ASME PTB-3-2022

Section VIII – Division 2

Example Problem Manual

ASME
SETTING THE STANDARD

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ASME Section VIII – Division 2 Example Problem Manual

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FOREWORD TO THE THIRD EDITION

This document is the third edition of the ASME Section VIII – Division 2 Example Problem Manual. The purpose of this third edition is to update the example problems to keep current with the changes incorporated into the 2021 edition of the ASME B&PV Code, Section VIII, Division 2. The example problems included in the second edition of the manual were based on the contents of the 2013 edition of the B&PV Code.

Known corrections to paragraph changes and references, design equations, and calculation results have been made in this third edition. Additionally, some formatting modifications were made to facilitate better use of the example manual, as applicable.

FOREWORD TO THE SECOND EDITION

This document is the second edition of the ASME Section VIII – Division 2 example problem manual. The purpose of this second edition is to update the example problems to keep current with the changes incorporated into the 2013 edition of the ASME B&PV Code, Section VIII, Division 2. The example problems included in the first edition of the manual were based on the contents of the 2010 edition of the B&PV Code. In 2011, ASME transitioned to a two year publishing cycle for the B&PV Code without the release of addenda. The release of the 2011 addenda to the 2010 edition was the last addenda published by ASME and numerous changes to the Code were since adopted.

Known corrections to design equations and results have also been made in this second edition. Additionally, some formatting modifications were made to facilitate better use of the example manual, as applicable.

FOREWORD

In 1998 the ASME Boiler and Pressure Vessel Standards Committee authorized a project to rewrite the ASME B&PV Code, Section VIII, Division 2. This decision was made shortly after the design margin on specified minimum tensile strength was lowered from 4.0 to 3.5 in Section I and Section VIII, Division 1. ASME saw the need to update Section VIII, Division 2 to incorporate the latest technologies and to be more competitive. In lieu of revising the existing standard, the decision was made to perform a clean sheet rewrite. By doing so it was felt that, not only could the standard be modernized with regard to the latest technical advances in pressure vessel construction, but it could be structured in a way to make it more user-friendly for both users and the committees that maintain it.

Much new ground was broken in the development of the new Section VIII, Division 2, including the process taken to write the new standard. Traditionally, development of new standards by ASME is carried out by volunteers who serve on the different committees responsible for any given standard. Depending upon the complexity of the standard, the development of the first drafts may take up to 15 years to complete based on past history. The prospect of taking 15 or more years to develop VIII-2 was unacceptable to ASME and the volunteer leadership. The decision was made to subcontract the development of the draft to the Pressure Vessel Research Council (PVRC) who in turn formed the Task Group on Continued Modernization of Codes to oversee the development of the new Section VIII, Division 2 Code. PVRC utilized professionals with both engineering and technical writing expertise to develop new technology and the initial drafts of the new Section VIII, Division 2.

A Steering Committee made up of ASME Subcommittee VIII members was formed to provide technical oversight and direction to the development team with the goal of facilitating the eventual balloting and approval process. ASME also retained a Project Manager to manage all the activities required to bring this new standard to publication.

The project began with the development of a detailed table of contents containing every paragraph heading that would appear in the new standard and identifying the source for the content that would be placed in this paragraph. In preparing such a detailed table of contents, the lead authors were able to quickly identify areas where major development effort was required to produce updated rules. A list of some of the new technology produced for VIII-2 rewrite includes:

- Adoption of a design margin on specified minimum tensile strength of 2.4,
- Toughness requirements,
- Design-by-rule for the creep range,
- Conical transition reinforcement requirements,
- Opening reinforcement rules,
- Local strain criteria for design-by-analysis using elastic-plastic analysis,
- Limit load and plastic collapse analysis for multiple loading conditions,
- Fatigue design for welded joints based on structural stress method, and
- Ultrasonic examination in lieu of radiographic examination

Users of the Section VIII, Division 2 Code (manufacturers and owner/operators) were surveyed at the beginning of the project to identify enhancements that they felt the industry wanted and would lead to increased use of the standard. Since the initial focus of the Code was for the construction of pressure equipment for the chemical and petrochemical industry, the people responsible for specifying equipment for this sector were very much interested in seeing that common requirements that are routinely found in vessel specifications would become a requirement within this standard. This was accomplished by close participation of the petrochemical industry during the development of this standard. Some of the enhancements included.

- Alternatives provided for U.S. and Canadian Registered Professional Engineer certification of the User Design Specification and Manufacturers Design Report,
- Consolidation of weld joint details and design requirements,
- Introduction of a weld joint efficiency and the use of partial radiographic and ultrasonic examination,
- Introduction of the concept of a Maximum Allowable Working Pressure (MAWP) identical to VIII-1,
- Significant upgrade to the design-by-rule and design-by-analysis procedures,
- Extension of the time-independent range for low chrome alloys used in heavy wall vessels,
- Extension of fatigue rules to 900°F (400°C) for low-chrome alloys used in heavy wall vessels,
- Adoption of new examination requirements and simplification of presentation of the rules,
- User-friendly extensive use of equations, tables, and figures to define rules and procedures, and
- ISO format; logical paragraph numbering system and single column format,
- Many of these enhancements identified by users were included in the first release of Section VIII, Division 2 in 2007.

After publication of Section VIII, Division 2, ASME contracted with the Equity Engineering Group, Inc. to develop the ASME Section VIII, Division 2 Example Problem Manual. This publication is provided to illustrate the design calculations used in the ASME B&PV Code, Section VIII, Division 2.

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PART 1

GENERAL REQUIREMENTS

1.1 Introduction

ASME B&PV Code, Section VIII, Division 2 contains mandatory requirements, specific prohibitions, and non-mandatory guidance for the design, materials, fabrication, examination, inspection, testing, and certification of pressure vessels and their associated pressure relief devices. The 2007 edition of the code has been re-written and reorganized and incorporates the latest technologies for pressure vessel design. Since this initial release, the code has undergone further development in all of its Parts, including refinement of its Part 4 Design-by-Rule (DBR) procedures and Part 5 Design-by-Analysis (DBA) methods. These modifications are captured in this PTB document.

1.2 Scope

Example problems illustrating the use of the Design-by-Rule and Design-by-Analysis methods in ASME B&PV Code, Section VIII, Division 2 are provided in this document. Example problems are provided for all calculation procedures primarily in US Customary units, however select problems are shown using SI units. As detailed in paragraph 1.5 of this document, ASME has introduced a two–class vessel structure. All example problems contained in this document are based on a Class 2 designation and applicable allowable stress values obtained in ASME Section II, Part D, Subpart 1, Table 5A and Table 5B.

1.3 Definitions

The following definition is used in this manual.

- VIII-2 – ASME B&PV Code, Section VIII, Division 2, 2021

1.4 Organization and Use

An introduction to the example problems is described in Part 2 of this document. The remaining Parts of this document contain the example problems. The Parts 3, 4, and 5 in this document coincide with the Parts 3, 4 and 5 in the ASME B&PV Code, Section VIII, Division 2. For example, example problems illustrating the design-by-rule calculations contained in Part 4 of Section VIII, Division 2 are provided in Part 4 of this document. Unless explicitly stated otherwise, all paragraph references are to the ASME B&PV Code, Section VIII, Division 2, 2021 Edition. Reference [1].

The example problems in this manual follow the calculation procedures in ASME B&PV Code, Section VIII, Division 2. It is recommended that users of this manual obtain a copy of ASME PTB-1-2013 [2] that contains criteria and commentary on the use of the design rules.

It should be noted that VIII-2 permits the use of API 579-1/ASME FFS-1 [3] for some calculation procedures. When reviewing certain example problems in this manual, it is recommended that users obtain a copy of this standard.

