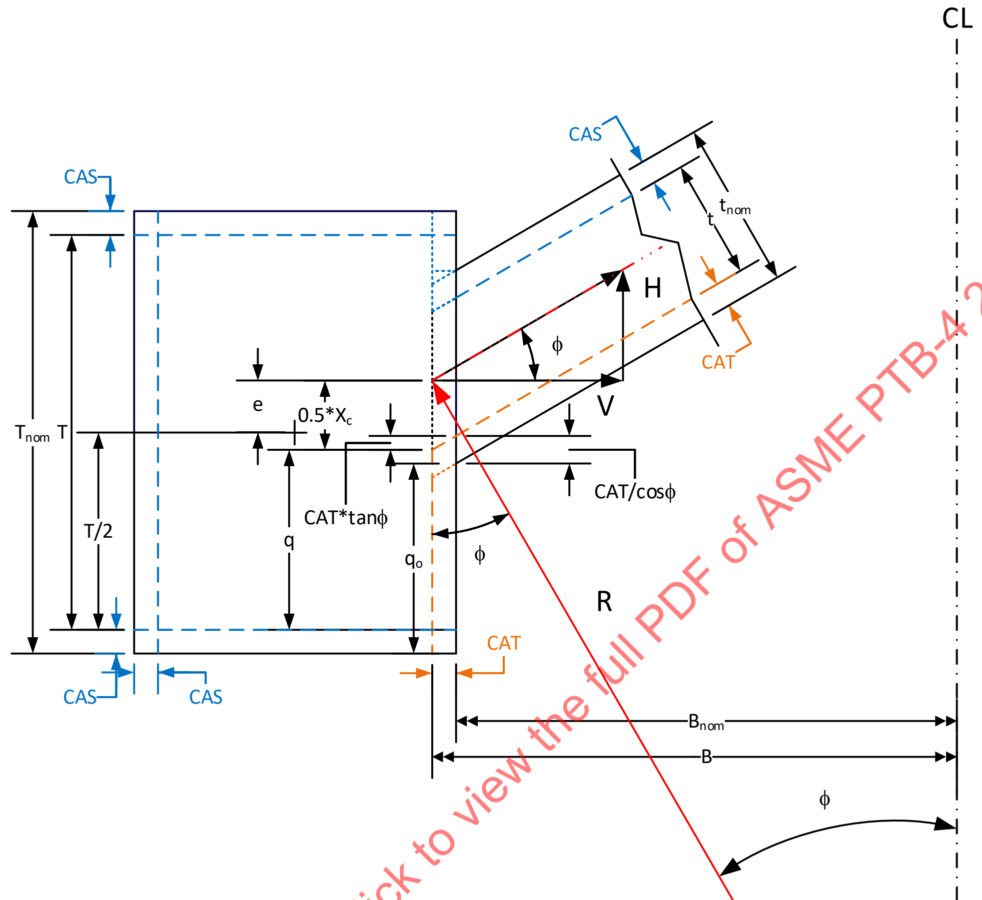

Figure AE4.7.2 – "q" Floating Head Geometry

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4.8 Quick-Actuating (Quick Opening) Closures

4.8.1 Example E4.8.1 – Review of Requirements for Quick-Actuating Closures

An engineer is tasked with developing a design specification for an air filter vessel to be equipped with a quick-actuating closure that is to be constructed in accordance with VIII-1, paragraph UG-35.2.

Design rules for quick–actuating (quick opening) closures are provided in UG-35.2. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.8.

As part of developing the design specification, the following items need to be considered.

a) Scope

Specific calculation methods are not given in paragraph UG-35.2. However, both general and specific design requirements are provided.

b) General Design Requirements

Quick-actuating closures shall be designed such that:

1) The locking elements are engaged prior to or upon application of pressure remain engaged until the pressure is released.

2) The failure of a single holding element while the vessel is pressurized will not:

i) Cause or allow the closure to be opened or leaked; or

ii) Result in the failure of any other locking element or holding element; or

iii) Increase the stress in any other holding element by more than 50% above the allowable stress of the element.

3) All locking elements can be verified to be fully engaged by visual observation or other means prior to application of pressure to the vessel.

4) The use of multilink component, such as a chain, is not permitted.

5) When installed:

i) It may be determined by visual external observation that the holding elements are in satisfactory condition.

ii) All vessels shall be provided with a pressure-indicating device visible from the operating area and suitable for detecting pressure at the closure.

6) When a quick-actuating closure is provided as a part, it shall be provided with a Partial Data Report as meeting the applicable requirements of VIII-1.

7) Supplementary information for the Manufacturer of the pressure vessel and guidance on installation, operation and maintenance for the owner and user is provided in VIII-1, Nonmandatory Appendix FF.

c) Specific Design Requirements

1) Quick-actuating closures that are held in position by positive locking devices and that are fully released by partial rotation or limited movement of the closure itself or the locking mechanism and any closure that is other than manually operated shall be so designed that when the vessel is installed the following conditions are met (see also VIII-1, Nonmandatory Appendix FF):

i) The closure and its holding elements are fully engaged in their intended operating position before pressure can be applied in the vessel.

ii) Pressure tending to force the closure open or discharge the vessel contents clear of the vessel shall be released before the closure can be fully opened for access.

iii) In the event compliance with the above conditions is not inherent in the design of the closure and its holding elements, provisions shall be made so that devices to accomplish this can be added when the vessel is installed.

2) The design rules of VIII-1, Mandatory Appendix 2 may not be applicable, see paragraph 2-1(e).

3) The design shall consider the effects of cyclic and other loadings and mechanical wear on the holding and locking elements and sealing surfaces.

4) Any device or devices that will provide the safeguards described above within these specific requirements will meet the intent of these rules.

5) If clamps used in the design meet the scope of VIII-1, Mandatory Appendix 24 then those requirements shall also be met.

6) The Manufacturer of a pressure vessel with a quick-actuating closure shall supply the user with an installation, operation, and maintenance manual. The manual should address the topics of VIII-1, Nonmandatory Appendix FF. The intent is for the manual to stay with the owner or operator of the pressure vessel.

d) Alternative Designs for Manually Operated Closures

1) Quick-actuating closures that are held in position by a locking mechanism designed for manual operation shall be designed such that if an attempt is made to open the closure when the vessel is under pressure, the closure will leak prior to full disengagement of the locking components and release of the closure. Any leakage shall be directed away from the normal position of the operator.

2) Manually operated closures need not satisfy specific design requirements found in (c)(1) above, but pressure vessels equipped with such closures shall be equipped with an audible or visible warning device that will warn the operator if pressure is applied to the vessel before the holding elements and locking elements are fully engaged in their intended position or if an attempt is made to disengage the locking mechanism before the pressure within the vessel is released.

4.9 Braced and Stayed Surfaces

4.9.1 Example E4.9.1 – Braced and Stayed Surfaces

Determine the required thickness for a flat plate with welded staybolts considering the following design condition. Verify that the welded staybolts are adequately designed. See Figure E4.9.1.

Vessel Data:

- Plate Material = $SA-516,\ Grade\ 70$
- Design Conditions = $100\ psig @ 300°F$
- Staybolt Material = $SA-675,\ Grade\ 70$
- Staybolt Diameter = $1.5\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress Plate Material = $20000\ psi$
- Allowable Stress Staybolt Material = $20000\ psi$
- Staybolt Pattern = $Equilateral\ Triangle$
- Staybolt Pitch = $p_s = p_{horizontal} = p_{diagonal} = 15.0\ in$

Design rules for braced and stayed surfaces are provided in UG-47, UG-48, UG-49, and UG-50. The rules in these paragraphs are the same as those provided in VIII-2, paragraph 4.9 with the exception that VIII-2 only includes rules for welded stays. UW-19 also provides requirements for welded-in stays.

a) STEP 1 – Evaluate per UG-47. Calculate the required thickness of the flat plate, the load carried by each staybolt, and the required diameter of the staybolt.

The minimum required thickness for braced and stayed flat plates and those parts that, by these rules, require staying as flat plates with braces or staybolts of uniform diameter symmetrically spaced, shall be calculated by the following equation.

Assume, $C = 2.2$ from UG-47 with the Welded Staybolt Construction per Figure UW-19.1 Sketch (e).

$$t = p_s\sqrt{\frac{P}{SC}} = 15.0\sqrt{\frac{100.0}{20000(2.2)}} = 0.7151\ in$$

b) STEP 2 – Evaluate per UG-50. UG-50(a) – The required area of a staybolt or stay at its minimum cross section, usually located at the root of the thread, exclusive of any corrosion allowance, is obtained by dividing the load on the staybolt computed in accordance with UG-50(b) by the allowable stress value for the staybolt material and multiplying the result by 1.10.

UG-50(b) – The area supported by a staybolt or stay shall be computed based on the full pitch dimensions, with a deduction for the area occupied by the stay. The load carried by a stay is the product of the area supported by the stay and the maximum allowable working pressure.

UG-50(c) – Stays made of parts joined by welding shall be checked for strength using a joint efficiency of 60% for the weld.

1) The area of the flat plate supported by the staybolt, A_p, is calculated as follows.

$$A_p = \left(p_{horizontal} \cdot p_{diagonal} \cdot \cos[\theta]\right) - A_{sb} = 15.0\left(15.0 \cdot \cos[30]\right) - 1.7671 = 193.0886\ in^2$$

where,

$$\theta = 30\ deg, \qquad See\ Figure\ E4.9.1$$

$$A_{sb} = \frac{\pi}{4}(1.5)^2 = 1.7671\ in^2$$

2) The load carried by the staybolt, L_{sb}, is calculated as follows.

$$L_{sb} = A_p \cdot P = 193.0886(100.0) = 19308.9\ lbs$$

3) The required area of the staybolt, A_{rsb}, is calculated as follows.

$$A_{rsb} = 1.10\left(\frac{L_{sb}}{S_{sb}}\right) = 1.10\left(\frac{19308.9}{20000}\right) = 1.0620\ in^2$$

Since $\left\{A_{sb} = 1.7671\ in^2\right\} > \left\{A_{rsb} = 1.0620\ in^2\right\}$, the staybolt is adequately designed.

4) If the stays are made of parts by welding, the allowable load on the welds shall not exceed the product of the weld area (based on the weld dimension parallel to the staybolt), the allowable stress of the material being welded, and a weld joint factor of 60%.

$$L_{sb} \le L_a$$

where,

$$L_a = E\left(t \cdot \pi d_{sb}\right)S_{sb} = 0.6\left(0.7151(\pi)(1.5)\right)20000 = 40438.0\ lbs$$

Since $\{L_{sb} = 19308.9\ lbs\} \le \{L_a = 40438.0\ lbs\}$, the staybolt is adequately designed.

c) STEP 3 – Evaluate per UW-19(a)(1). Welded-in staybolts shall meet the following requirements:

1) The configuration is in accordance with the typical arrangements shown in Figure UW-19.1.

$$Construction\ per\ Figure\ UW-19.1(e) \qquad Satisfied$$

2) The required thickness of the plate shall not exceed 1.5 in (38 mm), but if greater than 0.75 in (19 mm), the staybolt pitch shall not exceed 20 in (500 mm).

$$\{t = 0.7151\ in\} \le \{1.5\ in\} \qquad Satisfied$$

3) The provisions of UG-47 and UG-49 shall be followed.

Satisfied

4) The required area of the staybolt shall be determined in accordance with the requirements in UG-50.

Satisfied

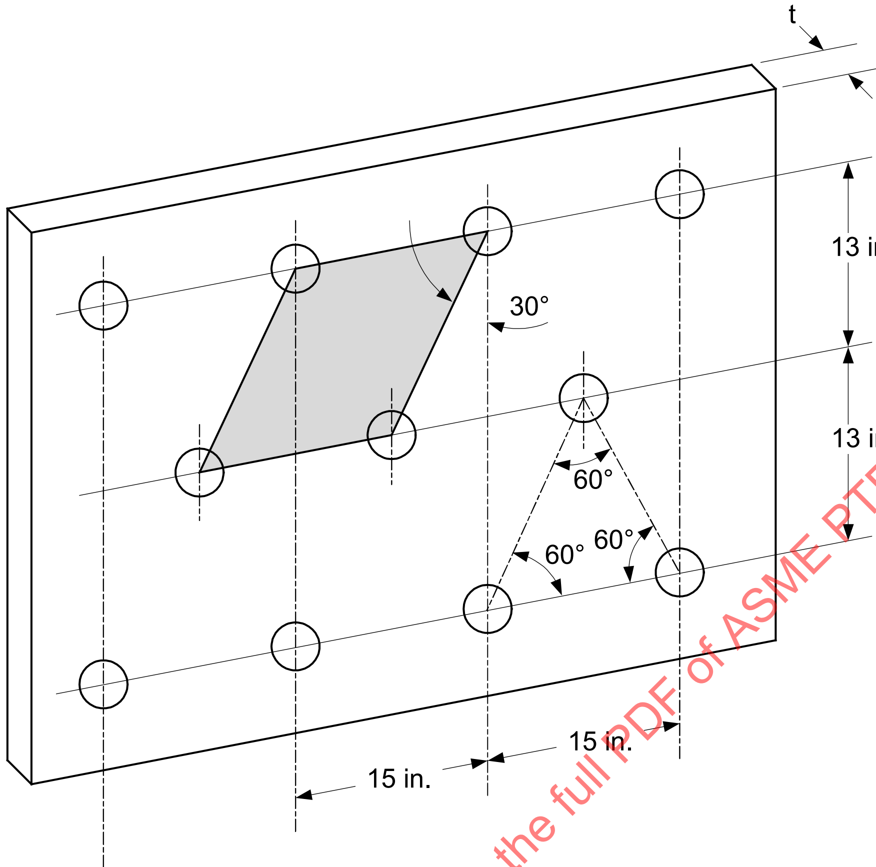

Figure E4.9.1 – Stayed Plate Detail

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4.10 Ligaments

4.10.1 Example E4.10.1 – Ligaments

Determine the ligament efficiency and corresponding efficiency to be used in the design equations of UG-27 for a group of tube holes in a cylindrical shell as shown in Figure E4.10.1.

Design rules for ligaments are provided in UG-53. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.10.

Evaluate the tube hole installation pattern in accordance with UG-53.

Commentary: As shown in Figure E4.10.1, three ligaments are produced; longitudinal, circumferential, and diagonal. UG-53(c) states that in addition to the longitudinal ligament, diagonal and circumferential ligaments shall also be examined with the least equivalent longitudinal ligament efficiency used to determine the minimum required wall thickness and the maximum allowable working pressure. Considering only pressure loading, the circumferential ligament can be half as strong as the longitudinal ligament. This is because the circumferential ligament is subject to longitudinal stress which is essentially half of circumferential stress. By inspection, the circumferential ligament is greater than the longitudinal ligament and thus will the not govern the design. Therefore, the circumferential ligament efficiency is not explicitly calculated.

Paragraph UG-53(d) – when a cylindrical shell is drilled for holes which form diagonal ligaments, as shown in Figure UG-53.4, the efficiency of these ligaments shall be determined by VIII-1, Figures UG-53.5 and UG-53.6. Figure UG-53.5 is used when either or both longitudinal and circumferential ligaments exist with diagonal ligaments. The procedure to determine the equivalent longitudinal ligament efficiency is described in UG-53(e).

a) STEP 1 – Compute the value of p'/p_1.

$$Diagonal\ Pitch,\ p' = 3.75\ in$$

$$Unit\ Length\ of\ Ligament,\ p_1 = 4.5\ in$$

$$\frac{p'}{p_1} = \frac{3.75}{4.5} = 0.8333$$

b) STEP 2 – Compute the efficiency of the longitudinal ligament in accordance with Figure UG-53.5, Note 4.

$$E_{long} = 100\left(\frac{p_1 - d}{p_1}\right) = 100\left(\frac{4.5 - 2.25}{4.5}\right) = 50\%$$

where,

$$Diameter\ of\ Tube\ Holes,\ d = 2.25\ in$$

c) STEP 3 – Compute the diagonal efficiency in accordance with Figure UG-53.5, Note 2.

$$E_{diag} = \frac{J + 0.25 - \left(1 - 0.01 \cdot E_{long}\right)\sqrt{0.75 + J}}{0.00375 + 0.005J}$$

$$E_{diag} = \frac{0.6944 + 0.25 - \left(1 - 0.01(50)\right)\sqrt{\left(0.75 + 0.6944\right)}}{0.00375 + 0.005\left(0.6944\right)} = 47.56\%$$

where,

$$J = \left(\frac{p'}{p_1} \right)^2 = \left(\frac{3.75}{4.5} \right)^2 = 0.6944$$

Alternatively, STEP 3 can be replaced with the following procedure.

STEP 3 (Alternate) – Enter Figure UG-53.5 at the vertical line corresponding to the value of the longitudinal efficiency, E_{long}, and follow this line vertically to the point where it intersects the diagonal line representing the ratio of the value of p'/p_1. Then project this point horizontally to the left, and read the diagonal efficiency of the ligament on the scale at the edge of the diagram.

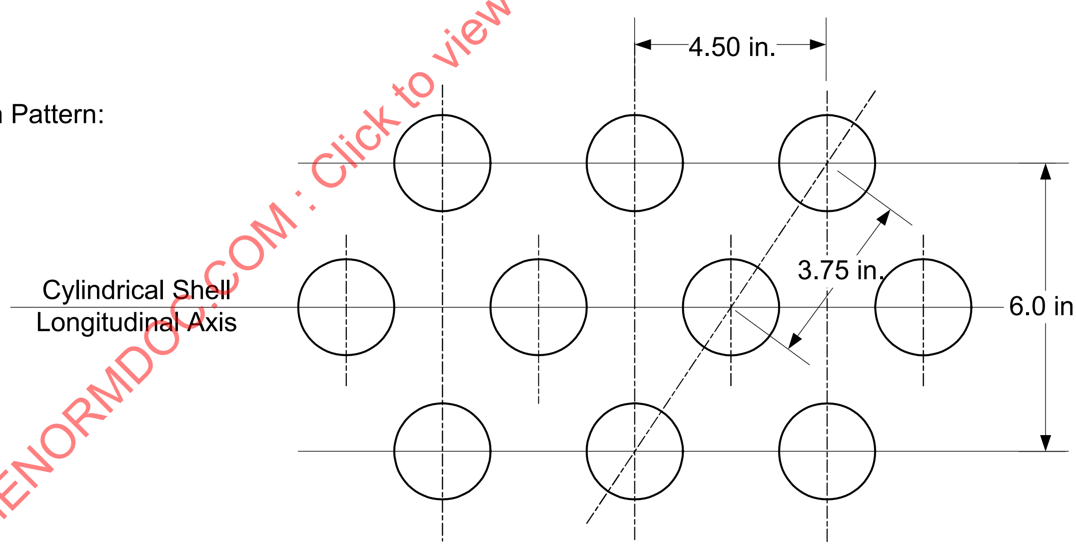
$$E_{diag} \approx 47.5\%$$

d) STEP 4 – The minimum shell thickness and the maximum allowable working pressure shall be based on the ligament that has the lower efficiency.

$$E = \min\left[E_{long}, E_{diag} \right] = \min\left[50\%, 47.5\% \right] = 47.5\%$$

In accordance with UG-53(i) when ligaments occur in cylindrical shells made from welded pipe or tubes and their calculated efficiency is less than 85% (longitudinal) or 50% (circumferential), the efficiency to be used in UG-27 to determine the minimum required thickness is the calculated ligament efficiency. In this case, the appropriate stress value in tension may be multiplied by the factor 1.18.

- Installation Pattern:



- All Finished Hole Diameters are 2.25 in.

Figure E4.10.1 – Installation Pattern

4.11 Jacketed Vessels

4.11.1 Example E4.11.1 – Partial Jacket

Design a partial jacket to be installed on the outside diameter of a section of a tower in accordance with Figure 9-2, Type 2.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $350\ psig @ 300°F$
- Vessel ID = $90.0\ in$
- Nominal Thickness = $1.125\ in$
- Allowable Stress = $20000\ psi$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0

Jacket Data:

- Jacket Type per Figure 9-2 = $Type\ 2$
- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $150\ psig @ 400°F$
- Jacket ID = $96.0\ in$
- Allowable Stress = $20000\ psi$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0

Notes:

1) Jacket closure will be made using closure members per Figure 9-5 of Mandatory Appendix 9.

2) Full penetration welds will be used in the closure.

Establish the corroded dimensions.

$$t_s = \begin{Bmatrix} t_s - Vessel\ Corrosion\ Allowance - Jacket\ Corrosion\ Allowance \\ 1.125 - 0.125 - 0.125 = 0.8750\ in \end{Bmatrix}$$

$$OD\ of\ Inner\ Shell = \begin{Bmatrix} Vessel\ ID + 2\left(t_s - Jacket\ Corrosion\ Allowance\right) \\ 90 + 2\left(1.125 - 0.125\right) = 92.0\ in \end{Bmatrix}$$

$$R_s = \frac{OD\ of\ Inner\ Shell}{2} = 46.0\ in$$

$$t_j = t_j - Jacket\ Corrosion\ Allowance = 0.5 - 0.125 = 0.375\ in$$

$$R_j = R_j + Jacket\ Corrosion\ Allowance = 48.0 + 0.125 = 48.125\ in$$

$$ID\ of\ Jacket = 2\left(48.125\right) = 96.25\ in$$

Design rules for jacketed vessels are provided in Mandatory Appendix 9. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.11.

Evaluate the partial jacket per Mandatory Appendix 9:

a) Determine required thickness of partial jacket per UG-27(c)(1).

$$t_{rj} = \frac{R_j P_j}{S_j E - 0.6P_j} = \frac{48.125(150)}{20000(1.0) - 0.6(150)} = 0.3626 \ in$$

$$t_{rj} = t_{rj} + Jacket \ \ Corrosion \ \ Allowance = 0.3626 + 0.125 = 0.4876 \ in$$

$$Select \ the \ next \ available \ plate \ thickness > 0.4876, \ use \ t_j = 0.5 \ in$$

b) Determine maximum jacket space, j, to ensure that proposed jacket is acceptable.

$$j_{specified} = \frac{(ID \ \ of \ \ Jacket) - (OD \ \ of \ \ Inner \ \ Shell)}{2} = \frac{(96.25 - 92.0)}{2} = 2.125 \ in$$

The maximum of j is determined from paragraph 9-5(c)(5).

$$j = \left(\frac{2S_c t_s^2}{P_j R_j}\right) - \left(\frac{(t_s + t_j)}{2}\right)$$

$$j = \left(\frac{2(20000)(0.875)^2}{150(48.125)}\right) - \left(\frac{0.875 + 0.375}{2}\right) = 3.6174 \ in$$

Since, $\{j_{specified} = 2.125 \ in\} \le \{j = 3.6174 \ in\}$, the design is acceptable.

c) Determine thickness of jacket closures. Use closure detail in Figure 9-5, Sketch (f-2).

$$t_{rc} = 1.414\sqrt{\frac{P_j R_s j}{S_c}} = 1.414\sqrt{\frac{150(46.0)(2.125)}{20000}} = 1.2107 \ in$$

$$t_{rc} = t_{rc} + Jacket \ \ Corrosion \ \ Allowance = 1.2107 + 0.125 = 1.3357 \ in$$

Use end closure plates with a wall thickness of 1.375 in.

d) Determine weld sizes for the closure to shell weld per Figure 9-5, Sketch (f-2).

Jacket to closure weld:

- To be full penetration with backing strip.
- Fillet weld to be equal to t_j as a minimum.

Closure to shell weld (a full penetration weld is to be used).

$$t_c = t_{rc} - Corrosion \ \ Allowance = 1.375 - 0.125 = 1.25 \ in$$

$$t_s = 0.875$$

$$Y = a + b \ge \min\left[1.5t_c, \ 1.5t_s\right] = \min\left[1.5(1.25), \ 1.5(0.875)\right] = 1.3125$$

$$Y = a + b \ge 1.3125 \ in$$

$$Z = Y - \frac{t_s}{2} = 1.3125 - \frac{0.875}{2} = 0.8750 \ in$$

4.11.2 Example E4.11.2 – Half-Pipe Jacket

Design a half-pipe jacket for a section of a tower using the information shown below.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $350\ psig @ 300°F$
- Vessel ID = $90.0\ in$
- Nominal Thickness = $1.125\ in$
- Allowable Stress = $20000\ psi$
- Corrosion Allowance = $0.125\ in$
- Applied Axial Force = $-78104.2\ lbs$
- Applied Net Section Bending Moment = $4.301E+06\ in-lbs$

Half-Pipe Jacket Data:

- Material = $SA-106,\ Grade\ B$
- Design Conditions = $150\ psig @ 400°F$
- Jacket ID = $NPS\ 4\ (STD\ WT) \rightarrow 0.237\ in$
- Allowable Stress = $20000\ psi$
- Yield Stress at Design Temperature = $29900\ psi$
- Minimum Ultimate Tensile Strength = $60000\ psi$
- Weld Joint Efficiency = 1.0
- Corrosion Allowance = $0.0\ in$

Establish the corroded dimensions.

Vessel:

$$D_0 = 90.0 + 2t_s = 90.0 + 2(1.125) = 92.25\ in$$

$$t_s = t_s - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

$$D = D + 2(Corrosion\ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

Half-Pipe Jacket:

$$d_J = 4.5 - 2t_j = 4.5 - 2(0.237) = 4.026\ in$$

$$r = \frac{dj}{2} = \frac{4.026}{2} = 2.013\ in$$

Design rules for half-pipe jackets are provided in Nonmandatory Appendix EE. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.11.6. However, the design rules from VIII-2 also provide specific requirements for the use of partial penetration and fillet welds. For informational purposes, this check is shown prior to the first step of the example problem procedure.

Supplementary check for the acceptability of half-pipe jackets per VIII-2, paragraph 4.11.6. Verify the acceptability of a half pipe jacket in accordance with the requirements VIII-2, paragraphs 4.11.6.1 and 4.11.6.2.

Specified nominal size of 4 NPS is acceptable per VIII-2, paragraph 4.11.6.1.

Material of construction is SA-106, Grade B per ASME Section II Part D, Table

$$S_{yT} = 29.9\ ksi\ \left(@\ 300°F\right)$$

$$S_u = 60\ ksi$$

$$\left\{\frac{S_{yT}}{S_u} = \frac{29.9}{60} = 0.498\right\} \le 0.625 \qquad True$$

Therefore, partial penetration welds can be used. In addition, the vessel is not in cyclic service; therefore, requirements of paragraph 4.11.6.2 are satisfied.

Note: This VIII-2, paragraph 4.11.3.3 requirement is not in VIII-1 Nonmandatory Appendix EE. This check is not required in VIII-1 because the above criteria will always be satisfied because of the allowable stress basis used in VIII-1.

a) Calculate the minimum required thickness for the NPS 4 STD WT half-pipe jacket.

$$T = \frac{P_1 r}{0.85S_1 - 0.6P_1} = \frac{150(2.0130)}{0.85(20000) - 0.6(150)} = 0.0179\ in$$

Since $\{t_j = 0.237\ in\} \ge \{T = 0.0179\ in\}$, the thickness of STD. WT pipe is acceptable for the half-pipe jacket.

b) Calculate maximum permissible pressure in the half-pipe, P', to verify that $P' \ge P_1$.

$$P' = \frac{F}{K}$$

where, $F = \min[(1.5S - S'), 1.5S]$. The value of K is determined from VIII-1, Figures EE-1, EE-2, or EE-3. For VIII-2 designs, the value of K or (K_p for VIII-2), is also provided below for informational purposes.

To compute P', the parameter S', defined as the actual longitudinal stress in the shell, must be computed. This stress may be computed using the following thin-wall equations for a cylindrical shell.

Note: Per VIII-1, paragraph EE-2, when the combination of axial forces and pressure stress is such that S' would be a negative number, then S' shall be taken as zero.

$$S' = Pressure\ Stress + Axial\ Stress \pm Bending\ Stress$$

$$S' = \frac{PD}{4t_s} + \frac{F}{A} \pm \frac{Mc}{I}$$

$$S' = \left\{\begin{array}{l} \dfrac{350(90.25)}{4(1.0)} + \dfrac{-78104.2}{286.6703} + \dfrac{4.301E+06\left(\dfrac{92.25}{2}\right)}{298408.1359} = 8289.2283\ psi \\ \dfrac{350(90.25)}{4(1.0)} + \dfrac{-78104.2}{286.6703} - \dfrac{4.301E+06\left(\dfrac{92.25}{2}\right)}{298408.1359} = 6959.6156\ psi \end{array}\right\}$$

where,

$$I = \frac{\pi}{64}\left(D_o^4 - D^4\right) = \frac{\pi}{64}\left(\left(92.25\right)^4 - \left(90.25\right)^4\right) = 298408.1359\ in^4$$

$$A = \frac{\pi}{4}\left(D_o^2 - D^2\right) = \frac{\pi}{4}\left(\left(92.25\right)^2 - \left(90.25\right)^2\right) = 286.6703\ in^2$$

therefore,

$$F = \min\left[\left(1.5S - S'\right),\ 1.5S\right]$$

$$F = \min\left[\left(1.5\left(20000\right) - 8289.2283\right),\ 1.5\left(20000\right)\right] = 21710.7717\ \ psi$$

The value of K is interpreted from Figure EE-3, with $D = 90.25\ in$ and $t_s = 1.0\ in$.

$$K \approx 11$$

For VIII-2 designs, the coefficients for the equation K_p are obtained from VIII-2, Table 4.11.3 for NPS 4 and shell nominal thickness of $1.0\ in$.

$$C_1 = -2.5016604E+02, \qquad C_2 = 1.7178270E+02, \qquad C_3 = -4.6844914E+01$$

$$C_4 = 6.6874346E+00, \qquad C_5 = -5.2507555E-01, \qquad C_6 = 2.1526948E-02$$

$$C_7 = -3.6091550E-04, \qquad C_8 = C_9 = C_{10} = 0.0$$

with a vessel diameter, $D = 90.25\ in$, the value of K_p is calculated as,

$$K_p = C_1 + C_2 D^{0.5} + C_3 D + C_4 D^{1.5} + C_5 D^2 + C_6 D^{2.5} + C_7 D^3 + C_8 D^{3.5} + C_9 D^4 + C_{10} D^{4.5} = 11.2903$$

Therefore, the maximum permissible pressure in the half-pipe is calculated as,

$$P' = \frac{F}{K} = \begin{Bmatrix} \dfrac{21710.7717}{11} = 1973.7\ \ psi & \quad VIII-1 \\ \dfrac{21710.7717}{11.2903} = 1923.0\ \ psi & \quad VIII-2 \end{Bmatrix} \geq \left\{P_1 = 150\ \ psi\right\} \qquad True$$

Since $P' \geq P_1$, the half-pipe design is acceptable.

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4.12 NonCircular Vessels

4.12.1 Example E4.12.1 – Unreinforced Vessel of Rectangular Cross Section

Using the data shown below, design a rectangular vessel per Appendix 13, Figure 13-2(a) Sketch (1)).

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $400\ psig @ 500°F$
- Inside Length (Short Side) = $7.125\ in$
- Inside Length (Long Side) = $9.25\ in$
- Overall Vessel Length = $40.0\ in$
- Thickness (Short Side) = $1.0\ in$
- Thickness (Long Side) = $1.0\ in$
- Thickness (End Plate) = $0.75\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency (Corner Joint) = 1.0
- Tube Outside Diameter = $1.0000\ in$
- Tube Pitch = $2.3910\ in$

Adjust variables for corrosion.

$$h = 9.25 + 2(Corrosion\ \ Allowance) = 9.25 + 2(0.125) = 9.50\ in$$

$$H = 7.125 + 2(Corrosion\ \ Allowance) = 7.125 + 2(0.125) = 7.375\ in$$

$$t_1 = 1.0 - Corrosion\ \ Allowance = 1.0 - 0.125 = 0.875\ in$$

$$t_2 = 1.0 - Corrosion\ \ Allowance = 1.0 - 0.125 = 0.875\ in$$

$$t_5 = 0.75 - Corrosion\ \ Allowance = 0.75 - 0.125 = 0.625\ in$$

Design rules for vessels of noncircular cross section are provided in Mandatory Appendix 13. The rules in this paragraph produce the same results as those provided in VIII-2, paragraph 4.12. However, the nomenclature and formatting of the equations in VIII-2 are significantly different. Therefore, the example problem will be shown twice, the first time using VIII-1 nomenclature and equations and secondly using the VIII-2 design procedure, nomenclature, and equations.

Section VIII, Division 1 Solution

Evaluate per Mandatory Appendix 13, Paragraph 13-4(h) – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. These equations are conservatively applicable to vessels of aspect ratio less than four. Vessels with aspect ratios less than four may be designed in accordance with the provisions of U-2(g).

$$Aspect\ \ Ratio = \frac{L_v}{h} = \frac{40.0}{9.5} = 4.21 \qquad\qquad Satisfied$$

Paragraph 13-4(g) – The ligament efficiencies e_m and e_b shall only be applied to the calculated stresses for the plates containing the ligaments. When e_m and e_b are less than the joint efficiency E, the membrane and bending stresses calculated on the gross area of the section shall be divided by e_m and e_b, respectively, to obtain the stresses based on the net area for the section. The allowable design stresses for membrane and membrane plus bending shall be calculated as described in paragraph 13-4(b) using $E = 1.0$. When e_m and e_b are greater than the joint efficiency E, the membrane and bending stresses shall be calculated as if there were no ligaments in the plate. The allowable design stresses for membrane and membrane plus bending shall be calculated as described in paragraph 13-4(b) using the appropriate E factor required by paragraph UW-12.

Paragraph 13-6 – It is assumed that the holes drilled in the long side plates (tube sheet and plug sheet) are of uniform diameter. Therefore, e_m and e_b shall be the same value and calculated in accordance with paragraph UG-53.

$$e_m = e_b = \frac{p-d}{p} = \frac{2.3910-1.0}{2.3910} = 0.5818$$

Paragraph 13-5 – Calculate the equation constants.

$$b = 1.0\ in$$

$$c_i = c_o = \frac{t_1}{2} = \frac{t_2}{2} = \frac{0.875}{2} = 0.4375\ in \rightarrow Note: \begin{Bmatrix} The\ sign\ of\ c_i\ is\ positive\ (+) \\ The\ sign\ of\ c_o\ is\ negative\ (-) \end{Bmatrix}$$

$$I_1 = \frac{bt_1^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558\ in^4$$

$$I_2 = \frac{bt_2^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558\ in^4$$

$$\alpha = \frac{H}{h} = \frac{7.375}{9.5} = 0.7763$$

$$K = \frac{I_2}{I_1}\alpha = \left(\frac{0.0558}{0.0558}\right)0.7763 = 0.7763$$

Paragraph 13-7(a) – Calculate the membrane and membrane plus bending stresses, short side plate.

The membrane stress on the short side plate, Equation (1):

$$S_m = \frac{Ph}{2(t_1)} = \frac{400(9.5)}{2(0.875)} = 2171.4\ psi$$

The bending stress at Location N, short side plate, Equation (3):

$$S_{bN} = \frac{Pc}{12I_1}\left[-1.5H^2 + h^2\left(\frac{1+\alpha^2 K}{1+K}\right)\right]$$

$$S_{bN} = \frac{400(\pm 0.4375)}{12(0.0558)}\left[\begin{array}{c} -1.5(7.375)^2 + (9.5)^2 \cdot \\ \left(\dfrac{1+(0.7763)^2(0.7763)}{1+0.7763}\right) \end{array}\right] = \begin{Bmatrix} -1831.7\ psi & Inside\ Surface \\ 1831.7\ psi & Outside\ Surface \end{Bmatrix}$$

The bending stress at Location Q, short side plate, Equation (4):

$$S_{bQ} = \frac{Ph^2 c}{12I_1}\left[\frac{1+\alpha^2 K}{1+K}\right]$$

$$S_{bQ} = \frac{400(9.5)^2(\pm 0.4375)}{12(0.0558)}\left[\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right]$$

$$S_{bQ} = \begin{Bmatrix} 19490.8 \ psi & Inside\ Surface \\ -19490.8 \ psi & Outside\ Surface \end{Bmatrix}$$

Paragraph 13-7(a) – Calculate the membrane and membrane plus bending stresses, long side plate.

The membrane stress on the long side plate, Equation (2):

$$S_m = \frac{PH}{2t_2 e_m} = \frac{400(7.375)}{2(0.875)(0.5818)} = 2897.4 \ psi$$

The bending stress at Location M, long side plate, Equation (5):

$$S_{bM} = \frac{Ph^2 c}{12I_2 e_b}\left[-1.5+\left(\frac{1+\alpha^2 K}{1+K}\right)\right]$$

$$S_{bM} = \frac{400(9.5)^2(\pm 0.4375)}{12(0.0558)(0.5818)}\left[-1.5+\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right]$$

$$S_{bM} = \begin{Bmatrix} -27310.9 \ psi & Inside\ Surface \\ 27310.9 \ psi & Ouside\ Surface \end{Bmatrix}$$

The bending stress at Location Q, long side plate, Equation (6):

$$S_{bQ} = \frac{Ph^2 c}{12I_2}\left[\frac{1+\alpha^2 K}{1+K}\right]$$

$$S_{bQ} = \frac{400(9.5)^2(\pm 0.4375)}{12(0.0558)}\left[\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right]$$

$$S_{bQ} = \begin{Bmatrix} 19490.8 \ psi & Inside\ Surface \\ -19490.8 \ psi & Outside\ Surface \end{Bmatrix}$$

Paragraphs 13-4(b), 13-4(c), and 13-7, Equations (7) through (10) – Acceptance Criteria:

Short side plate, Membrane Stress:

$$\{S_m = 2171.4 \ psi\} \le \{SE = 20000(1.0) = 20000 \ psi\} \qquad True$$

Short side plate at Location N, Membrane + Bending Stress:

$$\begin{Bmatrix} S_m + S_{bN} = 2171.4 + (-1831.7) = 339.7 \ psi \\ S_m + S_{bN} = 2171.4 + (1831.7) = 4003.1 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m + S_{bN} = 339.7 \ psi \\ S_m + S_{bN} = 4003.1 \ psi \end{Bmatrix} \le \{1.5SE = 1.5(20000)(1.0) = 30000 \ psi\} \qquad True$$

Short side plate at Location Q, Membrane + Bending Stress:

$$\begin{Bmatrix} S_m + S_{bQ} = 2171.4 + 19490.8 = 21662.2 \ psi \\ S_m + S_{bQ} = 2171.4 + (-19490.8) = -17319.4 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m + S_{bQ} = 21662.2 \ psi \\ S_m + S_{bQ} = -17319.4 \ psi \end{Bmatrix} \le \{1.5SE = 1.5(20000)(1.0) = 30000 \ psi\} \qquad True$$

Long side plate, Membrane Stress:

$$\{S_m = 2897.4 \ psi\} \le \{S = 20000 \ psi\} \qquad True$$

Long side plate at Location M, Membrane + Bending Stress:

$$\begin{Bmatrix} S_m + S_{bM} = 2897.4 + (-27310.9) = -24413.5 \ psi \\ S_m + S_{bM} = 2897.4 + 27310.9 = 30208.3 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m + S_{bM} = -24413.5 \ psi \\ S_m + S_{bM} = 30208.3 \ psi \end{Bmatrix} \le \{1.5SE = 1.5(20000)(1.0) = 30000 \ psi\} \qquad \begin{Bmatrix} True \\ False \end{Bmatrix}$$

Long side plate at Location Q, Membrane + Bending Stress:

$$\begin{Bmatrix} S_m + S_{bQ} = 2897.4 + 19490.8 = 22388.2 \ psi \\ S_m + S_{bQ} = 2897.4 + (19490.8) = -16593.4 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m + S_{bQ} = 22388.2 \ psi \\ S_m + S_{bQ} = -16593.4 \ psi \end{Bmatrix} \le \{1.5SE = 1.5(20000)(1.0) = 30000 \ psi\} \qquad True$$

The acceptance criteria for membrane and membrane plus bending stresses are satisfied at all locations except for the membrane plus bending stress at Location M on the long side plate, $\{S_m + S_{bM}\}$. However, the overstress is less than 1%.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

VIII-2, paragraph 4.12.2, General Design Requirements:

Paragraph 4.12.2.7 – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. These equations are conservatively applicable to vessels of aspect ratio less than four. Vessels with aspect ratios less than four may be designed in accordance with VIII-2, paragraph 4.12.5 and may be designed in accordance with the provisions of Part 5.

$$Aspect\ Ratio = \frac{L_v}{h} = \frac{40.0}{9.5} = 4.21 \qquad\qquad Satisfied$$

Paragraph 4.12.2.9 – The openings in this noncircular vessel meet the requirements of VIII-2, paragraph 4.5.2.