1.5 VIII-2 Vessel Classes

The 2017 edition of VIII-2 introduced a two–class vessel structure to attract more users to VIII–2. The format of the two–class structure assigns a Class 1 vessel a design margin of 3.0 on the Ultimate Tensile Strength (UTS), which is consistent with the philosophy of pre-2007 editions of VIII-2, while maintaining the 2.4 design margin on UTS for a Class 2 vessel. Class 1 vessels shall use the allowable stresses published in ASME B&PV Code, Section II, Part D, Subpart 1, Table 2A or Table 2B. Class 2 vessels shall use the allowable stresses published in ASME B&PV Code, Section II, Part D, Subpart 1, Table 5A or Table 5B.

The design and construction of a Class 1 vessel is permitted under the following limitations and relaxation of rules as compared to a Class 2 vessel.

- Design margin of 3.0 on UTS,
- The User's Design Specification (UDS) requires certification only when a fatigue analysis is mandated in the UDS,
- The Manufacturer's Design Report (MDR) requires certification only when the following are performed:
  - Fatigue analysis,
  - Use of Part 5 Design-by-Analysis to determine thickness of pressure parts when design rules are not provided in Part 4 Design-by-Rule,
  - Use of Part 4.8 to design quick-actuating closures, or
  - Dynamic seismic analysis.
- Part 5 DBA methods shall not be used in lieu of Part 4 DBR, and
- All other aspects of construction including materials, fabrication, examination, and testing shall be in accordance with the applicable parts of VIII–2.

1.6 References

[1] ASME B&PV Code, Section VIII, Division 2, *Rules for Construction of Pressure Vessels – Alternative Rules*, 2021, ASME, New York, NY, 2021.

[2] Osage, D., *ASME Section VIII – Division 2 Criteria and Commentary*, PTB-1-2013, ASME, New York, NY, 2013.

[3] API, API 579-1/ASME FFS-1 2021 *Fitness-For-Service*, American Petroleum Institute, Washington, D.C., 2021

PART 2

EXAMPLE PROBLEM DESCRIPTIONS

2.1 General

Example problems are provided for:

- Part 3 – Materials Requirements
- Part 4 – Design by Rule Requirements
- Part 5 – Design by Analysis
- Part 6 – Fabrication Requirements
- Part 7 – Examination Requirements
- Part 8 – Pressure Testing Requirements

A summary of the example problems provided is contained in the Table of Contents.

2.2 Example Problem Format

In all the example problems, except for tubesheet design rules in paragraph 4.18, the code equations are shown with symbols and with substituted numerical values to fully illustrate the use of the code rules. Because of the complexity of the tubesheet rules, only the results for each step in the calculation producer is shown.

2.3 Calculation Precision

The calculation precision used in the example problems is intended for demonstration proposes only; any intended precision is not implied. In general, the calculation precision should be equivalent to that obtained by computer implementation, rounding of calculations should only be done on the final results.

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PART 3

MATERIALS REQUIREMENTS

3.1 Commentary on Rules to Establish the Minimum Design Metal Temperature (MDMT)

Material toughness requirements are provided in paragraph 3.11.

Paragraph 3.11.1.1 – Charpy V-notch impact tests shall be made for materials used for shells, heads, nozzles, and other pressure containing parts, as well as for the structural members essential to structural integrity of the vessel, unless exempted by the rules of paragraph 3.11.

a) Toughness requirements for materials listed in Table 3-A.1 (carbon and low alloy steel materials except bolting materials) are given in paragraph 3.11.2.

b) Toughness requirements for materials listed in Table 3-A.2 (quenched and tempered steels with enhanced tensile properties) are given in paragraph 3.11.3.

c) Toughness requirements for materials listed in Table 3-A.3 (high alloy steels except bolting materials) are given in paragraph 3.11.4.

d) Toughness requirements for materials listed in Table 3-A.4 through 3-A.7 (nonferrous alloys) are given in paragraph 3.11.5.

e) Toughness requirements for all bolting materials are given in paragraph 3.11.6.

This commentary highlights the organization of paragraph 3.11.2, Carbon and Low Alloy Steels Except Bolting.

a) Paragraph 3.11.2.1 – Toughness Requirements for Carbon and Low Alloy Steels.

b) Paragraph 3.11.2.2 – Required Impact Testing Based on the MDMT, Thickness, and Yield Strength.

c) Paragraph 3.11.2.3 – Exemption from Impact Testing Based on the MDMT, Thickness, and Material Specification.

d) Paragraph 3.11.2.4 – Exemption from Impact Testing Based on Material Specification and Product Form.

e) Paragraph 3.11.2.5 – Exemption from Impact Testing Based on Design Stress Values.

f) Paragraph 3.11.2.6 – Adjusting the MDMT for Impact Tested Materials.

g) Paragraph 3.11.2.7 – Vessel or Components Operating Below the MDMT.

h) Paragraph 3.11.2.8 – Establishment of the MDMT Using a Fracture Mechanics Methodology.

i) Paragraph 3.11.2.9 – Postweld Heat Treatment Requirements for Materials in Low Temperature Service.

j) Paragraph 3.11.2.10 – Impact Tests of Welding Procedures.

There are significant changes to select paragraphs in the 2021 edition of VIII–2. A brief summary of these changes follows.

1) Paragraph 3.11.2.4(a) – The requirement on material properties and heat treatment for ferritic flanges produced to ASME B16.5 and ASME B16.47 standards has been changed to address the concern of Charpy Impact Test energy values when subject to ambient and low temperature. The paragraph was modified to coincide with the changes noted in Figure 3.7 (3.7M) and Figure 3.8 (3.8M).

   - Charpy Impact testing is not required for ferritic steel flanges when produced to fine grain practice and supplied in the heat-treated condition (normalized, normalized and tempered, or quenched and tempered after forging) when used at design temperatures no colder than –20°F.
   - Charpy Impact testing is not required for ferritic steel flanges supplied in the as-forged condition when used at temperatures no colder than 0°F.

2) Figure 3.7 (3.7M) and Figure 3.8 (3.8M) – The material classifications (Impact Test Exemption Curves) found in the NOTES of the applicable figures are summarized as follows. The figures were modified to coincide with the changes noted in paragraph 3.11.2.4(a).

   - Curve A applies to A/SA-105 forged flanges supplied in the as-forged condition, and
   - Curve B applies to A/SA-105 flanges produced to fine grain practice and normalized, normalized and tempered, or quenched and tempered after forging.

   VIII-2 does not provide specific guidance to the User as to what grain size constitutes a fine grain practice for an A/SA-105 forging specification. However, General Note (d)(2) of Figure 3.7 (3.7M) and Figure 3.8 (3.8M) states fine grain practice is defined as the procedure necessary to obtain a fine austenitic grain size as described in SA-20.

3) Paragraph 3.11.2.5(a) – Provides specific guidance for pressure vessel attachments that are exposed to tensile stresses from internal pressure (e.g., nozzle reinforcement pads, horizontal vessel saddle attachments, and stiffening rings), the coincident ratio shall be that of the shell or head to which each component is attached.

4) Paragraph 3.11.2.5(c) – The option to reduce the MDMT for a flange when the MDMT is established based on paragraph 3.11.2.4(a) was removed (see 1) above).

5) Paragraph 3.11.2.5(e) – Provides specific guidance that longitudinal stress in the vessel due to net-section bending that result in general primary membrane tensile stress (e.g., due to wind or earthquake in a vertical vessel, at mid-span and in the plane of the saddles of a saddle supported horizontal vessel) shall be considered when calculating the coincident ratio in Figure 3.12 (3.12M) or Figure 3.13 (3.13M).

6) Paragraph 3.11.2.6(a) – When adjusting the MDMT for impact tested materials, the same guidance as provided in paragraph 3.11.2.5(a) is repeated here. For pressure vessel attachments that are exposed to tensile stresses from internal pressure (e.g., nozzle reinforcement pads, horizontal vessel saddle attachments, and stiffening rings), the coincident ratio shall be that of the shell or head to which each component is attached.

3.2 Example E3.1 – Use of MDMT Exemptions Curves

Determine if impact testing is required for the proposed shell section using only the rules of paragraph 3.11.2.3. The shell is cylindrical with all Category A joints made up of Type 1 butt welds which have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Nominal Thickness = $1.8125\ in$
- PWHT = Yes
- MDMT = $-20°F$

Paragraph 3.11 Material Toughness Requirements. The procedure that is used to establish impact testing exemptions is shown below.

Paragraph 3.11.1.1 – Charpy V-notch impact tests shall be made for materials used for shells, heads, nozzles, and other pressure containing parts, as well as for the structural members essential to structural integrity of the vessel, unless exempted by the rules of paragraph 3.11.

Paragraph 3.11.2. for Carbon and Low Alloy Steel Except Bolting.

Paragraph 3.11.2.1 – Impact tests shall be performed on carbon and low alloy materials listed in Table 3-A.1 for all combinations of materials and MDMTs except as exempted by paragraphs 3.11.2.3, 3.11.2.4, 3.11.2.5, or 3.11.2.8.

Paragraph 3.11.2.3 – Exemption Based on the MDMT, Thickness, and Material Specification

Paragraph 3.11.2.4 – Exemption Based on Material Specification and Product Form

Paragraph 3.11.2.5 – Exemption Based on Design Stress Values

Paragraph 3.11.2.8 – Establishment of the MDMT Using a Fracture Mechanics Methodology

Paragraph 3.11.2.3:

a) STEP 1 – The vessel has been subject to PWHT; therefore, Figure 3.8 (or Table 3.15) shall be used to establish impact testing exemptions based on the impact test exemption curve for the subject material specification, MDMT, and governing thickness of a welded part. If an MDMT and thickness combination for the subject material is on or above the applicable impact test exemption curve in Figure 3.8, then impact testing is not required except as required by paragraph 3.11.8 for weld metal and heat-affected zones.

   From the Notes of Figure 3.8, the appropriate impact test exemption curve for the material specification SA-516, Grade 70 Normalized is designated a Curve D material.

b) STEP 2 – The governing thickness, t_g, of a welded part is determined from the criteria of paragraph 3.11.2.3.b. For a butt joint in a cylindrical shell, t_g, is equal to the nominal thickness of the thickest weld joint. In this example, the cylindrical shell is a welded part attached by a butt joint and the governing thickness is equal to the nominal thickness of the thickest welded joint see Figure 3.9 Sketch (a).

$$t_g = 1.8125\ in$$

c) STEP 3 – The MDMT is part of the design basis of the vessel and is defined in paragraph 4.1.5.2(e). The MDMT shall also be specified on the User's Design Specification as noted in paragraph 2.2.3.1.(d)(3) and is stated in the vessel data above.

$$MDMT = -20°F$$

d) STEP 4 – Interpreting the value of MDMT from Figure 3.8 is performed as follows. Enter the figure along the abscissa with a governing thickness of $t_g = 1.8125\ in$ and project upward until an intersection with the Curve D material is achieved. Project this point left to the ordinate and interpret the MDMT. Another approach to determine the MDMT with more consistency can be achieved by using the tabular values found in Table 3.15. Linear interpolation between thicknesses shown in the table is permitted.

$$y = \left(\frac{x - x_1}{x_2 - x_1}\right)(y_2 - y_1) + y_1$$

$$MDMT = \left(\frac{1.8125 - 1.75}{2.0 - 1.75}\right)\left[(-15.8) - (-20.2)\right] + (-20.2) = -19.1°F$$

Since the calculated $\{MDMT = -19.1°F\}$ is warmer than the required $\{MDMT = -20°F\}$, impact testing is required using only the rules in paragraph 3.11.2.3. However, impact testing may still be avoided by applying the rules of paragraph 3.11.2.5 and other noted impact test exemptions referenced in paragraph 3.11.2.1.

3.3 Example E3.2 – Use of MDMT Exemption Curves with Stress Reduction

Determine if impact testing is required for the proposed shell section in E3.1 by applying the rules of paragraph 3.11.2.5. The shell is cylindrical with all Category A joints made up of Type 1 butt welds which have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psi\ @\ 300°F$
- Inside Diameter = $150\ in$
- Nominal Thickness = $1.8125\ in$
- PWHT = Yes
- MDMT = $-20°F$
- Weld Joint Efficiency = 1.0
- Corrosion Allowance = $0.125\ in$
- Allowable Stress at Ambient Temperature = $25300\ psi$
- Allowable Stress at Design Temperature = $22400\ psi$
- Yield Stress at Ambient Temperature = $38000\ psi$

Paragraph 3.11.2.5 – Exemption Based on Design Stress Values.

Paragraph 3.11.2.5(a) – A colder MDMT for a component than that derived from paragraph 3.11.2.3 may be determined in accordance with the procedure outlined below.

a) STEP 1 – For the welded part under consideration, determine the nominal thickness of the part, t_n, and the required governing thickness of the part, t_g, using paragraph 3.11.2.3(b).

$$t_n = t_g = 1.8125\ in$$

b) STEP 2 – Determine the applicable material toughness curve to be used in Figure 3.8 for parts subject to PWHT. See paragraph 3.11.2.2(b) for materials having a specified minimum yield strength greater than 450 MPa (65 ksi.

From the Notes of Figure 3.8, the appropriate impact test exemption curve for the material specification SA-516, Grade 70 Normalized is designated a Curve D material.

c) STEP 3 – Determine the MDMT from Figure 3.8 for parts subject to PWHT based on the applicable toughness curve and the governing thickness, t_g. See Example E3.1.

$$MDMT = -19.1°F$$

d) STEP 4 – Based on the design loading conditions at the MDMT, determine the ratio, R_{ts}, using the thickness basis from equation (3.1). Note that this ratio can be computed in terms of required design thickness and nominal thickness, applied stress and allowable design stress, or applied pressure and maximum allowable working pressure based on the design rules in this Division or ASME/ANSI pressure–temperature ratings.

$$R_{ts} = \frac{t_r E^*}{t_n - CA}$$

where, t_r is the required thickness of the cylindrical shell at the specified $\{MDMT = -20°F\}$, using Part 4, equation 4.3.1.

$$t_r = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{150.25}{2}\left(\exp\left[\frac{356}{25300(1.0)}\right]-1\right) = 1.0646\ in$$

where,

$$D = 150.0 + 2(Corrosion\ Allowance) = 150.0 + 2(0.125) = 150.25\ in$$

and the variables E^*, t_n, and CA are defined as follows:

$$E^* = \max[E,\ 0.80] = \max[1.0, 0.8] = 1.0 \qquad \rightarrow paragraph\ 3.17$$

$$t_n = 1.8125\ in$$

$$CA = 0.125\ in$$

Therefore,

$$R_{ts} = \frac{t_r E^*}{t_n - CA} = \frac{1.0646(1.0)}{1.8125 - 0.125} = 0.6309$$

e) STEP 5 – Determine the final value of the MDMT and evaluate results. Since the computed value of the ratio $R_{ts} > 0.24$ from STEP 4, the specified minimum yield strength, $\{S_y = 38\ ksi\} > 65\ ksi$, and the shell was subject to PWHT, then the reduction in MDMT based on available thickness t_n is computed using Figure 3.13 (or Table 3.17). Interpreting the value of the temperature reduction, T_r, from Figure 3.13 is performed as follows. Enter the figure along the ordinate with a value of $R_{ts} = 0.6309$, project horizontally until an intersection with the provided curve is achieved. Project this point downward to the abscissa and interpret T_r. Another approach to determine the MDMT with more consistency can be achieved by using the tabular values found in Table 3.17. Linear interpolation between thicknesses shown in the table is permitted.

$$y = \left(\frac{x - x_1}{x_2 - x_1}\right)(y_2 - y_1) + y_1$$

$$T_R = \left(\frac{0.6309 - 0.610}{0.648 - 0.610}\right)(36.5 - 42.2) + (42.2) = 39.1°F$$

The final adjusted value of the MDMT is determined as follows.

$$MDMT = MDMT_{STEP3} - T_R = -19.1°F - 39.1°F = -58.2°F$$

Since the final value of $\{MDMT = -58.2°F\}$ is colder than the proposed $\{MDMT = -20°F\}$, impact testing is not required. However, if the MDMT is colder than -55°F, impact testing is required.