VIII-2, paragraphs 4.12.3, 4.12.4 and 4.12.5 – These paragraphs are not applicable to this design.

VIII-2, paragraph 4.12.6, Weld Joint Factors and Ligament Efficiency:

Paragraph 4.12.6.1 – The non-circular vessel is constructed with corner joints typical of VIII-2, paragraph 4.2. Therefore, the weld joint efficiencies E_m and E_b are set to 1.0 at stress calculation locations in the corners of the vessel. Since there are no welds or hole pattern in the short side plates of the vessel, the weld joint efficiencies E_m and E_b are set to 1.0 for these stress calculation locations. For the stress calculation locations on the long side plates that do not contain welded joints, but do contain a hole pattern, the weld joint efficiencies E_m and E_b are set equal to the ligament efficiencies e_m and e_b, respectively.

Paragraph 4.12.6.3 – It is assumed that the holes drilled in the long side plates (tube sheet and plug sheet) are of uniform diameter. Therefore, e_m and e_b shall be the same value and calculated in accordance with paragraph 4.10.

$$e_m = e_b = \frac{p-d}{p} = \frac{2.3910-1.0}{2.3910} = 0.5818$$

VIII-2, paragraph 4.12.7, Design Procedure:

a) STEP 1 – The design pressure and temperature are listed in the information given above.

b) STEP 2 – The vessel to be designed is a Type 1 vessel (VIII-1, Figure 13-2(a), Sketch (1)).

c) STEP 3 – The vessel configuration and wall thicknesses of the pressure containing plates are listed in the information given above.

d) STEP 4 – Determine the location of the neutral axis from the inside and outside surfaces. Since the section under evaluation does not have stiffeners, but has uniform diameter holes, then $c_i = c_o = t/2$ where t is the thickness of the plate.

$$c_i = c_o = \frac{t_1}{2} = \frac{t_2}{2} = \frac{0.875}{2} = 0.4375\ in$$

e) STEP 5 – Determine the weld joint factor and ligaments efficiencies as applicable, see VIII-2, paragraph 4.12.6, and determine the factors E_m and E_b.

f) STEP 6 – Complete the stress calculation for the selected noncircular vessel Type, see VIII-2, Table 4.12.1, and check the acceptance criteria.

For non-circular vessel Type 1, the applicable table for stress calculations is VIII-2, Table 4.12.2 and the corresponding details are shown in VIII-2, Figure 4.12.1

<u>Equation Constants:</u>

$$b = 1.0\ in$$

$$J_{2s} = 1.0$$

$$J_{3s} = 1.0$$

$$J_{2l} = 1.0$$

$$J_{3l} = 1.0$$

$$I_1 = \frac{bt_1^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558\ in^4$$

$$I_2 = \frac{bt_2^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558\ in^4$$

$$\alpha = \frac{H}{h} = \frac{7.375}{9.5} = 0.7763$$

$$K = \frac{I_2}{I_1}\alpha = \left(\frac{0.0558}{0.0558}\right)0.7763 = 0.7763$$

Nomenclature for Stress Results:

S_m^s membrane stress, short side.

S_{bi}^{sC}, S_{bo}^{sC} bending stress, short side at point C on the inside and outside surfaces, respectively.

S_{bi}^{sB}, S_{bo}^{sB} bending stress, short side at point B on the inside and outside surfaces, respectively.

S_m^l membrane stress, long side.

S_{bi}^{lA}, S_{bo}^{lA} bending stress, long side at point A on the inside and outside surfaces, respectively.

S_{bi}^{lB}, S_{bo}^{lB} bending stress, long side at point B on the inside and outside surfaces, respectively.

Membrane and Bending Stresses – Critical Locations of Maximum Stress:

$$S_m^s = \frac{Ph}{2(t_1)E_m} = \frac{400(9.5)}{2(0.875)(1.0)} = 2171.4\ psi$$

$$S_{bi}^{sC} = -S_{bo}^{sC}\left(\frac{c_i}{c_o}\right) = \frac{PbJ_{2s}c_i}{12I_1E_b}\left[-1.5H^2 + h^2\left(\frac{1+\alpha^2K}{1+K}\right)\right]$$

$$S_{bi}^{sC} = \frac{400(1.0)(1.0)(0.4375)}{12(0.0558)(1.0)}\left[-1.5(7.375)^2 + (9.5)^2\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right] = -1831.7\ psi$$

$$S_{bo}^{sC} = -S_{bi}^{sC}\left(\frac{c_o}{c_i}\right) = -(-1831.7)\left(\frac{0.4375}{0.4375}\right) = 1831.7\ psi$$

$$S_{bi}^{sB} = -S_{bo}^{sB}\left(\frac{c_i}{c_o}\right) = \frac{Pbh^2J_{3s}c_i}{12I_1E_b}\left[\frac{1+\alpha^2K}{1+K}\right]$$

$$S_{bi}^{sB} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(0.0558)(1.0)}\left[\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right] = 19490.8\ psi$$

$$S_{bo}^{sB} = -S_{bi}^{sB}\left(\frac{c_o}{c_i}\right) = -19490.8\left(\frac{0.4375}{0.4375}\right) = -19490.8\ psi$$

$$S_m^l = \frac{PH}{2t_2E_m} = \frac{400(7.375)}{2(0.875)(0.5818)} = 2897.4 \ psi$$

$$S_{bi}^{lA} = -S_{bo}^{lA}\left(\frac{c_i}{c_o}\right) = \frac{Pbh^2J_{2l}c_i}{12I_2E_b}\left[-1.5+\left(\frac{1+\alpha^2K}{1+K}\right)\right]$$

$$S_{bi}^{lA} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(1.0)(0.0558)(0.5818)}\left[-1.5+\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right] = -27310.9 \ psi$$

$$S_{bo}^{lA} = -S_{bi}^{lA}\left(\frac{c_o}{c_i}\right) = -(-27310.9)\left(\frac{0.4375}{0.4375}\right) = 27310.9 \ psi$$

$$S_{bi}^{lB} = -S_{bo}^{lB}\left(\frac{c_i}{c_o}\right) = \frac{Pbh^2J_{3l}c_i}{12I_2E_b}\left[\frac{1+\alpha^2K}{1+K}\right]$$

$$S_{bi}^{lB} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(0.0558)(1.0)}\left[\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right] = 19490.8 \ psi$$

$$S_{bo}^{lB} = -S_{bi}^{lB}\left(\frac{c_o}{c_i}\right) = -19490.8\left(\frac{0.4375}{0.4375}\right) = -19490.8 \ psi$$

Acceptance Criteria – Critical Locations of Maximum Stress:

$$\left\{S_m^s = 2171.4 \ psi\right\} \le \left\{S = 20000 \ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_m^s + S_{bi}^{sC} = 2171.4 + (-1831.7) = 339.7 \ psi \\ S_m^s + S_{bo}^{sC} = 2171.4 + 1831.7 = 4003.1 \ psi \end{array}\right\}$$

$$\left\{\begin{array}{l} S_m^s + S_{bi}^{sC} = 339.7 \ psi \\ S_m^s + S_{bo}^{sC} = 4003.1 \ psi \end{array}\right\} \le \left\{1.5S = 1.5(20000) = 30000 \ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_m^s + S_{bi}^{sB} = 2171.4 + 19490.8 = 21662.2 \ psi \\ S_m^s + S_{bo}^{sB} = 2171.4 + (-19490.8) = -17319.4 \ psi \end{array}\right\}$$

$$\left\{\begin{array}{l} S_m^s + S_{bi}^{sB} = 21662.2 \ psi \\ S_m^s + S_{bo}^{sB} = -17319.4 \ psi \end{array}\right\} \le \left\{1.5S = 1.5(20000) = 30000 \ psi\right\} \qquad True$$

$$\left\{S_m^l = 2897.4 \ psi\right\} \le \left\{S = 20000 \ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lA} = 2897.4 + (-27310.9) = -24413.5 \ psi \\ S_m^l + S_{bo}^{lA} = 2897.4 + 27310.9 = 30208.3 \ psi \end{array}\right\}$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lA} = -24413.5 \ psi \\ S_m^l + S_{bo}^{lA} = 30208.3 \ psi \end{array}\right\} \le \left\{1.5S = 1.5(20000) = 30000 \ psi\right\} \qquad \left\{\begin{array}{l} True \\ False \end{array}\right\}$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lB} = 2897.4 + 19490.8 = 22388.2 \ \ psi \\ S_m^l + S_{bo}^{lB} = 2897.4 + \left(-19490.8\right) = -16593.4 \ \ psi \end{array}\right\}$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lB} = 22388.2 \ \ psi \\ S_m^l + S_{bo}^{lB} = -16593.4 \ \ psi \end{array}\right\} \le \left\{1.5S = 1.5\left(20000\right) = 30000 \ \ psi\right\} \qquad True$$

The acceptance criteria for membrane and membrane plus bending stresses are satisfied at all locations except for the membrane plus bending stress at Location A on the long side plate, $\left\{S_m^l + S_{bo}^{lA}\right\}$. However, the overstress is less than 1%.

4.12.2 Example E4.12.2 – Reinforced Vessel of Rectangular Cross Section

Using the data shown below, design a rectangular vessel with reinforcement per Appendix 13 Figure 13-2(a) Sketch (4). The stiffeners are attached with continuous fillets welds on both sides of the member see Figure UG-30.

<u>Vessel Data:</u>

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $50\,psig\,@\,200°F$
- Inside Length (Short Side) = $30.0\,in$
- Inside Length (Long Side) = $60.0\ in$
- Overall Vessel Length = $240.0\ in$
- Unstiffened Span Length (pitch) = $12.0\ in$
- Thickness (Short Side) = $0.4375\ in$
- Thickness (Long Side) = $0.4375\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0
- Yield Stress at Design Temperature = $34800\ psi$
- Modulus of Elasticity at Design Temperature = $28.8E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$

<u>Stiffener Data (W4x13 I-Beam):</u>

- Material = $SA-36$
- Allowable Stress = $16600\ psi$
- Stiffener Yield Stress at Design Temperature = $33000\,psi$
- Modulus of Elasticity at Design Temperature = $28.8E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$
- Stiffener Cross Sectional Area = $3.83\ in^2$
- Stiffener Moment of Inertia = $11.3\ in^4$
- Stiffener Height = $4.125\ in$
- Stiffener Centerline Distance (Short Side) = $35.0\ in$
- Stiffener Centerline Distance (Long Side) = $65.0\ in$

Required variables.

$$h = 60.0\ in$$

$$H = 30.0\ in$$

$$t_1 = 0.4375\ in$$

$$t_2 = 0.4375\ in$$

Design rules for vessels of noncircular cross section are provided in Mandatory Appendix 13. The rules in this paragraph produce the same results as those provided in VIII-2, paragraph 4.12. However, the nomenclature and formatting of the equations in VIII-2 are significantly different. Therefore, the example

problem will be shown twice, the first time using VIII-1 nomenclature and equations and secondly using the VIII-2 design procedure, nomenclature, and equations.

Section VIII, Division 1 Solution

Evaluate per Appendix 13.

Paragraph 13-4(c) – For a vessel with reinforcement, when the reinforcing members and the shell plate does not have the same S and S_y values at the design temperature, the total stress shall be determined at the innermost and outermost fibers for each material. The appropriate c values (with proper signs, 13-5) for the composite section properties shall be used in the bending equations. The total stresses at the innermost and outermost fibers for each material shall be compared to the allowable design stress 13-4(b) for each material.

Paragraph 13-4(h) – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. These equations are conservatively applicable to vessels of aspect ratio less than four. Vessels with aspect ratios less than four may be designed in accordance with the provisions of U-2(g).

$$Aspect\ Ratio = \frac{L_v}{h} = \frac{240.0}{60.0} = 4.0$$

Paragraph 13-4(g) – In this example problem, there are no ligaments in either the short side or long side plates. Therefore, the allowable design stresses for membrane and membrane plus bending shall be calculated as described in paragraph 13-4(b) using the appropriate E factor required by paragraph UW-12.

Paragraph 13-8(b) – The rules of this paragraph cover only the types of reinforced rectangular cross section vessels shown in Figure 13-2(a) sketches (4), (5), and (6) where welded-on reinforcement members are in a plane perpendicular to the long axis of the vessel; however, the spacing between reinforcing members need not be uniform. All reinforcement members attached to two opposite plates shall have the same moment of inertia. Reinforcing members shall be placed on the outside of the vessel and shall be attached to the plates of the vessel by welding on each side of the reinforcing member. For continuous reinforcement, the welding may be continuous or intermittent.

Paragraph 13-8(d)(1) – The basic maximum distance between reinforcing member centerlines shall be determined by Equation (1) of UG-47. This distance is then used to calculate a value of β for the short side H and for the long side h. A value of J is then obtained for each value from Table 13-8(d). The values thus obtained are used in the applicable equations in paragraph 13-8(d)(5) to determine the values of p_1 and p_2. The maximum distance between any reinforcing member centerlines shall not be greater than the least of the values of p_1 and p_2.

For the short side plate, $\{H = 30.0\ in\} \geq \{p = 12.0\ in\}$, Equation (1a) and Table 13-8(d)(5).

Calculate the basic maximum distance between reinforcing members, per paragraph UG-47.

$$p = t_1\sqrt{\frac{SC}{P}} = 0.4375\sqrt{\frac{(20000)2.1}{50}} = 12.6800\ in$$

Determine β per paragraph 13-5.

$$\beta = \frac{H}{p} = \frac{30.0}{12.6800} = 2.3659$$

Calculate Stress Parameter J from Table 13-8(d).

$$\beta = \min\left[\max\left[\beta, \frac{1}{\beta}\right], 4.0\right]$$

$$\beta = \min\left[\max\left[2.3659, \frac{1}{2.3659}\right], 4.0\right] = 2.3659$$

$$From\ interpolation,\ J = 2.2206$$

Calculate p_1 from Equation 1(1a).

$$p_1 = t_1\sqrt{\frac{SJ}{P}} = 0.4375\sqrt{\frac{(20000)2.2206}{50}} = 13.0390\ in$$

For the long side plate, $\{h = 60.0\ in\} \geq \{p = 12.0\ in\}$, Equation (1c) and Table 13-8(d)(5).

Calculate the basic maximum distance between reinforcing members, per paragraph UG-47.

$$p = t_1\sqrt{\frac{SC}{P}} = 0.4375\sqrt{\frac{(20000)2.1}{50}} = 12.6800\ in$$

Determine β per paragraph 13-5.

$$\beta = \frac{h}{p} = \frac{60.0}{12.6800} = 4.7391$$

Calculate Stress Parameter J from Table 13-8(d).

$$\beta = \min\left[\max\left[\beta, \frac{1}{\beta}\right], 4.0\right]$$

$$\beta = \min\left[\max\left[4.7391, \frac{1}{4.7391}\right], 4.0\right] = 4.0$$

$$From\ direct\ observation,\ J = 2.0$$

Calculate p_2 from Equation (1c).

$$p_2 = t_2\sqrt{\frac{SJ}{P}} = 0.4375\sqrt{\frac{(20000)2.0}{50}} = 12.3744\ in$$

therefore,

$$p = \min\left[p_1,\ p_2\right] = \min\left[13.0390,\ 12.3744\right] = 12.3744\ in$$

Since $\{p_{design} = 12.0\} < \{p_{allow} = 12.3744\}$, the design is acceptable.

Paragraph 13-8(d)(3) – The allowable effective width of shell plate w shall not be greater than the least value of p computed in paragraph 13-8(d)(5) nor greater than the actual value of p if the actual value of p is less than that permitted in paragraph 13-8(d)(5). One half of w shall be considered effective on each side of the reinforcing member centerline, but the effective widths shall not overlap. The effective width shall not be greater than the actual width available.

For the short side plate, calculate w per paragraph 13-8(d)(5), Equation (2) and Table 13-8(e).

$$w = \frac{t_1 \Delta}{\sqrt{S_y}} \left(\frac{E_y}{E_{ya}} \right) = \frac{0.4375(6000)}{\sqrt{33000}} \left(\frac{28.8E+06}{29.4E+06} \right) = 14.1552 \ in$$

where,

$$\Delta = 6000 \ \sqrt{psi}$$

therefore,

$$w = \min\left[p, \ \min\left[w, \ p_1\right]\right] = \min\left[12.0, \min\left[14.1552, \ 13.0390\right]\right] = 12.0 \ in$$

For the long side plate, calculate w per paragraph 13-8(d)(5), Equation (2) and Table 13-8(e).

$$w = \frac{t_2 \Delta}{\sqrt{S_y}} \left(\frac{E_y}{E_{ya}} \right) = \frac{0.4375(6000)}{\sqrt{33000}} \left(\frac{28.8E+06}{29.4E+06} \right) = 14.1552 \ in$$

therefore,

$$w = \min\left[p, \ \min\left[w, \ p_2\right]\right] = \min\left[12.0, \ \min\left[14.1552, \ 12.3744\right]\right] = 12.0 \ in$$

Paragraph 13-8(d)(2) – Equation (2) of paragraph 13-8(d)(5) is used to compute the maximum effective width of the shell plate which can be used in computing the effective moment of inertia I_{11} and I_{21} of the composite section (reinforcement and shell plate acting together) at locations where the shell plate is in compression.

VIII-1 does not provide rules for computing the effective moment of inertia I_{11} and I_{21}. This example provides one possible method of satisfying U-2(g); however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

A composite structure may include the use of two or more different materials, each carrying a part of the load. Unless all the various materials used have the same Modulus of Elasticity, the evaluation of the composite section will need to consider the ratio of the moduli. Although the material specifications for the shell plate and stiffeners are different, their Moduli of Elasticity are the same; therefore, no adjustment to the procedure to calculate the composite section moment of inertia is required.

Calculate the short side stiffener/plate composite section neutral axis as follows, see Figure E4.12.2.

$$\bar{y} = \frac{A_{stif}\left(t_1 + \frac{h_s}{2}\right) + A_{plate}\left(\frac{t_1}{2}\right)}{\left(A_{stif} + A_{plate}\right)}$$

$$\bar{y} = \frac{3.83\left(0.4375 + \frac{4.125}{2}\right) + 0.4375(12.0)\left(\frac{0.4375}{2}\right)}{\left(3.83 + 0.4375(12.0)\right)} = 1.1810 \ in$$

Calculate the short side composite section moment of inertia, I_{11}, using parallel axis theorem.

$$I_{11} = I_{stif} + A_{stif}\left(t_1 + \frac{h_s}{2} - \overline{y}\right)^2 + \frac{w_1(t_1)^3}{12} + w_1(t_1)\left(\overline{y} - \frac{t_1}{2}\right)^2$$

$$I_{11} = \left\{ \begin{array}{l} 11.3 + 3.83\left(0.4375 + \dfrac{4.125}{2} - 1.1810\right)^2 + \\ \dfrac{12.0(0.4375)^3}{12} + 12.0(0.4375)\left(1.1810 - \dfrac{0.4375}{2}\right)^2 \end{array} \right\} = 22.9081\ in^4$$

Since the stiffener is continuous around the vessel with a consistent net section, the plate thicknesses of the short side and long side are equal, $t_1 = t_2$, the pitch of stiffeners are equal, $w_1 = w_2$, it follows that $\overline{y}$ for the short side and long side plates are equal and $I_{11} = I_{21}$.

Determine the location of the neutral axis from the inside and outside surfaces. If the section under evaluation has stiffeners, then c_i and c_o are determined from the cross section of the combined plate and stiffener section using strength of materials concepts.

For the short side plate,

$$c_i = \overline{y} = 1.1810\ in \rightarrow The\ sign\ of\ c_i\ is\ positive\ (+)$$

$$c_o = t_1 + h_s - \overline{y} = 0.4375 + 4.125 - 1.1810 = 3.3815\ in \rightarrow The\ sign\ of\ c_o\ is\ negative\ (-)$$

For the long side plate,

$$c_i = \overline{y} = 1.1810\ in \rightarrow The\ sign\ of\ c_i\ is\ positive\ (+)$$

$$c_o = t_1 + h_s - \overline{y} = 0.4375 + 4.125 - 1.1810 = 3.3815\ in \rightarrow The\ sign\ of\ c_o\ is\ negative\ (-)$$

The reinforcing member does not have the same allowable stress as the vessel; therefore, the stress at the interface of the components of the composite section shall be determined. Since the interface between components is oriented below the composite section neutral axis, the bending equation used to compute the stress shall be that considered acting on the inside surface. The distance between the composite section neutral axis and the interface of the components is calculated as follows.

For the short side and long side plates, respectively,

$$c_{i(interface)} = \overline{y} - t_1 = 1.1810 - 0.4375 = 0.7435\ in$$

$$c_{i(interface)} = \overline{y} - t_2 = 1.1810 - 0.4375 = 0.7435\ in$$

Paragraph 13-5 – Calculate the equation constants.

$$\alpha_1 = \frac{H_1}{h_1} = \frac{H + 2(t_1) + h_s}{h + 2(t_2) + h_s} = \frac{30 + 2(0.4375) + 4.125}{60 + 2(0.4375) + 4.125} = 0.5385$$

$$k = \frac{I_{21}}{I_{11}}\alpha_1 = \left(\frac{22.9081}{22.9081}\right)0.5385 = 0.5385$$

Paragraph 13-8(e) – Calculate the membrane and membrane plus bending stresses, short side plate.

The membrane stress on the short side plate, Equation (3):

$$S_m = \frac{Php}{2(A_1 + pt_1)} = \frac{50(60.0)12.0}{2(3.83 + 12.0(0.4375))} = 1982.4 \;\; psi$$

The bending stress at Location N, short side plate, Equation (5):

$$S_{bN} = \frac{Ppc}{24I_{11}}\left[-3H^2 + 2h^2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bN} = \begin{Bmatrix} \dfrac{50(12.0)(1.1810)}{24(22.9081)}\left[-3(30.0)^2 + 2(60.0)^2\left(\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right] \\ \dfrac{50(12.0)(-3.3815)}{24(22.9081)}\left[-3(30.0)^2 + 2(60.0)^2\left(\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right] \end{Bmatrix}$$

$$S_{bN} = \begin{Bmatrix} 3493.6 \;\; psi & Inside\ Surface \\ -10003.1 \;\; psi & Outside\ Surface \end{Bmatrix}$$

The bending stress at Location Q, short side plate, Equation (6):

$$S_{bQ} = \frac{Ph^2 pc}{12I_{11}}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bQ} = \begin{Bmatrix} \dfrac{50(60.0)^2(12.0)(1.1810)}{12(22.9081)}\left[\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] \\ \dfrac{50(60.0)^2(12.0)(-3.3815)}{12(22.9081)}\left[\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] \end{Bmatrix}$$

$$S_{bQ} = \begin{Bmatrix} 6973.5 \;\; psi & Inside\ Surface \\ -19966.9 \;\; psi & Outside\ Surface \end{Bmatrix}$$

Paragraph 13-8(e) – Calculate the membrane and membrane plus bending stresses, long side plate.

The membrane stress on the long side plate, Equation (4):

$$S_m = \frac{PHp}{2(A_2 + pt_2)} = \frac{50(30.0)(12.0)}{2(3.83 + 12.0(0.4375))} = 991.2 \;\; psi$$

The bending stress at Location M, long side plate, Equation (7):

$$S_{bM} = \frac{Ph^2 pc}{24I_{21}}\left[-3+2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bM} = \begin{Bmatrix} \dfrac{50(60.0)^2(12.0)(+1.1810)}{24(22.9081)}\left[-3+2\left(\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right] \\ \dfrac{50(60.0)^2(12.0)(-3.3815)}{24(22.9081)}\left[-3+2\left(\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right] \end{Bmatrix}$$

$$S_{bM} = \begin{Bmatrix} -6946.0\ psi & Inside\ Surface \\ 19888.2\ psi & Outside\ Surface \end{Bmatrix}$$

The bending stress at Location Q, long side plate, Equation (8):

$$S_{bQ} = \frac{Ph^2 pc}{12I_{21}}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bQ} = \begin{Bmatrix} \dfrac{50(60.0)^2(12.0)(1.1810)}{12(22.9081)}\left[\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] \\ \dfrac{50(60.0)^2(12.0)(-3.3815)}{12(22.9081)}\left[\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] \end{Bmatrix}$$

$$S_{bQ} = \begin{Bmatrix} 6973.5\ psi & Inside\ Surface \\ -19966.9\ psi & Outside\ Surface \end{Bmatrix}$$

Paragraph 13-8(e) – (re–visited) Calculate the membrane plus bending stresses at the interface.

The bending stress at Location N, short side plate, Equation (5):

$$S_{bN} = \frac{Ppc}{24I_{11}}\left[-3H^2+2h^2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bN} = \frac{50(12.0)(0.7435)}{24(22.9081)}\left[-3(30.0)^2+2(60.0)^2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bN} = 2199.4\ psi$$

The bending stress at Location Q, short side plate, Equation (6):

$$S_{bQ} = \frac{Ph^2 pc}{12I_{11}}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bQ} = \frac{50(60.0)^2(12.0)(0.7435)}{12(22.9081)}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right]$$

$$S_{bQ} = 4390.2\ psi$$

The bending stress at Location M, long side plate, Equation (7):

$$S_{bM} = \frac{Ph^2 pc}{24I_{21}}\left[-3+2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bM} = \frac{50(60.0)^2(12.0)(0.7435)}{24(22.9081)}\left[-3+2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bM} = -4372.9 \ psi$$

The bending stress at Location Q, long side plate, Equation (8):

$$S_{bQ} = \frac{Ph^2 pc}{12I_{21}}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bQ} = \frac{50(60.0)^2(12.0)(0.7435)}{12(22.9081)}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right]$$

$$S_{bQ} = 4390.2 \ psi$$

Paragraphs 13-4(b), 13-4(c), and 13-8, Equations (9) through (12) – Acceptance Criteria:

Paragraph 13-4(b) – Membrane stresses due to pressure and mechanical loads shall not exceed the design stress, S. NOTE – see paragraph 13-4(g) for the inclusion of the joint efficiency factor, E, as applicable.

Shell Material, $SA-516, Grade\ 70$: $S_{ms} = SE = 20000(1.0) = 20000\ psi$.

Reinforcement Material, $SA-36$: $S_{mr} = SE = 16600(1.0) = 16600\ psi$.

Note – membrane stress for a composite reinforced bar or shapes and plate sections, etc., shall not exceed the following limits.

$$S = \min\left[S_{ms},\ S_{mr}\right] = \min\left[20000,\ 16600\right] = 16600 \ psi$$

Paragraph 13-4(b)(2) – Any combination of membrane plus bending tension or compression stress for other cross sections (such as composite reinforced bar or shapes and plate sections, etc.), shall not exceed the following limits.

$$S = \min\left[1.5SE,\ 2/3S_y\right] = \min\left[1.5(16600)(1.0),\ 2/3(33000)\right] = 22000 \ psi$$

Paragraph 13-4(c)(2) – When the reinforcing members and the shell plate do not have the same S and S_y values at the design temperature, the total stress shall be determined at the innermost and outermost fibers for each material. The total stresses at the innermost and outermost fibers for each material shall be compared to the allowable design stress for each material.

Shell Material, $SA-516, Grade\ 70$:

$$S = \min\left[1.5(20000)(1.0) = 30000 \ psi,\ 2/3(34800) = 23200\right] = 23200 \ psi$$

Reinforcement Material, $SA-36$:

$$S = \min\left[1.5(16600)(1.0) = 24900 \ psi,\ 2/3(33000) = 22000\right] = 22000 \ psi$$

Short side plate, Membrane Stress:

$$\left\{S_m = 1982.4\ psi\right\} \le \left\{S = 16000\ psi\right\} \qquad True$$

Short side plate at Location N, Membrane + Bending Stress:

$$\left\{\begin{array}{l} S_m + S_{bN} = 1982.4 + 3493.6 = 5476.0\ psi \\ S_m + S_{bN} = 1982.4 + (-10003.1) = -8020.7\ psi \end{array}\right\} \le \left\{\begin{array}{l} S = 23200\ psi \\ S = 22000\ psi \end{array}\right\} \left\{\begin{array}{l} True \\ True \end{array}\right\}$$

Short side plate at Location Q, Membrane + Bending Stress:

$$\left\{\begin{array}{l} S_m + S_{bQ} = 1982.4 + 6973.5 = 8955.9\ psi \\ S_m + S_{bQ} = 1982.4 + (-19966.9) = -17984.5\ psi \end{array}\right\} \le \left\{\begin{array}{l} S = 23200\ psi \\ S = 22000\ psi \end{array}\right\} \left\{\begin{array}{l} True \\ True \end{array}\right\}$$

Long side plate, Membrane Stress:

$$\left\{S_m = 991.2\ psi\right\} \le \left\{16600\ psi\right\} \qquad True$$

Long side plate at Location M, Membrane + Bending Stress:

$$\left\{\begin{array}{l} S_m + S_{bM} = 991.2 + (-6946.0) = -5954.8\ psi \\ S_m + S_{bM} = 991.2 + 19888.2 = 20879.4\ psi \end{array}\right\} \le \left\{\begin{array}{l} S = 23200\ psi \\ S = 22000\ psi \end{array}\right\} \left\{\begin{array}{l} True \\ True \end{array}\right\}$$

Long side plate at Location Q, Membrane + Bending Stress:

$$\left\{\begin{array}{l} S_m + S_{bQ} = 991.2 + 6973.5 = 7964.7\ psi \\ S_m + S_{bQ} = 991.2 + (-19966.9) = -18975.7\ psi \end{array}\right\} \le \left\{\begin{array}{l} S = 23200\ psi \\ S = 22000\ psi \end{array}\right\} \left\{\begin{array}{l} True \\ True \end{array}\right\}$$

Paragraph 13-8(e) – Acceptance Criteria for the membrane plus bending stresses at the interface.

Short side plate at Location N, Membrane + Bending Stress:

$$\left\{S_m + S_{bN} = 1982.4 + 2199.4 = 4181.8\ psi\right\} \le \left\{S = 22000\ psi\right\} \qquad True$$

Short side plate at Location Q, Membrane + Bending Stress:

$$\left\{S_m + S_{bQ} = 1982.4 + 4390.2 = 6372.6\ psi\right\} \le \left\{S = 22000\ psi\right\} \qquad True$$

Long side plate at Location M, Membrane + Bending Stress:

$$\left\{S_m + S_{bM} = 991.2 + (-4372.9) = -3381.7\ psi\right\} \le \left\{S = 22000\ psi\right\} \qquad True$$

Long side plate at Location Q, Membrane + Bending Stress:

$$\left\{S_m + S_{bQ} = 991.2 + 4390.2 = 5381.4\ psi\right\} \le \left\{S = 22000\ psi\right\} \qquad True$$

The acceptance criteria for membrane and membrane plus bending stresses are satisfied at all locations; therefore, the design is complete.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraph 4.12.

VIII-2, paragraph 4.12.2, General Design Requirements.

Paragraph 4.12.2.3.c – For a vessel with reinforcement, when the reinforcing member does not have the same allowable stress as the vessel, the total stress shall be determined at the inside and outside surfaces of each component of the composite section. The total stresses at the inside and outside surfaces shall be compared to the allowable stress.

i) For locations of stress below the neutral axis, the bending equation used to compute the stress shall be that considered acting on the inside surface.

ii) For locations of stress above the neutral axis, the bending equation used to compute the stress shall be that considered acting on the outside surface.

Paragraph 4.12.2.7 – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. These equations are conservatively applicable to vessels of aspect ratio less than four. Vessels with aspect ratios less than four may be designed in accordance with VIII-2, paragraph 4.12.5 and may be designed in accordance with the provisions of Part 5.

$$Aspect\ Ratio = \frac{L_v}{h} = \frac{240.0}{60.0} = 4.0$$

Paragraph 4.12.2.9 – There are no specified openings for this example problem.

VIII-2, paragraph 4.12.3, Requirements for Vessels with Reinforcement.

Paragraph 4.12.3.1 – Design rules are provided for Type 4 configurations where the welded-on reinforcement members are in a plane perpendicular to the long axis of the vessel. All reinforcement members attached to two opposite plates shall have the same moment of inertia.

Paragraph 4.12.3.5 – Reinforcing members shall be placed on the outside of the vessel and shall be attached to the plates of the vessel by welding on each side of the reinforcing member. For continuous reinforcement, the welding may be continuous or intermittent.

Paragraph 4.12.3.6 – The maximum distance between reinforcing members is computed in VIII-2, paragraph 4.12.3 and are covered in STEP 3 of the Design Procedure in VIII-2, paragraph 4.12.7.

VIII-2, paragraphs 4.12.4 and 4.12.5.

These paragraphs are not applicable to this design.

VIII-2, paragraph 4.12.6, Weld Joint Factors and Ligament Efficiency.

Paragraph 4.12.6.1 – The non-circular vessel is constructed with corner joints typical of VIII-2, paragraph 4.2. Therefore, the weld joint efficiencies E_m and E_b are set to 1.0 at stress calculation locations in the corners of the vessel. Since there are no welds or hole pattern in either the short side plates or long side plates of the vessel, the weld joint efficiencies E_m and E_b are set to 1.0 for these stress calculation locations.

VIII-2, paragraph 4.12.7, Design Procedure.

a) STEP 1 – The design pressure and temperature are listed in the information given above.

b) STEP 2 – The vessel to be designed is a Type 4 vessel.

c) STEP 3 – The vessel configuration and wall thicknesses of the pressure containing plates are listed in the information given above. The vessel has stiffeners; therefore, calculate the maximum spacing and size of the stiffeners per VIII-2, paragraph 4.12.3.

VIII-2, paragraph 4.12.3.6.a – The maximum distance between reinforcing member centerlines is given by VIII-2, Equation (4.12.1). In the equations for calculating stresses for reinforced noncircular vessels, the value of p shall be the sum of one-half the distances to the next reinforcing member on each side.

For the short side plate, where $\{H = 30.0\ in\} \geq \{p = 12.0\ in\}$,

$$p_1 = t_1\sqrt{\frac{SJ_1}{P}} = 0.4375\sqrt{\frac{(20000)2.2206}{50}} = 13.0390\ in$$

where,

$$J_1 = -0.26667 + \frac{24.222}{(\beta_{1\max})} - \frac{99.478}{(\beta_{1\max})^2} + \frac{194.59}{(\beta_{1\max})^3} - \frac{169.99}{(\beta_{1\max})^4} + \frac{55.822}{(\beta_{1\max})^5}$$

$$J_1 = -0.26667 + \frac{24.222}{(2.3659)} - \frac{99.478}{(2.3659)^2} + \frac{194.59}{(2.3659)^3} - \frac{169.99}{(2.3659)^4} + \frac{55.822}{(2.3659)^5}$$

$$J_1 = 2.2206$$

$$\beta_{1\max} = \min\left[\max\left[\beta_1, \frac{1}{\beta_1}\right], 4.0\right]$$

$$\beta_{1\max} = \min\left[\max\left[2.3659, \frac{1}{2.3659}\right], 4.0\right] = 2.3659$$

$$\beta_1 = \frac{H}{p_{b1}} = \frac{30.0000}{12.680} = 2.3659 \qquad (\mathit{for\ rectangular\ vessels})$$

$$p_{b1} = t_1\sqrt{\frac{2.1S}{P}} = t_1\sqrt{\frac{2.1S}{P}} = 0.4375\sqrt{\frac{2.1(20000)}{50}} = 12.680\ in$$

For the long side plate, where $\{h = 60.0\ in\} \geq \{p = 12.0\ in\}$,

$$p_2 = t_2\sqrt{\frac{SJ_2}{P}} = 0.4375\sqrt{\frac{(20000)2.0000}{50}} = 12.3794\ in$$

where,

$$J_2 = -0.26667 + \frac{24.222}{(\beta_{1\max})} - \frac{99.478}{(\beta_{1\max})^2} + \frac{194.59}{(\beta_{1\max})^3} - \frac{169.99}{(\beta_{1\max})^4} + \frac{55.822}{(\beta_{1\max})^5}$$

$$J_2 = -0.26667 + \frac{24.222}{(4.0)} - \frac{99.478}{(4.0)^2} + \frac{194.59}{(4.0)^3} - \frac{169.99}{(4.0)^4} + \frac{55.822}{(4.0)^5} = 2.0000$$

$$\beta_{2\max} = \min\left[\max\left[\beta_2, \frac{1}{\beta_2}\right], 4.0\right]$$

$$\beta_{2\max} = \min\left[\max\left[4.7319, \frac{1}{4.7319}\right], 4.0\right] = 4.0$$

$$\beta_2 = \frac{h}{p_{b2}} = \frac{60.0000}{12.680} = 4.7319 \qquad (\mathit{for\ rectangular\ vessels})$$

$$p_{b2} = t_2\sqrt{\frac{2.1S}{P}} = t_2\sqrt{\frac{2.1S}{P}} = 0.4375\sqrt{\frac{2.1(20000)}{50}} = 12.680\ in$$

therefore,

$$p = \min\left[p_1,\ p_2\right] = \min\left[13.0390,\ 12.3744\right] = 12.3744\ in$$

Since $\{p_{design} = 12.0\} < \{p_{allow} = 12.3744\}$, the design is acceptable.

VIII-2, paragraph 4.12.3.6.b – The allowable effective widths of shell plate, w_1 and w_2 shall not be greater than the value given by VIII-2, Equation (4.12.16) or VIII-2, Equation (4.12.17), nor greater than the actual value of p if this value is less than that computed in VIII-2, paragraph 4.12.3.6.a. One half of w shall be considered effective on each side of the reinforcing member centerline, but the effective widths shall not overlap. The effective width shall not be greater than the actual width available.

$$w_1 = \min\left[p,\ \min\left[w_{\max},\ p_1\right]\right] = \min\left[12.0, \min\left[14.1552,\ 13.0390\right]\right] = 12.0\ in$$

$$w_2 = \min\left[p,\ \min\left[w_{\max},\ p_2\right]\right] = \min\left[12.0,\ \min\left[14.1552,\ 12.3744\right]\right] = 12.0\ in$$

where,

$$w_{\max} = \frac{t_1\Delta}{\sqrt{S_y}}\left(\frac{E_y}{E_{ya}}\right) = \frac{t_2\Delta}{\sqrt{S_y}}\left(\frac{E_y}{E_{ya}}\right) = \frac{0.4375(6000)}{\sqrt{33000}}\left(\frac{28.8E+06}{29.4E+06}\right) = 14.1552\ in$$

$$\Delta = 6000\ \sqrt{psi} \qquad \mathit{From\ Table}\ 4.12.14$$

VIII-2, paragraph 4.12.3.6.c – At locations, other than in the corner regions where the shell plate is in tension, the effective moments of inertia, I_{11} and I_{21}, of the composite section (reinforcement and shell plate acting together) shall be computed based on the values of w_1 and w_2 computed in VIII-2, paragraph 4.12.3.6.b.