3.4 Example E3.3 – Determine the MDMT for a Nozzle-to-Shell Welded Assembly

Determine if impact testing is required for the proposed nozzle assembly comprised of a shell and integrally reinforced nozzle using the rules of paragraph 3.11.2.3 and the rules of paragraph 3.11.2.5, as applicable. The shell is cylindrical with all Category A joints made up of Type 1 butt welds which have been 100% radiographically examined. The nozzle parameters used in the design procedure is shown in Figure E3.3.1.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psi\ @\ 300°F$
- Inside Diameter = $150\ in$
- Nominal Thickness = $1.8125\ in$
- PWHT = Yes
- MDMT = $-20°F$
- Weld Joint Efficiency = 1.0
- Corrosion Allowance = $0.125\ in$
- Allowable Stress at Ambient Temperature = $25300\ psi$
- Allowable Stress at Design Temperature = $22400\ psi$
- Yield Stress at Ambient Temperature = $38000\ psi$

Nozzle:

- Material = $SA-105,\ Normalized,\ Fine\ Grain$
- Outside Diameter = $25.5\ in$
- Thickness = $4.75\ in$
- Allowable Stress at Ambient Temperature = $24000\ psi$
- Allowable Stress at Design Temperature = $21200\ psi$
- Yield Stress at Ambient Temperature = $36000\ psi$

The nozzle is an integrally reinforced heavy barrel forging with flange dimensions in accordance with AME B16.5 Class 300 inserted through the shell, i.e., set–in type nozzle.

Paragraph 3.11.2.3 Material Toughness Requirements. The procedure that is used to establish impact testing exemptions is shown below.

a) STEP 1 – The vessel has been subject to PWHT; therefore, Figure 3.8 (or Table 3.15) shall be used to establish impact testing exemptions based on the impact test exemption curve for the subject material specification, MDMT, and governing thickness of a welded part. If an MDMT and thickness combination for the subject material is on or above the applicable impact test exemption curve in Figure 3.8, then impact testing is not required except as required by paragraph 3.11.8 for weld metal and heat-affected zones.

From the Notes of Figure 3.8, the appropriate impact test exemption curve for the cylindrical shell material specification SA-516, Grade 70 Normalized is designated a Curve D material. Similarly, the appropriate impact test exemption curve for the integrally reinforced nozzle material specification SA-105 Normalized with fine grain practice is designated a Curve B material.

b) STEP 2 – The governing thickness, t_g, of a welded part is determined from the criteria of paragraph 3.11.2.3.b. However, per paragraph 3.11.2.3(c), components such as shells and nozzles shall be treated as separate components. Each component shall be evaluated for impact test requirements based on its individual material classification, governing thickness, and the MDMT. For welded assemblies comprised of more than two components (e.g., nozzle-to shell joint with reinforcing pad), the governing thickness and permissible MDMT of each of the individual welded joints of the assembly shall be determined, and the warmest of the MDMT shall be used as the permissible MDMT of the welded assembly. The governing thickness of the full penetration corner joint, t_{g1} for the welded joint under consideration, was determined per Figure 3.11 Sketch (b) and Figure E3.3.1 of this example.

$$t_A = Shell\ thickness,\ 1.8125\ in$$

$$t_C = Nozzle\ thickness,\ 4.75\ in$$

$$t_{g1} = \min\left[t_A,\ t_C\right] = \min\left[1.8125,\ 4.75\right] = 1.8125\ in$$

c) STEP 3 – The MDMT is part of the design basis of the vessel and is defined in paragraph 4.1.5.2(e). The MDMT shall also be specified on the User's Design Specification as noted in paragraph 2.2.3.1.(d)(3) and is stated in the vessel data above.

$$MDMT = -20°F$$

d) STEP 4 – Interpreting the value of MDMT from Figure 3.8 for the welded joint requires that both the shell and nozzle material be evaluated, and the warmest minimum design metal temperature shall be used for the assembly. The general procedure is performed as follows. Enter the figure along the abscissa with a governing thickness and project upward until an intersection with the appropriate impact test exemption curve is achieved. Project this point left to the ordinate and interpret the MDMT. Another approach to determine the MDMT with more consistency can be achieved by using the tabular values found in Table 3.15. Linear interpolation between thicknesses shown in the table is permitted.

For the cylindrical shell with a $t_{g1} = 1.8125\ in$, Curve D material.

$$y = \left(\frac{x - x_1}{x_2 - x_1}\right)\left(y_2 - y_1\right) + y_1$$

$$MDMT_{curve\ D} = \left(\frac{1.8125 - 1.75}{2.0 - 1.75}\right)\left[\left(-15.8\right) - \left(-20.2\right)\right] + \left(-20.2\right) = -19.1°F$$

For the nozzle forging with a $t_{g1} = 1.8125\ in$, Curve B material.

$$y = \left(\frac{x - x_1}{x_2 - x_1}\right)\left(y_2 - y_1\right) + y_1$$

$$MDMT_{curve\ B} = \left(\frac{1.8125 - 1.75}{2.0 - 1.75}\right)\left[\left(48.2\right) - \left(43.8\right)\right] + \left(43.8\right) = 44.9°F$$

Therefore, the nozzle assembly minimum design metal temperature is determined as follows.

$$MDMT_{assembly} = Warmest\left[\left\{MDMT_{curve\ D} = -19.1\right\}, \left\{MDMT_{curve\ B} = 44.9\right\}\right] = 44.9°F$$

Since the calculated $\{MDMT = 44.9°F\}$ is warmer than the required $\{MDMT = -20°F\}$, impact testing is required using only the rules in paragraph 3.11.2.3. However, impact testing may still be avoided by applying the rules of paragraph 3.11.2.5 and other noted impact test exemptions referenced in paragraph 3.11.2.1.

Applying paragraph 3.11.2.5 – Exemption Based on Design Stress Values.

Paragraph 3.11.2.5(a): A colder MDMT for a component than that derived from paragraph 3.11.2.3 may be determined in accordance with the procedure outlined below.

a) STEP 1 – $t_{g1} = 1.8125\ in.$

b) STEP 2 – Cylindrical Shell: Curve D material, Nozzle Forging: Curve B.

c) STEP 3 – $MDMT = 44.9°F$.

d) STEP 4 – Based on the design loading conditions at the MDMT, determine the ratio, R_{ts}, using the thickness basis from equation (3.1). For pressure vessel attachments that are exposed to tensile stresses from internal pressure (e.g., nozzle reinforcement pads, horizontal vessel saddle attachments, and stiffening rings), the coincident ratio shall be that of the shell or head to which each component is attached.

Commentary: Although this nozzle-to-shell welded assembly does not contain a nozzle reinforcement pad, it is determined appropriate to apply this same logic to designate the coincident ratio to be that of the shell to which the nozzle is attached. This example provides one possible method of satisfying the requirement and is consistent with ASME Interpretation VIII-1-01-37.

$$R_{ts} = \frac{t_r E^*}{t_n - CA}$$

where, t_r is the required thickness of the cylindrical shell at the specified $\{MDMT = -20°F\}$, using Part 4, equation 4.3.1.

$$t_r = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{150.25}{2}\left(\exp\left[\frac{356}{25300(1.0)}\right]-1\right) = 1.0646\ in$$

where,

$$D = 150.0 + 2(Corrosion\ \ Allowance) = 150.0 + 2(0.125) = 150.25\ in$$

and the variables E^*, t_n, and CA are defined as follows:

$$E^* = \max\left[E,\ 0.80\right] = \max\left[1.0, 0.8\right] = 1.0 \qquad \rightarrow paragraph\ 3.17$$

$$t_n = 1.8125\ in$$

$$CA = 0.125\ in$$

Therefore,

$$R_{ts} = \frac{t_r E*}{t_n - CA} = \frac{1.0646(1.0)}{1.8125 - 0.125} = 0.6309$$

e) STEP 5 – Determine the final value of the MDMT and evaluate results. Since the computed value of the ratio $R_{ts} > 0.24$ from STEP 4, the specified minimum yield strength, $\{S_y = 38\ ksi\} > 65\ ksi$, and the assembly was subject to PWHT, then the reduction in MDMT based on available thickness t_n is computed using Figure 3.13 (or Table 3.17). Interpreting the value of the temperature reduction, T_r, from Figure 3.13 is performed as follows. Enter the figure along the ordinate with a value of $R_{ts} = 0.6309$, project horizontally until an intersection with the provided curve is achieved. Project this point downward to the abscissa and interpret T_r. Another approach to determine the MDMT with more consistency can be achieved by using the tabular values found in Table 3.17. Linear interpolation between thicknesses shown in the table is permitted.