NOTE – A composite structure may include the use of two or more different materials, each carrying a part of the load. Unless all the various materials used have the same Modulus of Elasticity, the evaluation of the composite section will need to consider the ratio of the moduli. Although the material specifications for the shell plate and stiffeners are different, their Moduli of Elasticity are the same; therefore, no adjustment to the procedure to calculate the composite section moment of inertia is required.

Calculate the short side stiffener/plate composite section neutral axis as follows, see Figure E4.12.2.

$$\overline{y}=\frac{A_{stif}\left(t_1+\dfrac{h_s}{2}\right)+A_{plate}\left(\dfrac{t_1}{2}\right)}{\left(A_{stif}+A_{plate}\right)}$$

$$\overline{y}=\frac{3.83\left(0.4375+\dfrac{4.125}{2}\right)+0.4375(12.0)\left(\dfrac{0.4375}{2}\right)}{\left(3.83+0.4375(12.0)\right)}=1.1810\ in$$

Calculate the short side composite section moment of inertia, I_{11}, using parallel axis theorem.

$$I_{11}=I_{stif}+A_{stif}\left(t_1+\frac{h_s}{2}-\overline{y}\right)^2+\frac{w_1\left(t_1\right)^3}{12}+w_1\left(t_1\right)\left(\overline{y}-\frac{t_1}{2}\right)^2$$

$$I_{11}=\left\{\begin{array}{l}11.3+3.83\left(0.4375+\dfrac{4.125}{2}-1.1810\right)^2+\\ \dfrac{12.0(0.4375)^3}{12}+12.0(0.4375)\left(1.1810-\dfrac{0.4375}{2}\right)^2\end{array}\right\}=22.9081\ in^4$$

Since the stiffener is continuous around the vessel with a consistent net section, the plate thicknesses of the short side and long side are equal, $t_1=t_2$, the pitch of stiffeners are equal, $w_1=w_2$, it follows that $\overline{y}$ for the short side and long side plates are equal and $I_{11}=I_{21}$.

d) STEP 4 – Determine the location of the neutral axis from the inside and outside surfaces. If the section under evaluation has stiffeners, then c_i and c_o are determined from the cross section of the combined plate and stiffener section using strength of materials concepts.

For the short side plate,

$$c_i=\overline{y}=1.1810\ in$$

$$c_o=t_1+h_s-\overline{y}=0.4375+4.125-1.1810=3.3815\ in$$

For the long side plate,

$$c_i=\overline{y}=1.1810\ in$$

$$c_o=t_2+h_s-\overline{y}=0.4375+4.125-1.1810=3.3815\ in$$

The reinforcing member does not have the same allowable stress as the vessel; therefore, the stress at the interface of the components of the composite section shall be determined. Since the interface between components is oriented below the composite section neutral axis, the bending equation used to compute the stress shall be that considered acting on the inside surface. The distance between the composite section neutral axis and the interface of the components is calculated as follows.

For the short side and long side plates, respectively,

$$c_{i(interface)}=\overline{y}-t_1=1.1810-0.4375=0.7435\ in$$

$$c_{i(interface)}=\overline{y}-t_2=1.1810-0.4375=0.7435\ in$$

e) STEP 5 – Determine the weld joint factor and ligaments efficiencies, as applicable, see VIII-2, paragraph 4.12.6, and determine the factors E_m and E_b.

$$E_m = E_b = 1.0$$

f) STEP 6 – Complete the stress calculation for the selected noncircular vessel Type, see VIII-2, Table 4.12.1, and check the acceptance criteria.

For non-circular vessel Type 4, the applicable table for stress calculations is VIII-2, Table 4.12.5 and the corresponding details are shown in VIII-2, Figure 4.12.4.

Equation Constants:

$$\alpha_1 = \frac{H_1}{h_1} = \frac{H + 2(t_1) + h_s}{h + 2(t_2) + h_s} = \frac{30 + 2(0.4375) + 4.125}{60 + 2(0.4375) + 4.125} = 0.5385$$

$$k = \frac{I_{21}}{I_{11}}\alpha_1 = \left(\frac{22.9081}{22.9081}\right)0.5385 = 0.5385$$

Nomenclature for Stress Results:

S_m^s membrane stress, short side.

S_{bi}^{sC}, S_{bo}^{sC} bending stress, short side at point C on the inside and outside surfaces, respectively.

S_{bi}^{sB}, S_{bo}^{sB} bending stress, short side at point B on the inside and outside surfaces, respectively.

S_m^l membrane stress, long side.

S_{bi}^{lA}, S_{bo}^{lA} bending stress, long side at point A on the inside and outside surfaces, respectively.

S_{bi}^{lB}, S_{bo}^{lB} bending stress, long side at point B on the inside and outside surfaces, respectively.

Membrane and Bending Stresses – Critical Locations of Maximum Stress:

$$S_m^s = \frac{Php}{2(A_1 + t_1 p)E_m} = \frac{50(60.0)12.0}{2(3.83 + 0.4375(12.0))1.0} = 1982.4 \ psi$$

$$S_{bi}^{sC} = -S_{bo}^{sC}\left(\frac{c_i}{c_o}\right) = \frac{Ppc_i}{24I_{11}E_b}\left[-3H^2 + 2h^2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{sC} = \frac{50(12.0)1.1810}{24(22.9081)1.0}\left[-3(30.0)^2 + 2(60.0)^2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bi}^{sC} = 3493.6 \ psi$$

$$S_{bo}^{sC} = -S_{bi}^{sC}\left(\frac{c_o}{c_i}\right) = -3493.6\left(\frac{3.3815}{1.1810}\right) = -10003.1 \ psi$$

$$S_{bi}^{sB} = -S_{bo}^{sB}\left(\frac{c_i}{c_o}\right) = \frac{Ph^2pc_i}{12I_{11}E_b}\left[\frac{1+\alpha_1^2k}{1+k}\right]$$

$$S_{bi}^{sB} = \frac{50(60.0)^2(12.0)(1.1810)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right]$$

$$S_{bi}^{sB} = 6973.5\ psi$$

$$S_{bo}^{sB} = -S_{bi}^{sB}\left(\frac{c_o}{c_i}\right) = -6973.5\left(\frac{3.3815}{1.1810}\right) = -19966.9\ psi$$

$$S_m^l = \frac{PHp}{2(A_2+t_2p)E_m} = \frac{50(30.0)(12.0)}{2(3.83+0.4375(12.0))1.0} = 991.2\ psi$$

$$S_{bi}^{lA} = -S_{bo}^{lA}\left(\frac{c_i}{c_o}\right) = \frac{Ph^2pc_i}{24I_{21}E_b}\left[-3+2\left(\frac{1+\alpha_1^2k}{1+k}\right)\right]$$

$$S_{bi}^{lA} = \frac{50(60.0)^2(12.0)(1.1810)}{24(22.9081)1.0}\left[-3+2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bi}^{lA} = -6946.0\ psi$$

$$S_{bo}^{lA} = -S_{bi}^{lA}\left(\frac{c_o}{c_i}\right) = -(-6946.0)\left(\frac{3.3815}{1.1810}\right) = 19888.2\ psi$$

$$S_{bi}^{lB} = -S_{bo}^{lB}\left(\frac{c_i}{c_o}\right) = \frac{Ph^2pc_i}{12I_{21}E_b}\left[\frac{1+\alpha_1^2k}{1+k}\right]$$

$$S_{bi}^{lB} = \frac{50(60.0)^2(12.0)(1.1810)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right]$$

$$S_{bi}^{lB} = 6973.5\ psi$$

$$S_{bo}^{lB} = -S_{bi}^{lB}\left(\frac{c_o}{c_i}\right) = -6973.5\left(\frac{3.3815}{1.1810}\right) = -19966.9\ psi$$

Calculate the bending stresses at the interface of the shell plate and stiffener at the Critical Locations of Maximum Stress.

$$S_{bi}^{sC} = \frac{Ppc_i}{24I_{11}E_b}\left[-3H^2+2h^2\left(\frac{1+\alpha_1^2k}{1+k}\right)\right]$$

$$S_{bi}^{sC} = \frac{50(12.0)(0.7435)}{24(22.9081)1.0}\left[-3(30.0)^2+2(60.0)^2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bi}^{sC} = 2199.4\ psi$$

$$S_{bi}^{sB} = \frac{Ph^2 pc_i}{12I_{11}E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{sB} = \frac{50(60.0)^2(12.0)(0.7435)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] = 4390.2\ psi$$

$$S_{bi}^{lA} = \frac{Ph^2 pc_i}{24I_{21}E_b}\left[-3+2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{lA} = \frac{50(60.0)^2(12.0)(0.7435)}{24(22.9081)1.0}\left[-3+2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right] = -4372.9\ psi$$

$$S_{bi}^{lB} = \frac{Ph^2 pc_i}{12I_{21}E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{lB} = \frac{50(60.0)^2(12.0)(0.7435)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] = 4390.2\ psi$$

<u>Acceptance Criteria – Critical Locations of Maximum Stress:</u> The stiffener allowable stress, S_{stif}, is used for the membrane stress and membrane plus bending stress for the outside fiber stress acceptance criteria, while the plate allowable stress, S, is used for the membrane plus bending stress for inside fiber allowable stress criteria.

$$\left\{S_m^s = 1982.4\ psi\right\} \le \left\{S = 20000\ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_m^s + S_{bi}^{sC} = 1982.4 + 3493.5 = 5476.0\ psi \\ S_m^s + S_{bo}^{sC} = 1982.4 + (-10003.1) = -8020.7\ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(20000) = 30000\ psi \\ 1.5S = 1.5(16600) = 24900\ psi \end{array}\right\} \left\{\begin{array}{l} True \\ True \end{array}\right\}$$

$$\left\{\begin{array}{l} S_m^s + S_{bi}^{sB} = 1982.4 + 6973.5 = 8955.9\ psi \\ S_m^s + S_{bo}^{sB} = 1982.4 + (-19966.9) = -17984.5\ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(20000) = 30000\ psi \\ 1.5S = 1.5(16600) = 24900\ psi \end{array}\right\} \left\{\begin{array}{l} True \\ True \end{array}\right\}$$

$$\left\{S_m^l = 991.2\ psi\right\} \le \left\{S = 16600\ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lA} = 991.2 + (-6946.0) = -5954.8\ psi \\ S_m^l + S_{bo}^{lA} = 991.2 + 19888.2 = 20879.4\ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(20000) = 30000\ psi \\ 1.5S = 1.5(16600) = 24900\ psi \end{array}\right\} \left\{\begin{array}{l} True \\ True \end{array}\right\}$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lB} = 991.2 + 6973.5 = 7964.7\ psi \\ S_m^l + S_{bo}^{lB} = 991.2 + (-19966.9) = -18975.7\ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(20000) = 30000\ psi \\ 1.5S = 1.5(16600) = 24900\ psi \end{array}\right\} \left\{\begin{array}{l} True \\ True \end{array}\right\}$$

The allowable stress of the shell plate and stiffener is limited by the stiffener. Therefore, at the interface of the shell plate and stiffener, the allowable stress used in the acceptance criteria is that of the stiffener.

$$\left\{S_m^s + S_{bi}^{sC} = 1982.4 + 2199.4 = 4181.8\ psi\right\} \le \left\{1.5S = 1.5(16600) = 24900\ psi\right\} \qquad True$$

$$\left\{S_m^s + S_{bi}^{sB} = 1982.4 + 4390.2 = 6372.6\ psi\right\} \le \left\{1.5S = 1.5(16600) = 24900\ psi\right\} \qquad True$$

$$\left\{S_m^l + S_{bi}^{lA} = 991.2 + (-4372.9) = -3381.7\ psi\right\} \le \left\{1.5S = 1.5(16600) = 24900\ psi\right\} \quad True$$

$$\left\{S_m^l + S_{bi}^{lB} = 991.2 + 4390.2 = 5381.4\ psi\right\} \le \left\{1.5S = 1.5(16600) = 24900\ psi\right\} \quad True$$

The acceptance criteria for membrane and membrane plus bending stresses are satisfied at all locations; therefore, the design is complete.

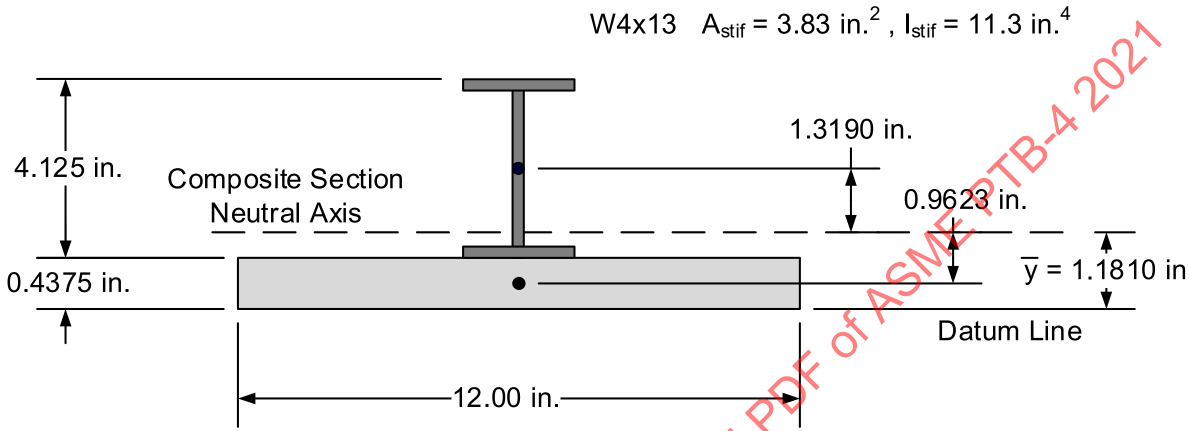

Figure E4.12.2 – Composite Section Details

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4.13 Layered Vessels

4.13.1 Example E4.13.1 – Layered Cylindrical Shell

Determine the required total thickness of the layered cylindrical shell for the following design conditions. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with ULW-50 through ULW-57.

Vessel Data:

- Material = $SA-724,\ Grade\ B$
- Design Conditions = $3600\ psig\ @\ 300°F$
- Inside Diameter = $84.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $26800\ psi$
- Weld Joint Efficiency = 1.0
- Thickness of each layer = $0.3125\ in$

Section VIII, Division 1 Solution

Evaluate per ULW-16 and UG-27(c)(1).

$$t = \frac{PR}{SE - 0.6P}$$

$$Note: \{P = 3600\ \ psi\} \le \{0.385SE = 0.385(26800)(1.0) = 10318\ \ psi\}$$

$$R = \frac{84.0}{2} = 42.0\ in$$

$$t = \frac{3600(42.0)}{26800(1.0) - 0.6(3600)} = 6.1364\ in$$

The required thickness for all layers is $6.1364\ in$.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

In accordance with Part 4, paragraph 4.13.4.1, determine the total thickness of the layered cylindrical shell using Part 4, paragraph 4.3.1.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right] - 1\right) = \frac{84}{2}\left(\exp\left[\frac{3600}{26800(1.0)}\right] - 1\right) = 6.0383\ in$$

The required thickness for all layers is $6.0383\ in$.

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4.13.2 Example E4.13.2 – Layered Hemispherical Head

Determine the required total thickness of the layered hemispherical head for the following design conditions. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with ULW-50 through ULW-57.

Vessel Data:

- Material = $SA-724,\ Grade\ B$
- Design Conditions = $3600\ psig\ @\ 300°F$
- Inside Diameter = $84.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $26800\ psi$
- Weld Joint Efficiency = 1.0
- Thickness of each layer = $0.3125\ in$

Section VIII, Division 1 Solution

Evaluate per ULW-16 and UG-32(f).

$$t = \frac{PL}{2SE - 0.2P}$$

$$Note: \left\{P = 3600\ \ psi\right\} \le \left\{0.665SE = 0.665(26800)(1.0) = 17822\ \ psi\right\}$$

$$L = \frac{D}{2} = \frac{84.0}{2} = 42.0\ in$$

$$t = \frac{PL}{2SE - 0.2P} = \frac{3600(42.0)}{2(26800)(1.0) - 0.2(3600)} = 2.8593\ in$$

The required thickness is 2.8593 in.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

In accordance with Part 4, paragraph 4.13.4.1, determine the total thickness of the layered hemispherical head using Part 4, paragraph 4.3.3.

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right] - 1\right) = \frac{84.0}{2}\left(\exp\left[\frac{0.5(3600)}{26800(1.0)}\right] - 1\right) = 2.9178\ in$$

The required thickness for all layers is 2.9178 in.

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4.13.3 Example E4.13.3 – Maximum Permissible Gap in a Layered Cylindrical Shell

Determine if the anticipated maximum permissible gap between any layers, in accordance with ULW-77, for the cylindrical shell in Example Problem E4.13.1 is adequate, given the specified design cycles. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with ULW-50 through ULW-57.

Vessel Data:

- Material = $SA-724,\ Grade\ B$
- Design Conditions = $3600\ psig\ @\ 300°F$
- Inside Diameter = $84.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $26800\ psi$
- Weld Joint Efficiency = 1.0
- Thickness per Layer = $0.3125\ in$
- Number of Layers = 20
- Specified Design Cycles = $1.0E+06$
- Anticipated Maximum Gap Height = $0.015\ in$
- Minimum Ultimate Tensile Strength = $95\ ksi$
- Elastic Modulus at Temperature = $28.3E+06\ psi$

Section VIII, Division 1 Solution

Per ULW-77(d), after weld preparation and before welding circumferential seams, the height of the radial gaps between any two adjacent layers shall be measured at the ends of the layered section or layered head section at right angles to the vessel axis and the length of the relevant radial gap, see Figure ULW-77.

The gap area A_g shall not exceed the thickness of a layer, $A_g \le 0.3125\ in$.

The maximum length of any gap shall not exceed the inside diameter of the vessel, $b \le 84.0\ in$.

Where more than one gap exists between any two adjacent layers, the sum of the gap lengths shall not exceed the inside diameter of the vessel.

The maximum height of any gap shall not exceed 0.1875 in.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Commentary: In layered vessel design, the permissible gap height between layers is limited by the number of pressure cycles applied during operation. The opening and closure of the gap due to the applied pressure cycle creates a bending moment from which a bending stress and resulting total stress range and stress amplitude can be calculated. Using an appropriate fatigue curve with the calculated stress amplitude, a permissible number of pressure cycles can be determined. A comparison between the specified design cycles and the permissible cycles, based on the gap height, is made to determine acceptance.

The procedure to determine the maximum gap height is iterative as an initial value of the gap height is assumed, the resulting bending stress and stress amplitude are calculated, and the permissible number of cycles is determined using as appropriate fatigue curve. If the specified design cycles are less than the determined permissible cycles, the gap can be increased until the specified design cycles equal the permissible number of cycles. Conversely, if the specified design cycles are greater than the calculated permissible cycles, the gap can be reduced until the specified design cycles equal the permissible number of cycles.

In accordance with VIII-2, paragraph 4.13.12.3.a.1, the maximum gap between any layers shall be evaluated as follows. For this example, conservatively consider the gap between layers to be located at the outermost layer of the vessel.

The circumferential stress of the shell and the bending stress due to the gap can be calculated as follows.

$$S = \frac{R_o^2 + R_1^2}{R_o^2 - R_1^2} P = \frac{(48.25)^2 + (42.0)^2}{(48.25)^2 - (42.0)^2}(3600) = 26116.7\ psi$$

where,

$$R_o = \begin{Bmatrix} Inside\ Radius + Number\ of\ Layers(Layer\ Thickness) \\ 42.0 + 20(0.3125) = 48.25\ in \end{Bmatrix}$$

$$R_1 = R_i = 42.0\ in$$

and,

$$S_b = \frac{1.812 E_y h}{R_g} = \frac{1.812(28.3E+06)(0.015)}{48.25} = 15941.8\ psi$$

where,

$$R_g = R_o = 48.25\ in$$

Since, $\{S_b = 15941.8\,psi\} < \{0.71S = 0.71(26800) = 19028\,psi\}$, the total stress range, ΔS_n, is calculated as follows.

$$\Delta S_n = S + 0.3S_b + P = 26116.7 + 0.3(15941.8) + 3600 = 34499.2\ psi$$

The stress amplitude for fatigue analysis at the gap is calculated as follows.

$$S_{ag} = \frac{K_e \Delta S_n}{2} = \frac{1.0(34499.2)}{2} = 17249.6\ psi$$

Since, $\{\Delta S_n = 34499.2\} < \{3S_m = 3(26800) = 80400\,psi\}$, the fatigue penalty factor, K_e, is calculated as follows.

$$K_e = 1.0$$

The fatigue analysis to determine the permissible number of cycles is in accordance with Annex 3-F, using the smooth bar design fatigue models per paragraph 3-F.1.2(a) for carbon steel not exceeding 700°F and paragraph 3-F.1.3. Since, $UTS = 95\ ksi$, interpolation of the permissible number of cycles is required using the equations found in paragraph 3-F.1.2(a)(1) and paragraph 3-F.1.2(a)(2).

Paragraph 3-F.1.2(a) and substituting $S_a = S_{ag} = 17249.6\,psi \rightarrow 17.2496\,ksi$,

$$Y = \log\left[28.3E+03\left(\frac{S_a}{E_T}\right)\right] = \log\left[28.3E+03\left(\frac{17.2496}{28.3E+03}\right)\right] = 1.2368$$

Paragraph 3-F.1.2(a)(1), with $\{10^Y = 10^{1.2368} = 17.2504\} < 20$, the number of permissible cycles is determined as follows.

$$X = \frac{38.1309 - 60.1705Y^2 + 25.0352Y^4}{1 + 1.80224Y^2 - 4.68904Y^4 + 2.26536Y^6}$$

$$X = \frac{38.1309 - 60.1705(1.2368)^2 + 25.0352(1.2368)^4}{1 + 1.80224(1.2368)^2 - 4.68904(1.2368)^4 + 2.26536(1.2368)^6}$$

$$X = 5.2272$$

Paragraph 3-F.1.3, the number of design cycles, N can be computed based on the parameter X calculated for the applicable materials as follows.

$$N_{80} = 10^X = 10^{5.2272} = 168733\ cycles$$

Paragraph 3-F.1.2(a)(2), with $\{10^Y = 10^{1.2368} = 17.2504\} < 43$, the number of permissible cycles is determined as follows.

$$X = \frac{-9.41749 + 14.7982Y - 5.94Y^2}{1 - 3.46282Y + 3.63495Y^2 - 1.21849Y^3}$$

$$X = \frac{-9.41749 + 14.7982(1.2368) - 5.94(1.2368)^2}{1 - 3.46282(1.2368) + 3.63495(1.2368)^2 - 1.21849(1.2368)^3}$$

$$X = 7.2454$$

Paragraph 3-F.1.3, the number of design cycles, N can be computed based on the parameter X calculated for the applicable materials as follows.

$$N_{115} = 10^X = 10^{7.2454} = 17595435\ cycles$$

Performing linear interpolation to determine the permissible number of cycles for $UTS = 95\ ksi$.

$$N_{95} = N_{80} + (UTS_{95} - UTS_{80})\frac{(N_{115} - N_{80})}{(UTS_{115} - UTS_{80})}$$

$$N_{95} = 168733 + (95 - 80)\frac{(17595435 - 168733)}{(115 - 80)}$$

$$N_{95} = 7.6E+06\ cycles$$

Since the estimated permissible number of cycles is greater than the specified design cycles, $\{N_{95} = 7.6E+06\} \geq N_D = 1.0E+06$, the anticipated gap at the outermost layer, $h = 0.015\ in.$, is acceptable.

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4.14 Evaluation of Vessels Outside of Tolerance

4.14.1 Example E4.14.1 – Shell Tolerances

A pressure vessel is constructed from NPS 30 long seam welded pipe. During construction, examination of the vessel shell indicates peaking at the long seam weld. Peaking is known to lead to in-service damage. Determine if the design is acceptable.

Vessel Data

- Material = $SA\text{-}333,\ Grade\ 6$
- Design Conditions = $325\ psig @ 600\ ^\circ F$
- Pipe Outside Diameter = $30\ in$
- Wall Thickness = $0.5\ in$
- Joint Efficiency = $100\ \%$
- Corrosion Allowance = $0.063\ in$
- Allowable Stress = $17100\ psi @ 600\ ^\circ F$

Examination Data

- Peaking distortion δ = $0.33\ in$

Section VIII, Division 1 Solution

Evaluate per UG-80.

The maximum inside diameter is:

$$ID_{\max} = 30 - 2(0.5) + 0.33 = 29.33\ in$$

The minimum inside diameter is:

$$ID_{min} = 30 - 2(0.5) = 29\ in$$

In accordance with UG-80

$$\frac{ID_{\max} - ID_{min}}{ID_{nom}} = \frac{0.33\ in}{29.0} \cdot 100.0 = 1.14\% \geq 1\%$$

The out of roundness is not acceptable in accordance with UG-80.

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4.14.2 Example E4.14.2 – Local Thin Area

For the vessel in Example Problem 1, an arc strike was removed during fabrication by blend grinding that has resulted in a region of local metal loss. Determine whether the local thin area is acceptable using Appendix 32.

Vessel Data

- Material = $SA\text{-}333,\ Grade\ 6$
- Design Conditions = $325\ psig @ 600\ °F$
- Pipe Outside Diameter = $30\ in$
- Wall Thickness = $0.5\ in$
- Joint Efficiency = $100\ \%$
- Future Corrosion Allowance = $0.063\ in$
- Allowable Stress = $17100\ psi$
- Supplemental Loads = $Negligible$
- The vessel is not in cyclic service (subject to less than 150 cycles).

Examination Data

Based on inspection data, the thickness profile measured in the longitudinal direction has a length of $4.0\ in.$ and measured in the circumferential direction has a length of $2.0\ in.$ The minimum measured thickness within the LTA is $0.36\ in.$ The region of local metal loss is located $45\ in$ away from the nearest structural discontinuity and is the only region of local metal loss found during inspection.

Section VIII, Division 1 Solution

Evaluate per Mandatory Appendix 32.

Paragraph UG-32-3, Nomenclature.

$$L = 4.0\ in$$

$$C = 2.0\ in$$

$$t_L = 0.36 - CA = 0.36 - 0.63 = 0.2970\ in$$

The required thickness of the vessel is determined in accordance with paragraph UG-27(c).

$$t = \frac{PR}{SE - 0.6P} = \frac{325(14.563)}{17100(1.0) - 0.6(325)} = 0.2800\ in$$

where,

$$R = \frac{OD}{2} - t_{nom} + CA = \frac{30}{2} - 0.5 + 0.063 = 14.563\ in$$

Paragraph 32-4(h), the edge of an LTA shall not be closer than $2.5\sqrt{Rt}$ to the centerline of a stiffening ring or structural support.

$$Spacing\ of\ 45\ in > \left\{2.5\sqrt{Rt} = 2.5\sqrt{(15 \cdot 0.5)} = 6.847\ in\right\} \rightarrow Satisfied$$

Paragraph 32-6(a), a single LTA shall satisfy the following equations, where

$$\left\{\frac{t_L}{t} = \frac{0.2970}{0.2800} = 1.06\right\} \geq 0.9 \qquad True$$

$$\{L = 4.0\ in\} \leq \left\{\sqrt{Rt} = \sqrt{14.563(0.2800)} = 2.0193\ in\right\} \qquad False$$

$$\{C = 2.0\ in\} \leq \{2L = 2(4.0) = 8.0\ in\} \qquad True$$

$$\{t - t_L = 0.2800 - 0.2970 = -0.017\ in\} \leq 0.1875\ in \qquad True$$

Paragraph 32-6(b), the longitudinal stress on the LTA from mechanical loads other than internal pressure shall not exceed $0.3S$.

Supplemental loads were noted as negligible.

Paragraph 32-6(c), the thickness of the LTA shall meet the requirements of UG-23(b) and/or UG-28 as applicable.

The vessel is not subject to external pressure or loads which produce compressive stresses.

The LTA must be repaired in accordance with Appendix 32.

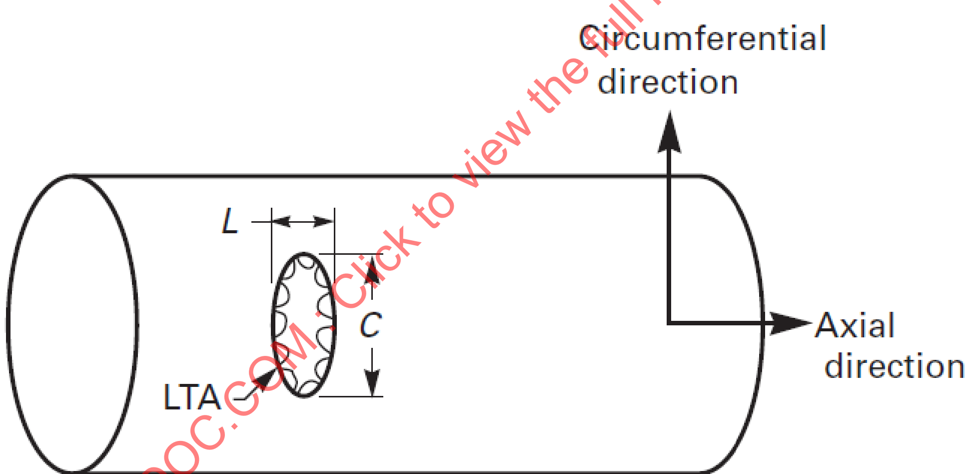

Figure E4.14.1 – Figure 32-3(a) Nomenclature

4.15 Supports and Attachments

4.15.1 Example E4.15.1 – Horizontal Vessel Supported by Two Saddles

Determine if the stresses in the horizontal vessel induced by the proposed saddle supports are with acceptable limits. The vessel is supported by two symmetric equally spaced saddles welded to the vessel, without reinforcing plates or stiffening rings. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined. See Figure E4.15.1

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $1800\ psig@175°F$
- Inside Cylinder Diameter = $60.0\ in$
- Cylinder Thickness = $3.0\ in$
- Corrosion Allowance = $0.125\ in$
- Formed Head Type = $2:1\ Elliptical$
- Head Thickness = $3.0\ in$
- Allowable Stress = $20000\ psi$
- Yield Stress at Design Temperature = $35250\ psi$
- Weld Joint Efficiency = 1.0
- Shell Tangent to Tangent Length = $292.0\ in$

Saddle Data:

- Material = $SA-516,\ Grade\ 70$
- Saddle Center Line to Head Tangent Line = $41.0\ in$
- Saddle Contact Angle = $123.0\ deg$
- Width of Saddles = $8.0\ in$
- Vessel Load per Saddle = $50459.0\ lbs$

Adjust the vessel inside diameter and thickness by the corrosion allowance.

$$ID = ID_{uc} + 2(Corrosion\ \ Allowance) = 60.0 + 2(0.125) = 60.25\ in$$

$$t = t - Corrosion\ \ Allowance = 3.0 - 0.125 = 2.875\ in$$

$$t_h = t_h - Corrosion\ \ Allowance = 3.0 - 0.125 = 2.875\ in$$

$$R_m = \frac{OD + ID}{4} = \frac{66.0 + 60.25}{4} = 31.5625\ in$$

$$R_{mh} = \frac{OD + ID}{4} = \frac{66.0 + 60.25}{4} = 31.5625\ in$$

$$h_m = \left(0.25(ID_{uc}) + Corrosion\ \ Allowance\right) + 0.5t_h$$

$$h_m = \left(0.25(60.0) + 0.125\right) + 0.5(2.875) = 16.5625\ in$$

Section VIII, Division 1 Solution

VIII-1 does not provide rules for saddle supported vessels. However, UG-22 requires consideration of such loadings and the provisions of U-2(g) apply. This example provides one possible method of satisfying U-2(g) via Mandatory Appendix 46 as referenced in U-2(g)(1)(a); however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

The design rules for supports and attachments provided in VIII-2, paragraph 4.15.3 is one of the accepted analysis procedures for determining the stresses in the shell of a horizontal drum support on two saddle supports. This procedure will be used in this example problem.

VIII-2, paragraph 4.15.3.1, Application of Rules:

a) The stress calculation method is based on linear elastic mechanics and covers modes of failure by excessive deformation and elastic instability.

b) Saddle supports for horizontal vessels shall be configured to provide continuous support for at least one-third of the shell circumference, or $\theta = 120.0\ deg$.

   Since $\{\theta = 123.0\ deg\} \geq \{\theta_{req} = 120.0\ deg\}$ the geometry is acceptable.

VIII-2, paragraph 4.15.3.2, Moment and Shear Force:

The vessel is composed of a cylindrical shell with formed heads at each end that is supported by two equally spaced saddle supports. The moment at the saddle, M_1, the moment at the center of the vessel, M_2, and the shear force at the saddle, T, may be computed if the distance between the saddle centerline and head tangent line satisfies the following limit.

$$\{a = 41.0\ in\} \leq \{0.25L = 0.25(292.0) = 73.0\ in\} \qquad Satisfied$$

Bending Moment at the Saddle:

$$M_1 = -Qa\left(1 - \frac{1 - \dfrac{a}{L} + \dfrac{R_m^2 - h_m^2}{2aL}}{1 + \dfrac{4h_m}{3L}}\right)$$

$$M_1 = -(50459.0)(41.0)\left(1 - \frac{1 - \left(\dfrac{41.0}{292.0}\right) + \dfrac{(31.5625)^2 - (16.5625)^2}{2(41.0)(292.0)}}{1 + \dfrac{4(16.5625)}{3(292.0)}}\right)$$

$$M_1 = -357533.9\ in-lbs$$

Bending Moment at the Center of the Vessel:

$$M_2 = \frac{QL}{4}\left(\frac{1+\dfrac{2\left(R_m^2-h_m^2\right)}{L^2}}{1+\dfrac{4h_m}{3L}}-\frac{4a}{L}\right)$$

$$M_2 = \frac{50459.0(292.0)}{4}\left(\frac{1+\dfrac{2\left[(31.5625)^2-(16.5625)^2\right]}{(292.0)^2}}{1+\dfrac{4(16.5625)}{3(292.0)}}-\frac{4(41.0)}{292.0}\right)$$

$$M_2 == 1413685.4\ in-lbs$$

Shear Force at the Saddle

$$T = \frac{Q(L-2a)}{L+\dfrac{4h_m}{3}} = \frac{50459.0\left[292.0-2(41.0)\right]}{292.0+\dfrac{4(16.5625)}{3}} = 33737.5\ lbs$$

VIII-2, paragraph 4.15.3.3, Longitudinal Stress:

a) The longitudinal membrane plus bending stresses in the cylindrical shell between the supports are given by the following equations.

At the top of shell:

$$\sigma_1 = \frac{PR_m}{2t}-\frac{M_2}{\pi R_m^2 t} = \frac{1800(31.5625)}{2(2.875)}-\frac{1413685.4}{\pi(31.5625)^2(2.875)} = 9723.3\ psi$$

Note: A load combination that includes zero internal pressure and the vessel full of contents would provide the largest compressive stress at the top of the shell and should be checked as part of the design.

At the bottom of the shell:

$$\sigma_2 = \frac{PR_m}{2t}+\frac{M_2}{\pi R_m^2 t} = \frac{1800(31.5625)}{2(2.875)}+\frac{1413685.4}{\pi(31.5625)^2(2.875)} = 10037.6\ psi$$

b) The longitudinal stresses in the cylindrical shell at the support location are given by the following equations. The values of these stresses depend on the rigidity of the shell at the saddle support. The cylindrical shell may be considered as suitably stiffened if it incorporates stiffening rings at, or on both sides of the saddle support, or if the support is sufficiently close defined as $a \le 0.5R_m$ to the elliptical head.

Since $\{a = 41.0\ in\} > \{0.5R_m = 0.5(31.5625) = 15.7813\ in\}$, the criterion is not satisfied.

Therefore, for an unstiffened shell, calculate the maximum values of longitudinal membrane plus bending stresses at the saddle support as follows.

At points A and B in VIII-2, Figure 4.15.5:

$$\sigma_3^* = \frac{PR_m}{2t} - \frac{M_1}{K_1 \pi R_m^2 t} = \frac{1800(31.5625)}{2(2.875)} - \frac{-357533.9}{0.1114(\pi)(31.5625)^2(2.875)} = 10237.1\ psi$$

where the coefficient K_1 is found in VIII-2, Table 4.15.1,

$$K_1 = \frac{\Delta + \sin\Delta \cdot \cos\Delta - \dfrac{2\sin^2\Delta}{\Delta}}{\pi\left(\dfrac{\sin\Delta}{\Delta} - \cos\Delta\right)} = \frac{1.4181 + \sin[1.4181]\cdot\cos[1.4181] - \dfrac{2\sin^2[1.4181]}{1.4181}}{\pi\left(\dfrac{\sin[1.4181]}{1.4181} - \cos[1.4181]\right)}$$

$$K_1 = 0.1114$$

$$\Delta = \frac{\pi}{6} + \frac{5\theta}{12} = \frac{\pi}{6} + \frac{5\left[(123.0)\left(\dfrac{\pi}{180}\right)\right]}{12} = 1.4181\ rad$$

At the bottom of the shell:

$$\sigma_4^* = \frac{PR_m}{2t} + \frac{M_1}{K_1^* \pi R_m^2 t} = \frac{1800(31.5625)}{2(2.875)} + \frac{-357533.9}{0.2003(\pi)(31.5625)^2(2.875)} = 9682.1\ psi$$

where the coefficient K_1^* is found in VIII-2, Table 4.15.1,

$$K_1^* = \frac{\Delta + \sin\Delta \cdot \cos\Delta - \dfrac{2\sin^2\Delta}{\Delta}}{\pi\left(1 - \dfrac{\sin\Delta}{\Delta}\right)} = \frac{1.4181 + \sin[1.4181]\cdot\cos[1.4181] - \dfrac{2\sin^2[1.4181]}{1.4181}}{\pi\left(1 - \dfrac{\sin[1.4181]}{1.4181}\right)}$$

$$K_1^* = 0.2003$$

c) Acceptance Criteria:

$$\left\{|\sigma_1| = |9723.3|\ psi\right\} \le \left\{SE = 20000(1.0) = 20000\ psi\right\} \qquad True$$

$$\left\{|\sigma_2| = |10037.6|\ psi\right\} \le \left\{SE = 20000(1.0) = 20000\ psi\right\} \qquad True$$

$$\left\{|\sigma_3^*| = |10237.1|\ psi\right\} \le \left\{SE = 20000(1.0) = 20000\ psi\right\} \qquad True$$

$$\left\{|\sigma_4^*| = |9682.1|\ psi\right\} \le \left\{SE = 20000(1.0) = 20000\ psi\right\} \qquad True$$

Since all calculated stresses are positive (tensile), the compressive stress check per VIII-2, paragraph 4.15.3.3.c.2 is not required.

VIII-2, paragraph 4.15.3.4, Shear Stresses:

The shear stress in the cylindrical shell without stiffening ring(s) that is not stiffened by a formed head, $\{a = 41.0\ in\} > \{0.5R_m = 0.5(31.5625) = 15.7813\ in\}$, is calculated as follows.

$$\tau_2 = \frac{K_2 T}{R_m t} = \frac{1.1229(33737.5)}{31.5625(2.875)} = 417.5\ psi$$

where the coefficient K_2 is found in VIII-2, Table 4.15.1,

$$K_2 = \frac{\sin\alpha}{\pi - \alpha + \sin\alpha \cdot \cos\alpha} = \frac{\sin[1.9648]}{\pi - (1.9648) + \sin[1.9648] \cdot \cos[1.9648]} = 1.1229$$

$$\alpha = 0.95\left(\pi - \frac{\theta}{2}\right) = 0.95\left(\pi - \frac{123.0\left(\frac{\pi}{180}\right)}{2}\right) = 1.9648\ rad$$

Acceptance Criteria:

$$|\tau_2| \le \min\left[0.8S, 0.533S_y\right]$$

$$|417.5|\ psi \le \left\{\min\left[0.8(20000), 0.533(35250)\right] = 16000\ psi\right\} \qquad True$$

VIII-2, paragraph 4.15.3.5, Circumferential Stress:

a) Maximum circumferential bending moment – the distribution of the circumferential bending moment at the saddle support is dependent on the use of stiffeners at the saddle location. For a cylindrical shell without a stiffening ring, the maximum circumferential bending moment is shown in VIII-2, Figure 4.15.6 Sketch (a) and is calculated as follows.

$$M_\beta = K_7 Q R_m = (0.0504)(50459.0)(31.5625) = 80267.7\ in-lbs$$

where the coefficient K_7 is found in VIII-2, Table 4.15.1,

when $a/R_m \ge 1.0$, $K_7 = K_6$,

$$\left\{\frac{a}{R_m} = \frac{41.0}{31.5625} = 1.2990\right\} \ge 1.0 \rightarrow K_7 = K_6 = 0.0504$$

$$K_6 = \left(\frac{\frac{3\cos\beta}{4}\left(\frac{\sin\beta}{\beta}\right)^2 - \frac{5\sin\beta\cos^2\beta}{4\beta} + \frac{\cos^3\beta}{2} - \frac{\sin\beta}{4\beta} + \frac{\cos\beta}{4} - \beta\sin\beta\left[\left(\frac{\sin\beta}{\beta}\right)^2 - \frac{1}{2} - \frac{\sin 2\beta}{4\beta}\right]}{2\pi\left[\left(\frac{\sin\beta}{\beta}\right)^2 - \frac{1}{2} - \frac{\sin 2\beta}{4\beta}\right]}\right)$$

$$K_6 = \left(\frac{\begin{array}{l} \dfrac{3\cos[2.0682]}{4}\left(\dfrac{\sin[2.0682]}{2.0682}\right)^2 - \dfrac{5\sin[2.0682]\cos^2[2.0682]}{4(2.0682)} + \\ \dfrac{\cos^3[2.0682]}{2} - \dfrac{\sin[2.0682]}{4(2.0682)} + \dfrac{\cos[2.0682]}{4} - \\ (2.0682)\sin[2.0682]\left[\left(\dfrac{\sin[2.0682]}{2.0682}\right)^2 - \dfrac{1}{2} - \left(\dfrac{\sin[2(2.0682)]}{4(2.0682)}\right)\right] \end{array}}{2\pi\left[\left(\dfrac{\sin[2.0682]}{2.0682}\right)^2 - \dfrac{1}{2} - \left(\dfrac{\sin[2(2.0682)]}{4(2.0682)}\right)\right]} \right) = 0.0504$$

$$\beta = \pi - \frac{\theta}{2} = \pi - \frac{123.0\left(\dfrac{\pi}{180}\right)}{2} = 2.0682 \ rad$$

b) Width of cylindrical shell – the width of the cylindrical shell that contributes to the strength of the cylindrical shell at the saddle location shall be determined as follows.

$$\{x_1, x_2\} \le \left\{0.78\sqrt{R_m t} = 0.78\sqrt{31.5625(2.875)} = 7.4302 \ in\right\}$$

If the width $(0.5b + x_1)$ extends beyond the limit of a, as shown in VIII-2, Figure 4.15.2, then the width x_1 shall be reduced such as not to exceed a.

$$\left\{(0.5b + x_1) = 0.5(8.0) + 7.4302 = 11.4302 \ in\right\} \le \{a = 41.0 \ in\} \qquad \textit{Satisfied}$$

c) Circumferential stresses in the cylindrical shell without stiffening ring(s).

The maximum compressive circumferential membrane stress in the cylindrical shell at the base of the saddle support shall be calculated as follows.

$$\sigma_6 = \frac{-K_5 Q k}{t(b + x_1 + x_2)} = \frac{-0.7492(50459.0)(0.1)}{2.875(8.0 + 7.4302 + 7.4302)} = -57.5 \ psi$$

where the coefficient K_5 is found in Table 4.15.1,

$$K_5 = \frac{1 + \cos\alpha}{\pi - \alpha + \sin\alpha \cdot \cos\alpha} = \frac{1 + \cos[1.9648]}{\pi - (1.9648) + \sin[1.9648] \cdot \cos[1.9648]} = 0.7492$$

$$k = 0.1 \qquad \textit{when the vessel is welded to the saddle support}$$

The circumferential compressive membrane plus bending stress at Points G and H of VIII-2, Figure 4.15.6 Sketch (a) is determined as follows.

If $L \geq 8R_m$, then the circumferential compressive membrane plus bending stress shall be computed using VIII-2, Equation (4.15.24).

Since $\{L = 292.0\ in\} \geq \{8R_m = 8(31.5625) = 252.5\ in\}$, the criterion is satisfied.

$$\sigma_7 = \frac{-Q}{4t\left(b+x_1+x_2\right)} - \frac{3K_7Q}{2t^2}$$

$$\sigma_7 = \frac{-(50459.0)}{4(2.875)(8+7.4302+7.4302)} - \frac{3(0.0504)(50459.0)}{2(2.875)^2} = -653.4\ psi$$

The stresses at σ_6 and σ_7 may be reduced by adding a reinforcement or wear plate at the saddle location that is welded to the cylindrical shell.

A wear plate was not specified in this problem.

Acceptance Criteria:

$$\left\{|\sigma_6| = |57.5|\ psi\right\} \leq \left\{S = 20000\ psi\right\} \qquad True$$

$$\left\{|\sigma_7| = |653.4|\ psi\right\} \leq \left\{1.25S = 1.25(20000) = 25000\ psi\right\} \qquad True$$

VIII-2, paragraph 4.15.3.6, Horizontal Splitting Force:

The horizontal force at the minimum section at the low point of the saddle is given by VIII-2, Equation (4.15.42). The saddle shall be designed to resist this force.

$$F_h = Q\left(\frac{1+\cos\beta - 0.5\sin^2\beta}{\pi - \beta + \sin\beta \cdot \cos\beta}\right)$$

$$F_h = (50459.0)\left(\frac{1+\cos[2.0682] - 0.5\sin^2[2.0682]}{\pi - (2.0682) + \sin[2.0682]\cdot\cos[2.0682]}\right) = 10545.1\ lbs$$

Commentary: The horizontal splitting force is equal to the sum of all the horizontal reactions at the saddle due to the weight loading of the vessel. The splitting force is used to calculate tension stress and bending stress in the web of the saddle. The following provides one possible method of calculating the tension and bending stress in the web and its acceptance criteria. However, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

The membrane stress is given by,

$$\left\{\sigma_t = \frac{F_h}{A_s}\right\} \leq \left\{0.6S_y\right\}$$

where A_2 is the cross-sectional area of the web at the low point of the saddle with units of in^2, and S_y is the yield stress of the saddle material with units of psi.

The bending stress is given by,

$$\left\{ \sigma_b = \frac{F_h \cdot d \cdot c}{I} \right\} \leq \left\{ 0.66 S_y \right\}$$

where d is the moment arm of the horizontal splitting force, measured from the center of gravity of the saddle arc to the bottom of the saddle baseplate with units of in, c is the distance from the centroid of the saddle composite section to the extreme fiber with units of in, I is the moment of inertia of the composite section of the saddle with units of in^4, and S_y is the yield stress of the saddle material with units of psi.

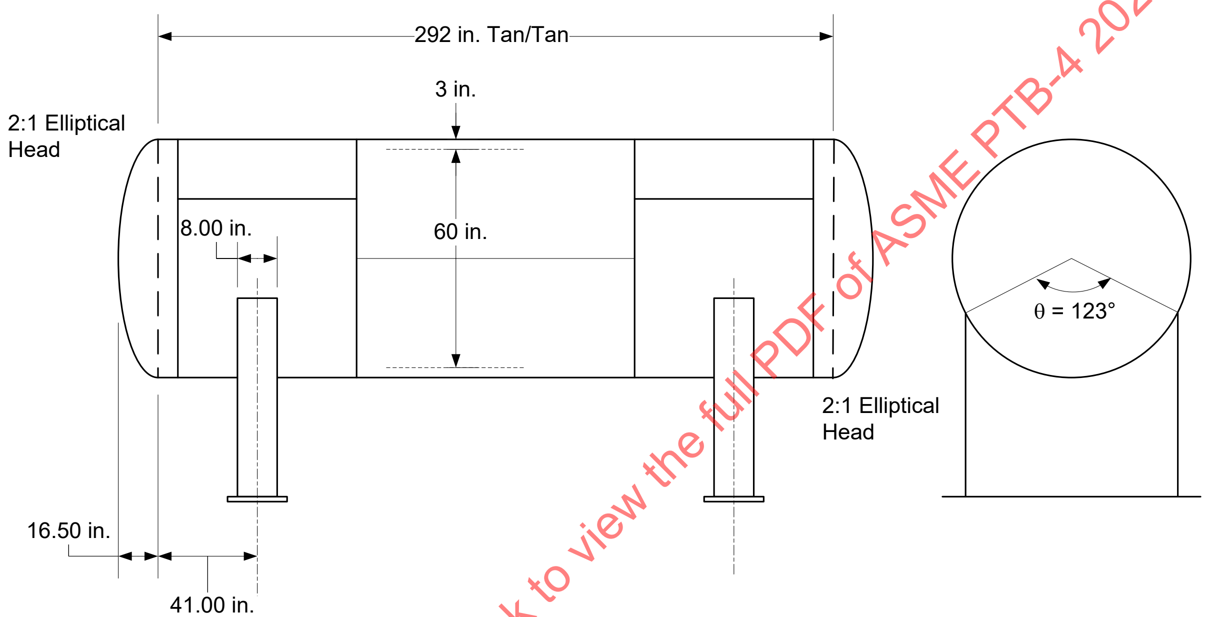

Figure E4.15.1 – Saddle Details

4.15.2 Example E4.15.2 – Vertical Vessel, Skirt Design

Determine if the proposed cylindrical vessel skirt is adequately designed considering the following loading conditions.

Skirt Data:

- Material = $SA-516,\ Grade\ 70$
- Design Temperature = $300°F$
- Skirt Inside Diameter = $150.0\ in$
- Thickness = $0.625\ in$
- Corrosion Allowance = $0.0\ in$
- Length of Skirt = $147.0\ in$
- Allowable Stress at Design Temperature = $20000\ psi$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength at Design Temperature = $33600\ psi$
- Design Loads = See Table E4.15.2.3

Adjust variable for corrosion and determine outside dimensions.

$$D = 150.0 + 2(Corrosion\ Allowance) = 150.0 + 2(0.0) = 150.0\ in$$

$$R = 0.5D = 0.5(150.0) = 75.0\ in$$

$$t = 0.625 - Corosion\ Allowance = 0.625 - 0.0 = 0.625\ in$$

$$D_o = 150.0 + 2(Uncorroded\ Thickness) = 150.0 + 2(0.625) = 151.25\ in$$

$$R_o = 0.5D_o = 0.5(151.25) = 75.625\ in$$

Section VIII, Division 1 Solution

VIII-1 does not provide rules on the loadings to be considered in the design of a vessel. However, UG-22 requires consideration of such loadings and the provisions of U-2(g) apply. This example provides one possible method of satisfying U-2(g) via Mandatory Appendix 46 as referenced in U-2(g)(1)(a); however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

This example uses VIII-2, paragraph 4.1.5.3 which provides specific requirements to account for both design loads and design load combinations used in the design of a vessel. These design loads and design load combinations (Table 4.1.1 and Table 4.1.2 of VIII-2, respectively) are shown in this example problem in Table E4.15.2.1 and Table E4.15.2.2 for reference. The load factor, Ω_p, shown in Table 4.1.2 of VIII-2 is used to simulate the maximum anticipated operating pressure acting simultaneously with the occasional loads specified. For this example, problem, $\Omega_p = 1.0$.

Additionally, VIII-1 does not provide a procedure for the calculation of combined stresses. VIII-2, paragraph 4.3.10.2 provides a procedure, and this procedure is used in this example problem with modifications to address specific requirements of VIII-1.

In accordance with VIII-2, paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment.

Determine applicability of the rules of VIII-2, paragraph 4.3.10 based on satisfaction of the following requirements.

- The section of interest is at least $2.5\sqrt{Rt}$ away from any major structural discontinuity.

$$2.5\sqrt{Rt} = 2.5\sqrt{(75.0)(0.625)} = 17.1163 \; in \qquad True$$

- Shear force is not applicable.
- The shell R/t ratio is greater than 3.0, or:

$$\left\{R/t = \frac{75.0}{0.625} = 120.0\right\} > 3.0 \qquad\qquad True$$

By inspection of the results shown in Table E4.15.2.3 and Table E4.15.2.4, Design Load Combination 5 is determined to be the governing load combination. The pressure, net section axial force, and bending moment at the location of interest for Design Load Combination 5 are:

$$\Omega P + P_s = 1.0P + P_s = 0.0 \; psi$$
$$F_5 = -363500 \; lbs$$
$$M_5 = 29110000 \; in-lbs$$

a) STEP 1 – Calculate the membrane stress for the cylindrical shell. Note that the circumferential membrane stress, $\sigma_{\theta m}$, is determined based on the equations in UG-27(c)(1) and the longitudinal membrane stress due to internal pressure, σ_{sm}, is determined based on the equations in UG-27(c)(2). The shear stress is computed based on the known strength of materials solution. For the skirt, weld joint efficiency is set as $E = 1.0$.

Note: θ is defined as the angle measured around the circumference from the direction of the applied bending moment to the point under consideration. For this example, problem $\theta = 0.0 \; deg$ to maximize the bending stress.

$$\sigma_{\theta m} = \frac{1}{E}\left(\frac{PR}{t} + 0.6P\right) = \frac{1}{1.0}\left(\frac{0.0(75.0)}{0.625} + 0.6(0.0)\right) = 0.0 \; psi$$

$$\sigma_{sm} = \frac{1}{E}\left(\left(\frac{PR}{2t} - 0.2P\right) + \frac{4F}{\pi\left(D_o^2 - D^2\right)} \pm \frac{32MD_o\cos[\theta]}{\pi\left(D_o^4 - D^4\right)}\right)$$

$$\sigma_{sm} = \frac{1}{1.0}\left(\begin{gathered}\left(\frac{0.0(75.0)}{2(0.625)} - 0.2(0.0)\right) + \frac{4(-363500)}{\pi\left((151.25)^2 - (150.0)^2\right)} \pm \\ \frac{32(29110000)(151.25)\cos[0.0]}{\pi\left((151.25)^4 - (150.0)^4\right)}\end{gathered}\right)$$

$$\sigma_{sm} = \begin{Bmatrix} 0.0 + (-1229.0724) + 2624.6357 = 1395.5633 \; psi \\ 0.0 + (-1229.0724) - 2624.6357 = -3853.7081 \; psi \end{Bmatrix}$$

$$\tau = \frac{16M_t D_o}{\pi\left(D_o^4 - D^4\right)} = \frac{16(0.0)(151.25)}{\pi\left((151.25)^4 - (150.0)^4\right)} = 0.0 \ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4(\tau)^2}\right)$$

$$\sigma_1 = \begin{cases} 0.5\left(0 + (1395.5633) + \sqrt{\left(0 - (1395.5633)\right)^2 + 4(0)^2}\right) = 1395.5633 \ psi \\ 0.5\left(0 + (-3853.7081) + \sqrt{\left(0 - (-3853.7081)\right)^2 + 4(0)^2}\right) = 0.0 \ psi \end{cases}$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2}\right)$$

$$\sigma_2 = \begin{cases} 0.5\left(0 + (1395.5633) - \sqrt{\left(0 - (1395.5633)\right)^2 + 4(0)^2}\right) = 0.0 \ psi \\ 0.5\left(0 + (-3853.7081) - \sqrt{\left(0 - (-3853.7081)\right)^2 + 4(0)^2}\right) = -3853.7081 \ psi \end{cases}$$

$$\sigma_3 = \sigma_r = 0.0 \ psi \qquad \textit{For stress on the outside surface}$$

c) STEP 3 – Check the allowable stress acceptance criteria.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]^{0.5}$$

$$\sigma_e = \begin{cases} \frac{1}{\sqrt{2}}\begin{bmatrix} \left(0 - (1395.5633)\right)^2 + \left((1395.5633) - 0\right)^2 + \\ \left((0) - (0)\right)^2 \end{bmatrix}^{0.5} = 1395.6 \ psi \\ \frac{1}{\sqrt{2}}\begin{bmatrix} \left(0 - (-3853.7081)\right)^2 + \left((-3853.7081) - 0\right)^2 + \\ \left((0) - (0)\right)^2 \end{bmatrix}^{0.5} = 3853.7 \ psi \end{cases}$$

$$\begin{Bmatrix} \sigma_e = 1395.6 \\ \sigma_e = 3853.7 \end{Bmatrix} \le \left\{S = 20000 \ psi\right\} \qquad \textit{True}$$

Note that VIII-2 uses an acceptance criterion based on von Mises Stress. Per Mandatory Appendix 46, the acceptance criteria for tensile stress in VIII-1 is in accordance with UG-23. Therefore,

$$\max\left[\sigma_1, \sigma_2, \sigma_3\right] \le S$$

$$\left\{\max\left[1395.6, \left|-3853.7\right|, 0.0\right] = 3853.7 \ psi\right\} \le \left\{S = 20000 \ psi\right\} \qquad \textit{True}$$

Since the maximum tensile principal stress is less than the acceptance criteria, the shell section is adequately designed.

d) STEP 4 – For cylindrical and conical shells, if the meridional stress, σ_{sm} is compressive, then check the allowable compressive stress using paragraph 4.4.12.2 with $\lambda = 0.15$. Per Mandatory Appendix 46, the maximum allowable compressive stress shall be limited as prescribed in VIII-2, paragraph 4.4.12 in lieu of the rules of UG-23(b).

Since σ_{sm} is compressive, $\{\sigma_{sm} = -3853.7\ psi < 0\}$, a compressive stress check is required.

In accordance with VIII-2, paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

1) STEP 4.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$$F_{xe} = \frac{C_x E_y t}{D_o} = \frac{0.6482(28.3E+06)(0.625)}{151.25} = 75801.9008\ psi$$

where,

$$\frac{D_o}{t} = \frac{151.25}{0.625} = 242.0$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{147.0}{\sqrt{75.625(0.625)}} = 21.3818$$

Since $D_o/t < 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389+\frac{D_o}{t}\right)},\ 0.9\right] = \min\left[\frac{409(1.0)}{389+\frac{151.25}{0.625}},\ 0.9\right] = 0.6482$$

Since $M_x \geq 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

2) STEP 4.2 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{xe}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 4.2.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$F_{xe} = 75801.9008\ psi$ (as determined in STEP 2 above)

ii) STEP 4.2.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{xe}}{E} = \frac{75801.9008}{28.3E+06} = 0.00267851$$

iii) STEP 4.2.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 24624.7292 \ psi$$

3) STEP 4.3 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{24624.7292}{33600}\right) = 1.8639$$

4) STEP 4.4 – Calculate the allowable axial compressive membrane stress as follows:

$$F_{xa} = \frac{F_{ic}}{FS} = \frac{24624.7292}{1.8639} = 13211.4004 \ psi$$

5) STEP 4.5 – Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria.

$$\left\{\sigma_{sm} = 3853.7 \ psi\right\} \le \left\{F_{xa} = 13211.4 \ psi\right\} \qquad True$$

The allowable compressive stress criterion is satisfied.

Table E4.15.2.1 – Design Loads from VIII-2

Table 4.1.1 – Design Loads	
Design Load Parameter	**Description**
P	Internal of External Specified Design Pressure (see paragraph 4.1.5.2.a)
P_s	Static head from liquid or bulk materials (e.g., catalyst)
D	Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following: • Weight of vessel including internals, supports (e.g., skirts, lugs, saddles, and legs), and appurtenances (e.g., platforms, ladders, etc.) • Weight of vessel contents under design and test conditions • Refractory linings, insulation • Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping • Transportation loads (the static forces obtained as equivalent to the dynamic loads experienced during normal operation of a transport vessel [see paragraph 1.2.1.2(b)]
L	• Appurtenance live loading • Effects of fluid flow, steady state or transient • Loads resulting from wave action
E	Earthquake loads [see paragraph 4.1.5.3(b)]
W	Wind loads [see paragraph 4.1.5.3(b)]
S_s	Snow loads
F	Loads due to deflagration

Table E4.15.2.2 – Design Load Combinations from VIII-2

Table 4.1.2 – Design Load Combinations	
Design Load Combination [Note (1) and (2)]	**General Primary Membrane Allowable Stress [Note (3)]**
$P+P_s+D$	S
$P+P_s+D+L$	S
$P+P_s+D+S_s$	S
$\Omega P+P_s+D+0.75L+0.75S_s$	S
$\Omega P+P_s+D+(0.6W\ or\ 0.7E)$	S
$\Omega P+P_S+D+0.75(0.6W\ or\ 0.7E)+0.75L+0.75S_s$	S
$0.6D+(0.6W\ or\ 0.7E)$ [Note (4)]	S
P_s+D+F	See Annex 4-D
Other load combinations as defined in the UDS	S

Notes:

1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.
2) See paragraph 4.1.5.3 for additional requirements.
3) S is the allowable stress for the load case combination [see paragraph 4.1.5.3(c)].
4) This load combination addresses an overturning condition for foundation design. It does not apply to design of anchorage (if any) to the foundation. Refer to ASCE/SEI 7, 2.4.1 Exception 2 for an additional reduction to W that may be applicable.

Table E4.15.2.3 – Design Loads (Net-Section Axial Force and Bending Moment) at the Base of The Skirt

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a); The skirt is not pressurized.	$P = 0.0$
P_s	Static head from liquid or bulk materials (e.g., catalyst); The skirt does not contain liquid head.	$P_s = 0.0$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest.	$D_F = -363500\ lbs$ $D_M = 0.0\ in - lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = -85700\ lbs$ $L_M = 90580\ in - lbs$
E	Earthquake loads	$E_F = 0.0\ lbs$ $E_M = 18550000\ in - lbs$
W	Wind Loads	$W_F = 0.0\ lbs$ $W_M = 48516667\ in - lbs$
S_s	Snow Loads	$S_{sF} = 0.0\ lbs$ $S_{sM} = 0.0\ in - lbs$
F	Loads due to Deflagration	$F_F = 0.0\ lbs$ $F_M = 0.0\ in - lbs$

Based on these loads, the skirt is required to be designed for the design load combinations shown in Table E4.15.2.4. Note that this table is given in terms of the design load combinations shown in VIII-2, Table 4.1.2 (Table E4.15.2.2 of this example).

Table E4.15.2.4 – Design Load Combination at the Base of the Skirt

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P+P_s+D$	$P=P_s=0.0\ psi$ $F_1=-363500\ lbs$ $M_1=0.0\ in-lbs$	S
2	$P+P_s+D+L$	$P=P_s=0.0\ psi$ $F_2=-449200\ lbs$ $M_2=90580\ in-lbs$	S
3	$P+P_s+D+S_s$	$P=P_s=0.0\ psi$ $F_3=-363500\ lbs$ $M_3=0.0\ in-lbs$	S
4	$\Omega P+P_s+D+0.75L+0.75S_s$	$\Omega P=P_s=0.0\ psi$ $F_4=-427775\ lbs$ $M_4=67935\ in-lbs$	S
5	$\Omega P+P_s+D+\left(0.6W\ or\ 0.7E\right)$	$\Omega P=P_s=0.0\ psi$ $F_5=-363500\ lbs$ $M_5=29110000\ in-lbs$	S
6	$\Omega P+P_S+D+0.75\left(0.6W\ or\ 0.7E\right)+$ $0.75L+0.75S_s$	$\Omega P=P_s=0.0\ psi$ $F_6=-427775\ lbs$ $M_6=21900435\ in-lbs$	S
7	$0.6D+\left(0.6W\ or\ 0.7E\right)$	$F_7=-218100\ lbs$ $M_7=29110000\ in-lbs$	S
8	P_s+D+F	$P=P_s=0.0\ psi$ $F_8=-363500\ lbs$ $M_8=0.0\ in-lbs$	See Annex 4-D

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4.16 Flanged Joints

4.16.1 Example E4.16.1 – Integral Type

Determine if the stresses in the heat exchanger girth flange are with acceptable limits, considering the following design conditions. The flange is of an integral type and is attached to a cylindrical shell with a Category C, Type 1 butt weld and has been 100% radiographically examined. See Figure E4.16.1.

General Data:

- Cylinder Material = $SA-516,\ Grade\ 70$
- Design Conditions = $135\ psig @ 650°F$
- Allowable Stress at Design Temperature = $18800\ psi$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Corrosion Allowance = $0.125\ in$

Flange Data:

- Material = $SA-105$
- Allowable Stress at Design Temperature = $17800\ psi$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Modulus of Elasticity at Design Temperature = $26.0E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$

Bolt Data:

- Material = $SA-193,\ Grade\ B7$
- Allowable Stress at Design Temperature = $25000\ psi$
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Diameter = $0.75\ in$
- Number of Bolts = 44
- Root area = $0.302\ in^2$

Gasket Data:

- Material = Flat Metal Jacketed (Iron/Soft Steel)
- Gasket Factor = 3.75
- Seating Stress = $7600\ psi$
- Inside Diameter = $29.0\ in$
- Outside Diameter = $30.0\ in$

Design rules for bolted flange connections with ring type gaskets are provided in VIII-1 Mandatory Appendix 2. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.16. However, there are differences to be noted, including a step-by-step design procedure, nomenclature

regarding operating and gasket seating bolt loads, the inclusion of a flange moment due to externally applied axial forces and bending moment, and minor differences in bolt spacing criteria. Therefore, while the example problem will be presented for use with VIII-1, Appendix 2, references to VIII-2 paragraphs will be provided, as applicable.

Evaluate the girth flange in accordance with VIII-1, Appendix 2.

Establish the design conditions and gasket reaction diameter, (VIII-2, paragraph 4.16.6).

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

$$P = 135\ psig\ at\ 650°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 2-5.1 (VIII-2, Table 4.16.1).

$$m = 3.75$$
$$y = 7600\ psi$$

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5(GOD - GID) = 0.5(30.0 - 29.0) = 0.500\ in$$

from Table 2-5.2 (VIII-2, Table 4.16.3), Facing Sketch Detail 2, Column II,

$$b_o = \frac{w + 3N}{8} = \frac{0.125 + 3(0.500)}{8} = 0.2031\ in$$

where,

$$w = raised\ nubbin\ width = 0.125\ in$$

for $b_o \le 0.25\ in$,

$$b = b_o = 0.2031\ in$$

therefore, from paragraph 2-3 the location of the gasket reaction is calculated as follows.

$$G = mean\ diameter\ of\ the\ gasket\ contact\ face$$
$$G = 0.5(30.0 + 29.0) = 29.5\ in$$

Paragraph 2-5 – Calculate the design bolt load for the operating and gasket seating conditions, (VIII-2, paragraph 4.16.6).

a) STEP 1 – Paragraph 2-5(c)(1), determine the design bolt load for the operating condition.

$$W_{m1} = H + H_p = 0.785G^2P + (2b \cdot 3.14GmP) \quad for\ non-self-energized\ gaskets$$
$$W_{m1} = 0.785(29.5)^2(135) + 2(0.2031)(3.14)(29.5)(3.75)(135) = 111273.1\ lbs$$

b) STEP 2 – Paragraph 2-5(c)(2), determine the design bolt load for the gasket seating condition.

$$W_{m2} = 3.14bGy \qquad for\ non-self-energized\ gaskets$$
$$W_{m2} = 3.14(0.2031)(29.5)(7600) = 142980.0\ lbs$$

c) STEP 3 – Paragraph 2-5(d), determine the total required and actual bolt areas.

The total cross-sectional area of bolts A_m required for both the operating conditions and gasket seating is determined as follows.

$$A_m = \max\left[A_{m1},\, A_{m2}\right] = \max\left[4.4532,\, 5.7192\right] = 5.7192\ in^2$$

where,

$$A_{m1} = \frac{W_{m1}}{S_b} = \frac{111273.1}{25000} = 4.4509\ in^2$$

$$A_{m2} = \frac{W_{m2}}{S_a} = \frac{142980.0}{25000} = 5.7192\ in^2$$

The actual bolt area A_b is calculated as follows.

$$A_b = (Number\ \ of\ \ bolts)(Root\ area\ of\ \ one\ bolt) = 44(0.302) = 13.2880\ in^2$$

Verify that the actual bolt area is equal to or greater than the total required area.

$$\left\{A_b = 13.2880\ in^2\right\} \geq \left\{A_m = 5.7192\ in^2\right\} \qquad True$$

d) STEP 4 – Paragraph 2-5(e), determine the flange design bolt load.

For operating conditions,

$$W = W_{m1} = 111273.1\ lbs$$

For gasket seating,

$$W = \frac{(A_m + A_b)S_a}{2} = \frac{(5.7192 + 13.2880)25000}{2} = 237590.0\ lbs$$

Commentary: VIII-2 design procedure to determine the design bolt load, paragraph 4.16.6:

1) The nomenclature differences include:

W_o, bolt load for operating conditions and flange design load bolt load, (VIII-1 W_{m1}, W).

$$W_{m1} = H + H_p = 0.785G^2P + (2b \cdot 3.14GmP) \quad \rightarrow ASME\ VIII-1$$

$$W_o = H + H_p = 0.785G^2P + (2b \cdot \pi GmP) \quad \rightarrow ASME\ VIII-2$$

W_{gs}, design bolt load for the gasket seating condition, (VIII-1, W_{m2}).

$$W_{m2} = 3.14bGy \rightarrow ASME\ VIII-1$$

$$W_{gs} = \pi bGy \quad \rightarrow ASME\ VIII-2$$

W_g, flange design bolt load for gasket seating, (VIII-1, W).

2) When calculating the required total cross-sectional area of bolts using the VIII-2 procedure, the designer has the ability to add an externally applied net-section axial force, F_A and bending moment, M_E to the bolt load W_o for the operating condition. This is shown in the following equation.

$$A_m = \max\left[\left(\frac{W_o + F_A + \frac{4M_E}{G}}{S_{bo}}\right), \left(\frac{W_{gs}}{S_{bg}}\right)\right]$$

3) The bolt spacing criteria of VIII-1 and VIII-2 are similar, but there are some minor differences regarding the application of the two. This will be further discussed later in the example problem.

Commentary: VIII-1, Appendix 2 does not include an overall step-by-step procedure to design a flange. However, an organized procedure of the steps taken when designing a flange is presented in VIII-2, paragraph 4.16.7. The procedure is applicable to VIII-1, Appendix 2 and is presented in this example problem to assist the designer.

<u>VIII-2, paragraph 4.16.7, Flange Design Procedure</u>. The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings. The procedure incorporates both a strength check and a rigidity check for flange rotation.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

$$P = 135 \ psig \ at \ 650°F$$

b) STEP 2 – Determine the design bolt loads for operating condition W, and the gasket seating condition W, and the corresponding actual bolt load area A_b, (VIII-1, paragraph 2-5).

$$W = 111273.9 \ lbs \qquad Operating \ Condition$$

$$W = 237590.0 \ lbs \qquad Gasket \ Seating$$

$$A_b = 13.2880 \ in^2$$

c) STEP 3 – Determine an initial flange geometry (see Figure E4.16.1) in addition to the information required to determine the bolt load, the following geometric parameters are required, (VIII-1, paragraph 2-3).

1) Flange bore

$$B = \left[26.0 + 2(Corrosion \ Allowance)\right] = \left[26.0 + 2(0.125)\right] = 26.25 \ in$$

2) Bolt circle diameter

$$C = 31.25 \ in$$

3) Outside diameter of the flange

$$A = 32.875 \ in$$

4) Flange thickness

$$t = 1.625 - 0.1875 = 1.4375 \ in$$

5) Thickness of the hub at the large end

$$g_1 = \left[0.5(Hub \ OD \ at \ Back \ of \ Flange - Uncorroded \ Bore) - Corrosion \ Allowance\right]$$

$$g_1 = \left[0.5(27.625 - 26.0) - 0.125\right] = 0.6875 \ in$$

6) Thickness of the hub at the small end

$$g_0 = \left(Hub\ Thickness\ at\ Cylinder\ Attachment - Corrosion\ Allowance \right)$$

$$g_0 = \left(0.4375 - 0.125\right) = 0.3125\ in$$

7) Hub length

$$h = 2.125\ in$$

d) STEP 4 – Determine the flange stress factors using the equations/direct interpretation from Table 2-7.1 and Figure 2-7.1 – Figure 2-7.6 and paragraph 2-3.

Figure 2-7.1:

$$K = \frac{A}{B} = \frac{32.875}{26.25} = 1.2524$$

$$Y = \frac{1}{K-1}\left[0.66845 + 5.71690\left(\frac{K^2 \log_{10} K}{K^2 - 1}\right)\right]$$

$$Y = \frac{1}{1.2524-1}\left[0.66845 + 5.71690\frac{(1.2524)^2 \log_{10}\left[1.2524\right]}{(1.2524)^2 - 1}\right] = 8.7565$$

$$T = \frac{K^2\left(1 + 8.55246\log_{10} K\right) - 1}{\left(1.04720 + 1.9448K^2\right)\left(K-1\right)} = \frac{(1.2524)^2\left(1 + 8.55246\log_{10}\left[1.2524\right]\right) - 1}{\left(1.04720 + 1.9448(1.2524)^2\right)\left(1.2524 - 1\right)} = 1.8175$$

$$U = \frac{K^2\left(1 + 8.55246\log_{10} K\right) - 1}{1.36136\left(K^2 - 1\right)\left(K-1\right)} = \frac{(1.2524)^2\left(1 + 8.55246\log_{10}\left[1.2524\right]\right) - 1}{1.36136\left((1.2524)^2 - 1\right)\left(1.2524 - 1\right)} = 9.6225$$

$$Z = \frac{\left(K^2 + 1\right)}{\left(K^2 - 1\right)} = \frac{\left((1.2524)^2 + 1\right)}{\left((1.2524)^2 - 1\right)} = 4.5180$$

Figure 2-7.2:

$$\frac{h}{h_o} = \frac{h}{\sqrt{Bg_0}} = \frac{2.125}{\sqrt{(26.25)(0.3125)}} = \frac{2.125}{2.8641} = 0.7419$$

$$\frac{g_1}{g_0} = \frac{0.6875}{0.3125} = 2.2000$$

Interpretation of Figure 2-7.2, $F \approx 0.77$. From the equations of Table 2-7.1, $F = 0.7677$.

Figure 2-7.3:

with $h/h_o = 0.7419$ and $g_1/g_o = 2.2000$:

Interpretation of Figure 2-7.3, $V \approx 0.16$. From the equations of Table 2-7.1, $V = 0.1577$.

Figure 2-7.6:

with $h/h_o = 0.7419$ and $g_1/g_o = 2.2000$:

Interpretation of Figure 2-7.6, $f = 1.0$. From the equations of Table 2-7.1, $f = 1.0$.