$$y = \left(\frac{x - x_1}{x_2 - x_1}\right)(y_2 - y_1) + y_1$$

$$T_R = \left(\frac{0.6309 - 0.610}{0.648 - 0.610}\right)(36.5 - 42.2) + (42.2) = 39.1°F$$

The final adjusted value of the MDMT is determined as follows.

$$MDMT = MDMT_{STEP3} - T_R = 44.9°F - 39.1°F = 5.8°F$$

Since the calculated $\{MDMT = 5.8°F\}$ is warmer than the required $\{MDMT = -20°F\}$, impact testing is required using the rules in paragraph 3.11.2.3 and paragraph 3.11.2.5. However, an MDMT colder than the determined in this example would be possible if the nozzle forging were fabricated from a material specification that includes the provisions of impact testing, such as SA-350. See paragraph 3.11.2.4(b).

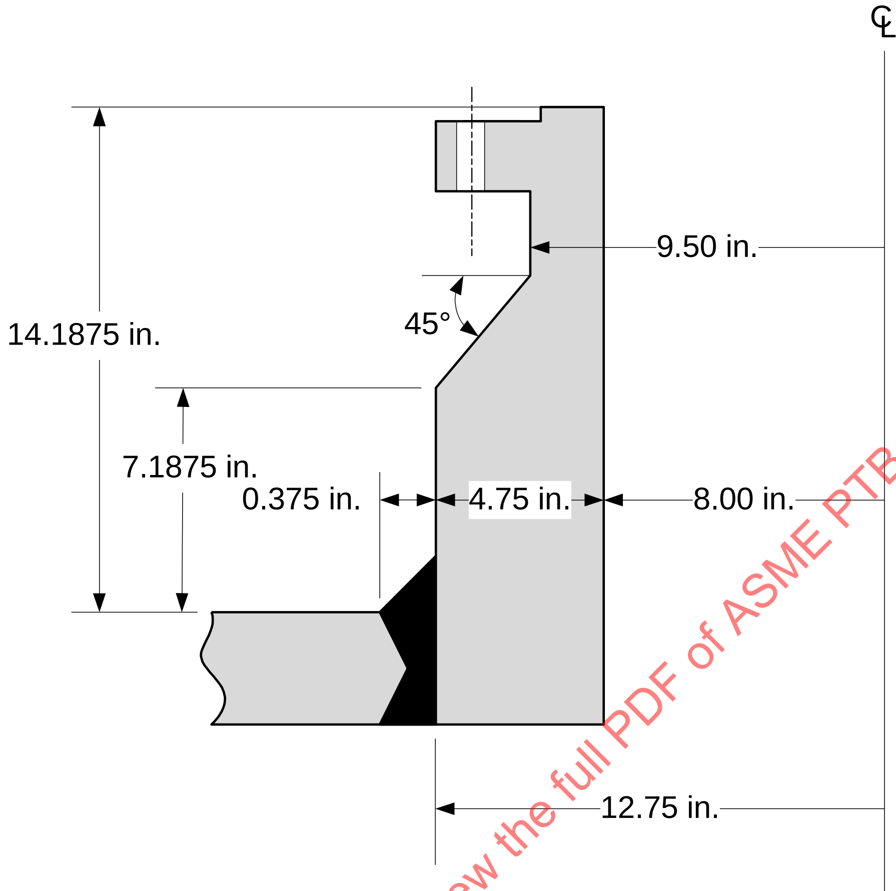

Figure E3.3.1 – Nozzle-Shell Detail

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PART 4

DESIGN BY RULE REQUIREMENTS

4.1 General Requirements

4.1.1 Example E4.1.1 – Review of General Requirements for a Vessel Design

a) General Requirements

An engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with ASME B&PV Code, Section VIII, Division 2 (VIII-2). The VIII-2 Code is being considered because the vessel in question is to be constructed of carbon steel with a specified corrosion allowance and a design pressure and temperature of 1650 psig at 200°F. As part of developing the design specification, the following items need to be evaluated.

b) Scope

1) The vessel may be designed with either a Class 1 or Class 2 designation. The following differences need to be considered when selecting the class designation of the vessel.

i) Class 1 Vessel – a vessel that is designed using the allowable stresses from Section II, Part D, Subpart 1, Table 2A or Table 2B.

1. The User's Design Specification shall be certified by a Certifying Engineer meeting the requirements described in Annex 2-A when the user provides the data required by 2.2.3.1(f)(1) and 2.2.3.1(f)(2) to perform a fatigue analysis.

2. When design rules are not provided in Part 4 for a vessel or vessel part, the Manufacturer shall either perform a stress analysis in accordance with Part 5 considering all of the loadings specified in the User's Design Specification, or, with acceptance by the Authorized Inspector, use a recognized and accepted design-by-rule method that meets the applicable design allowable stress criteria given in paragraph 4.1.6. If the design cannot be performed using Part 5 or a design-by-rule method (e.g., creep-fatigue), a design method consistent with the overall design philosophy of Class 1 and acceptable to the Authorized Inspector shall be used.

3. Design-by-analysis methods of Part 5 shall not be used in lieu of the design-by-rules of Part 4.

ii) Class 2 Vessel – a vessel that is designed using the allowable stresses from Section II, Part D, Subpart 1, Table 5A or Table 5B.

1. The User's Design Specification shall be certified by a Certifying Engineer meeting the requirements described in Annex 2-A.

2. When design rules are not provided for a vessel or vessel part, the Manufacturer shall perform a stress analysis in accordance with Part 5 considering all of the loadings specified in the User's Design Specification.

3. A design-by-analysis in accordance with Part 5 may be used to establish the design thicknessand/or configuration (i.e., nozzle reinforcement configuration) in lieu of the design-by-rules in Part 4 for any geometry or loading conditions.

4. Components of the same pressure vessel may be designed (thickness and configuration) using a combination of Part 4 design-by-rules or any of the three methods of Part 5 design-by-analysis in 5.2.1.1.

2) The user of the vessel is responsible for defining all applicable loads and operating conditions that the vessel will be subject to. All loads and conditions must be specified on the User's Design Specification, see Part 2 paragraph 2.2.

3) A fatigue screening shall be applied to all vessel part designed in accordance with this Division to determine if a formal fatigue analysis is required, see Part 5 paragraph 5.5.2.

c) Minimum Thickness Requirements

Based on product form and process service, the parts of the vessel must meet the minimum thickness requirements presented in Part 4, paragraph 4.1.2.

d) Material Thickness Requirements

Fabrication tolerances must be considered in the design of the vessel components, based on forming, heat treatment and product form.

e) Corrosion Allowance in Design Equations

The equations used in a design-by-rule procedure of Part 4 or the thicknesses used in a design-by-analysis of Part 5 must be performed in a corroded condition. The term corrosion allowance is representative of loss of metal due to corrosion, erosion, mechanical abrasion, or other environmental effects. The corrosion allowance must be documented in the User's Design Specification.

f) Design Basis

1) The vessel may be designed with either a Class 1 or Class 2 designation. The following differences need to be considered when selecting the class designation of the vessel.

i) Class 1 Vessel. The design-by-rule methods of Part 4 shall be applied using the load and load case combinations specified in paragraph 4.1.5.3.

ii) Class 2 Vessel – The design-by-rule methods of Part 4 shall be applied using the load and load case combinations specified in paragraph 4.1.5.3. Alternatively, the design thickness may be established using the design-by-analysis procedures and the load and load case combinations specified in Part 5. The design thickness established using Part 5 may be less than that established using Part 4.