Paragraph 2-3:

$$d = \frac{Ug_0^2 h_o}{V} = \frac{(9.6225)(0.3125)^2(2.8641)}{0.1577} = 17.0665\ in^3$$

$$e = \frac{F}{h_o} = \frac{0.7677}{2.8641} = 0.2680\ in^{-1}$$

$$L = \frac{te+1}{T} + \frac{t^3}{d} = \frac{1.4375(0.2680)+1}{1.8175} + \frac{(1.4375)^3}{17.0665} = 0.9362$$

Commentary: VIII-2, Table 4.16.4 provides the equations to determine the flange stress factors Y, T, U, Z as a function of K as provided for in VIII-1, Figure 2-7.1, and the flange factors d, e, L located in VIII-1, paragraph 2-3. Similarly, VIII-2, Table 4.16.5 provides regressed curve-fit equations of the flange stress factors F, V, f as provided in VIII-1, Figure 2-7.2, Figure 2-7.3, and Figure 2-7.6, respectively. The curve-fit equations are shown for informational purposes with the variable substitution of $X_h = h/h_o$ and $X_g = g_1/g_o$.

$$F = \begin{pmatrix} 0.897697 - 0.297012\ln\left[X_g\right] + 9.5257\left(10^{-3}\right)\ln\left[X_h\right] + \\ 0.123586\left(\ln\left[X_g\right]\right)^2 + 0.0358580\left(\ln\left[X_h\right]\right)^2 - \\ 0.194422\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right) - 0.0181259\left(\ln\left[X_g\right]\right)^3 + \\ 0.0129360\left(\ln\left[X_h\right]\right)^3 - 0.0377693\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right)^2 + \\ 0.0273791\left(\ln\left[X_g\right]\right)^2\left(\ln\left[X_h\right]\right) \end{pmatrix}$$

$$F = \begin{pmatrix} 0.897697 - 0.297012\ln[2.20] + 9.5257\left(10^{-3}\right)\ln[0.7419] + \\ 0.123586\left(\ln[2.20]\right)^2 + 0.0358580\left(\ln[0.7419]\right)^2 - \\ 0.194422\left(\ln[2.20]\right)\left(\ln[0.7419]\right) - 0.0181259\left(\ln[2.20]\right)^3 + \\ 0.0129360\left(\ln[0.7419]\right)^3 - 0.0377693\left(\ln[2.20]\right)\left(\ln[0.7419]\right)^2 + \\ 0.0273791\left(\ln[2.20]\right)^2\left(\ln[0.7419]\right) \end{pmatrix}$$

$$F = 0.7695$$

$$For\ 0.5 \le X_h \le 2.0,$$

$$V = \left(\begin{array}{l} 0.0144868 - \dfrac{0.135977}{X_g} - \dfrac{0.0461919}{X_h} + \dfrac{0.560718}{X_g^2} + \dfrac{0.0529829}{X_h^2} + \\ \dfrac{0.244313}{X_g X_h} + \dfrac{0.113929}{X_g^3} - \dfrac{0.00929265}{X_h^3} - \dfrac{0.0266293}{X_g X_h^2} - \dfrac{0.217008}{X_g^2 X_h} \end{array} \right)$$

$$V = \left(\begin{array}{l} 0.0144868 - \dfrac{0.135977}{2.20} - \dfrac{0.0461919}{0.7419} + \dfrac{0.560718}{(2.20)^2} + \dfrac{0.0529829}{(0.7419)^2} + \\ \dfrac{0.244313}{(2.20)(0.7419)} + \dfrac{0.113929}{(2.20)^3} - \dfrac{0.00929265}{(0.7419)^3} - \dfrac{0.0266293}{(2.20)(0.7419)^2} - \dfrac{0.217008}{(2.20)^2(0.7419)} \end{array} \right)$$

$$V = 0.1577$$

$$f = \max\left[1.0, \left(\frac{\begin{array}{l} 0.0927779 - 0.0336633 X_g + 0.964176 X_g^2 + \\ 0.0566286 X_h + 0.347074 X_h^2 - 4.18699 X_h^3 \end{array}}{\begin{array}{l} 1 - 5.96093\left(10^{-3}\right) X_g + 1.62904 X_h + \\ 3.49329 X_h^2 + 1.39052 X_h^3 \end{array}} \right) \right]$$

$$f = \max\left[1.0, \left(\frac{\begin{array}{l} 0.0927779 - 0.0336633(2.20) + 0.964176(2.20)^2 + \\ 0.0566286(0.7419) + 0.347074(0.7419)^2 - 4.18699(0.7419)^3 \end{array}}{\begin{array}{l} 1 - 5.96093\left(10^{-3}\right)(2.20) + 1.62904(0.7419) + \\ 3.49329(2.20)^2 + 1.39052(0.7419)^3 \end{array}} \right) \right]$$

$$f = 1.0$$

e) STEP 5 – Determine the flange forces, (VIII-1, paragraph 2-3).

$$H_D = 0.785 B^2 P = 0.785(26.25)^2(135) = 73023.4\ lbs$$

$$H = 0.785 G^2 P = 0.785(29.5)^2(135) = 92224.7\ lbs$$

$$H_T = H - H_D = 92224.7 - 73023.4 = 19201.3\ lbs$$

$$H_G = W - H = 111273.9 - 92224.7 = 19049.2\ lbs \qquad Operating$$

f) STEP 6 – Determine the flange moment for the operating condition using paragraph 2-6. In these equations, h_D, h_T, and h_G are determined from Table 2-6.

$$h_D = R + 0.5 g_1 = \frac{C - B - g_1}{2} = \frac{31.25 - 26.25 - 0.6875}{2} = 2.1563\ in$$

$$h_G = \frac{C - G}{2} = \frac{31.25 - 29.5}{2} = 0.875\ in$$

$$h_T = \frac{R + g_1 + h_G}{2} = \frac{1}{2}\left[\frac{C - B}{2} + h_G\right] = \frac{1}{2}\left[\frac{31.25 - 26.25}{2} + 0.875\right] = 1.6875\ in$$

Paragraph 2-6:

$$M_o = H_D h_D + H_T h_T + H_G h_G$$
$$M_o = 73023.4(2.1563) + 19201.3(1.6875) + 19049.2(0.875)$$
$$M_o = 206530.6 \; in-lbs$$

For vessels in lethal service or when specified by the user or his designated agent, the bolt spacing correction factor B_{SC} shall be applied in calculating the flange stress in paragraph 2-7. The flange moment M_o without correction for bolt spacing is used for the calculation of the rigidity index in paragraph 2-14. When the bolt spacing exceeds $2a + t$, multiply M_o by the bolt spacing correction factor B_{SC} for calculating flange stress, where:

$$B_{SC} = \sqrt{\frac{B_s}{2a+t}}$$

Commentary:

1) The equations for maximum bolt spacing B_{smax} and bolt spacing correction factor B_{SC} used in the VIII-2 are the same as in VIII-1 and are in Table 4.16.11.

2) As is the case with VIII-1 design, VIII-2 also notes that the flange moment M_o without correction for bolt spacing is used for the calculation of the rigidity index located in Table 4.16.10.

3) The bolt spacing correction factor B_{SC} and the split loose flange factor of VIII-1, paragraph 2-9, F_s, are directly incorporated into the calculations for determining the flange moments for both operating and gasket seating.

$$M_o = abs\left[\left(\left(H_D h_D + H_T h_T + H_G h_G\right)B_{SC} + M_{oe}\right)F_s\right] \quad Internal \;\; Pressure$$

$$M_g = \frac{W_g\left(C-G\right)B_{SC}F_s}{2} \qquad Gasket \;\; Seating$$

4) As previously noted, the VIII-2 procedure provides the designer the ability to add an externally applied net-section axial force and bending moment to the bolt load for the operating condition. These externally applied loads induce a bending moment, referenced as M_{oe}, which is calculated from Equation 4.16.16,

$$M_{oe} = 4M_E\left[\frac{I}{0.3846I_p + I}\right]\cdot\left[\frac{h_D}{\left(C-2h_D\right)}\right] + F_A h_D$$

g) STEP 7 – Determine the flange moment for gasket seating condition using paragraph 2-6.

$$M_o = W\frac{(C-G)}{2} = 237590.0\left(\frac{(31.25-29.5)}{2}\right) = 207891.3 \; in-lbs$$

h) STEP 8 – Determine the flange stresses for the operating and gasket seating conditions using the equations in paragraph 2-7.

Note: Paragraph 2-3 – If $B < 20g_1$, the designer may substitute the value of B_1 for B in the equation for S_H, where, For integral flanges when $f \geq 1.0$,

$$B_1 = B + g_o$$

For integral flanges when $f < 1.0$ and for loose type flanges,

$$B_1 = B + g_1$$

Since $\{B = 26.25\ in\} \geq \{20g_1 = 20(0.6875) = 13.75\ in\}$, the value of B shall be used to determine the value of S_H.

Operating Condition:

$$S_H = \frac{fM_o}{Lg_1^2 B} = \frac{(1.0)(206530.6)}{(0.9362)(0.6875)^2(26.25)} = 17780.4\ psi$$

$$S_R = \frac{(1.33te+1)M_o}{Lt^2 B} = \frac{\left[(1.33)(1.4375)(0.2680)+1\right](206530.6)}{(0.9362)(1.4375)^2(26.25)} = 6150.8\ psi$$

$$S_T = \frac{YM_o}{t^2 B} - ZS_R = \frac{(8.7565)(206530.6)}{(1.4375)^2(26.25)} - 4.5180(6150.8) = 5551.0\ psi$$

Gasket Seating Condition:

$$S_H = \frac{fM_o}{Lg_1^2 B} = \frac{(1.0)(207891.3)}{(0.9362)(0.6875)^2(26.25)} = 17897.5\ psi$$

$$S_R = \frac{(1.33te+1)M_o}{Lt^2 B} = \frac{\left[(1.33)(1.4375)(0.2680)+1\right](207891.3)}{(0.9362)(1.4375)^2(26.25)} = 6191.3\ psi$$

$$S_T = \frac{YM_o}{t^2 B} - ZS_R = \frac{(8.7565)(207891.3)}{(1.4375)^2(26.25)} - 4.5180(6191.3) = 5587.7\ psi$$

i) STEP 9 – Check the flange stress acceptance criteria. The criteria below shall be evaluated. If the stress criteria are satisfied, go to STEP 10. If the stress criteria are not satisfied, re-proportion the flange dimensions and go to STEP 4.

Allowable normal stress – The criteria to evaluate the normal stresses for the operating and gasket seating conditions are shown in paragraph 2-8, for integral type flanges with hub welded to the neck, pipe, or vessel wall.

Operating Condition:

$$S_H \leq \min\left[1.5S_f,\ 2.5S_n\right]$$

$$\{S_H = 17780.4\ psi\} \leq \left\{\min\left[1.5(17800),\ 2.5(18800)\right] = 26700\ psi\right\} \qquad \mathit{True}$$

$$\{S_R = 6150.8\ psi\} \leq \{S_f = 17800\ psi\} \qquad \mathit{True}$$

$$\{S_T = 5551.0\ psi\} \leq \{S_f = 17800\ psi\} \qquad \mathit{True}$$

$$\left\{\frac{(S_H + S_R)}{2} = \frac{(17780.4 + 6150.8)}{2} = 11965.6 \ psi\right\} \le \left\{S_{fo} = 17800 \ psi\right\} \qquad True$$

$$\left\{\frac{(S_H + S_T)}{2} = \frac{(17780.4 + 5551.0)}{2} = 11665.7 \ psi\right\} \le \left\{S_{fo} = 17800 \ psi\right\} \qquad True$$

Gasket Seating Condition:

$$S_H \le \min\left[1.5S_f,\ 2.5S_n\right]$$

$$\left\{S_H = 17897.5 \ psi\right\} \le \left\{\min\left[1.5(20000),\ 2.5(20000)\right] = 30000 \ psi\right\} \qquad True$$

$$\left\{S_R = 6191.3 \ psi\right\} \le \left\{S_f = 20000 \ psi\right\} \qquad True$$

$$\left\{S_T = 5587.7 \ psi\right\} \le \left\{S_f = 20000 \ psi\right\} \qquad True$$

$$\left\{\frac{(S_H + S_R)}{2} = \frac{(17897.5 + 6191.3)}{2} = 12044.4 \ psi\right\} \le \left\{S_f = 20000 \ psi\right\} \qquad True$$

$$\left\{\frac{(S_H + S_T)}{2} = \frac{(17897.5 + 5587.7)}{2} = 11742.6 \ psi\right\} \le \left\{S_f = 20000 \ psi\right\} \qquad True$$

j) STEP 10 – Check the flange rigidity criterion in Table 2-14. If the flange rigidity criterion is satisfied, then the design is complete. If the flange rigidity criterion is not satisfied, then re-proportion the flange dimensions and go to STEP 3.

Operating Condition:

$$J = \frac{52.14VM_o}{LE_y g_0^2 K_I h_o} \le 1.0$$

$$\left\{J = \frac{52.14(0.1577)(206530.6)}{(0.9362)(26.0E+06)(0.3125)^2(0.3)(2.8641)} = 0.8314\right\} \le 1.0 \qquad True$$

where,

$$K_I = 0.3 \ for \ integral \ flanges$$

Gasket Seating Condition:

$$J = \frac{52.14VM_o}{LE_y g_o^2 K_I h_o} \le 1.0$$

$$\left\{J = \frac{52.14(0.1577)(207891.3)}{(0.9362)(29.4E+06)(0.3125)^2(0.3)(2.8641)} = 0.7401\right\} \le 1.0 \qquad True$$

where,

$$K_I = 0.3 \ for \ integral \ flanges$$

Since the acceptance criteria are satisfied, the design is complete.

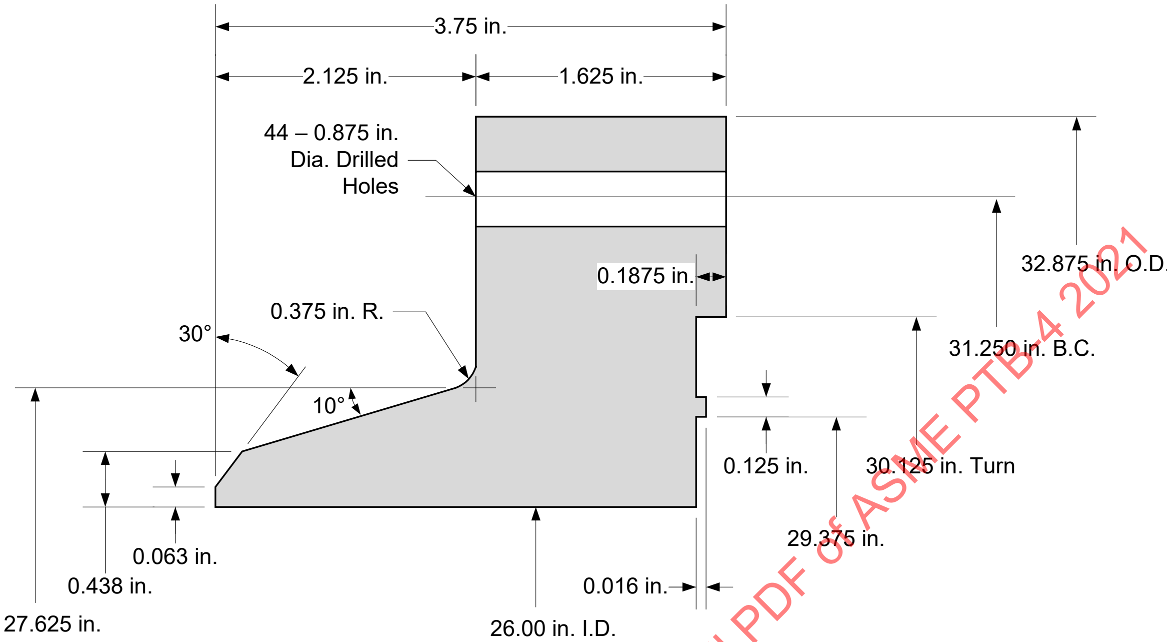

Figure E4.16.1 – Flanged Joints

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4.16.2 Example E4.16.2 – Loose Type

Determine if the stresses in the ASME B16.5, Class 300, NPS 20 Slip–on Flange are with acceptable limits, considering the following design conditions. The flange is of a loose type with hub and is attached to a cylindrical shell with Category C fillet welds, see VIII-1 Appendix 2, Figure 2-4 Sketch 3.

General Data:

- Cylinder Material = $SA-516,\ Grade\ 70$
- Design Conditions = $450\ psig @ 650°F$
- Allowable Stress at Design Temperature = $18800\ psi$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Corrosion Allowance = $0.0\ in$

Flange Data

- Material = $SA-105$
- Allowable Stress at Design Temperature = $17800\ psi$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Modulus of Elasticity at Design Temperature = $26.0E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$

Bolt Data

- Material = $SA-193,\ Grade\ B7$
- Allowable Stress at Design Temperature = $25000\ psi$
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Diameter = $1.25\ in$
- Number of Bolts = 24
- Root area = $0.929\ in^2$

Gasket Data

- Material = Kammprofile
- Gasket Factor = 2.0
- Seating Stress = $2500\ psi$
- Inside Diameter = $20.875\ in$
- Outside Diameter = $22.875\ in$

Design rules for bolted flange connections with ring type gaskets are provided in VIII-1 Mandatory Appendix 2. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.16. However, there are differences to be noted, including a step-by-step design procedure, nomenclature regarding operating and gasket seating bolt loads, the inclusion of a flange moment due to externally applied axial forces and bending moment, and minor differences in bolt spacing criteria. Therefore, while the example problem will be presented for use with VIII-1, Appendix 2, references to VIII-2 paragraphs will be provided, as applicable.

Evaluate the flange in accordance with VIII-1, Appendix 2.

Establish the design conditions and gasket seating reaction diameter, (VIII-2, paragraph 4.16.6).

a) STEP 1 – Determine the design pressure and temperature of the flanged joint

$$P = 450 \ psig \ at \ 650°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 2-5.1 (VIII-2, Table 4.16.1).

$$m = 2.0$$

$$y = 2500 \ psi$$

Note: Table 2-5.1 (VIII-2, Table 4.16.1) provides a list of many commonly used gasket materials and contact facings with suggested design values of m and y that have generally proved satisfactory in actual service when using effective seating width b given in Table 2-5.2 (VIII-2, Table 4.16.3). The design values and other details given in this table are suggested only and are not mandatory.

For this example, gasket manufacturer's suggested m and y values were used.

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5\left(GOD - GID\right) = 0.5\left(22.875 - 20.875\right) = 1.0 \ in$$

from Table 2-5.2 (VIII-2, Table 4.16.3), Facing Sketch Detail 1a, Column II,

$$b_o = \frac{N}{2} = \frac{1.0}{2} = 0.500 \ in$$

for $b_o > 0.25 \ in$,

$$b = C_b\sqrt{b_o} = \left(0.5\right)\sqrt{0.500} = 0.3536 \ in$$

where,

$$C_b = 0.5, \ for \ US \ Customary \ Units$$

therefore, from paragraph 2-3 the location of the gasket reaction is calculated as follows.

$$G = outside \ diameter \ of \ gasket \ contact \ face \ less \ 2b$$

$$G = G_C - 2b = 22.875 - 2\left(0.3536\right) = 22.1678 \ in$$

where,

$$G_C = \min\left[Gasket \ OD, \ Flange \ Face \ OD\right] = \min\left[22.875, \ 23.0\right] = 22.875 \ in$$

Paragraph 2-5 – Calculate the design bolt load for the operating and gasket seating conditions, (VIII-2, paragraph 4.16.6).

a) STEP 1 – Paragraph 2-5(c)(1), determine the design bolt load for the operating condition.

$$W_{m1} = H + H_p = 0.785G^2P + \left(2b \cdot 3.14GmP\right) \ \ for \ non-self-energized \ gaskets$$

$$W_{m1} = 0.785\left(22.1678\right)^2\left(450\right) + 2\left(0.3536\right)\left(3.14\right)\left(22.1678\right)\left(2.0\right)\left(450\right) = 217894.5 \ lbs$$

b) STEP 2 – Paragraph 2-5(c)(2), determine the design bolt load for the gasket seating condition.

$$W_{m2} = 3.14bGy \qquad for\ non-self-energized\ gaskets$$
$$W_{m2} = 3.14(0.3536)(22.1678)(2500) = 61532.5\ lbs$$

c) STEP 3 – Paragraph 2-5(d), determine the total required and actual bolt areas.

The total cross-sectional area of bolts A_m required for both the operating conditions and gasket seating is determined as follows.

$$A_m = \max[A_{m1}, A_{m2}] = \max[8.7158, 2.4613] = 8.7158\ in^2$$

where,

$$A_{m1} = \frac{W_{m1}}{S_b} = \frac{217894.5}{25000} = 8.7158\ in^2$$
$$A_{m2} = \frac{W_{m2}}{S_a} = \frac{61532.5}{25000} = 2.4613\ in^2$$

The actual bolt area A_b is calculated as follows.

$$A_b = (Number\ of\ bolts)(Root\ area\ of\ one\ bolt) = 24(0.929) = 22.2960\ in^2$$

Verify that the actual bolt area is equal to or greater than the total required area.

$$\{A_b = 22.2960\ in^2\} \geq \{A_m = 8.7158\ in^2\} \qquad True$$

d) STEP 4 – Paragraph 2-5(e), determine the flange design bolt load.

For operating conditions,

$$W = W_{m1} = 217894.5\ lbs$$

For gasket seating,

$$W = \frac{(A_m + A_b)S_a}{2} = \frac{(8.7158 + 22.2960)25000}{2} = 387647.5\ lbs$$

Commentary: VIII-2 design procedure to determine the design bolt load, paragraph 4.16.6:

1) The nomenclature differences include:

W_o, bolt load for operating conditions and flange design load bolt, (VIII-1 W_{m1}, W).

$$W_{m1} = H + H_p = 0.785G^2P + (2b \cdot 3.14GmP) \rightarrow ASME\ VIII-1$$
$$W_o = H + H_p = 0.785G^2P + (2b \cdot \pi GmP) \rightarrow ASME\ VIII-2$$

W_{gs}, design bolt load for the gasket seating condition, (VIII-1, W_{m2}).

$$W_{m2} = 3.14bGy \rightarrow ASME\ VIII-1$$
$$W_{gs} = \pi bGy \rightarrow ASME\ VIII-2$$

W_g, flange design bolt load for gasket seating (VIII-1, W).

2) When calculating the required total cross-sectional area of bolts using the VIII-2 procedure, the designer has the ability to add an externally applied net-section axial force, F_A and bending moment, M_E to the bolt load W_o for the operating condition. This is shown in the following equation.

$$A_m = \max\left[\left(\frac{W_o + F_A + \frac{4M_E}{G}}{S_{bo}}\right), \left(\frac{W_{gs}}{S_{bg}}\right)\right]$$

3) The bolt spacing criteria of VIII-1 and VIII-2 are similar, but there are some minor differences regarding the application of the two. This will be further discussed later in the example problem.

Commentary: VIII-1, Appendix 2 does not include an overall step-by-step procedure to design a flange. However, an organized procedure of the steps taken when designing a flange is presented in VIII-2, paragraph 4.16.7. The procedure is applicable to VIII-1, Appendix 2 and is presented in this example problem to assist the designer.

VIII-2, paragraph 4.16.7, Flange Design Procedure. The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings. The procedure incorporates both a strength check and a rigidity check for flange rotation.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

$$P = 450 \ psig \ at \ 650°F$$

b) STEP 2 – Determine the design bolt loads for operating condition W, and the gasket seating condition W, and the corresponding actual bolt load area A_b, (VIII-1, paragraph 2-5).

$$W = 217894.5 \ lbs \qquad Operating \ Condition$$

$$W = 387647.5 \ lbs \qquad Gasket \ Seating$$

$$A_b = 22.2960 \ in^2$$

c) STEP 3 – Determine an initial flange geometry, in addition to the information required to determine the bolt load, the following geometric parameters are required, (VIII-1, paragraph 2-3). The flange is an ASME B16.5, Class 300, NPS 20 Slip–on Flange.

1) Flange bore

$$B = 20.20 \ in$$

2) Bolt circle diameter

$$C = 27.0 \ in$$

3) Outside diameter of the flange

$$A = 30.5 \ in$$

4) Flange thickness

$$t = 2.44 \ in$$

5) Thickness of the hub at the large end

$$g_1 = 1.460 \ in$$

6) Thickness of the hub at the small end

$$g_0 = 1.460\ in$$

7) Hub length

$$h = 1.25\ in$$

d) STEP 4 – Determine the flange stress factors using the equations/direct interpretation from Table 2-7.1 and Figure 2-7.1 – Figure 2-7.6 and paragraph 2-3.

Figure 2-7.1:

$$K = \frac{A}{B} = \frac{30.5}{20.20} = 1.5099$$

$$Y = \frac{1}{K-1}\left[0.66845 + 5.71690\left(\frac{K^2 \log_{10} K}{K^2 - 1}\right)\right]$$

$$Y = \frac{1}{1.5099-1}\left[0.66845 + 5.71690\frac{(1.5099)^2 \log_{10}[1.5099]}{(1.5099)^2 - 1}\right] = 4.8850$$

$$T = \frac{K^2(1+8.55246\log_{10} K)-1}{(1.04720+1.9448K^2)(K-1)} = \frac{(1.5099)^2(1+8.55246\log_{10}[1.5099])-1}{(1.04720+1.9448(1.5099)^2)(1.5099-1)} = 1.7064$$

$$U = \frac{K^2(1+8.55246\log_{10} K)-1}{1.36136(K^2-1)(K-1)} = \frac{(1.5099)^2(1+8.55246\log_{10}[1.5099])-1}{1.36136((1.5099)^2-1)(1.5099-1)} = 5.3681$$

$$Z = \frac{(K^2+1)}{(K^2-1)} = \frac{((1.5099)^2+1)}{((1.5099)^2-1)} = 2.5627$$

Figure 2-7.4:

$$\frac{h}{h_o} = \frac{h}{\sqrt{Bg_0}} = \frac{1.25}{\sqrt{(20.20)(1.46)}} = \frac{1.25}{5.4307} = 0.2302$$

$$\frac{g_1}{g_0} = \frac{1.460}{1.460} = 1.0$$

Interpretation of Figure 2-7.4, $F_L \approx 3.3$. From the equations of Table 2-7.1, $F_L = 3.2609$.

Figure 2-7.5:

with $h/h_o = 0.2302$ and $g_1/g_o = 1.0$:

Interpretation of Figure 2-7.5, $V_L \approx 11.4$. From the equations of Table 2-7.1, $V_L = 11.3725$.

Figure 2-7.6:

with $h/h_o = 0.2302$ and $g_1/g_o = 1.0$:

Interpretation of Figure 2-7.6, $f = 1.0$. From the equations of Table 2-7.1, $f = 1.0$.

Paragraph 2-3:

$$d = \frac{Ug_0^2 h_o}{V_L} = \frac{(5.3681)(1.460)^2(5.4307)}{11.3725} = 5.4642\ in^3$$

$$e = \frac{F_L}{h_o} = \frac{3.2609}{5.4307} = 0.6005\ in^{-1}$$

$$L = \frac{te+1}{T} + \frac{t^3}{d} = \frac{2.44(0.6005)+1}{1.7064} + \frac{(2.44)^3}{5.4642} = 4.1032$$

Commentary: VIII-2, Table 4.16.4 provides the equations to determine the flange stress factors Y, T, U, Z as a function of K as provided for in VIII-1, Figure 2-7.1, and the flange factors d, e, L located in VIII-1, paragraph 2-3. Similarly, VIII-2, Table 4.16.5 provides regressed curve-fit equations of the flange stress factors F_L, V_L, f as provided in VIII-1, Figure 2-7.2, Figure 2-7.3, and Figure 2-7.6, respectively. The curve-fit equations are shown for informational purposes with the variable substitution of $X_h = h/h_o$ and $X_g = g_1/g_o$.

$$F_L = \left(\frac{\left\{ \begin{array}{l} 0.941074 + 0.176139\left(\ln\left[X_g\right]\right) - 0.188556\left(\ln\left[X_h\right]\right) + \\ 0.0689847\left(\ln\left[X_g\right]\right)^2 + 0.523798\left(\ln\left[X_h\right]\right)^2 - \\ 0.513894\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right) \end{array} \right\}}{\left\{ \begin{array}{l} 1 + 0.379392\left(\ln\left[X_g\right]\right) + 0.184520\left(\ln\left[X_h\right]\right) - \\ 0.00605208\left(\ln\left[X_g\right]\right)^2 - 0.00358934\left(\ln\left[X_h\right]\right)^2 + \\ 0.110179\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right) \end{array} \right\}} \right)$$

$$F_L = \left(\frac{\left\{ \begin{array}{l} 0.941074 + 0.176139\left(\ln\left[1.0\right]\right) - 0.188556\left(\ln\left[0.2302\right]\right) + \\ 0.0689847\left(\ln\left[1.0\right]\right)^2 + 0.523798\left(\ln\left[0.2302\right]\right)^2 - \\ 0.513894\left(\ln\left[1.0\right]\right)\left(\ln\left[0.2302\right]\right) \end{array} \right\}}{\left\{ \begin{array}{l} 1 + 0.379392\left(\ln\left[1.0\right]\right) + 0.184520\left(\ln\left[0.2302\right]\right) - \\ 0.00605208\left(\ln\left[1.0\right]\right)^2 - 0.00358934\left(\ln\left[0.2302\right]\right)^2 + \\ 0.110179\left(\ln\left[1.0\right]\right)\left(\ln\left[0.2302\right]\right) \end{array} \right\}} \right)$$

$$F_L = 3.2556$$

$$For\ 0.1 \le X_h \le 0.25,$$

$$\ln\left[V_L\right] = \left(\begin{array}{l} 6.57683 - 0.115516 X_g + 1.39499\sqrt{X_g}\left(\ln\left[X_g\right]\right) + \\ 0.307340\left(\ln\left[X_g\right]\right)^2 - 8.30849\sqrt{X_g} + 2.62307\left(\ln\left[X_g\right]\right) + \\ 0.239498 X_h\left(\ln\left[X_h\right]\right) - 2.96125\left(\ln\left[X_h\right]\right) + \dfrac{7.035052\left(10^{-4}\right)}{X_h} \end{array} \right)$$

$$\ln\left[V_L\right] = \left(\begin{array}{l} 6.57683 - 0.115516(1.0) + 1.39499\sqrt{1.0}\left(\ln 1.0\right) + \\ 0.307340\left(\ln\left[1.0\right]\right)^2 - 8.30849\sqrt{1.0} + 2.62307\left(\ln\left[1.0\right]\right) + \\ 0.239498(0.2302)\left(\ln\left[0.2302\right]\right) - 2.96125\left(\ln\left[0.2302\right]\right) + \dfrac{7.035052\left(10^{-4}\right)}{0.2302} \end{array} \right)$$

$$\ln\left[V_L\right] = 2.4244$$

$$V_L = \exp\left[2.4244\right] = 11.2955$$

$$f = 1.0$$

e) STEP 5 – Determine the flange forces, paragraph 2-3.

$$H_D = 0.785 B^2 P = 0.785(20.20)^2(450) = 144140.1\ lbs$$

$$H = 0.785 G^2 P = 0.785(22.1678)^2(450) = 173591.1\ lbs$$

$$H_T = H - H_D = 173591.1 - 144140.1 = 29451.0\ lbs$$

$$H_G = W - H = 217894.5 - 173591.1 = 44303.4\ lbs \quad Operating$$

f) STEP 6 – Determine the flange moment for the operating condition using paragraph 2-6. In these equations, h_D, h_T, and h_G are determined from Table 2-6.

$$h_D = \frac{C - B}{2} = \frac{27.0 - 20.20}{2} = 3.40\ in$$

$$h_G = \frac{C - G}{2} = \frac{27.0 - 22.1678}{2} = 2.4161\ in$$

$$h_T = \frac{h_D + h_G}{2} = \frac{3.40 + 2.4161}{2} = 2.9081\ in$$

Paragraph 2-6:

$$M_o = H_D h_D + H_T h_T + H_G h_G$$

$$M_o = 144140.1(3.40) + 29451.0(2.9081) + 44303.4(2.4161)$$

$$M_o = 682764.2\ in - lbs$$

For vessels in lethal service or when specified by the user or his designated agent, the bolt spacing correction factor B_{SC} shall be applied in calculating the flange stress in paragraph 2-7. The flange moment M_o without correction for bolt spacing is used for the calculation of the rigidity index in paragraph 2-14. When the bolt spacing exceeds $2a+t$, multiply M_o by the bolt spacing correction factor B_{SC} for calculating flange stress, where:

$$B_{SC}=\sqrt{\frac{B_s}{2a+t}}$$

Commentary:

1) The equations for maximum bolt spacing B_{smax} and bolt spacing correction factor B_{SC} used in the VIII-2 are the same as in VIII-1 and are in Table 4.16.11.

2) As is the case with VIII-1 design, VIII-2 also notes that the flange moment M_o without correction for bolt spacing is used for the calculation of the rigidity index located in Table 4.16.10.

3) The bolt spacing correction factor B_{SC} and the split loose flange factor of VIII-1, paragraph 2-9, F_s, are directly incorporated into the calculations for determining the flange moments for both operating and gasket seating.

$$M_o=abs\left[\left(\left(H_D h_D+H_T h_T+H_G h_G\right)B_{SC}+M_{oe}\right)F_s\right] \quad Internal\ Pressure$$

$$M_g=\frac{W_g\left(C-G\right)B_{SC}F_s}{2} \qquad Gasket\ Seating$$

4) As previously noted, the VIII-2 procedure provides the designer the ability to add an externally applied net-section axial force and bending moment to the bolt load for the operating condition. These externally applied loads induce a bending moment, referenced as M_{oe}, which is calculated from Equation 4.16.16,

$$M_{oe}=4M_E\left[\frac{I}{0.3846I_p+I}\right]\left[\frac{h_D}{\left(C-2h_D\right)}\right]+F_A h_D$$

g) STEP 7 – Determine the flange moment for gasket seating condition using paragraph 2-6.

$$M_o=W\frac{\left(C-G\right)}{2}=387647.5\frac{\left(27.0-22.1678\right)}{2}=936595.1\ in-lbs$$

h) STEP 8 – Determine the flange stresses for the operating and gasket seating conditions using the equations in paragraph 2-7.

Note: Paragraph 2-3 – If $B<20g_1$, the designer may substitute the value of B_1 for B in the equation for S_H, where,

For integral flanges when $f\geq 1.0$,

$$B_1=B+g_o$$

For integral flanges when $f<1.0$ and for loose type flanges,

$$B_1=B+g_1$$

Although, $\{B=20.20\ in\}<\{20g_1=20(1.46)=29.2\ in\}$, the value of B will be used to determine the value of S_H.

Operating Condition:

$$S_H = \frac{fM_o}{Lg_1^2 B} = \frac{(1.0)(682764.2)}{(4.1032)(1.460)^2(20.20)} = 3864.5\ psi$$

$$S_R = \frac{(1.33te+1)M_o}{Lt^2 B} = \frac{\left[(1.33)(2.44)(0.6005)+1\right](682764.2)}{(4.1032)(2.44)^2(20.20)} = 4079.9\ psi$$

$$S_T = \frac{YM_o}{t^2 B} - ZS_R = \frac{(4.8850)(682764.2)}{(2.44)^2(20.20)} - 2.5627(4079.9) = 17277.9\ psi$$

Gasket Seating Condition:

$$S_H = \frac{fM_o}{Lg_1^2 B} = \frac{(1.0)(936595.1)}{(4.1032)(1.460)^2(20.20)} = 5301.2\ psi$$

$$S_R = \frac{(1.33te+1)M_o}{Lt^2 B} = \frac{\left[(1.33)(2.44)(0.6005)+1\right](682764.2)}{(4.1032)(2.44)^2(20.20)} = 5596.7\ psi$$

$$S_T = \frac{YM_o}{t^2 B} - ZS_R = \frac{(4.8850)(682764.2)}{(2.44)^2(20.20)} - 2.5627(5596.7) = 23701.3\ psi$$

i) STEP 9 – Check the flange stress acceptance criteria. The criteria below shall be evaluated. If the stress criteria are satisfied, go to STEP 10. If the stress criteria are not satisfied, re-proportion the flange dimensions and go to STEP 4.

Allowable normal stress – The criteria to evaluate the normal stresses for the operating and gasket seating conditions are shown in VIII-1, paragraph 2-8, for loose type flanges with a hub.

Operating Condition:

$$S_H \le 1.5 S_f$$

$$\{S_H = 3864.5\ psi\} \le \{1.5(17800) = 26700\ psi\} \qquad True$$

$$\{S_R = 4079.9\ psi\} \le \{S_f = 17800\ psi\} \qquad True$$

$$\{S_T = 17277.9\ psi\} \le \{S_f = 17800\ psi\} \qquad True$$

$$\left\{\frac{(S_H + S_R)}{2} = \frac{(3864.5 + 4079.9)}{2} = 3972.2\ psi\right\} \le \{S_f = 17800\ psi\} \qquad True$$

$$\left\{\frac{(S_H + S_T)}{2} = \frac{(3864.5 + 17277.9)}{2} = 10571.2\ psi\right\} \le \{S_f = 17800\ psi\} \qquad True$$

Gasket Seating Condition:

$$S_H \le 1.5 S_f$$

$$\{S_H = 5301.2\ psi\} \le \{1.5(20000) = 30000\ psi\} \qquad True$$

$$\{S_R = 5596.7 \ psi\} \le \{S_f = 20000 \ psi\} \qquad True$$

$$\{S_T = 23701.3 \ psi\} \le \{S_f = 20000 \ psi\} \qquad False$$

$$\left\{\frac{(S_H + S_R)}{2} = \frac{(5301.2 + 5596.7)}{2} = 5449.0 \ psi\right\} \le \{S_f = 20000 \ psi\} \qquad True$$

$$\left\{\frac{(S_H + S_T)}{2} = \frac{(5301.2 + 23701.3)}{2} = 14501.3 \ psi\right\} \le \{S_f = 20000 \ psi\} \qquad True$$

Since the acceptance criteria is not satisfied for the Tangential Flange Stress, S_T re-proportion the flange dimensions and go to STEP 4 to re-evaluate the design.

j) STEP 10 – Check the flange rigidity criterion in Table 2-14. If the flange rigidity criterion is satisfied, then the design is complete. If the flange rigidity criterion is not satisfied, then re-proportion the flange dimensions and go to STEP 3.

Operating Condition:

$$J = \frac{52.14 V_L M_o}{L E_y g_0^2 K_L h_o} \le 1.0$$

$$\left\{J = \frac{52.14(11.3725)(682764.2)}{(4.1032)(26.0E+06)(1.460)^2(0.2)(5.4307)} = 1.6391\right\} \le 1.0 \quad Not\ Satisfied$$

where,

$$K_L = 0.2 \ for \ loose \ type \ flanges$$

Gasket Seating Condition:

$$J = \frac{52.14 V_L M_o}{L E_y g_o^2 K_L h_o} \le 1.0$$

$$\left\{J = \frac{52.14(11.3725)(936595.1)}{(4.1032)(29.4E+06)(1.460)^2(0.2)(5.4307)} = 1.9885\right\} \le 1.0 \quad Not\ Satisfied$$

where,

$$K_L = 0.2 \ for \ loose \ type \ flanges$$

Since the flange rigidity criterion is not satisfied for either the operating condition or the gasket seating condition, the flange dimensions should be re-proportioned, and the design procedure shall be performed beginning with STEP 3.

NOTE: Although the proposed ASME B16.5 slip-on flange is shown not to satisfy the Tangential Flange Stress, S_T, in the gasket seating condition and the flange rigidity acceptance criteria of VIII-1 Appendix 2 Rules for Bolted Flange Connections, ASME B16.5–2009, Table II–2–1.1, Pressure–Temperature Ratings for Group 1.1 Materials, states an ASME Class 300 flange is permitted to operate at a pressure of 550 psi for a coincident temperature of 650°F.

4.17 Clamped Connections

4.17.1 Example E4.17.1 – Flange and Clamp Design Procedure

Using the data shown below, determine if the clamp design meets the design requirements of Section VIII, Division 1.

Data (Refer to Figure E4.17.1):

- Design Conditions = $3000\ psi @ 200°F$
- Corrosion Allowance = $0.0\ in$

Clamp:

- Material = $SA-216,\ Grade\ WCB$
- Inside Diameter = $43.75\ in$
- Thickness = $7.625\ in$
- Width = $28.0\ in$
- Gap = $14.0\ in$
- Lug height = $15.0\ in$
- Lug Width = $28.0\ in$
- Lip Length = $2.75\ in$
- Radial Distance from Connection Centerline to Bolts = $32.25\ in$
- Distance from W to the point where the clamp lug joins the clamp body = $3.7\ in$
- Allowable Stress @ Design Temperature = $20000\ psi$
- Allowable Stress @ Ambient Temperature = $20000\ psi$

Hub:

- Material = $SA-105$
- Inside Diameter = $18.0\ in$
- Pipe End Neck Thickness = $12.75\ in$
- Shoulder End Neck Thickness = $12.75\ in$
- Shoulder Thickness = $7.321\ in$
- Shoulder Height = $2.75\ in$
- Friction Angle = $5\ deg$
- Shoulder Transition Angle = $10\ deg$
- Allowable Stress @ Design Temperature = $20000\ psi$
- Allowable Stress @ ambient Temperature = $20000\ psi$

Bolt Data:

- Material = $SA-193,\ Grade\ B7$
- Allowable Stress @ Design Temperature = $23000\ psi$

- Allowable Stress @ Gasket Temperature = $23000\ psi$
- Diameter = $1.75\ in$
- Number of Bolts = 2
- Root area = $1.980\ in^2$

Gasket Data:

- Material = Self Energizing O-ring Type
- Gasket Reaction Location = $19.0\ in$
- Gasket Factor = 0
- Seating Stress = $0\ psi$

Design rules for clamped connections are provided in VIII-1 Mandatory Appendix 24. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.17. However, there are differences to be noted, including a step-by-step design procedure and nomenclature about operating and gasket seating bolt loads. Therefore, while the example problem will be presented for use with VIII-1, Appendix 24, references to VIII-2 paragraphs will be provided, as applicable.

Evaluate the clamp in accordance with VIII-1, Appendix 24.

Establish the design conditions and gasket reaction diameter.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint

$$P = 3000\ psig\ at\ 200°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 2-5.1.

$$m = 0.0 \qquad for\ self-energized\ gaskets$$

$$y = 0.0$$

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.0 \qquad for\ self-energized\ gaskets$$

From Table 2-5.2, Facing Sketch Detail (not required because gasket is self-energized)

$$b_o = \frac{N}{2} = \frac{0.0}{2} = 0.0\ in$$

For $b_o \leq 0.25\ in.$,

$$b = b_o = 0.0\ in$$

Therefore, paragraph 2-3, the location of the gasket reaction is calculated as follows.

$$G = mean\ diameter\ of\ the\ gasket\ contact\ face$$

$$G = 19.0\ in$$

Commentary: VIII-1, Appendix 24 does not include an overall step-by-step procedure to determine the bolt loads for the operating, gasket seating, and assembly conditions. However, an organized procedure of the steps taken when determining the bolt loads is presented in VIII-2, paragraph 4.17.4. The procedure is applicable to VIII-1, Appendix 24 and is presented in this example problem in effort to assist the designer.

The procedure to determine the bolt loads for operating, gasket seating, and assembly conditions.

a) STEP 1 – Paragraph 24-3, determine the flange forces for the bolt load calculation.

$$H = 0.785G^2 P = 0.785(19.0)^2(3000) = 850155.0\ lbs$$
$$H_p = 0.0 \qquad (for\ self-energized\ gaskets)$$
$$H_m = 0.0 \qquad (for\ self-energized\ gaskets)$$

b) STEP 2 – Paragraph 24-4(b)(1), determine the required bolt load for the operating condition.

$$W_{m1} = 0.637(H + H_p)\tan[\phi - \mu] = 0.637(850155 + 0)\cdot\tan[10 - 5] = 47379.4\ lbs$$

c) STEP 3 – Paragraph 24-4(b)(2), determine the minimum required total bolt load for the gasket seating conditions.

$$W_{m2} = 0.637H_m \tan[\phi + \mu] = 0.637(0.0)\cdot\tan[10 + 5] = 0.0\ lbs$$

d) STEP 4 – Paragraph 24-4(b)(3), determine the minimum required total bolt load for the assembly condition.

$$W_{m3} = 0.637(H + H_p)\tan[\phi + \mu] = 0.637(850155 + 0)\cdot\tan[10 + 5] = 145107.5\ lbs$$

e) STEP 5 – Paragraph 24-4(c), determine the total required and actual bolt areas.

The total cross-sectional area of bolts A_{mL} required for the operating conditions, gasket seating, and assembly condition is determined as follows.

$$A_{mL} = \max\left[\frac{W_{m1}}{2S_b}, \frac{W_{m2}}{2S_a}, \frac{W_{m3}}{2S_a}\right] = \max[1.0300,\ 0.0,\ 3.1545] = 3.1545\ in^2$$

where,

$$A_{m1} = \frac{W_{m1}}{2S_b} = \frac{47379.4}{2(23000)} = 1.0300\ in^2$$
$$A_{m2} = \frac{W_{m2}}{2S_a} = \frac{0}{2(23000)} = 0.0\ in^2$$
$$A_{m3} = \frac{W_{m3}}{2S_a} = \frac{145107.5}{2(23000)} = 3.1545\ in^2$$

The actual bolt area is calculated as follows (using two 1.75 in diameter bolts).

$$A_{bL} = (Number\ of\ bolts)(Root\ area\ of\ one\ bolt) = 2(1.980) = 3.96\ in^2$$

Verify that the actual bolt area is equal to or greater than the total required area.

$$\{A_{bL} = 3.96\ in^2\} \geq \{A_{mL} = 3.1545\ in^2\} \qquad True$$

f) STEP 6 – Paragraph 24-4(d), determine the clamp connection design bolt load, W.

Operating conditions:

$$W = W_{m1} = 47379.4 \ lbs$$

Assembly conditions:

$$W = \left(A_{mL} + A_{bL}\right)S_a = \left(3.1545 + 3.96\right)23000 = 163633.5 \ lbs$$

Alternatively, if controlled bolting (e.g., bolt tensioning or torque control) techniques are used to assemble the clamp, assembly design bolt load may be calculated as follows.

$$W = 2A_{mL} \cdot S_a = 2\left(3.1545\right)23000 = 145107.0 \ lbs$$

Note: This calculation is shown for informational purposes only and will not be used in the example problem.

Commentary: VIII-1, Appendix 24 does not include an overall step-by-step procedure to design a clamp. However, an organized procedure of the steps taken when designing a clamp is presented in VIII-2, paragraph 4.17.5. The procedure is applicable to VIII-1, Appendix 24 and is presented in this example problem to assist the designer.

The procedure to design a clamp connection is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flange joint.

See above data.

b) STEP 2 – Determine an initial flange and clamp geometry see Figures 24-1 Sketch (a) and Sketch (c) and Figure E4.17.1 of this example.

c) STEP 3 – Determine the design bolt loads for operating condition, W, and the gasket seating and assembly condition, W, from VIII-1, paragraph 24-4(d).

$$W = 47379.4 \ lbs \qquad Operating\ Condition$$
$$W = 163633.5 \ lbs \qquad Assembly\ Condition$$

d) STEP 4 – Determine the flange forces, H_D, H_G, and H_T from VIII-1, paragraph 24-3.

$$H_D = 0.785B^2P = 0.785\left(18.0\right)^2\left(3000\right) = 763020.0 \ lbs$$

$$H_G = \frac{1.571W}{\tan\left[\phi + \mu\right]} - \left(H + H_p\right) = \frac{1.571(47379.4)}{\tan\left[10+5\right]} - \left(850155.0 + 0\right) = -572367.1 \ lbs$$

$$H_T = H - H_D = 850155.0 - 763020.0 = 87135.0 \ lbs$$

e) STEP 5 – Determine the flange moment for the operating condition paragraphs 24-3 and 24-5. In these equations, the moment arms h_D, h_G, and h_T and constants $A, C, N, \overline{h}$, and h_2 are determined from paragraph 24-3.

$$M_o = M_D + M_G + M_T + M_F + M_P + M_R$$

$$M_o = 5961093.8 + 0.0 + 1214444.1 + 0.0 + 25944.2 + \left(-255151.2\right) = 6946330.9 \ in-lbs$$

where,

$$h_D = \left[\frac{C-(B+g_1)}{2}\right] = \left[\frac{46.375-(18.0+12.75)}{2}\right] = 7.8125\ in$$

$$h_G = 0.0 \qquad for\ \ full\ \ face\ contact\ geometries$$

$$h_T = \frac{\left[C-\dfrac{(B+G)}{2}\right]}{2} = \left[\frac{\left[46.375-\dfrac{(18.0+19.0)}{2}\right]}{2}\right] = 13.9375\ in$$

$$A = B+2(g_1+g_2) = 18.0+2(12.75+2.75) = 49.0\ in$$

$$C = \frac{(A+C_i)}{2} = \frac{(49.0+43.75)}{2} = 46.375\ in$$

$$N = B+2g_1 = 18.0+2(12.75) = 43.5\ in$$

$$\bar{h} = \frac{T^2 g_1 + h_2^2 g_2}{2(Tg_1+h_2 g_2)} = \frac{(7.321)^2\,12.75+(7.0785)^2\,2.75}{2(7.321(12.75)+7.0785(2.75))} = 3.6396\ in$$

$$h_2 = T-\frac{g_2\tan[\phi]}{2} = 7.321-\frac{2.75\tan[10]}{2} = 7.0786\ in$$

therefore,

$$M_D = H_D h_D = 763020.0(7.8125) = 5961093.8\ lbs$$

$$M_G = H_G h_G = -572367.1(0.0) = 0.0$$

$$M_T = H_T h_T = 87135.0(13.9375) = 1214444.1\ in-lbs$$

$$M_F = H_D\left(\frac{g_1-g_0}{2}\right) = 763020.0\left(\frac{12.75-12.75}{2}\right) = 0.0$$

$$M_P = (3.14)PBT\left(\frac{T}{2}-\bar{h}\right)$$

$$M_P = (3.14)(3000)(18.0)(7.321)\left(\frac{7.321}{2}-3.6396\right) = 25944.2\ lbs$$

$$M_R = 1.571W\left(\bar{h}-T+\frac{(C-N)\tan[\phi]}{2}\right)$$

$$M_R = 1.571(47379.4)\left(3.6396-7.321+\frac{(46.375-43.5)\tan[10]}{2}\right) = -255151.2\ lbs$$

f) STEP 6 – Determine the flange moment for the assembly condition paragraph 24-5.

$$M_o = \frac{0.785W(C-G)}{\tan[\phi+\mu]} = \frac{0.785(163633.5)(46.375-19.0)}{\tan[10+5]} = 13123315.0 \ \ in-lbs$$

g) STEP 7 – Determine the hub factors, paragraph 24-3.

$$I_h = \frac{g_1 T^3}{3} + \frac{g_2 h_2^3}{3} - (g_2 h_2 + g_1 T)\bar{h}^2$$

$$I_h = \frac{12.75(7.321)^3}{3} + \frac{2.75(7.0786)^3}{3} - (2.75(7.0786) + 12.75(7.321))(3.6396)^2$$

$$I_h = 498.4148 \ in^4$$

$$\bar{g} = \frac{Tg_1^2 + h_2 g_2(2g_1 + g_2)}{2(Tg_1 + h_2 g_2)} = \frac{7.321(12.75)^2 + 7.0786(2.75)(2(12.75)+2.75)}{2(7.321(12.75)+7.0786(2.75))}$$

$$\bar{g} = 7.7123 \ in$$

h) STEP 8 – Determine the reaction moment, M_H and reaction shear force, Q at the hub neck for the operating condition, paragraph 24-3.

$$M_H = \frac{M_o}{1 + \frac{1.818}{\sqrt{Bg_1}}\left[T - \bar{h} + \frac{3.305 I_h}{g_1^2(0.5B+\bar{g})}\right]}$$

$$M_H = \frac{6946330.9}{1 + \frac{1.818}{\sqrt{18.0(12.75)}}\left[7.321 - 3.6396 + \frac{3.305(498.4148)}{(12.75)^2(0.5(18.0)+7.7123)}\right]}$$

$$M_H = 4586392.3 \ in-lbs$$

$$Q = \frac{1.818 M_H}{\sqrt{Bg_1}} = \frac{1.818(4586392.3)}{\sqrt{18(12.75)}} = 550394.1 \ lbs$$

i) STEP 9 – Determine the reaction moment, M_H and reaction shear force, Q at the hub neck for the assembly condition, paragraph 24-3.

$$M_H = \frac{M_o}{1 + \frac{1.818}{\sqrt{Bg_1}}\left[T - \bar{h} + \frac{3.305 I_h}{g_1^2(0.5B+\bar{g})}\right]}$$

$$M_H = \frac{13123315.0}{1 + \frac{1.818}{\sqrt{18.0(12.75)}}\left[7.321 - 3.6396 + \frac{3.305(498.4148)}{(12.75)^2(0.5(18.0)+7.7123)}\right]}$$

$$M_H = 8664814.8 \ in-lbs$$

$$Q = \frac{1.818 M_H}{\sqrt{B g_1}} = \frac{1.818(8664814.8)}{\sqrt{18.0(12.75)}} = 1039828.7 \; lbs$$

j) STEP 10 – Determine the clamp factors paragraph 24-3.

$$A_c = A_1 + A_2 + A_3 = 97.2188 + 91.3389 + 38.5 = 227.0577 \; in^2$$

where,

$$A_1 = \left(C_w - 2C_t\right)C_t = \left(28.0 - 2\left(7.625\right)\right)7.625 = 97.2188 \; in^2$$

$$A_2 = 1.571 C_t^2 = 1.571\left(7.625\right)^2 = 91.3389 \; in^2$$

$$A_3 = \left(C_w - C_g\right)l_c = \left(28.0 - 14.0\right)2.75 = 38.5 \; in^2$$

$$e_b = B_c - \frac{C_i}{2} - l_c - X = 32.25 - \frac{43.75}{2} - 2.75 - 2.7009 = 4.9241 \; in$$

where,

$$X = \frac{\left(\frac{C_w}{2} - \frac{C_t}{3}\right)C_t^2 - 0.5\left(C_w - C_g\right)l_c^2}{A_c} = \frac{\left(\frac{28}{2} - \frac{7.625}{3}\right)\left(7.625\right)^2 - 0.5\left(28 - 14\right)\left(2.75\right)^2}{227.0577}$$

$$X = 2.7009 \; in$$

$$I_c = \left(\frac{A_1}{3} + \frac{A_2}{4}\right)C_t^2 + \frac{A_3 l_c^2}{3} - A_c X^2$$

$$I_c = \left(\frac{97.2188}{3} + \frac{91.3389}{4}\right)\left(7.625\right)^2 + \frac{38.5\left(2.75\right)^2}{3} - 227.0577\left(2.7009\right)^2 = 1652.4435 \; in^4$$

k) STEP 11 – Determine the hub stress correction factor, f, based on g_1, g_0, h, and B using Figure 2-7.6 and l_m using the following equation in paragraph 24-3.

$$X_g = \frac{g_1}{g_0} = \frac{12.75}{12.75} = 1.0$$

$$X_h = \frac{h}{h_o} = \frac{h}{\sqrt{B g_o}} = \frac{0.0}{\sqrt{18(12.75)}} = 0.0$$

Since $X_g = 1.0$, $f = 1.0$ per Figure 2-7.6.

$$l_m = l_c - 0.5(C - C_i) = 2.75 - 0.5\left(46.375 - 43.75\right) = 1.4375 \; in$$

l) STEP 12 – Determine the hub and clamp stresses for the operating and assembly conditions using the equations in paragraphs 24-6 and 24-7.

Operating Condition – Location: Hub:

Longitudinal Stress:

$$S_1 = f\left[\frac{PB^2}{4g_1(B+g_1)} + \frac{1.91M_H}{g_1^2(B+g_1)}\right]$$

$$S_1 = 1.0\left[\frac{3000(18.0)^2}{4(12.75)(18.0+12.75)} + \frac{1.91(4586392.3)}{(12.75)^2(18.0+12.75)}\right] = 2372.2\ psi$$

Hoop Stress:

$$S_2 = P\left(\frac{N^2+B^2}{N^2-B^2}\right) = 3000\left(\frac{(43.5)^2+(18.0)^2}{(43.5)^2-(18.0)^2}\right) = 4239.6\ psi$$

Axial Shear Stress:

$$S_3 = \frac{0.75W}{T(B+2g_1)\tan[\phi-\mu]} = \frac{0.75(47379.4)}{7.321(18.0+2(12.75))\tan[10-5]} = 1275.4\ psi$$

Radial Shear Stress:

$$S_4 = \frac{0.477Q}{g_1(B+g_1)} = \frac{0.477(550394.1)}{12.75(18.0+12.75)} = 669.6\ psi$$

Operating Condition – Location: Clamp:

Longitudinal Stress:

$$S_5 = \frac{W}{2C\tan[\phi-\mu]}\left[\frac{1}{C_t} + \frac{3(C_t+2l_m)}{C_t^2}\right]$$

$$S_5 = \frac{47379.4}{2\times 46.375\tan[10-5]}\left[\frac{1}{7.625} + \frac{3(7.625+2(1.4375))}{(7.625)^2}\right] = 3932.2\ psi$$

Tangential Stress:

$$S_6 = \frac{W}{2}\left[\frac{1}{A_c} + \frac{|e_b|\cdot(C_t-X)}{I_c}\right]$$

$$S_6 = \frac{47379.4}{2}\left[\frac{1}{227.0577} + \frac{|4.9241|\cdot(7.625-2.7009)}{1652.4435}\right] = 451.9\ psi$$

Lip Shear Stress:

$$S_7 = \frac{1.5W}{(C_w-C_g)C\tan[\phi-\mu]} = \frac{1.5(47379.4)}{(28.0-14.0)(46.375)\tan[10-5]} = 1251.2\ psi$$

Lug Bending Stress:

$$S_8 = \frac{3WL_a}{L_w L_h^2} = \frac{3(47379.4)(3.7)}{28.0(15.0)^2} = 83.5 \ psi$$

Bearing Stress at clamp-to-hub contact:

$$S_9 = \frac{W}{(A-C_i)C\tan[\phi-\mu]} = \frac{47379.4}{(49.0-43.75)(46.375)\tan[10-5]} = 2224.3 \ psi$$

Gasket Seating/Assembly Condition – Location: Hub:

Longitudinal Stress:

$$S_1 = f\left[\frac{1.91M_H}{g_1^2(B+g_1)}\right] = 1.0\left[\frac{1.91(8664814.8)}{(12.75)^2(18.0+12.75)}\right] = 3310.8 \ psi$$

Hoop Stress:

$$S_2 = 0.0$$

Axial Shear Stress:

$$S_3 = \frac{0.75W}{T(B+2g_1)\tan[\phi+\mu]} = \frac{0.75(163633.5)}{7.321(18.0+2(12.75))\tan[10+5]} = 1438.2 \ psi$$

Radial Shear Stress:

$$S_4 = \frac{0.477Q}{g_1(B+g_1)} = \frac{0.477(1039828.7)}{12.75(18.0+12.75)} = 1265.1 \ psi$$

Gasket Seating/Assembly Condition – Location: Clamp:

Longitudinal Stress:

$$S_5 = \frac{W}{2C\tan[\phi+\mu]}\left[\frac{1}{C_t} + \frac{3(C_t+2l_m)}{C_t^2}\right]$$

$$S_5 = \frac{163633.5}{2(46.375)\tan[10+5]}\left[\frac{1}{7.625} + \frac{3(7.625+2(1.4375))}{7.625^2}\right] = 4430.8 \ psi$$

Tangential Stress:

$$S_6 = \frac{W}{2}\left[\frac{1}{A_c} + \frac{|e_b|\cdot(C_t-X)}{I_c}\right]$$

$$S_6 = \frac{163633.5}{2}\left[\frac{1}{227.0577} + \frac{|4.9241|\cdot(7.625-2.7009)}{1652.4435}\right] = 1560.9 \ psi$$

Lip Shear Stress:

$$S_7 = \frac{1.5W}{(C_w - C_g)C\tan[\phi + \mu]} = \frac{1.5(163633.5)}{(28.0 - 14.0)(46.375)\tan[10+5]} = 1410.9 \ psi$$

Lug Bending Stress:

$$S_8 = \frac{3WL_a}{L_w L_h^2} = \frac{3(163633.5)(3.7)}{28.0(15.0)^2} = 288.3 \ psi$$

Bearing Stress at clamp-to-hub contact:

$$S_9 = \frac{W}{(A - C_i)C\tan[\phi + \mu]} = \frac{163633.5}{(49.0 - 43.75)(46.375)\tan[10+5]} = 2508.3 \ psi$$

m) STEP 13 – Check the flange stress acceptance criteria for the operating and gasket seating conditions are shown in Table 24-8.

Operating Condition – Location: Hub:

$$\{S_1 = 2372.2 \, psi\} \le \{1.5S_{OH} = 30000 \, psi\} \qquad True$$

$$\{S_2 = 4239.6 \ psi\} \le \{S_{OH} = 20000 \ psi\} \qquad True$$

$$\{S_3 = 1275.4 \ psi\} < \{0.8S_{OH} = 16000 \ psi\} \qquad True$$

$$\{S_4 = 669.6 \ psi\} \le \{0.8S_{OH} = 16000 \ psi\} \qquad True$$

Operating Condition – Location: Clamp:

$$\{S_5 = 3932.2 \ psi\} \le \{1.5S_{OC} = 30000 \ psi\} \qquad True$$

$$\{S_6 = 451.9 \ psi\} \le \{1.5S_{OC} = 30000 \ psi\} \qquad True$$

$$\{S_7 = 1251.2 \ psi\} < \{0.8S_{OC} = 16000 \ psi\} \qquad True$$

$$\{S_8 = 83.5 \ psi\} \le \{S_{OC} = 20000 \ psi\} \qquad True$$

$$\{S_9 = 2224.3 \ psi\} \le \{1.6 \cdot \min[S_{OH}, S_{OC}] = 32000 \, psi\} \qquad True$$

Gasket Seating/Assembly Condition – Location: Hub:

$$\{S_1 = 3310.8 \ psi\} \le \{1.5S_{AH} = 30000 \ psi\} \qquad True$$

$$\{S_2 = 0 \ psi\} \le \{S_{AH} = 20000 \ psi\} \qquad True$$

$$\{S_3 = 1438.2 \ psi\} \le \{0.8S_{AH} = 16000 \ psi\} \qquad True$$

$$\{S_4 = 1265.1 \ psi\} \le \{0.8S_{AH} = 16000 \ psi\} \qquad True$$

Gasket Seating/Assembly Condition – Location: Clamp:

$$\{S_5 = 4430.8\ psi\} \le \{1.5S_{AC} = 30000\ psi\} \qquad True$$

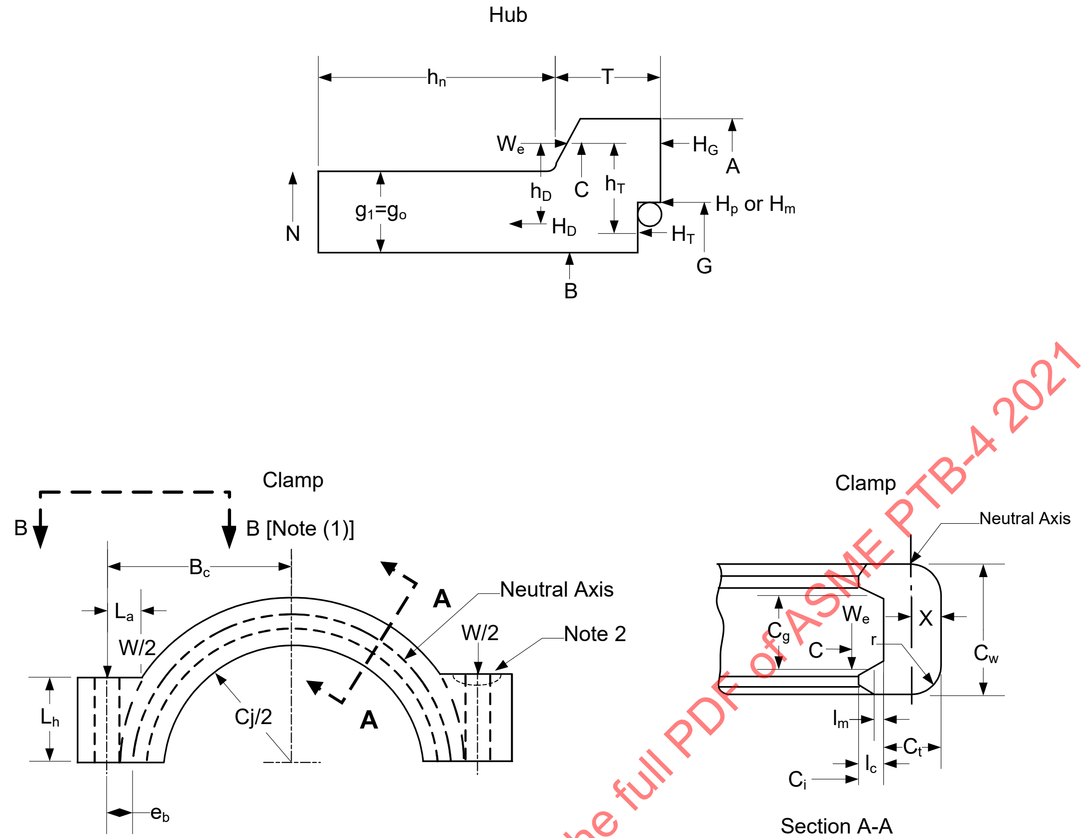
$$\{S_6 = 1560.9\ psi\} \le \{1.5S_{AC} = 30000\ psi\} \qquad True$$

$$\{S_7 = 1410.9\ psi\} \le \{0.8S_{AC} = 16000\ psi\} \qquad True$$

$$\{S_8 = 288.3\ psi\} < \{S_{AC} = 20000\ psi\} \qquad True$$

$$\{S_9 = 2508.3\ psi\} < \{1.6\min[S_{AH}, S_{AC}] = 32000\ psi\} \qquad True$$

Since the acceptance criteria are satisfied, the design is complete.



Notes:
1) See Figure 4.17.2 for sction B-B
2) Clamp may have spherical depressions at bolt holes to facilitate the use of spherical nuts

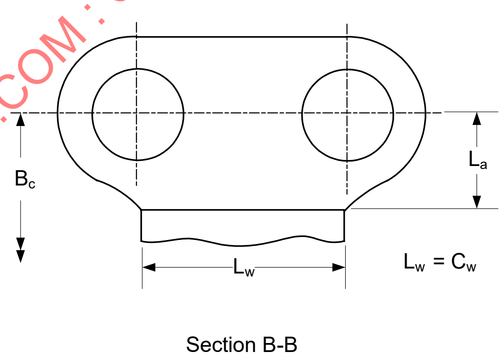


Figure E4.17.1 – Typical Hub and Clamp Configuration

4.18 Tubesheets in Shell and Tube Heat Exchangers

4.18.1 Example E4.18.1 – U-Tube Tubesheet Integral with Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration a as shown in VIII-1, Figure UHX-12.1, Configuration a.

- The shell side design condition is -10 to 60 psig at 500°F, and the tube side design condition is -15 to 140 psig at 500°F.
- The tube material is SA-249, Type 316 (S31600). The tubes are 0.75 in. outside diameter and 0.065 in. thick.
- The tubesheet material is SA-240, Type 316 (S31600). The tubesheet diameter is 12.939 in. The tubesheet has 76 tube holes on a 1.0 in. square pattern with one centerline pass lane and no pass partition grooves. The largest center-to-center distance between adjacent tube rows is 2.25 in., the length of the untubed lane is 11.626 in., and the radius to the outermost tube hole center is 5.438 in. There is no corrosion allowance on the tubesheet. The tubes are full-strength welded to the tubesheet with no credit taken for expansion.
- The shell material is SA-312, Type 316 (S31600) welded pipe. The shell inside diameter is 12.39 in. and the shell thickness is 0.18 in.
- The channel material is SA-240, Type 316 (S31600). The channel inside diameter is 12.313 in. and the channel thickness is 0.313 in.

Data Summary

The data summary consists of those variables from the nomenclature (see VIII-1, paragraph UHX-5.1) that are applicable to this configuration.

Design Conditions:

$P_{sd,\max} = 60\ psig$

$P_{sd,\min} = -10\ psig$

$P_{td,\max} = 140\ psig$

$P_{td,\min} = -15\ psig$

$T = 500^oF$

$T_c = 500^oF$

$T_s = 500^oF$

Tubes:

$d_t = 0.75\ in.$

$E_{tT} = 25.9E6\ psi$ from Table TM-1 of Section II, Part D at T

$S_{tT} = 18{,}000\ psi$ from Table 1A of Section II, Part D at T (for seamless tube, SA-213)

$t_t = 0.065\ in.$

The tubes are SA-249, Type 316 (welded). VIII-1, paragraph UHX-5.2 requires the use of the allowable stress for the equivalent seamless product, which is SA-213, Type 316.

Tubesheet:

Tube Pattern: Square

$A = 12.939\ in.$

$A_L = 26.16\ in.^2$

$c_t = 0\ in.$

$E = 25.9E6\ psi$ from Table TM-1 of Section II, Part D at T

$h = 0.521\ in.$ (assumed)

$h_g = 0\ in.$

$L_{L1} = 11.626\ in.$

$p = 1.0\ in.$

$r_o = 5.438\ in.$

$S = 18{,}000\ psi$ from Table 1A of Section II, Part D at T

$S_y = 20{,}000\ psi$ from Table Y-1 of Section II, Part D at T

$U_{L1} = 2.25\ in.$

$\rho = 0$ for no tube expansion

Shell:

$D_s = 12.39\ in.$

$E_s = 25.9E6\ psi$ from Table TM-1 of Section II, Part D at T_s

$S_s = 18{,}000\ psi$ from Table 1A of Section II, Part D at T_s (for seamless pipe, SA-312)

$t_s = 0.180\ in.$

$\nu_s = 0.31$ from Table PRD of Section II, Part D for High Alloy Steels (300 Series)

The shell is SA-312, Type 316 welded pipe. VIII-1, paragraph UHX-5.2 requires the use of the allowable stress for the equivalent seamless product, which is SA-312, Type 316 seamless pipe.

Channel:

$D_c = 12.313\ in.$

$E_c = 25.9E6\ psi$ from Table TM-1 of Section II, Part D at T_c

$S_c = 18{,}000\ psi$ from Table 1A of Section II, Part D at T_c

$t_c = 0.313\ in.$

$\nu_c = 0.31$ from Table PRD of Section II, Part D for High Alloy Steels (300 Series)

Calculation Procedure

The calculation procedure for a U-tube heat exchanger tubesheet is given in VIII-1, paragraph UHX-12.5. The calculation results are shown for the loading cases required to be analyzed (see paragraph UHX-12.4).

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-1, paragraph UHX-11.5.1.

$D_o = 11.626\ in.$

$\mu = 0.2500$

$d^* = 0.7500\ in.$

$p^* = 1.152\ in.$

$\mu^* = 0.3489$

$h'_g = 0\ in.$

b) STEP 2 – Calculate ρ_s and ρ_c. For each loading case, list the tubesheet loads and the calculated value of M_{TS} for configuration a.

$\rho_s = 1.066$

$\rho_c = 1.059$

Summary Table for Tubesheet Loads and STEP 2				
Loading Case	P_s (psi)	P_t (psi)	W^* (lbf)	M_{TS} (in.-lb/in.)
1	-10	140	0	-160.1
2	60	-15	0	87.03
3	60	140	0	-77.14
4	-10	-15	0	4.030

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII, paragraph UHX-11.5.2 and calculate E^*.

$h/p = 0.5210$

$E^*/E = 0.4452$

$\nu^* = 0.2539$

$E^* = 11.53E6\ psi$

d) STEP 4 – For configuration a, calculate shell coefficients β_s, k_s, λ_s, δ_s and ω_s.

$\beta_s = 1.206\ in.^{-1}$

$k_s = 33.60E3\ lb$

$\lambda_s = 32.26E6\ psi$

$\delta_s = 6.956E\text{-}6\ in.^3/lb$

$\omega_s = 0.4894\ in.^2$

For configuration a, calculate channel coefficients β_c, k_c, λ_c, δ_c and ω_c.

$\beta_c = 0.9129\ in.^{-1}$

$k_c = 0.1337E6\ lb$

$\lambda_c = 111.0E6\ psi$

$\delta_c = 3.951E\text{-}6\ in.^3/lb$

$\omega_c = 0.7535\ in.^2$

e) STEP 5 – Calculate K and F for configuration a.

$K = 1.113$

$F = 9.446$

f) STEPS 6 thru 8 – For each loading case, calculate M^*, M_p, M_o, M and tubesheet bending stress σ.

Summary Table for STEPS 6 thru 8						
Loading Case	M^* (in.-lb/in.)	M_p (in.-lb/in.)	M_o (in.-lb/in.)	M (in.-lb/in.)	σ (psi)	$2S$ (psi)
1	-49.75	568.2	-462.6	568.2	35,990	36,000
2	46.36	-282.0	233.4	282.0	17,870	36,000
3	-1.014	305.5	-244.3	305.5	19,350	36,000
4	-2.378	-19.33	15.04	19.33	1,224	36,000

For Loading Cases 1-4 $|\sigma| \le 2S$. The bending stress criterion for the tubesheet is satisfied.

g) STEP 9 – Check the criterion below for the largest value of $|P_s - P_t|$ and calculate the shear stress, if required.

$$\left\{ |P_s - P_t| = 150.0\ psi \right\} \le \left\{ \frac{4\mu h}{D_o} \cdot \min\left[0.8S, 0.533S_y\right] = 477.7\ psi \right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

h) STEP 10 – For each loading case, calculate the stresses in the shell and channel for configuration a, and check the acceptance criterion. The shell thickness shall be 0.18 in. for a minimum length of 2.688 in. adjacent to the tubesheet, and the channel thickness shall be 0.313 in. for a minimum length of 3.534 in. adjacent to the tubesheet.

Summary Table for STEP 10, Shell Results				
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)
1	-169.6	-17,600	17,770	27,000
2	1,018	12,210	13,230	27,000
3	1,018	-5,336	6,354	27,000
4	-169.6	-53.91	223.5	27,000

For Design Loading Cases 1 - 4 $|\sigma_s| \le 1.5S_s$. The stress criterion for the shell is satisfied.

Summary Table for STEP 10, Channel Results				
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)
1	1,343	25,290	26,640	27,000
2	-143.9	-11,690	11,830	27,000
3	1,343	14,630	15,970	27,000
4	-143.9	-1,023	1,167	27,000

For Design Loading Cases 1-4 $|\sigma_c| \le 1.5S_c$. The stress criterion for the channel is satisfied.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

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4.18.2 Example E4.18.2 – U-Tube Tubesheet Gasketed with Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration d as shown in VIII-1, Figure UHX-12.1, Configuration d.

- The shell side design condition is -15 to 10 psig at 300°F and the tube side design condition is 0 to 135 psig at 300°F.
- The tube material is SB-111, Admiralty (C44300). The tubes are 0.625 in. outside diameter and 0.065 in. thick.
- The tubesheet material is SA-285, Grade C (K02801). The tubesheet diameter is 20.0 in. The tubesheet has 386 tube holes on a 0.75 in. equilateral triangular pattern with one centerline pass lane and no pass partition grooves. The largest center-to-center distance between adjacent tube rows is 1.75 in., the length of the untubed lane is 16.813 in. and the radius to the outermost tube hole center is 8.094 in. There is a 0.125 in. corrosion allowance on the tube side. The tubes are expanded for the full thickness of the tubesheet.
- The channel flange gasket consists of a ring gasket with a centerline rib. The ring gasket outside diameter is 19.375 in., the inside diameter is 18.625 in., and the gasket factors are y = 10,000 psi and m = 3.0. The rib gasket width is 0.375 in., the length is 18.625 in., and the rib gasket factors are y = 9,000 psi and m = 3.75. The shell flange gasket outside diameter is 19.375 in., the inside diameter is 18.625 in., and the gasket factors are y = 10,000 psi and m = 3.0. The effective gasket width for both gaskets is per VIII-1 Appendix 2, Table 2-5.2, sketch (1a). There are (24) 0.75 in. diameter SA-193-B7 bolts on a 20.875 in. bolt circle.

<u>**Data Summary**</u>

The data summary consists of those variables from the nomenclature (see VIII-1, paragraph UHX-5.1) that are applicable to this configuration.

<u>**Design Conditions:**</u>

$P_{sd,\max} = 10\ psig$

$P_{sd,\min} = \text{-}15\ psig$

$P_{td,\max} = 135\ psig$

$P_{td,\min} = 0\ psig$

$T = 300^{o}F$

$T_{fe} = 300^{o}F$

<u>**Tubes:**</u>

$d_t = 0.625\ in.$

$E_{tT} = 15.3E6\ psi$ from Table TM-3 of Section II, Part D at T

$S_{tT} = 10{,}000\ psi$ from Table 1B of Section II, Part D at T

$t_t = 0.065\ in.$

Tubesheet:

Tube Pattern: Triangular

Assume an uncorroded tubesheet thickness of 1.405 inches.

$A = 20\ in.$

$A_L = 29.42\ in.^2$

$c_t = 0.125\ in.$

$D_E = 18.625\ in.$

$E = 28.3E6\ psi$ from Table TM-1 of Section II, Part D at T

$G_c = 19.00\ in.$ (G per VIII-1 Appendix 2)

$G_s = 19.00\ in.$ (G per VIII-1 Appendix 2)

$h = 1.405\ in. - 0.125\ in. = 1.28\ in.$ (assumed)

$h_g = 0\ in.$

$L_{L1} = 16.813\ in.$

$p = 0.75\ in.$

$r_o = 8.094\ in.$

$S = 15{,}700\ psi$ from Table 1A of Section II, Part D at T

$S_{fe} = 15{,}700\ psi$ from Table 1A of Section II, Part D at T_{fe}

$S_y = 26{,}500\ psi$ from Table Y-1 of Section II, Part D at T

$U_{L1} = 1.75\ in.$

$W_{m1c} = 50{,}854\ lb$ (W_{m1} per VIII-1 Appendix 2)

$W_{m1s} = 3{,}505\ lb$ (W_{m1} per VIII-1 Appendix 2)

$\rho = 1.0$ for full length tube expansion

Calculation Procedure

The tubesheet is not extended as a flange but has an unflanged extension. The calculation procedure for this extension is given in VIII-1, paragraph UHX-9-5(c). The minimum required thickness of this extension calculated at T_{fe} is:

$h_r = 0.05561\ in.$

The calculation procedure for a U-tube heat exchanger tubesheet is given in VIII-1, paragraph UHX-12.5. The calculation results are shown for the loading cases required to be analyzed (see paragraph UHX-12.4).

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-1, paragraph UHX-11.5.1.

$D_o = 16.813\ in.$

$\mu = 0.1667$

$d^* = 0.5802\ in.$

$p^* = 0.8053\ in.$

$\mu^* = 0.2794$

$h'_g = 0\ in.$

b) STEP 2 – Calculate ρ_s and ρ_c. For each loading case, list the tubesheet loads and the calculated value of M_{TS} for configuration d.

$\rho_s = 1.130$

$\rho_c = 1.130$

Summary Table for Tubesheet Loads and STEP 2				
Loading Case	P_s (psi)	P_t (psi)	W^* (lbf)	M_{TS} (in.-lb/in.)
1	-15	135	50,854	-785.0
2	10	0	3,505	52.33
3	10	135	50,854	-654.1
4	-15	0	0	-78.50

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII, paragraph UHX-11.5.2 and calculate E^*.

$h/p = 1.707$

$E^*/E = 0.2647$

$\nu^* = 0.3579$

$E^* = 7.491E6\ psi$

d) STEP 4 – For configuration d, skip STEP 4 and proceed to STEP 5.

e) STEP 5 – Calculate the diameter ratio K and the coefficient F for configuration d.

$K = 1.190$

$F = 0.4211$

f) STEPS 6 thru 8 – For each loading case, calculate M^*, M_p, M_o, M and tubesheet bending stress σ.

Summary Table for STEPS 6 thru 8						
Loading Case	M^* (in.-lb/in.)	M_p (in.-lb/in.)	M_o (in.-lb/in.)	M (in.-lb/in.)	σ (psi)	$2S$ (psi)
1	-785.0	-159.8	-2,384	2,384	31,250	31,400
2	52.33	10.65	159.0	159.0	2,083	31,400
3	-654.1	-133.1	-1,987	1,987	26,040	31,400
4	-78.50	-15.98	-238.4	238.4	3,125	31,400

For Loading Cases 1-4 $|\sigma| \le 2S$. The bending stress criterion for the tubesheet is satisfied.

g) STEP 9 – Check the criterion below for the largest value of $|P_s - P_t|$ and calculate the shear stress, if required.

$$\{|P_s - P_t| = 150.0\ psi\} \le \left\{\frac{4\mu h}{D_o} \cdot \min\left[0.8S, 0.533S_y\right] = 637.5\ psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

4.18.3 Example E4.18.3 – U-Tube Tubesheet Gasketed with Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration d as shown in VIII-1, Figure UHX-12.1 Configuration d.