2) The pressure used in the design of a vessel component together with the coincident design metal temperature must be specified. Where applicable, the pressure resulting from static head and other static or dynamic loads shall be included in addition to the specified design pressure.

3) The specified design temperature shall not be less than the mean metal temperature expected coincidentally with the corresponding maximum pressure.

4) A minimum design metal temperature shall be determined and shall consider the coldest operating temperature, operational upsets, auto refrigeration, atmospheric temperature, and any other source of cooling.

5) All applicable loads and load case combinations shall be considered in the design to determine the minimum required wall thickness for a vessel part. The loads and load case combinations that shall be considered in the design shall include, but not be limited to, those shown in Part 4 Table 4.1.1 and Table 4.1.2 or Part 5, Tables 5.1 through Table 5.5.

6) All applicable loads and load case combinations shall be included in the User's Design Specification.

g) Design Allowable Stress

Specifications for all materials of construction are provided in Part 3, Annex 3.A. Class 1 vessels are designed using the allowable stresses from Section II, Part D, Subpart 1, Table 2A or Table 2B. Class 2 vessels are designed using the allowable stresses from Section II, Part D, Subpart 1, Table 5A or Table 5B.

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4.1.2 Example E4.1.2 – Required Wall Thickness of a Hemispherical Head

Determine the required thickness for a hemispherical head at the bottom of a vertical vessel considering the following design conditions. All Category A joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $1650\ psig@300°F$
- Liquid Head = $60\ ft$
- Liquid Specific Gravity = 0.89
- Inside Diameter = $96.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0

The design pressure used to establish the wall thickness must be adjusted for the liquid head in accordance with Part 4, paragraph 4.1.5.2.a.

$$Design\ Pressure = Specified\ \ Design\ \ Pressure + \gamma\rho h$$

$$Design\ Pressure = 1650 + \frac{0.89(62.4)(60)}{144} = 1673.140\ psig$$

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the bottom hemispherical head.

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right] - 1\right)$$

$$D = 96.0 + 2(Corrosion\ \ Allowance) = 96.0 + 2(0.125) = 96.25\ in$$

$$t = \frac{96.25}{2}\left(\exp\left[\frac{0.5(1673.14)}{22400(1.0)}\right] - 1\right) = 1.8313\ in$$

$$t = 1.8313 + Corrosion\ \ Allowance = 1.8313 + 0.125 = 1.9563\ in$$

The required thickness of the bottom head is $1.9563\ in$.

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4.1.3 Example E4.1.3 – Required Wall Thickness of a Hemispherical Head – Higher Strength Material

Determine the required thickness for a hemispherical head at the bottom of a vertical vessel in example E.4.1.2 considering the following design conditions. Note that a higher strength material is being used. All Category A joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-537,\ Class\ 1$
- Design Conditions = $1650\ psig @ 300°F$
- Liquid Head = $60\ ft$
- Liquid Specific Gravity = 0.89
- Inside Diameter = $96.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $29000\ psi$
- Weld Joint Efficiency = 1.0

The design pressure used to establish the wall thickness must be adjusted for the liquid head in accordance with Part 4, paragraph 4.1.5.2.a

$$Design\ Pressure = Specified\ Design\ Pressure + \gamma\rho h$$

$$Design\ Pressure = 1650 + \frac{0.89(62.4)(60)}{144} = 1673.140\ psig$$

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the bottom hemispherical head.

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right] - 1\right)$$

$$D = 96.0 + 2(Corrosion\ Allowance) = 96.0 + 2(0.125) = 96.25\ in$$

$$t = \frac{96.25}{2}\left(\exp\left[\frac{0.5(1673.14)}{29000(1.0)}\right] - 1\right) = 1.4085\ in$$

$$t = 1.4085 + Corrosion\ Allowance = 1.4085 + 0.125 = 1.5335\ in$$

The required thickness of the bottom head constructed with a material with a higher yield-to-tensile strength ratio, $S_y/S_{uts} = 50\,ksi/70\,ksi = 0.71$, is $1.5335\,in.$ This represents a savings in material costs of approximately 22%. Additional costs in welding and NDE are also expected. Similar cost savings can be achieved by using a higher strength material for the cylinder shell. The design margins in Section VIII, Division 2 will typically result in a more efficient design when higher strength materials are used as shown in this example. For many fluid service environments, higher strength materials may be prone to cracking. However, if PWHT is specified for fluid service, as opposed to wall thickness requirements in accordance with Part 6, the use of higher strength materials may be justified and result in significant cost savings.

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4.2 Welded Joints

4.2.1 Example E4.2.1 – Nondestructive Examination Requirement for Vessel Design

A notable change to VIII-2 is the introduction of rules for the examination of welded joints, which introduce Examination Groups. These Examination Groups permit partial radiography and require surface examination as part of the nondestructive examination (NDE) requirements. Part 7, paragraph 7.4.2, and Table 7.1 of VIII-2 define the different Examination Groups assigned to welded joints based on the manufacturing complexity of the material group, the maximum thickness, the welding process, and the selected joint efficiency. Once the Examination Group is selected, Table 7.2 provides the required NDE, joint category designation, joint efficiency, and acceptable joint types.

An engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with ASME B&PV Code, Section VIII, Division 2 (VIII-2). Based on the anticipated fabrication data provided, the engineer compares the Examination Groups to aid in the decision for NDE requirements for vessel design as follows.

<u>**Vessel Data:**</u>

- Material = $SA-516,\ Grade\ 70$
- Welding Process = $SAW\ \ and\ \ SMAW$
- Weld Categories = $A,\ B,\ C,\ D$
- Weld Joint Types = $Type\ 1\ \rightarrow Category\ \ A,\ B,\ and\ \ C$
  $Type\ 7\ \rightarrow Category\ \ D$

The definitions of Weld Categories and Weld Joint Types are provided in Part 4, Tables 4.2.1 and 4.2.2, respectively. Acceptable weld joints for main shell weld seams, formed heads, flange attachments, and nozzles are provided in Tables 4.2.4, 4.2.5, 4.2.9, and 4.2.10, respectively.

Per Part 7, Table 7.1, a comparison of Examination Group 1b and Examination Group 3b for carbon steel material, $SA-516, Grade\ 70$ is as follows.

Parameter	Examination Group	
	1b	3b
Permitted Material	Groups 1.1, 1.2, 8.1, 11.1	Groups 1.1, 1.2, 8.1, 11.1
Maximum Thickness	Unlimited	50 mm (2 in) Groups 1.1, 8.1, 11.1 30 mm (1.1875 in) Group 1.2
Welding Process	Unrestricted	Unrestricted
Design Basis	Part 4 or Part 5	Part 4

From the results of the comparison above, there are two parameters that will require a decision to be made by the engineer prior to assigning an Examination Group, maximum thickness of the vessel components and design basis. A preliminary check of the required wall thickness for the main cylinder and heads can be performed in accordance with the rules of Part 4. However, if there is one or several components that may require their design to be based on numerical analysis, i.e., finite element analysis per Part 5, only Examination Group 1b would be permitted.

Per Part 7, Table 7.2, a comparison of the required NDE for Examination Group 1b and Examination Group 3b for carbon steel material, $SA-516, Grade\ 70$ is as follows.

Examination Group			1b	3b
Joint Efficiency			1.0	0.85
Joint Category	Type of Weld	Type of NDE	Extent of NDE	
A	Type 1: Full Penetration Longitudinal Seam	RT or UT MT or PT	100% 10%	10% 0%
B	Type 1: Full Penetration Circumferential Seam	RT or UT MT or PT	10% 10%	5% 0%
C	Type 1: Full Penetration Flange/Nozzle Attachment	RT or UT MT or PT	10% 10%	5% 10%
D	Type 7: Full Penetration Corner Joint, Nozzle $d > 150\ mm(NPS\ 6) or$ $t > 16\ mm(0.625\ in)$	RT or UT MT or PT	10% 10%	5% 10%
D	Type 7: Full Penetration Corner Joint, Nozzle $d \leq 150\ mm(NPS\ 6) and$ $t \leq 16\ mm(0.625\ in)$	MT or PT	10%	10%

A review of the above table indicates that more inspection is required for Examination Group 1b when compared to 3b. However, the increased costs for examination may be offset by the materials and fabrication savings. Consider the following comparison for a cylindrical shell.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $725\ psig @ 300°F$
- Inside Diameter = $60.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$

For examination Group 1b, consider the requirements for a Category A Type 1 weld. The required wall thickness in accordance with Part 4, paragraph 4.3.3 is computed as shown below.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right)$$

$$D = 60.0 + 2(Corrosion\ \ Allowance) = 60.0 + 2(0.125) = 60.25\ in$$

$$t = \frac{60.25}{2}\left(\exp\left[\frac{725}{22400(1.0)}\right]-1\right) = 0.9910\ in$$

$$t = 0.9910 + Corrosion\ \ Allowance = 0.9910 + 0.125 = 1.1160\ in$$

Alternatively, for examination Group 3b, the required wall thickness for a Category A Type 1 weld in accordance with Part 4, paragraph 4.3.3 is computed as shown below.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right)$$

$$D = 60.0 + 2(Corrosion\ \ Allowance) = 60.0 + 2(0.125) = 60.25\ in$$

$$t = \frac{60.25}{2}\left(\exp\left[\frac{725}{22400(0.85)}\right]-1\right) = 1.1692\ in$$

$$t = 1.1692 + Corrosion\ \ Allowance = 1.1692 + 0.125 = 1.2942\ in$$

Examination Group 1b when compared to 3b results in an approximate 14% reduction in wall thickness. Cost savings for this reduction in wall thickness will include less material and less welding, and these reductions may offset the increased examination costs. It should also be noted that many refining and petrochemical companies invoke additional examination requirements in their associated corporate standards based on fluid service. Therefore, in some cases the increased examination requirements for Examination Group 1b may already be required based on fluid service.

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4.2.2 Example E4.2.2 – Nozzle Detail and Weld Sizing

Determine the required fillet weld size and inside corner radius of a set-in type of nozzle as shown in Table 4.2.10, Detail 4 (Figure E4.2.2). The vessel and nozzle were designed such that their nominal thicknesses were established as follows.

Vessel Data:

- Cylinder Thickness = $0.625\ in$
- Nozzle Diameter = $NPS\ 10$
- Nozzle Thickness = $Schedule\ XS \rightarrow 0.500\ in$
- Corrosion Allowance = $0.125\ in$

Adjust variables for corrosion.

$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$
$$t_n = 0.500 - Corrosion\ \ Allowance = 0.500 - 0.125 = 0.375\ in$$

The minimum fillet weld throat dimension, t_c, is calculated as follows.

$$t_c \geq \min\left[0.7t_n,\ 0.25\ in\right]$$
$$t_n = 0.375\ in$$
$$t_c \geq \min\left[0.7(0.375), 0.25\ in\right]$$
$$t_c \geq 0.25\ in$$

The resulting fillet weld leg size is determined as, $t_c/0.7 = 0.357\ in$. Therefore, a fillet weld leg size of $0.375\ in$ would be acceptable.

The minimum inside corner radius, r_1, is calculated as follows.

$$0.125t \leq r_1 \leq 0.5t$$
$$t = 0.500\ in$$
$$0.125(0.500) \leq r_1 \leq 0.5(0.500)$$
$$0.0625 \leq r_1 \leq 0.250\ in$$

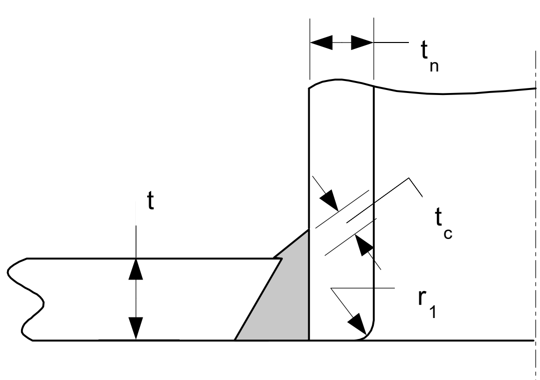

Figure E4.2.2 – Weld Details

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4.2.3 Example E4.2.3 – Nozzle Detail with Reinforcement Pad and Weld Sizing

Determine the required fillet weld sizes and inside corner radius of a set-in type of nozzle with added reinforcement pad as shown in Table 4.2.11, Detail 2 (Figure E4.2.3). The vessel and nozzle were designed such that their nominal thicknesses were established as follows.

Vessel Data:

- Cylinder Thickness = $0.625\ in$
- Nozzle Diameter = $NPS\ 10$
- Nozzle Thickness = $Schedule\ XS \rightarrow 0.500\ in$
- Reinforcement Pad Thickness = $0.625\ in$
- Corrosion Allowance = $0.125\ in$

Adjust variables for corrosion.

$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$

$$t_n = 0.500 - Corrosion\ \ Allowance = 0.500 - 0.125 = 0.375\ in$$

$$t_e = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.0 = 0.625\ in$$

Note: The corrosion allowance specified is for internal corrosion, not external corrosion. Therefore, the corrosion allowance of the reinforcement pad thickness is set equal to zero.

The minimum fillet weld throat dimension, t_c, is calculated as follows.

$$t_c \geq \min\left[0.7t_n, 6\,mm\ (0.25\ in)\right]$$

$$t_n = 0.375\ in$$

$$t_c \geq \min\left[0.7(0.375), 0.25\right]$$

$$t_c \geq 0.25\ in$$

The resulting fillet weld leg size is determined as, $t_c/0.7 = 0.357\ in$. Therefore, a fillet weld leg size of $0.375\ in$ would be acceptable.

The minimum fillet weld throat dimension, t_{f1}, is calculated as follows.

$$t_{f1} \geq \min\left[0.6t_e,\ 0.6t\right]$$

$$t_e = 0.625\ in$$

$$t = 0.500\ in$$

$$t_{f1} \geq \min\left[0.6(0.625), 0.6(0.500)\right]$$

$$t_{f1} \geq 0.300\ in$$

The resulting fillet weld leg size is determined as, $t_{f1}/0.7 = 0.429\ in$. Therefore, a fillet weld leg size of $0.4375\ in$ would be acceptable.

The minimum inside corner radius, r_1, is calculated as follows.

$$0.125t \le r_1 \le 0.5t$$

$$t = 0.500\ in$$

$$0.125(0.500) \le r_1 \le 0.5(0.500)$$

$$0.0625 \le r_1 \le 0.250\ in$$

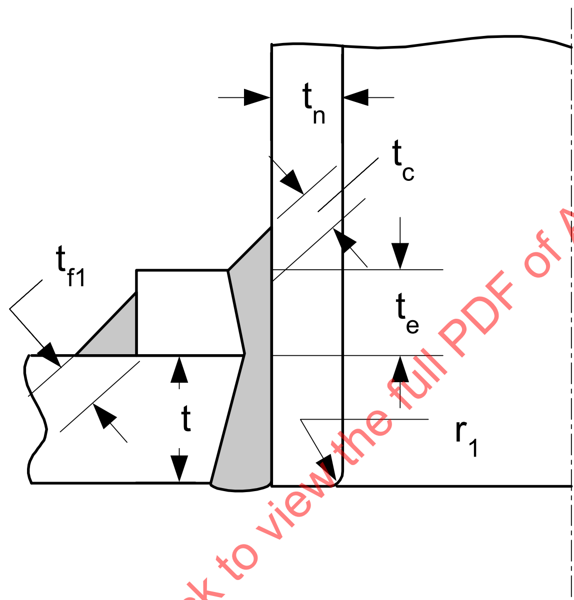

Figure E4.2.3 – Weld Details

4.3 Internal Design Pressure

4.3.1 Example E4.3.1 – Cylindrical Shell

Determine the required thickness for a cylindrical shell considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psig@300°F$
- Inside Diameter = $90.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0

Adjust for corrosion allowance.

$$D = 90.0 + 2(Corrosion\ \ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

Evaluate per paragraph 4.3.3.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right] - 1\right) = \frac{90.25}{2}\left(\exp\left[\frac{356}{22400(1.0)}\right] - 1\right) = 0.7229\ in$$

$$t = 0.7229 + Corrosion\ \ Allowance = 0.7229 + 0.125 = 0.8479\ in$$

The required thickness is $0.8479\ in$.