- The shell side design condition is 0 to 375 psig at 500°F and the tube side design condition is 0 to 75 psig at 500°F.
- The tube material is SB-111, 90/10 Copper-Nickel (C70600). The tubes are 0.75 in. outside diameter and 0.049 in. thick.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet diameter is 48.88 in. The tubesheet has 1,534 tube holes on a 0.9375 in. equilateral triangular pattern with one centerline pass lane and a 0.1875 in. deep pass partition groove. The largest center-to-center distance between adjacent tube rows is 2.25 in., the length of the untubed lane 41.75 in., and the radius to the outermost tube hole center is 20.5 in. There is a 0.125 in. corrosion allowance on the tube side. The tubes are expanded for one-half of the tubesheet thickness.
- The channel flange gasket consists of a ring gasket with a centerline rib. The ring gasket outside diameter is 45.38 in., the inside diameter is 44.38 in., and the gasket factors are y = 10,000 psi and m = 3.0. The rib gasket width is 0.50 in., the length is 44.38 in., and the rib gasket factors are y = 9,000 psi and m = 3.75. The shell flange gasket outside diameter is 44.0 in., the inside diameter is 43.0 in., and the gasket factors are y = 10,000 psi and m = 3.0. The effective gasket width for both gaskets is per VIII-1 Appendix 2, Table 2-5.2, sketch (1a). There are (52) 1.0 in. diameter SA-193-B7 bolts on a 46.75 in. bolt circle.
- The tubesheet shall be designed for a differential design pressure of 300 psi.

Data Summary

The data summary consists of those variables from the nomenclature (see VIII-1, paragraph UHX-5.1) that are applicable to this configuration.

Design Conditions:

$P_{sd,\max} = 375\ psig$

$P_{td,\max} = 75\ psig$

$T = 500^oF$

$T_{fe} = 500^oF$

Tubes:

$d_t = 0.75\ in.$

$E_{tT} = 16.5E6\ psi$ from Table TM-3 of Section II, Part D at T

$S_{tT} = 8{,}000\ psi$ from Table 1B of Section II, Part D at T

$t_t = 0.049\ in.$

Tubesheet:

Tube Pattern: Triangular

Assume an uncorroded tubesheet thickness of 4.275 inches.

$A = 48.88\ in.$

$A_L = 93.94\ in.^2$

$c_t = 0.125\ in.$

$D_E = 44.38\ in.$

$E = 27.1E6\ psi$ from Table TM-1 of Section II, Part D at T

$G_c = 44.88\ in.$ (G per VIII-1 Appendix 2)

$G_s = 43.50\ in.$ (G per VIII-1 Appendix 2)

$h = 4.275\ in. - 0.125\ in. = 4.15\ in.$ (assumed)

$h_g = 0.1875\ in.$

$L_{L1} = 41.75\ in.$

$p = 0.9375\ in.$

$r_o = 20.5\ in.$

$S = 20{,}000\ psi$ from Table 1A of Section II, Part D at T

$S_{fe} = 20{,}000\ psi$ from Table 1A of Section II, Part D at T_{fe}

$S_y = 31{,}000\ psi$ from Table Y-1 of Section II, Part D at T

$U_{L1} = 2.25\ in.$

$W_{m1c} = 140{,}682\ lb$ (W_{m1} per VIII-1 Appendix 2)

$W_{m1s} = 633{,}863\ lb$ (W_{m1} per VIII-1 Appendix 2)

$\rho = 0.50$

Calculation Procedure

The tubesheet is extended as a flange but has no bolt loads applied to the extension. The calculation procedure for this extension is given in VIII-1, paragraph UHX-9-5(c). The minimum required thickness of the extension calculated at T_{fe} is:

$h_r = 0.2080\ in.$

The calculation procedure for a U-tube heat exchanger tubesheet is given in VIII-1, paragraph UHX-12.5. The calculation results are shown for the loading cases required to be analyzed (see paragraph UHX-12.4).

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-1, paragraph UHX-11.5.1.

$D_o = 41.75\ in.$

$\mu = 0.2000$

$d^* = 0.7381\ in.$

$p^* = 0.9714\ in.$

$\mu^{*} = 0.2402$

$h'_g = 0.06250\ in.$

b) STEP 2 – Calculate ρ_s and ρ_c. For Loading Case 3, list the tubesheet loads and the calculated value of M_{TS} for configuration d.

$\rho_s = 1.042$

$\rho_c = 1.075$

Summary Table for Tubesheet Loads and STEP 2				
Loading Case	P_s (psi)	P_t (psi)	W^{*} (lbf)	M_{TS} (in.-lb/in.)
1	N/A	N/A	N/A	N/A
2	N/A	N/A	N/A	N/A
3	375	75	633,863	2,251
4	N/A	N/A	N/A	N/A

c) STEP 3 – Calculate h/p. Determine E^{*}/E and ν^{*} from VIII, paragraph UHX-11.5.2 and calculate E^{*}.

$h/p = 4.427$

$E^{*}/E = 0.2043$

$\nu^{*} = 0.4069$

$E^{*} = 5.537E6\ psi$

d) STEP 4 – For configuration d, skip STEP 4 and proceed to STEP 5.

e) STEP 5 – Calculate the diameter ratio K and the coefficient F for configuration d.

$K = 1.171$

$F = 0.4577$

f) STEPS 6 thru 8 – For Loading Case 3, calculate M^{*}, M_p, M_o, M and tubesheet bending stress σ.

Summary Table for STEPS 6 thru 8						
Loading Case	M^{*} (in.-lb/in.)	M_p (in.-lb/in.)	M_o (in.-lb/in.)	M (in.-lb/in.)	σ (psi)	$2S$ (psi)
1	N/A	N/A	N/A	N/A	N/A	N/A
2	N/A	N/A	N/A	N/A	N/A	N/A
3	5,586	-1,299	26,540	26,540	39,670	40,000
4	N/A	N/A	N/A	N/A	N/A	N/A

For Loading Cases 1-4 $|\sigma| \le 2S$. The bending stress criterion for the tubesheet is satisfied.

g) STEP 9 – Check the criterion below for the largest value of $|P_s - P_t|$ and calculate the shear stress, if required.

$$\{|P_s - P_t| = 300.0\ psi\} \le \left\{\frac{4\mu h}{D_o} \cdot \min\left[0.8S, 0.533S_y\right] = 1272\ psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

4.18.4 Example E4.18.4 – U-Tube Tubesheet Gasketed with Shell and Integral with Channel, Extended as a Flange

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration e as shown in VIII-1, Figure UHX-12.1, Configuration e.

- The shell side design condition is 0 to 650 psig at 400°F, and the tube side design condition is 0 to 650 psig at 400°F.
- The tube material is SA-179 (K10200). The tubes are 0.75 in. outside diameter and 0.085 in. thick.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet diameter is 37.25 in. The tubesheet has 496 tube holes on a 1.0 in. square pattern with one centerline pass lane and no pass partition grooves. The largest center-to-center distance between adjacent tube rows is 1.375 in., the length of the untubed lane is 26.25 in., and the radius to the outermost tube hole center is 12.75 in. There is a 0.125 in. corrosion allowance on the tube side. The tubes are expanded for the full thickness of the tubesheet.
- The shell flange gasket outside diameter is 32.875 in., the inside diameter is 31.875 in., and the gasket factors are y = 10,000 psi and m = 3.0. The effective gasket width is per VIII-1 Appendix 2, Table 2-5.2, sketch (1a). There are (36) 1.125 in. diameter SA-193-B7 bolts on a 35.0 in. bolt circle.
- The channel material is SA-516, Grade 70 (K02700). The channel inside diameter is 31 in. and the channel thickness is 0.625 in.

<u>Data Summary</u>

The data summary consists of those variables from the nomenclature (see VIII-1, paragraph UHX-5.1) that are applicable to this configuration.

<u>Design Conditions:</u>

$P_{sd,\max} = 650\ psig$

$P_{sd,\min} = 0\ psig$

$P_{td,\max} = 650\ psig$

$P_{td,\min} = 0\ psig$

$T = 400^{o}F$

$T_a = 70^{o}F$

$T_c = 400^{o}F$

$T_{fe} = 400^{o}F$

<u>Tubes:</u>

$d_t = 0.75\ in.$

$E_{tT} = 27.9E6\ psi$ from Table TM-1 of Section II, Part D at T

$S_{tT} = 13{,}400\ psi$ from Table 1A of Section II, Part D at T

$t_t = 0.085\ in.$

Tubesheet:

Tube Pattern: Square

Assume an uncorroded tubesheet thickness of 3.625 inches.

$A = 37.25$ *in.*

$A_L = 36.09$ *in.*^{2}

$C = 35.0$ *in.*

$c_t = 0.125$ *in.*

$E = 27.9E6$ *psi* from Table TM-1 of Section II, Part D at T

$G_s = 32.375$ *in.* (G per VIII-1 Appendix 2)

$h = 3.625$ *in.* $- 0.125$ *in.* $= 3.50$ *in.* (assumed)

$h_g = 0$ *in.*

$L_{L1} = 26.25$ *in.*

$p = 1.0$ *in.*

$r_o = 12.75$ *in.*

$S = 20{,}000$ *psi* from Table 1A of Section II, Part D at T

$S_{fe} = 20{,}000$ *psi* from Table 1A of Section II, Part D at T_{fe}

$S_y = 32{,}500$ *psi* from Table Y-1 of Section II, Part D at T

$U_{L1} = 1.375$ *in.*

$W_{m1s} = 633{,}930$ *lb* (W_{m1} per VIII-1 Appendix 2)

$W_s = 644{,}565$ *lb* (W per VIII-1 Appendix 2)

$\rho = 1.0$ for full length tube expansion

Channel:

$D_c = 31.0$ *in.*

$E_c = 27.9E6$ *psi* from Table TM-1 of Section II, Part D at T_c

$S_c = 20{,}000$ *psi* from Table 1A of Section II, Part D at T_c

$S_{y,c} = 32{,}500$ *psi* from Table Y-1 of Section II, Part D at T_c

$S_{PS,c} = 65{,}000$ *psi* [$2S_{y,c}$ per UG-23(e)] at T_c

$t_c = 0.625$ *in.*

$\nu_c = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Calculation Procedure

The tubesheet is extended as a flange. The calculation procedure for a flanged extension is given in VIII-1, paragraph UHX-9-5(a). The minimum required thickness of the flanged extension is the maximum of required thicknesses for the operating condition (at T_{fe}) and gasket seating condition (at T_a):

$$h_r = \max\left(1.563,\ 1.576\right) = 1.576\ in.$$

The calculation procedure for a U-tube heat exchanger tubesheet is given in VIII-1, paragraph UHX-12.5. The calculation results are shown for the loading cases required to be analyzed (see paragraph UHX-12.4).

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-1, paragraph UHX-11.5.1.

$$D_o = 26.25\ in.$$

$$\mu = 0.2500$$

$$d^* = 0.6361\ in.$$

$$p^* = 1.035\ in.$$

$$\mu^* = 0.3855$$

$$h'_g = 0\ in.$$

b) STEP 2 – Calculate ρ_s and ρ_c. For each loading case, list the tubesheet loads and the calculated value of M_{TS} for configuration e.

$$\rho_s = 1.233$$

$$\rho_c = 1.181$$

Summary Table for Tubesheet Loads and STEP 2				
Loading Case	P_s (psi)	P_t (psi)	W^* (lbf)	M_{TS} (in.-lb/in.)
1	0	650	0	-12,130
2	650	0	633,930	16,470
3	650	650	633,930	4,337

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII, paragraph UHX-11.5.2 and calculate E^*.

$$h/p = 3.500$$

$$E^*/E = 0.4413$$

$$\nu^* = 0.3179$$

$$E^* = 12.31E6\ psi$$

d) STEP 4 – For configuration e, shell coefficients $\beta_c = 0$, $k_s = 0$, $\lambda_s = 0$, $\delta_s = 0$ and $\omega_s = 0$.

For configuration e, calculate channel coefficients β_c, k_c, λ_c, δ_c and ω_s.

$$\beta_c = 0.4089\ in.^{-1}$$

$$k_c = 0.5101E6\ lb$$

$$\lambda_c = 7.646E6\ psi$$

$\delta_c = 11.71E\text{-}6\ in.^3/lb$

$\omega_c = 7.013\ in.^2$

e) STEP 5 – Calculate the diameter ratio K and the coefficient F for configuration e.

$K = 1.419$

$F = 0.9645$

f) STEPS 6 thru 8 – For each loading case, calculate M^*, M_p, M_o, M and tubesheet bending stress σ.

Summary Table for STEPS 6 thru 8						
Loading Case	M^* (in.-lb/in.)	M_p (in.-lb/in.)	M_o (in.-lb/in.)	M (in.-lb/in.)	σ (psi)	$2S$ (psi)
1	-7,572	3,018	-20,200	20,200	25,670	40,000
2	26,560	6,646	29,870	29,870	39,750	40,000
3	18,980	9,664	9,664	9,664	12,280	40,000

For Loading Cases 1-3 $|\sigma| \le 2S$. The bending stress criterion for the tubesheet is satisfied.

g) STEP 9 – Check the criterion below for the largest value of $|P_s - P_t|$ and calculate the shear stress, if required.

$$\{|P_s - P_t| = 650.0\ psi\} \le \left\{\frac{4\mu h}{D_o} \cdot \min\left[0.8S, 0.533S_y\right] = 2133\ psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

h) STEP 10 – For each loading case, calculate the stresses in the channel for configuration e, and check the acceptance criterion. The channel thickness shall be 0.625 in. for a minimum length of 7.923 in. adjacent to the tubesheet.

Summary Table for STEP 10, Channel Results					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	$S_{PS,c}$ (psi)
1	7,901	54,420	62,320	30,000	65,000
2	0	-56,470	56,470	30,000	65,000
3	7,901	-2,048	9,948	30,000	65,000

For Design Loading Cases 1 and 2 $|\sigma_c| > 1.5S_c$. For Design Loading Case 3 $|\sigma_c| \le 1.5S_c$. The stress criterion for the channel is not satisfied. For Design Loading Cases 1 and 2, since $|\sigma_c| \le S_{PS,c}$, Option 3 in STEP 11 is permitted.

i) STEP 11 – The design shall be reconsidered by using one or a combination of the following options.

- Option 1 – Increase the tubesheet thickness and return to STEP 1.
- Option 2 – Increase the integral channel thickness and return to STEP 1.
- Option 3 – Perform the simplified elastic-plastic calculation procedures as defined in VIII-1, paragraph UHX-12.5.11.

Since the total axial stress in the channel σ_c is between $1.5S_c$ and $S_{PS,c}$ for Design Condition Loading Cases 1 and 2, the procedure of VIII-1, paragraph UHX-12.5.11, Option 3 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the channel occurs.

The results for the effect of plasticity for Design Condition Loading Cases 1 and 2 are shown below.

Summary Results for STEP 11, Elastic-Plastic Iteration Results per VIII-1, paragraph UHX-12.5.11, Option 3		
Design Condition Loading Case	1	2
$1.5S_c$, *psi*	30,000	30,000
σ_c, *psi*	62,320	56,470
E_c^*, *psi*	19.36*E*6	20.34*E*6
k_c, *lb*	0.3539*E6*	0.3718*E6*
λ_c	5.305*E*6	5.573*E*6
F	0.8348	0.8497
M_p, *in.-lb/in.*	2,242	7,928
M_o, *in.-lb/in.*	-20,980	31,150
M, *in.-lb/in.*	20,980	31,150
$\lvert\sigma\rvert$, *psi*	26,660	39,580

The final calculated tubesheet bending stresses of 26,660 psi (Loading Case 1) and 39,580 psi (Loading Case 2) are less than the allowable tubesheet bending stress of 40,000 psi. As such, this geometry meets the requirement of VIII, paragraph UHX. The intermediate results for the elastic-plastic iteration are shown above.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

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4.18.5 Example E4.18.5 – Fixed Tubesheet Exchanger, Configuration b, Tubesheet Integral with Shell, Extended as a Flange and Gasketed on the Channel Side

A fixed tubesheet heat exchanger is to be designed with the tubesheet construction in accordance with configuration b as shown in VIII-1, Figure UHX-13.1, Configuration b.

- For the Design Condition, the shell side design pressure is 0 to 150 psig at 700°F, and the tube side design pressure is 0 to 400 psig at 700°F.
- There is one operating condition. For Operating Condition 1, the operating pressures and operating metal temperatures are assumed to be the same as the Design Condition. The shell mean metal temperature is 550°F, and the tube mean metal temperature is 510°F.
- The tube material is SA-214 Welded (K01807). The tubes are 1 in. outside diameter and 0.083 in. thick. The unsupported tube span under consideration is between 2 tube supports, and the length of the unsupported tube span is 59 inches.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet diameter is 40.5 in. The tubesheet has 649 tube holes on a 1.25 in. equilateral triangular pattern. There is no pass partition lane, and the outermost tube radius from the tubesheet center is 16.625 in. The distance between the outer tubesheet faces is 168 in. The option for the effect of differential radial expansion is not required. There is no corrosion allowance on the tubesheet. The tubes are expanded to 95% of the tubesheet thickness.
- The shell material is SA-516, Grade 70 (K02700). The shell inside diameter is 34.75 in. and the shell thickness is 0.1875 in. There is no corrosion allowance on the shell. The shell contains an expansion joint that has an inside diameter of 38.5 in. and an axial rigidity of 11,388 lb/in. The efficiency of the shell circumferential welded joint (Category B) is 1.0.
- The channel flange gasket outside diameter is 37.3125 in., the inside diameter is 36.3125 in., and the gasket factors are y = 7,600 psi and m = 3.75. The effective gasket width is per VIII-1 Appendix 2, Table 2-5.2, sketch (1a). There are (68) 0.75 in. diameter SA-193-B7 bolts on a 38.875 in. bolt circle.

<u>Data Summary</u>

The data summary consists of those variables from the nomenclature (see VIII-1, paragraph UHX-5.1) that are applicable to this configuration.

<u>Design Conditions:</u>

$P_{sd,\max} = 150\ psig$

$P_{sd,\min} = 0\ psig$

$P_{td,\max} = 400\ psig$

$P_{td,\min} = 0\ psig$

$T = 700^oF$

$T_a = 70^oF$

$T_{fe} = 700^oF$

$T_s = 700^oF$

$T_t = 700^oF$

Operating Conditions:

$P_{so1,\max} = 150\ psig$

$P_{so1,\min} = 0\ psig$

$P_{to1,\max} = 400\ psig$

$P_{to1,\min} = 0\ psig$

$T_1 = 700^oF$

$T_{s1} = 700^oF$

$T_{t1} = 700^oF$

$T_{s,m1} = 550^oF$

$T_{t,m1} = 510^oF$

Tubes:

$d_t = 1.0\ in.$

$E_t = 25.5E6\ psi$ from Table TM-1 of Section II, Part D at T_t & T_{t1}

$E_{tT} = 25.5E6\ psi$ from Table TM-1 of Section II, Part D at T

$k = 1$ for an unsupported tube span between two tube supports

$\ell = 59\ in.$

$\ell_t = 59\ in.$

$S_t = 10{,}500\ psi$ from Table 1A of Section II, Part D at T_t & T_{t1} (see explanation below)

$S_{tT} = 10{,}500\ psi$ from Table 1A of Section II, Part D at T (see explanation below)

$S_{y,t} = 18{,}600\ psi$ from Table Y-1 of Section II, Part D at T_t & T_{t1}

$t_t = 0.083\ in.$

$\alpha_{t,m1} = 7.3E\text{-}6\ in./in./^oF$ from Table TE-1 of Section II, Part D at $T_{t,m1}$

$\nu_t = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Since the tubes are SA-214 (welded), VIII-1, paragraph UHX-5.2 requires that the allowable stress for the welded tubes be divided by 0.85 if the equivalent seamless product is not available. In this case, SA-556, Grade A2 (K01807) could be used as the equivalent seamless product, but the alternative will be illustrated in this example. When the welded tube allowable stresses are divided by 0.85, the resulting allowable stresses are $S_t = 12{,}353\ psi$ and $S_{tT} = 12{,}353\ psi$.

Tubesheet:

Tube Pattern: Triangular

$A = 40.5\ in.$

$A_L = 0\ in.^2$ for no pass lanes

$C = 38.875\ in.$

$c_t = 0\ in.$

$E = 25.5E6\ psi$ from Table TM-1 of Section II, Part D at T & T_1

$G_c = 36.8125\ in.$ (G per VIII-1 Appendix 2)

$a_c = 18.41\ in.$

$h = 3.0625\ in.$ (assumed)

$h_g = 0\ in.$

$L_t = 168\ in.$

$L = 161.875\ in.$

$N_t = 649$

$p = 1.25\ in.$

$r_o = 16.625\ in.$

$S = 18{,}100\ psi$ from Table 1A of Section II, Part D at T & T_1

$S_a = 20{,}000\ psi$ from Table 1A of Section II, Part D at T_a

$S_{fe} = 18{,}100\ psi$ from Table 1A of Section II, Part D at T_{fe}

$S_y = 27{,}200\ psi$ from Table Y-1 of Section II, Part D at T & T_1

$S_{PS} = 54{,}400\ psi$ [$2S_y$ per UG-23(e)] at T & T_1

$W_{m1c} = 512{,}473\ lb$ (W_{m1} per VIII-1 Appendix 2)

$W_c = 512{,}937\ lb$ (W per VIII-1 Appendix 2)

$\rho = 0.9500$

Shell:

$D_J = 38.5\ in.$

$D_s = 34.75\ in.$

$a_s = 17.38\ in.$

$E_s = 25.5E6\ psi$ from Table TM-1 of Section II, Part D at T_s & T_{s1}

$E_{s,w} = 1.0$

$K_J = 11{,}388\ lb/in.$

$S_s = 18{,}100\ psi$ from Table 1A of Section II, Part D at T_s & T_{s1}

$S_{y,s} = 27{,}200\ psi$ from Table Y-1 of Section II, Part D at T_s & T_{s1}

$S_{PS,s} = 54{,}400\ psi$ [$2S_{y,s}$ per UG-23(e)] at T_s & T_{s1}

$t_s = 0.1875\ in.$

$\alpha_{s,m1} = 7.30E\text{-}6\ in./in./^oF$ from Table TE-1 of Section II, Part D at $T_{s,m1}$

$\nu_s = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Calculation Procedure

The tubesheet is extended as a flange. The calculation procedure for a flanged extension is given in VIII-1, paragraph UHX-9-5(a). The minimum required thickness of the flanged extension is the maximum of required thicknesses for the operating condition (at T_{fe}) and gasket seating condition (at T_a):

$h_r = \max(1.228, 1.168) = 1.228\ in.$

The calculation procedure for a tubesheets of a fixed tubesheet heat exchanger is given in VIII-1, paragraph UHX-13.5. The following results are for the design and operating loading cases required to be analyzed (see paragraph UHX-13.4). This example illustrates the calculation of both the elastic and elastic-plastic solutions.

a) STEP 1 – Calculate Do, m, m* and h'_g from VIII-1, paragraph UHX-11.5.1.

$D_o = 34.25\ in.$

$\mu = 0.2000$

$d^* = 0.8924\ in.$

$p^* = 1.250\ in.$

$\mu^* = 0.2861$

$h'_g = 0\ in.$

Calculate a_o, ρ_σ, ρ_c, x_s and x_t.

$a_o = 17.13\ in.$

$\rho_s = 1.015$

$\rho_c = 1.075$

$x_s = 0.4467$

$x_t = 0.6152$

b) STEP 2 – Calculate the shell axial stiffness K_s, tube axial stiffness K_t and stiffness factors $K_{s,t}$ and J.

$K_s = 3.242E6\ lb/in.$

$K_t = 37.67E3\ lb/in.$

$K_{s,t} = 0.1326$

$J = 3.500E\text{-}3$

For configuration b, calculate shell coefficients β_s, k_s, λ_s and δ_s.

$\beta_s = 0.7102\ in.^{-1}$

$k_s = 21.87E3\ lb$

$\lambda_s = 0.8794E6\ psi$

$\delta_s = 53.67E\text{-}6\ in.^3/lb$

For configuration b, channel coefficients $\beta_c = 0$, $k_c = 0$, $\lambda_c = 0$ and $\delta_c = 0$.

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII, paragraph UHX-11.5.2 and calculate E^*.

$h/p = 2.450$

$E^*/E = 0.2630$

$n^* = 0.3640$

$E^* = 6.706E6\ psi$

Calculate X_a and parameters Z_a, Z_d, Z_v, Z_w and Z_m from VIII-1, Table UHX-13.1.

$X_a = 3.963$

$Z_a = 6.547$

$Z_d = 0.02461$

$Z_v = 0.06426$

$Z_w = 0.06426$

$Z_m = 0.3715$

d) STEP 4 – Calculate the diameter ratio K and the coefficient F.

$K = 1.182$

$F = 0.4888$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} and U.

$\Phi = 0.6667$

$Q_1 = -0.02264$

$Q_{z1} = 2.856$

$Q_{z2} = 6.888$

$U = 13.78$

e) STEP 5 – Calculate γ, $\omega_s, \omega_s^*, \omega_c, \omega_c^*$ and γ_b. Use the loads listed in the table below to calculate the results for an elastic solution in the corroded condition.

$\omega_s = 2.685\ in.^2$ $\qquad$ $\omega_s^* = -2.654\ in.^2$

$\omega_c = 0\ in^2$ $\qquad$ $\omega_c^* = 9.682\ in^2$

$\gamma_b = -0.06022$

Summary Table for Step 5 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	0	400	0	512,473
2	150	0	0	0
3	150	400	0	512,473

Summary Table for Step 5 – Operating Condition 1				
Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	0	400	-0.04727	512,937
2	150	0	-0.04727	512,937
3	150	400	-0.04727	512,937
4	0	0	-0.04727	512,937

f) STEP 6 – For each loading case, calculate P_s', P_t', P_γ, P_ω, P_W, P_{rim} and effective pressure P_e.

Summary Table for STEP 6 – Design Condition							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	0.8620*E*6	0	0	230.7	181.9	-399.4
2	-46,390	0	0	0	0	18.70	-21.50
3	-46,390	0.8620*E*6	0	0	230.7	200.6	-420.9

Summary Table for STEP 6 – Operating Condition 1							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	0.8620*E*6	-1,254	0	230.9	181.9	-400.0
2	-46,390	0	-1,254	0	230.9	18.70	-21.97
3	-46,390	0.8620*E*6	-1,254	0	230.9	200.6	-421.5
4	0	0	-1,254	0	230.9	0	-0.4744

g) STEP 7 – Elastic Iteration: Calculate Q_2, Q_3 and F_m, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (lbf)	Q_3	F_m	$h - h_g'$ (in)	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-7,041	0.09758	0.09751	3.0625	25,540	27,150
2	-319.0	0.07858	0.09006	3.0625	1,269	27,150
3	-7,360	0.09661	0.09713	3.0625	26,810	27,150

Summary Table for STEP 7 – Operating Condition 1						
Loading Case	Q_2 (lbf)	Q_3	F_m	h (in)	$\lvert\sigma\rvert$ (psi)	S_{PS} (psi)
1	-7,044	0.09747	0.09747	3.0625	25,570	54,400
2	-4,259	1.300	0.6705	3.0625	9,660	54,400
3	-7,363	0.09650	0.09709	3.0625	26,830	54,400
4	-3,940	56.63	28.42	3.0625	8,840	54,400

For Design Loading Cases 1-3 $|\sigma| \le 1.5S$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma| \le S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Check the criterion below for the largest value of P_e and calculate the shear stress, if required.

$$\{|P_e| = 421.5\ psi\} \le \left\{\frac{2\mu h}{a_o} \cdot \min\left[0.8S, 0.533S_y\right] = 1040\ psi\right\} \quad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

i) STEP 9 – For each loading case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$r_t = 0.3255\ in$

$F_t = 181.2$

$C_t = 164.5$

Summary Table for STEP 9 – Design Condition				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	-1.081	-4,024	3.808	7,570
2	-1.011	268.9	3.658	864.7
3	-1.077	-3,755	3.801	8,434

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	7,570	12,353	-4,024	1.346	5,693
2	864.7	12,353	---	---	---
3	8,434	12,353	-3,755	1.350	5,677

Summary Table for STEP 9 – Operating Condition 1				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	-1.081	-4,207	3.807	7,581
2	-5.520	-322.2	13.33	2,137
3	-1.077	-3,758	3.800	8,445
4	-213.2	-600.4	451.8	1,272

Summary Table for STEP 9 – Operating Condition 1 (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	$2S_t$ (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	7,581	24,706	-4,207	1.346	5,691
2	2,137	24,706	-322.2	1.250	6,129
3	8,445	24,706	-3,758	1.350	5,675
4	1,272	24,706	-600.4	1.250	6,129

For Design Loading Cases 1-3 $\sigma_{t,\max} \le S_t$, and for Operation Condition 1, Loading Cases 1-4 $\sigma_{t,\max} \le 2S_t$. The axial tension stress criterion for the tube is satisfied.

For all Loading Cases, $|\sigma_{t,\min}| \le S_{tb}$. The buckling criterion for the tube is satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Design Condition			
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{s,b}$ (psi)
1	26.08	18,100	---
2	-764.8	18,100	8,505
3	-738.7	18,100	8,505

Summary Table for STEP 10 – Operating Condition 1			
Loading Case	$\sigma_{s,m}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	0.05859	54,400	---
2	-786.1	54,400	8,505
3	-764.8	54,400	8,505
4	-21.24	54,400	8,505

For Design Loading Cases 1-3 $|\sigma_{s,m}| \leq S_s E_{s,w}$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma_{s,m}| \leq S_{PS,s}$. The axial membrane stress criterion for the shell is satisfied.

For all Loading Cases where the value of $\sigma_{s,m}$ is negative, $|\sigma_{s,m}| \leq S_{s,b}$. The longitudinal compressive stress criterion for the shell is satisfied.

k) STEP 11 – For each loading case, calculate the stresses in the shell for configuration b, and check the acceptance criterion. The shell thickness shall be 0.1875 in. for a minimum length of 4.595 in. adjacent to the tubesheet.

Summary Table for STEP 11 – Design Condition					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)
1	26.08	-42,440	42,470	27,150	54,400
2	-764.8	19,210	19,980	27,150	54,400
3	-738.7	-23,230	23,970	27,150	54,400

Summary Table for STEP 11 – Operating Condition 1				
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$S_{PS,s}$ (psi)
1	0.05859	-42,480	42,480	54,400
2	-786.1	8,633	9,419	54,400
3	-764.8	-23,270	24,040	54,400
4	-21.24	-10,580	10,600	54,400

For Design Loading Case 1 $|\sigma_s| > 1.5S_s$. For Design Loading Cases 2 and 3 $|\sigma_s| \leq 1.5S_s$ and for Operation Condition 1, Loading Cases 1-4 $|\sigma_s| \leq S_{PS,s}$. The stress criterion for the shell is not satisfied. For Design Loading Case 1, since $|\sigma_s| \leq S_{PS,s}$, Option 3 in STEP 12 is permitted.

l) STEP 12 – The design shall be reconsidered by using one or a combination of the following options.

- Option 1 – Increase the tubesheet thickness and return to STEP 1.
- Option 2 – Increase the integral shell and/or channel thickness and return to STEP 1.
- Option 3 – Perform the elastic-plastic calculation procedures as defined in VIII-1, paragraph UHX-13.7.

Since the total axial stress in the shell $\sigma_{s,1}$ is between $1.5S_{s,1}$ and $S_{PS,s,1}$ for Design Condition Loading Case 1, the procedure of VIII-1, paragraph UHX-13.7 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

The results for the effect of plasticity for Design Condition Loading Case 1 are shown below.

Summary Results for STEP 12, Elastic-Plastic Iteration Results per VIII-1, paragraph UHX-13.7.3	
Design Condition Loading Case	1
S_s^*, psi	27,200
$fact_s$	0.7759
E_s^*, psi	19.78*E*6
k_s, *lb*	16,960
λ_s	0.6823*E*6
F	0.4701
ϕ	0.6412
Q_1	-0.02149
Q_{Z1}	2.865
Q_{Z2}	6.941
U	13.88
P_W, *psi*	232.5
P_{rim}, *psi*	183.3
P_e, *psi*	-399.4
Q_2, *lb*	-7,095
Q_3	0.09965
F_m	0.09832
$\lvert\sigma\rvert$, *psi*	25,750

The final calculated tubesheet bending stress of 25,750 psi (Loading Case 1) is less than the allowable tubesheet bending stress of 27,150 psi. As such, this geometry meets the requirement of VIII, paragraph UHX. The intermediate results for the elastic-plastic iteration are shown above.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

4.18.6 Example E4.18.6 – Fixed Tubesheet Exchanger, Configuration b, Tubesheet Integral with Shell, Extended as a Flange and Gasketed on the Channel Side

A fixed tubesheet heat exchanger is to be designed with the tubesheet construction in accordance with configuration b as shown in VIII-1, Figure UHX-13.1, Configuration b.

- For the Design Condition, the shell side design pressure is 0 to 335 psig at 675°F, and the tube side design pressure is 0 to 1040 psig at 650°F.
- There is one operating condition. For Operating Condition 1, the operating pressures and operating metal temperatures are assumed to be the same as the Design Condition. The shell mean metal temperature is 550°F, and the tube mean metal temperature is 490°F.
- The tube material is SA-214 (K01807). The tubes are 0.75 in. outside diameter and 0.083 in. thick. The unsupported tube span under consideration is between 2 tube supports, and the length of the unsupported tube span is 34 inches.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet diameter is 32.875 in. The tubesheet has 434 tube holes on a 0.9375 in. triangular pattern. There is no pass partition lane and the outermost tube radius from the tubesheet center is 10.406 in. The distance between the outer tubesheet faces is 144.375 in. The option for the effect of differential radial expansion is not required. There is a 0.125 in. corrosion allowance on both sides of the tubesheet. The tubes are expanded for a length of 4.374 in.
- The shell material is SA-516, Grade 70 (K02700). The shell outside diameter is 24 in. and the thickness is 0.5 in. There is a 0.125 in. corrosion allowance on the shell. There is also a shell band adjacent to each tubesheet. The shell bands are 1.25 in. thick and 9.75 in. long with a 0.125 in. corrosion allowance. The shell and shell band materials are the same. The shell contains an expansion joint that has an inside diameter of 29.46 in. and an axial rigidity of 14,759 lb/in. The efficiency of shell circumferential welded joint (Category B) is 0.85.
- The channel flange gasket outside diameter is 26.125 in., the inside diameter is 25.125 in., and the gasket factors are y = 26,000 psi and m = 6.5. The effective gasket width is per VIII-1 Appendix 2, Table 2-5.2, sketch (1a). There are (28) 1.375 in. diameter SA-193-B7 bolts on a 30.125 in. bolt circle.

<u>**Data Summary**</u>

The data summary consists of those variables from the nomenclature (see VIII-1, paragraph UHX-5.1) that are applicable to this configuration.

<u>**Design Conditions:**</u>

$$P_{sd,\max} = 335\ psig$$

$$P_{sd,\min} = 0\ psig$$

$$P_{td,\max} = 1040\ psig$$

$$P_{td,\min} = 0\ psig$$

$$T = 675^{o}F$$

$$T_a = 70^{o}F$$

$$T_{fe} = 675^{o}F$$

$T_s = 675^oF$

$T_t = 675^oF$

Operating Conditions:

$P_{so1,\max} = 335\ psig$

$P_{so1,\min} = 0\ psig$

$P_{to1,\max} = 1040\ psig$

$P_{to1,\min} = 0\ psig$

$T_1 = 675^oF$

$T_{s1} = 675^oF$

$T_{t1} = 675^oF$

$T_{s,m1} = 550^oF$

$T_{t,m1} = 490^oF$

Tubes:

$d_t = 0.75\ in.$

$E_t = 25.75E6\ psi$ from Table TM-1 of Section II, Part D at T_t & T_{t1}

$E_{tT} = 25.75E6\ psi$ from Table TM-1 of Section II, Part D at T

$k = 1$ for an unsupported tube span between two tube supports

$\ell = 34\ in.$

$\ell_t = 34\ in.$

$S_t = 10{,}700\ psi$ from Table 1A of Section II, Part D at T_t & T_{t1} (see explanation below)

$S_{tT} = 10{,}700\ psi$ from Table 1A of Section II, Part D at T (see explanation below)

$S_{y,t} = 18{,}950\ psi$ from Table Y-1 of Section II, Part D at T_t & T_{t1}

$t_t = 0.083\ in.$

$\alpha_{t,m1} = 7.28E\text{-}6\ in./in./^oF$ from Table TE-1 of Section II, Part D at $T_{t,m1}$

$\nu_t = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Since the tubes are SA-214 (welded), VIII-1, paragraph UHX-5.2 requires that the allowable stress for the welded tubes be divided by 0.85 if the equivalent seamless product is not available. In this case, SA-556, Grade A2 (K01807) could be used as the equivalent seamless product, but the alternative will be illustrated in this example. When the welded tube allowable stresses are divided by 0.85, the resulting allowable stresses are $S_t = 12{,}588\ psi$ and $S_{tT} = 12{,}588\ psi$.

Tubesheet:

Tube Pattern: Triangular

Assume an uncorroded tubesheet thickness of 4.75 inches.

$A = 32.875\ in.$

$A_L = 0\ in.^2$ for no pass lanes

$C = 30.125\ in.$

$c_t = 0.125\ in.$

$E = 25.575E6\ psi$ from Table TM-1 of Section II, Part D at T & T_1

$G_c = 25.625\ in.$ (G per VIII-1 Appendix 2)

$a_c = 12.81\ in.$

$h = 4.75\ in. - 0.125\ in. - 0.125\ in. = 4.500\ in.$ (assumed)

$h_g = 0\ in.$

$L_t = 144.125\ in.$

$L = 144.125\ in. - 2(4.50\ in.) = 135.125\ in.$

$\ell_{tx} = 4.374\ in.$

$N_t = 434$

$p = 0.9375\ in.$

$r_o = 10.406\ in.$

$S = 18{,}450\ psi$ from Table 1A of Section II, Part D at T & T_1

$S_a = 20{,}000\ psi$ from Table 1A of Section II, Part D at T_a

$S_{fe} = 18{,}450\ psi$ from Table 1A of Section II, Part D at T_{fe}

$S_y = 27{,}700\ psi$ from Table Y-1 of Section II, Part D at T & T_1

$S_{PS} = 55{,}400\ psi$ [$2S_y$ per UG-23(e)] at T & T_1

$W_{m1c} = 808{,}456\ lb$ (W_{m1} per VIII-1 Appendix 2)

$W_c = 808{,}478\ lb$ (W per VIII-1 Appendix 2)

$\rho = 0.9720$

Shell Band (Adjacent to Tubesheet):

$D_s = 23.25\ in.$

$a_s = 11.63\ in.$

$E_{s,1} = 25.75E6\ psi$ from Table TM-1 of Section II, Part D at T_s & T_{s1}

$E_{s,w,1} = 0.85$

$\ell_1 = 9.75\ in. + 0.125\ in. = 9.875\ in.$

$\ell'_1 = 9.75\ in. + 0.125\ in. = 9.875\ in.$

$S_{s,1} = 18{,}450\ psi$ from Table 1A of Section II, Part D at T_s & T_{s1}

$S_{y,s,1} = 27{,}700\ psi$ from Table Y-1 of Section II, Part D at T_s & T_{s1}

$S_{PS,s,1} = 55{,}400\ psi$ [$2S_{y,s,1}$ per UG-23(e)] at T_s & T_{s1}

$t_{s,1} = 1.125\ in.$

$\alpha_{s,m1,1} = 7.30E\text{-}6\ in./in./^oF$ from Table TE-1 of Section II, Part D at $T_{s,m1}$

$\nu_{s,1} = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Shell:

$D_J = 29.46\ in.$

$D_s = 23.25\ in.$

$E_s = 25.75E6\ psi$ from Table TM-1 of Section II, Part D at T_s & T_{s1}

$E_{s,w} = 0.85$

$K_J = 14{,}759\ lb/in.$

$S_s = 18{,}450\ psi$ from Table 1A of Section II, Part D at T_s & T_{s1}

$S_{y,s} = 27{,}700\ psi$ from Table Y-1 of Section II, Part D at T_s & T_{s1}

$S_{PS,s} = 55{,}400\ psi$ [$2S_{y,s}$ per UG-23(e)] at T_s & T_{s1}

$t_s = 0.375\ in.$

$\alpha_{s,m1} = 7.30E\text{-}6\ in./in./^oF$ from Table TE-1 of Section II, Part D at $T_{s,m1}$

$\nu_s = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Calculation Procedure

The tubesheet is extended as a flange. The calculation procedure for a flanged extension is given in VIII-1, paragraph UHX-9-5(a). The minimum required thickness of the flanged extension is the maximum of required thicknesses for the operating condition (at T_{fe}) and gasket seating condition (at T_a):

$h_r = \max(2.704, 2.597) = 2.704\ in.$

The calculation procedure for a tubesheets of a fixed tubesheet heat exchanger is given in VIII-1, paragraph UHX-13.5. The following results are for the design and operating loading cases required to be analyzed (see paragraph UHX-13.4). This example illustrates the calculation of both the elastic and elastic-plastic solutions for a shell that has a thickened shell band adjacent to the tubesheet.

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-1, paragraph UHX-11.5.1.

$D_o = 21.562\ in.$

$\mu = 0.2000$

$d^* = 0.6392\ in.$

$p^* = 0.9375\ in.$

$\mu^* = 0.3182$

$h'_g = 0\ in.$

Calculate a_o, ρ_s, ρ_c, x_s and x_t.

$a_o = 10.78\ in.$

$\rho_s = 1.078$

$\rho_c = 1.188$

$x_s = 0.4749$

$x_t = 0.6816$

b) STEP 2 – Calculate the shell axial stiffness K_s^*, tube axial stiffness K_t and stiffness factors $K_{s,t}$ and J.

K_s^* = 5.876E6 lb/in.

K_t = 33.14E3 $lb/in.$

$K_{s,t}$ = 0.4085

J = 2.505E-3

For configuration b, calculate shell coefficients β_s, k_s, λ_s and δ_s.

β_s = 0.3471 $in.^{-1}$

k_s = 2.331E6 lb

λ_s = 13.50E6 psi

δ_s = 3.965E-6 $in.^3/lb$

For configuration b, channel coefficients $\beta_c = 0$, $k_c = 0$, $\lambda_c = 0$ and $\delta_c = 0$.

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII, paragraph UHX-11.5.2 and calculate E^*.

$h/p = 4.800$

$E^*/E = 0.3051$

$\nu^* = 0.3423$

$E^* = 7.804E6\ psi$

Calculate X_a and parameters Z_a, Z_d, Z_v, Z_w and Z_m from VIII-1, Table UHX-13.1.

$X_a = 1.995$

$Z_a = 0.8092$

$Z_d = 0.1745$

$Z_v = 0.1605$

$Z_w = 0.1605$

$Z_m = 0.6679$

d) STEP 4 – Calculate the diameter ratio K and the coefficient F.

K = 1.525

F = 2.047

Calculate Φ, Q_1, Q_{z1}, Q_{z2} and U.

$\Phi = 2.747$

$Q_1 = -0.1280$

$Q_{z1} = 1.221$

$Q_{z2} = 0.5952$

$U = 1.190$

e) STEP 5 – Calculate γ, $\omega_s, \omega_s^*, \omega_c, \omega_c^*$ and γ_b. Use the loads listed in the table below to calculate the results for an elastic solution in the corroded condition.

$\omega_s = 8.865\ in.^2$ $\qquad \omega_s^* = -8.495\ in.^2$

$\omega_c = 0\ in^2$ $\qquad \omega_c^* = 8.659\ in^2$

$\gamma_b = -0.2087$

Summary Table for Step 5 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	0	1040	0	808,456
2	335	0	0	0
3	335	1040	0	808,456

Summary Table for Step 5 – Operating Condition 1				
Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	0	1040	-0.06032	808,478
2	335	0	-0.06032	808,478
3	335	1040	-0.06032	808,478
4	0	0	-0.06032	808,478

f) STEP 6 – For each loading case, calculate P_s', P_t', P_γ, P_ω, P_W, P_{rim} and effective pressure P_e.

Summary Table for STEP 6 – Design Condition							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	1.017*E*6	0	0	275.0	92.23	-1,039
2	-0.1674*E*6	0	0	0	0	29.14	-171.0
3	-0.1674*E*6	1.017*E*6	0	0	275.0	121.4	-1,210

Summary Table for STEP 6 – Operating Condition 1							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	1.017*E*6	-2,376	0	275.0	92.23	-1,042
2	-0.1674*E*6	0	-2,376	0	275.0	29.14	-173.2
3	-0.1674*E*6	1.017*E*6	-2,376	0	275.0	121.4	-1,213
4	0	0	-2,376	0	275.0	0	-2.148

g) STEP 7 – Elastic Iteration: Calculate Q_2, Q_3 and F_m, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (lbf)	Q_3	F_m	$h - h_g'$ (in)	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-12,650	0.08150	0.1986	4.500	22,330	27,675
2	-1,004	-0.02696	0.1574	4.500	2,913	27,675
3	-13,650	0.06617	0.1927	4.500	25,240	27,675

Summary Table for STEP 7 – Operating Condition 1						
Loading Case	Q_2 (lbf)	Q_3	F_m	h (in)	$\lvert\sigma\rvert$ (psi)	S_{PS} (psi)
1	-12,650	0.08101	0.1984	4.500	22,360	55,400
2	-10,480	0.9131	0.5333	4.500	9,995	55,400
3	-13,650	0.06578	0.1926	4.500	25,280	55,400
4	-9,473	75.77	37.94	4.500	8,817	55,400

For Design Loading Cases 1-3 $|\sigma| \leq 1.5S$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma| \leq S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Check the criterion below for the largest value of P_e and calculate the shear stress, if required.

$$\{|P_e| = 1213\ psi\} \leq \left\{\frac{2\mu h}{a_o} \cdot \min\left[0.8S, 0.533S_y\right] = 2464\ psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

i) STEP 9 – For each loading case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$r_t = 0.2376\ in$

$F_t = 143.1$

$C_t = 163.8$

Summary Table for STEP 9 – Design Condition				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	0.4598	-1,118	1.487	4,047
2	0.5904	1,258	1.349	1,886
3	0.4782	140.2	1.468	5,933

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	4,047	12,588	-1,118	2.000	5,336
2	1,886	12,588	---	---	---
3	5,933	12,588	---	---	---

Summary Table for STEP 9 – Operating Condition 1				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	0.4604	-1,110	1.487	4,061
2	-0.5417	315.9	2.545	2,902
3	0.4787	148.5	1.467	5,947
4	-90.69	-942.2	97.82	1,016

Summary Table for STEP 9 – Operating Condition 1 (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	$2S_t$ (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	4,061	25,176	-1,110	2.000	5,336
2	2,902	25,176	---	---	---
3	5,947	25,176	---	---	---
4	1,016	25,176	-942.2	1.250	8,538

For Design Loading Cases 1-3 $\sigma_{t,\max} \le S_t$, and for Operation Condition 1, Loading Cases 1-4 $\sigma_{t,\max} \le 2S_t$. The axial tension stress criterion for the tube is satisfied.

For all Loading Cases $|\sigma_{t,\min}| \le S_{tb}$. The buckling criterion for the tube is satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Shell Band – Design Condition			
Loading Case	$\sigma_{s,m,1}$ (psi)	$S_{s,1}E_{s,w}$ (psi)	$S_{s,b,1}$ (psi)
1	3.369	15,683	---
2	-493.9	15,683	12,580
3	-490.5	15,683	12,580

Summary Table for STEP 10 – Shell Band – Operating Condition 1			
Loading Case	$\sigma_{s,m,1}$ (psi)	$S_{PS,s,1}$ (psi)	$S_{s,b,1}$ (psi)
1	-6.926	55,400	12,580
2	-503.0	55,400	12,580
3	-500.8	55,400	12,580
4	-9.103	55,400	12,580

For Design Loading Cases 1-3 $|\sigma_{s,m,1}| \leq S_{s,1}E_{s,w}$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma_{s,m,1}| \leq S_{PS,s,1}$. The axial membrane stress criterion for the shell band is satisfied.

For all Loading Cases where the value of $\sigma_{s,m,1}$ is negative, $|\sigma_{s,m,1}| \leq S_{s,b,1}$. The longitudinal compressive stress criterion for the shell band is satisfied.

Summary Table for STEP 10 – Main Shell – Design Condition			
Loading Case	$\sigma_{s,m}$ (psi)	$S_sE_{s,w}$ (psi)	$S_{s,b}$ (psi)
1	10.43	15,683	---
2	-1,529	15,683	10,800
3	-1,518	15,683	10,800

Summary Table for STEP 10 – Main Shell – Operating Condition 1			
Loading Case	$\sigma_{s,m}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	-21.44	55,400	10,800
2	-1,557	55,400	10,800
3	-1,550	55,400	10,800
4	-28.18	55,400	10,800

For Design Loading Cases 1-3 $|\sigma_{s,m}| \leq S_sE_{s,w}$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma_{s,m}| \leq S_{PS,s}$. The axial membrane stress criterion for the main shell is satisfied.

For all Loading Cases where the value of $\sigma_{s,m}$ is negative, $|\sigma_{s,m}| \leq S_{s,b}$. The longitudinal compressive stress criterion for the main shell is satisfied.

k) STEP 11 – For each loading case, calculate the stresses in the shell band for configuration b, and check the acceptance criterion. The shell band thickness shall be 1.125 in. for a minimum length of 9.206 in adjacent to the tubesheet.

Summary Table for STEP 11, Shell Band Results – Design Condition					
Loading Case	$\sigma_{s,m,1}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_{s,1}$ (psi)	$S_{PS,s,1}$ (psi)
1	3.369	-41,040	41,040	27,675	55,400
2	-493.9	617.7	1,112	27,675	55,400
3	-490.5	-40,420	40,910	27,675	55,400

Summary Table for STEP 11, Shell Band Results – Operating Condition 1				
Loading Case	$\sigma_{s,m,1}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$S_{PS,s,1}$ (psi)
1	-6.926	-41,070	41,080	55,400
2	-503.0	-19,410	19,920	55,400
3	-500.8	-40,460	40,960	55,400
4	-9.103	-20,030	20,040	55,400

For Design Loading Cases 1 and 3 $|\sigma_s| > 1.5S_{s,1}$. For Design Loading Case 2 $|\sigma_s| \le 1.5S_{s,1}$ and for Operation Condition 1, Loading Cases 1-4 $|\sigma_s| \le S_{PS,s,1}$. The stress criterion for the shell band is not satisfied. For Design Loading Cases 1 and 3, since $|\sigma_s| \le S_{PS,s,1}$, Option 3 in STEP 12 is permitted.

l) STEP 12 – The design shall be reconsidered by using one or a combination of the following options.

- Option 1 – Increase the tubesheet thickness and return to STEP 1.
- Option 2 – Increase the integral shell and/or channel thickness and return to STEP 1.
- Option 3 – Perform the elastic-plastic calculation procedures as defined in VIII-1, paragraph UHX-13.7.

Since the total axial stress in the shell band $\sigma_{s,1}$ is between $1.5S_{s,1}$ and $S_{PS,s,1}$ for Design Condition Loading Cases 1 and 3, the procedure of VIII-1, paragraph UHX-13.7 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

The results for the effect of plasticity for Design Condition Loading Cases 1 and 3 are shown below.

Summary Results for STEP 12, Elastic-Plastic Iteration Results per VIII-1, paragraph UHX-13.7.3		
Design Condition Loading Case	1	3
S_S^*, psi	27,700	27,700
$fact_s$	0.8074	0.8163
E_S^*, psi	20.79E6	21.02E6
k_s, lb	1.882$E6$	1.903E6
λ_s	10.90E6	11.02E6
F	1.828	1.838
ϕ	2.453	2.467
Q_1	-0.1196	-0.1200
Q_{Z1}	1.231	1.231
Q_{Z2}	0.6395	0.6373
U	1.279	1.275
P_W, psi	295.5	294.5
$P_{\rm rim}$, psi	99.10	130.0
P_e, psi	-1,039	-1,210
Q_2, lb	-13,590	-14,620
Q_3	0.1055	0.08787
F_m	0.2077	0.2010
$\|\sigma\|$, psi	23,350	26,320

The final calculated tubesheet bending stresses of 23,350 psi (Loading Case 1) and 26,320 psi (Loading Case 3) are less than the allowable tubesheet bending stress of 27,675 psi. As such, this geometry meets the requirement of VIII, paragraph UHX. The intermediate results for the elastic-plastic iteration are shown above.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

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4.18.7 Example E4.18.7 – Fixed Tubesheet Exchanger, Configuration a

A fixed tubesheet heat exchanger with the tubesheet construction in accordance with configuration a as shown in VIII-1, Figure UHX-13.1, Configuration a.

- For the Design Condition, the shell side design pressure is 0 to 325 psig at 400°F, and the tube side design pressure is 0 to 200 psig at 300°F. The tube design temperature is 300°F.
- There is one operating condition. For Operating Condition 1, the operating pressures and operating metal temperatures are assumed to be the same as those for the Design Condition. The shell mean metal temperature is 151°F, and the tube mean metal temperature is 113°F.
- The tube material is SA-249, Type 304L (S30403). The tubes are 1 in. outside diameter and 0.049 in. thick. The unsupported tube span under consideration is between 2 tube supports, and the length of the unsupported tube span is 48 inches.
- The tubesheet material is SA-240, Type 304L (S30403). The tubesheet diameter is 43.125 in. The tubesheet has 955 tube holes on a 1.25 in. equilateral triangular pattern. There is no pass partition lane and the outermost tube radius from the tubesheet center is 20.125 in. The distance between the outer tubesheet faces is 240 in. The option for the effect of differential radial expansion is not required. There is no corrosion allowance on the tubesheet. The tubes are expanded from the tube side face of the tubesheet to 0.125 in. from the shell side face of the tubesheet.
- The shell material is SA-240, Type 304L (S30403). The shell inside diameter is 42 in. and the shell thickness is 0.5625 in. There is no corrosion allowance on the shell and no expansion joint in the shell. The efficiency of shell circumferential welded joint (Category B) is 0.85.
- The channel material is SA-516, Grade 70 (K02700). The inside diameter of the channel is 42.125 in. and the channel thickness is 0.375 in. There is no corrosion allowance on the channel.

For this example, first assume a value of 1.375 in. for the tubesheet thickness and perform the calculation procedure described below starting at STEP 1. The data shown below will be the same except as follows:

$$h = 1.375\ in.$$

$$\ell_{tx} = 1.25\ in.$$

$$\rho = 0.9091$$

$$L = 237.25\ in.$$

In STEP 7, the calculated bending stresses for the tubesheet are less than the allowable stresses for all the Design Loading Cases and for all the Operating Condition 1 Loading Cases. The maximum Design Loading Case bending stress is 23,480 psi, which is less than the allowable of 23,700 psi and the maximum Operating Case bending stress 40,360 psi, which is less than the allowable of 47,400 psi. The bending stress criterion for the tubesheet is satisfied.

In STEP 8, check the criterion below for the largest value of P_e and calculate the shear stress, if required.

$$\{|P_e| = 262\ psi\} \le \left\{\frac{2\mu h}{a_o} \cdot \min\left[0.8S, 0.533S_y\right] = 249\ psi\right\} \qquad False$$

The above criterion is not satisfied.

Calculate the shear stress τ for Operating Condition 1, Loading Case 1 in accordance with UHX-13.5.8(b).

$A_p = 1406.25\ in^2$

$C_p = 135\ in$

$\tau = -9906\ psi$

$$\{|\tau| = 9906\ psi\} \leq \{\min[0.8S, 0.533S_y] = 9330\ psi\} \qquad False$$

The tubesheet is overstressed for Operating Condition 1, Loading Case 1. Increase the tubesheet thickness to 1.500 in and return to STEP 1 of the calculation procedure in UHX-13.5.

Data Summary

The data summary consists of those variables from the nomenclature (see VIII-1, paragraph UHX-5.1) that are applicable to this configuration.

Design Conditions:

$P_{sd,max} = 325\ psig$

$P_{sd,min} = 0$ psig

$P_{td,max} = 200\ psig$

$P_{td,min} = 0$ psig

$T = 400^\circ F$

$T_a = 70^\circ F$

$T_c = 300^\circ F$

$T_s = 400^\circ F$

$T_t = 300^\circ F$

Operating Conditions:

$P_{so1,max} = 325\ psig$

$P_{so1,min} = 0$ psig

$P_{to1,max} = 200\ psig$

$P_{to1,min} = 0$ psig

$T_1 = 400^\circ F$

$T_{c1} = 300^\circ F$

$T_{s1} = 400^\circ F$

$T_{t1} = 300^\circ F$

$T_{s,m1} = 151^\circ F$

$T_{t,m1} = 113^\circ F$

Tubes:

d_t = 1.0 *in.*

E_t = 27.0E6 *psi* from Table TM-1 of Section II, Part D at T_t & T_{t1}

E_{tT} = 26.4E6 *psi* from Table TM-1 of Section II, Part D at T

k = 1 for an unsupported tube span between two tube supports

ℓ = 48 *in.*

ℓ_t = 48 in.

S_t = 14,200 *psi* from Table 1A of Section II, Part D at T_t & T_{t1} (see explanation below)

S_{tT} = 13,400 *psi* from Table 1A of Section II, Part D at T (see explanation below)

$S_{y,t}$ = 19,200 *psi* from Table Y-1 of Section II, Part D at T_t & T_{t1}

t_t = 0.049 *in.*

$\alpha_{t,m1}$ = 8.652E-6 *in./in./°F* from Table TE-1 of Section II, Part D at $T_{t,m1}$

ν_t = 0.31 from Table PRD of Section II, Part D for High Alloy Steels (300 Series)

Since the tubes are SA-249, Type 304L (welded), VIII-1, paragraph UHX-5.2 requires that the allowable stress for the welded tubes be divided by 0.85 if the equivalent seamless product is not available. In this case, SA-213, Type 304L could be used as the equivalent seamless product, but the alternative will be illustrated in this example. When the welded tube allowable stresses are divided by 0.85, the resulting allowable stresses are $S_t = 16{,}706\ psi$ and $S_{tT} = 15{,}765\ psi$.

Tubesheet:

Tube Pattern: Triangular

A = 43.125 *in.*

A_L = 0 *in.*^{2} for no pass lanes

c_t = 0 in.

E = 26.4E6 *psi* from Table TM-1 of Section II, Part D at T & T_1

h = 1.500 *in.* (assumed)

h_g = 0 in.

L_t = 240 in.

L = 237 *in.*

ℓ_{tx} = 1.375 *in.*

N_t = 955

p = 1.25 *in.*

r_o = 20.125 in.

S = 15,800 *psi* from Table 1A of Section II, Part D at T & T_1

S_y = 17,500 *psi* from Table Y-1 of Section II, Part D at T & T_1

S_{PS} = 47,400 [3S per UG-23(e)] *psi* at T & T_1

ρ = 0.9167

Shell:

D_s = 42 in.

a_s = 21.00 *in.*

E_s = 26.4E6 *psi* from Table TM-1 of Section II, Part D at T_s & T_{s1}

$E_{s,w}$ = 0.85

S_s = 15,800 *psi* from Table 1A of Section II, Part D at T_s & T_{s1}

$S_{y,s}$ = 17,500 *psi* from Table Y-1 of Section II, Part D at T_s & T_{s1}

$S_{PS,s}$ = 47,400 *psi* [3S_s per UG-23(e)] at T_s & T_{s1}

t_s = 0.5625 *in.*

$\alpha_{s,m1}$ = 8.802E-6 *in./in./°F* from Table TE-1 of Section II, Part D at $T_{s,m1}$

ν_s = 0.31 from Table PRD of Section II, Part D for High Alloy Steels (300 Series)

Channel:

D_c = 42.125 *in.*

a_c = 21.06 *in.*

E_c = 28.3E6 *psi* from Table TM-1 of Section II, Part D at T_c & T_{c1}

S_c = 20,000 *psi* from Table 1A of Section II, Part D at T_c & T_{c1}

$S_{y,c}$ = 33,600 *psi* from Table Y-1 of Section II, Part D at T_c & T_{c1}

$S_{PS,c}$ = 67,200 *psi* [2$S_{y,c}$ per UG-23(e)]at T_c & T_{c1}

t_c = 0.375 *in.*

ν_c = 0.30 from Table PRD of Section II, Part D for Carbon Steels

Calculation Procedure

The calculation procedure for the tubesheets of a fixed tubesheet heat exchanger is given in VIII-1, paragraph UHX-13.5. The following results are for the design and operating loading cases required to be analyzed (see VIII-1, paragraph UHX-13.4). This example illustrates the calculation of both the elastic and elastic-plastic solutions.

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-1, paragraph UHX-11.5.1.

D_o = 41.25 *in.*

μ = 0.2000

d^* = 0.9104 *in.*

p^* = 1.250 *in.*

μ^* = 0.2717

h'_g = 0 in.

Calculate a_o, ρ_s, ρ_c, x_s and x_t.

$a_o = 20.63\ in.$

$\rho_s = 1.018$

$\rho_c = 1.021$

$x_s = 0.4388$

$x_t = 0.5434$

b) STEP 2 – Calculate the shell axial stiffness K_s, tube axial stiffness K_t and stiffness factors $K_{s,t}$ and J.

$K_s = 8.378E6\ lb/in.$

$K_t = 16.68E3\ lb/in.$

$K_{s,t} = 0.5260$

$J = 1.0$

For configuration a, calculate shell coefficients β_s, k_s, λ_s and δ_s.

$\beta_s = 0.3709\ in.^{-1}$

$k_s = 0.3213E6\ lb$

$\lambda_s = 41.05E6\ psi$

$\delta_s = 25.09E\text{-}6\ in.^3/lb$

For configuration a, calculate channel coefficients β_c, k_c, λ_c and δ_c.

$\beta_c = 0.4554\ in.^{-1}$

$k_c = 0.1245E6\ lb$

$\lambda_c = 17.86E6\ psi$

$\delta_c = 35.53E\text{-}6\ in.^3/lb$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII, paragraph UHX-11.5.2 and calculate E^*.

$h/p = 1.200$

$E^*/E = 0.2723$

$\nu^* = 0.3439$

$E^* = 7.188E6\ psi$

Calculate X_a and parameters Z_a, Z_d, Z_v, Z_w and Z_m from VIII-1, Table UHX-13.1.

$X_a = 6.586$

$Z_a = 170.6$

$Z_d = 5.246E\text{-}3$

$Z_v = 0.02339$

$Z_w = 0.02339$

$Z_m = 0.2203$

d) STEP 4 – Calculate the diameter ratio K and the coefficient F.

$K = 1.045$

$F = 5.484$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} and U.

$\Phi = 7.371$

$Q_1 = -0.05879$

$Q_{z1} = 3.641$

$Q_{z2} = 9.822$

$U = 19.64$

e) STEP 5 – Calculate γ, ω_s, ω_s^*, ω_c, ω_c^* and γ_b. Use the loads listed in the table below to calculate the results for an elastic solution in the corroded condition.

$\omega_s = 4.739\ in.^2$ $\qquad \omega_s^* = -4.668\ in.^2$

$\omega_c = 3.461\ in^2$ $\qquad \omega_c^* = -2.720\ in^2$

$\gamma_b = 0.0$

Summary Table for Step 5 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	0	200	0	0
2	325	0	0	0
3	325	200	0	0

Summary Table for Step 5 – Operating Condition 1				
Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	0	200	-0.08080	0
2	325	0	-0.08080	0
3	325	200	-0.08080	0
4	0	0	-0.08080	0

f) STEP 6 – For each loading case, calculate P_s', P_t', P_γ, P_ω, P_W, P_{rim} and effective pressure P_e.

Summary Table for STEP 6 – Design Condition							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	545.5	0	0	0	-25.12	-99.74
2	630.1	0	0	0	0	70.06	122.4
3	630.1	545.5	0	0	0	44.94	22.65

Summary Table for STEP 6 – Operating Condition 1							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	545.5	-963.0	0	0	-25.12	-268.1
2	630.1	0	-963.0	0	0	70.06	-45.93
3	630.1	545.5	-963.0	0	0	44.94	-145.7
4	0	0	-963.0	0	0	0	-168.3

g) STEP 7 – Elastic Iteration: Calculate Q_2, Q_3 and F_m, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (lbf)	Q_3	F_m	$h - h_g'$ (in)	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	207.3	-0.06856	0.03428	1.500	14,280	23,700
2	-578.3	-0.08100	0.04050	1.500	20,700	23,700
3	-371.0	-0.1358	0.06790	1.500	6,420	23,700

Summary Table for STEP 7 – Operating Condition 1						
Loading Case	Q_2 (lbf)	Q_3	F_m	h (in)	$\lvert\sigma\rvert$ (psi)	S_{PS} (psi)
1	207.3	-0.06242	0.03121	1.500	34,930	47,400
2	-578.3	0.4075E-3	0.03703	1.500	7,101	47,400
3	-371.0	-0.04681	0.02341	1.500	14,240	47,400
4	0	-0.05879	0.02939	1.500	20,660	47,400

For Design Loading Cases 1-3 $|\sigma| \le 1.5S$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma| \le S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Check the criterion below for the largest value of P_e and calculate the shear stress, if required.

$$\{|P_e| = 268.1\ psi\} \le \left\{\frac{2\mu h}{a_o} \cdot \min\left[0.8S, 0.533S_y\right] = 271.3\ psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

i) STEP 9 – For each loading case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$r_t = 0.3367\ in$

$F_t = 142.6$

$C_t = 166.6$

Summary Table for STEP 9 – Design Condition				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	-0.2819	-1,308	3.426	2,228
2	-0.2569	1,664	3.152	-2,325
3	-0.2187	371.6	2.117	-134.1

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	2,228	16,706	-1,308	1.537	7,148
2	2,325	16,706	-2,325	1.674	6,563
3	371.6	16,706	-134.1	2.000	5,493

Summary Table for STEP 9 – Operating Condition 1				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	-0.2967	-1,799	3.561	8,087
2	-0.5149	1,137	4.944	3,534
3	-0.3401	-149.3	3.905	5,762
4	-0.3059	-492.3	3.641	5,859

Summary Table for STEP 9 – Operating Condition 1 (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	$2S_t$ (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	8,087	33,412	-1,799	1.469	7,476
2	3,534	33,412	---	---	---
3	5,762	33,412	-149.3	1.298	8,465
4	5,859	33,412	-492.3	1.429	7,685

For Design Loading Cases 1-3 $\sigma_{t,\max} \le S_t$, and for Operation Condition 1, Loading Cases 1-4 $\sigma_{t,\max} \le 2S_t$. The axial tension stress criterion for the tube satisfied.

For all Loading Cases, $|\sigma_{t,\min}| \le S_{tb}$. The buckling criterion for the tube satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Main Shell – Design Condition			
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{s,b}$ (psi)
1	1,781	13,430	---
2	2,387	13,430	---
3	4,168	13,430	---

Summary Table for STEP 10 – Main Shell – Operating Condition 1			
Loading Case	$\sigma_{s,m}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	-1,210	47,400	6,730
2	-604.3	47,400	6,730
3	1,177	47,400	---
4	-2,991	47,400	6,730

For Design Loading Cases 1-3 $|\sigma_{s,m}| \leq S_s E_{s,w}$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma_{s,m}| \leq S_{PS,s}$. The axial membrane stress criterion for the shell satisfied.

For all Loading Cases where the value of $\sigma_{s,m}$ is negative, $|\sigma_{s,m}| \leq S_{s,b}$. The longitudinal compressive stress criterion for the shell satisfied.

k) STEP 11 – For each loading case, calculate the stresses in the shell and channel for configuration a, and check the acceptance criterion. The shell thickness shall be 0.5625 in. for a minimum length of 8.749 in. adjacent to the tubesheet, and the channel thickness shall be 0.375 in. for a minimum length of 7.154 in. adjacent to the tubesheet.

Summary Table for STEP 11, Shell Results – Design Condition					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)
1	1,781	-12,320	14,100	23,700	47,400
2	2,387	28,550	30,940	23,700	47,400
3	4,168	16,230	20,400	23,700	47,400

Summary Table for STEP 11, Shell Results – Operating Condition 1				
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$S_{PS,s}$ (psi)
1	-1,210	-38,510	39,720	47,400
2	-604.3	2,360	2,964	47,400
3	1,177	-9,961	11,140	47,400
4	-2,991	-26,190	29,180	47,400

For Design Loading Case 2 $|\sigma_s| > 1.5S_s$. For Design Loading Cases 1 and 3 $|\sigma_s| \leq 1.5S_s$ and for Operation Condition 1, Loading Cases 1-4 $|\sigma_s| \leq S_{PS,s}$. The stress criterion for the shell is not satisfied. For Design Loading Case 2, since $|\sigma_s| \leq S_{PS,s}$, Option 3 in Step 12 is permitted.

Summary Table for STEP 11, Channel Results – Design Condition					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	$S_{PS,c}$ (psi)
1	5,567	28,450	34,020	30,000	67,200
2	0	-9,257	9,257	30,000	67,200
3	5,567	19,200	24,760	30,000	67,200

Summary Table for STEP 11, Channel Results – Operating Condition 1				
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$S_{PS,c}$ (psi)
1	5,567	52,410	57,980	67,200
2	0	14,700	14,700	67,200
3	5,567	43,150	48,720	67,200
4	0	23,960	23,960	67,200

For Design Loading Case 1 $|\sigma_c| > 1.5S_c$. For Design Loading Cases 2 and 3 $|\sigma_c| \leq 1.5S_c$ and for Operation Condition 1, Loading Cases 1-4 $|\sigma_c| \leq S_{PS,c}$. The stress criterion for the channel is not satisfied. For Design Loading Case 1, since $|\sigma_c| \leq S_{PS,c}$, Option 3 in Step 12 is permitted.

l) STEP 12 – The design shall be reconsidered by using one or a combination of the following options.

- Option 1 – Increase the tubesheet thickness and return to STEP 1.
- Option 2 – Increase the integral shell and/or channel thickness and return to STEP 1.
- Option 3 – Perform the elastic-plastic calculation procedures as defined in VIII-1, paragraph UHX-13.7.

Since the total axial stress in the shell σ_s is between $1.5S_s$ and $S_{PS,s}$ for Design Condition Loading Case 2, the procedure of VIII-1, paragraph UHX-13.7 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

Since the total axial stress in the channel σ_c is between $1.5S_c$ and $S_{PS,c}$ for Design Condition Loading Case 1, the procedure of UHX-13.7 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the channel occurs. The results are not presented for Design Condition Loading Case 1, because the calculated values of $fact_s$ and $fact_c$ equal 1.0 for this case and further plasticity calculations are not required.

The results for the effect of plasticity for Design Condition Loading Case 2 are shown below.

Summary Results for STEP 12, Elastic-Plastic Iteration Results per VIII-1, paragraph UHX-13.7.3	
Design Condition Loading Case	2
S_s^*, psi	17,500
S_c^*, psi	33,600
$fact_s$	0.7474
$fact_c$	1.000
E_s^*, psi	19.73E6
E_c^*, psi	28.30E6
k_s, lb	0.2402$E6$
λ_s	30.68E6
k_c, lb	0.1245$E6$
λ_c	17.86E6
F	4.538
ϕ	6.098
Q_1	-0.05312
Q_{Z1}	3.766
Q_{Z2}	11.00
U	21.99
P_W, psi	0
P_{rim}, psi	78.44
P_e, psi	120.8
Q_2, lb	-647.5
Q_3	-0.07832
F_m	0.03916
$\lvert\sigma\rvert, psi$	19,750

The final calculated tubesheet bending stress is 19,750 psi (Design Loading Case 1) is less than the allowable tubesheet bending stress of 23,700 psi. As such, this geometry meets the requirement of VIII, paragraph UHX. The intermediate results for the elastic-plastic iteration are shown above.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

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4.18.8 Example E4.18.8 – Floating Tubesheet Heat Exchanger with an Immersed Floating Head

A floating tubesheet exchanger with an immersed floating head is to be designed as shown in VIII-1, Figure UHX-14.1, sketch (a). The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in VIII-1, Figure UHX-14.2, sketch (d) and not extended as a flange. The floating tubesheet is not extended as a flange in accordance with configuration C as shown in VIII-1, Figure UHX-14.3, sketch (c).

- For the Design Condition, the shell side design pressure is 0 to 250 psig at 550°F, and the tube side design pressure is 0 to 150 psig at 550°F.
- For these configurations, the operating conditions are not required to be considered.
- The tube material is SA-179 (K01200). The tubes are 0.75 in. outside diameter and 0.083 in. thick. The largest equivalent unsupported buckling length of the tube is 15.375 in.
- The tubesheet material is SA-105 (K03504). The stationary tubesheet diameter is 29.875 in. and the floating tubesheet diameter is 26.875 in. The tubesheet has 466 tube holes on a 1.0 in. triangular pattern with one centerline pass lane. There is a 0.197 in. deep pass partition groove in the stationary tubesheet only. The largest center-to-center distance between adjacent tube rows is 2.5 in., the length of the untubed lane is 25.75 in., and the radius to the outermost tube hole center is 12.5 in. The distance between the outer tubesheet faces is 256 in. There is no corrosion allowance on the tubesheet. The tubes are expanded to 80% of the tubesheet.
- The channel flange gasket consists of a ring gasket with a centerline rib. The ring gasket outside diameter is 29.875 in., the inside diameter is 28.875 in., and the gasket factors are y = 4,000 psi and m = 3.0. The rib gasket width is 0.50 in., the length is 28.875 in., and the rib gasket factors are y = 4,000 psi and m = 3.0. The shell flange gasket outside diameter is 29.875 in., the inside diameter is 28.875 in., and the gasket factors are y = 4,000 psi and m = 3.0. The effective gasket width for both gaskets is per VIII-1 Appendix 2, Table 2-5.2, sketch (1a). There are (32) 0.75 in. diameter SA-193-B7 bolts on a 31.417 in. bolt circle.
- The floating head flange gasket outside diameter is 26.875 in., the inside diameter is 26.125 in., and the gasket factors are y = 4,000 psi and m = 3.0. The effective gasket width is per VIII-1 Appendix 2, Table 2-5.2, sketch (1a). There are (28) 0.625 in. diameter SA-193-B7 bolts on a 27.625 in. bolt circle.

For this example, first assume a value of 1.75 in. for the tubesheet thickness and perform the calculation procedure described below starting at STEP 1. The data shown below will be the same except as follows:

$$h = 1.75\ in.$$

$$L = 252.5\ in.$$

In STEP 7, the calculated bending stress of 29,830 psi for the stationary tubesheet exceeds the allowable stress of 28,500 psi for Design Loading Case 2.

The tubesheet is overstressed for Design Loading Case 2. Increase the tubesheet thickness to 1.8125 in and return to Step 1 of the calculation procedure in UHX-14.5.

Data Summary

The data summary consists of those variables from the nomenclature (see VIII-1, paragraph UHX-5.1) that are applicable to these configurations.

Design Conditions:

$P_{sd,\max} = 250\ psig$

$P_{sd,\min} = 0\ psig$

$P_{td,\max} = 150\ psig$

$P_{td,\min} = 0\ psig$

$T = 550^oF$

$T_{fe} = 550^oF$

$T_t = 550^oF$

Tubes:

$d_t = 0.75\ in.$

$E_t = 26.9E6\ psi$ from Table TM-1 of Section II, Part D at T_t

$E_{tT} = 26.9E6\ psi$ from Table TM-1 of Section II, Part D at T

$\ell_t = 15.375\ in.$

$S_t = 13{,}350\ psi$ from Table 1A of Section II, Part D at T_t

$S_{tT} = 13{,}350\ psi$ from Table 1A of Section II, Part D at T

$S_{y,t} = 20{,}550\ psi$ from Table Y-1 of Section II, Part D at T_t

$t_t = 0.083\ in.$

Stationary and Floating Tubesheets (Common Data):

Tube Pattern: Triangular

$A_L = 26.38\ in.^2$

$c_t = 0\ in.$

$E = 26.75E6\ psi$ from Table TM-1 of Section II, Part D at T

$h = 1.8125\ in.$ (assumed)

$L_{L1} = 25.75\ in.$

$L_t = 256\ in.$

$L = 252.375\ in.$

$N_t = 466$

$p = 1.0\ in.$

$r_o = 12.5\ in.$

$S = 19{,}000\ psi$ from Table 1A of Section II, Part D at T

$S_{fe} = 19{,}000\ psi$ from Table 1A of Section II, Part D at T_{fe}

$S_y = 28{,}450\ psi$ from Table Y-1 of Section II, Part D at T

$U_{L1} = 2.5\ in.$

$\rho = 0.80$

Stationary Tubesheet:

$A = 29.875\ in.$

$C = 31.417\ in.$

$D_E = 29.375\ in.$

$G_c = 29.375\ in.$ (G per VIII-1 Appendix 2)

$a_c = 14.69\ in.$

$G_s = 29.375\ in.$ (G per VIII-1 Appendix 2)

$a_s = 14.69\ in.$

$h_g = 0.197\ in.$

$W_{m1c} = 128{,}856\ lb$ (W_{m1} per VIII-1 Appendix 2)

$W_{m1s} = 203{,}931\ lb$ (W_{m1} per VIII-1 Appendix 2)

Floating Tubesheet:

$A = 26.875\ in.$

$C = 27.625\ in.$

$D_E = 26.125\ in.$

$G_1 = 26.5\ in.$

$G_c = 26.5\ in.$ (G per VIII-1 Appendix 2)

$a_c = 13.25\ in.$

$a_s = 13.25\ in.$

$h_g = 0\ in.$

$W_{m1c} = 96{,}732\ lb$ (W_{m1} per VIII-1 Appendix 2)

Calculation Procedure – Stationary Tubesheet

The stationary tubesheet is not extended as a flange but has an unflanged extension. The calculation procedure for this extension is given in VIII-1, paragraph UHX-9-5(c). The minimum required thickness of this extension calculated at T_{fe} is:

$h_r = 0.1208\ in.$

The calculation procedure for the stationary tubesheet of a floating tubesheet heat exchanger is given in VIII-1, paragraph UHX-14.5. The following results are for the design loading cases required to be analyzed (see VIII-1, paragraph UHX-14.4).

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-1, paragraph UHX-11.5.1.

$D_o = 25.75\ in.$

$\mu = 0.2500$