Combined Loadings – cylindrical shells subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this example problem, the cylindrical shell is only subject to internal pressure.

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4.3.2 Example E4.3.2 – Conical Shell

Determine the required thickness for a conical shell considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psig @ 300°F$
- Inside Diameter (Large End) = $150.0\ in$
- Inside Diameter (Small End) = $90.0\ in$
- Length of Conical Section = $78.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0

Adjust for corrosion allowance and determine the cone angle.

$$D_L = 150.0 + 2\left(Corrosion\ \ Allowance\right) = 150.0 + 2\left(0.125\right) = 150.25\ in$$

$$D_s = 90.0 + 2\left(Corrosion\ \ Allowance\right) = 150.0 + 2\left(0.125\right) = 90.25\ in$$

$$L_C = 78.0\ in$$

$$\alpha = \arctan\left[\frac{0.5\left(D_L - D_S\right)}{L_C}\right] = \arctan\left[\frac{0.5\left(150.25 - 90.25\right)}{78.0}\right] = 21.0375\ deg$$

Evaluate per paragraph 4.3.4 using the large end diameter of the conical shell.

$$t = \frac{D}{2\cos\left[\alpha\right]}\left(\exp\left[\frac{P}{SE}\right] - 1\right) = \frac{150.25}{2\cos\left[21.0375\right]}\left(\exp\left[\frac{356}{22400\left(1.0\right)}\right] - 1\right) = 1.2894\ in$$

$$t = 1.2894 + Corrosion\ \ Allowance = 1.2894 + 0.125 = 1.4144\ in$$

The required thickness is $1.4144\ in$.

Combined Loadings – conical shells subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this example problem, the conical shell is only subject to internal pressure.

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4.3.3 Example E4.3.3 – Spherical Shell

Determine the required thickness for a spherical shell considering the following design conditions. All Category A joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-542,\ Type\,D,\ Class\ 4a$
- Design Conditions = $2080\ psig@850°F$
- Inside Diameter = $149.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $28900\ psi$
- Weld Joint Efficiency = 1.0

Evaluate per paragraph 4.3.5.

$$t=\frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right]-1\right)=\frac{149.0}{2}\left(\exp\left[\frac{0.5(2080)}{28900(1.0)}\right]-1\right)=2.7298\ in$$

The required thickness is $2.7298\ in$.

Combined Loadings – spherical shells and hemispherical heads subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this example problem, the spherical shell is only subject to internal pressure.

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4.3.4 Example E4.3.4 – Torispherical Head

Determine the Maximum Allowable Working Pressure (MAWP) for the proposed seamless torispherical head. The Category B joint joining the head to the shell is a Type 1 butt weld and has been 100% radiographically examined.

Vessel Data:

- Material = $SA-387,\ Grade\ 11,\ Class\ 1$
- Design Temperature = $650°F$
- Inside Diameter = $72.0\ in$
- Crown Radius = $72.0\ in$
- Knuckle Radius = $4.375\ in$
- Thickness = $0.625\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $18000\ psi$
- Weld Joint Efficiency = 1.0
- Modulus of Elasticity at Design Temperature = $26.55E+06\ psi$
- Yield Strength at Design Temperature = $26900\ psi$

Evaluate per paragraph 4.3.5.

a) STEP 1 – Determine, D, assume values for L, r and t (known variables from above) and adjust for corrosion.

$$D = 72.0 + 2(Corrosion\ \ Allowance) = 72.0 + 2(0.125) = 72.25\ in$$

$$L = 72.0 + Corrosion\ \ Allowance = 72.0 + 0.125 = 72.125\ in$$

$$r = 4.375 + Corrosion\ \ Allowance = 4.375 + 0.125 = 4.5\ in$$

$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.5\ in$$

b) STEP 2 – Compute the head L/D, r/D, and L/t ratios and determine if the following equations are satisfied.

$$0.7 \le \left\{ \frac{L}{D} = \frac{72.125}{72.25} = 0.9983 \right\} \le 1.0 \qquad True$$

$$\left\{ \frac{r}{D} = \frac{4.5}{72.25} = 0.0623 \right\} \ge 0.06 \qquad True$$

$$20 \le \left\{ \frac{L}{t} = \frac{72.125}{0.5} = 144.25 \right\} \le 2000 \qquad True$$

c) STEP 3 – Calculate the geometric constants β_{th}, ϕ_{th}, R_{th}.

$$\beta_{th} = \arccos\left[\frac{0.5D - r}{L - r}\right] = \arccos\left[\frac{0.5(72.25) - 4.5}{72.125 - 4.5}\right] = 1.0842\ rad$$

$$\phi_{th} = \frac{\sqrt{Lt}}{r} = \frac{\sqrt{72.125(0.5)}}{4.5} = 1.3345\ rad$$

Since $\phi_{th} \geq \beta_{th}$, calculate R_{th} as follows:

$$R_{th} = 0.5D = 0.5(72.25) = 36.125\ in$$

d) STEP 4 – Compute the coefficients C_1 and C_2.

Since $r/D = 0.0623 \leq 0.08$, calculate C_1 and C_2 as follows:

$$C_1 = 9.31\left(\frac{r}{D}\right) - 0.086 = 9.31(0.0623) - 0.086 = 0.4940$$

$$C_2 = 1.25$$

e) STEP 5 – Calculate the value of internal pressure expected to produce elastic buckling of the knuckle, P_{eth}.

$$P_{eth} = \frac{C_1 E_T t^2}{C_2 R_{th}\left(\frac{R_{th}}{2} - r\right)} = \frac{(0.4940)(26.55E+06)(0.5)^2}{1.25(36.125)\left(\frac{36.125}{2} - 4.5\right)} = 5353.9445\ psi$$

f) STEP 6 – Calculate the value of internal pressure that will result in a maximum stress in the knuckle equal to the material yield strength, P_y.

$$P_y = \frac{C_3 t}{C_2 R_{th}\left(\frac{R_{th}}{2r} - 1\right)}$$

Since the allowable stress at design temperature is governed by time-independent properties, is C_3 the material yield strength at the design temperature, or $C_3 = S_y$.

$$P_y = \frac{26900(0.5)}{1.25(36.125)\left(\frac{36.125}{2(4.5)} - 1\right)} = 98.8274\ psi$$

g) STEP 7 – Calculate the value of internal pressure expected to result in a buckling failure of the knuckle, P_{ck}.

Calculate variable G:

$$G = \frac{P_{eth}}{P_y} = \frac{5353.9445}{98.8274} = 54.1747$$

Since $G > 1.0$, calculate P_{ck} as follows:

$$P_{ck} = \left(\frac{0.77508G - 0.20354G^2 + 0.019274G^3}{1 + 0.19014G - 0.089534G^2 + 0.0093965G^3} \right)(P_y)$$

$$P_{ck} = \left(\frac{0.77508(54.1747) - 0.20354(54.1747)^2 + 0.019274(54.1747)^3}{1 + 0.19014(54.1747) - 0.089534(54.1747)^2 + 0.0093965(54.1747)^3} \right)(98.8274)$$

$$P_{ck} = 199.5671\, psi$$

h) STEP 8 – Calculate the allowable pressure based on a buckling failure of the knuckle, P_{ak}.

$$P_{ak} = \frac{P_{ck}}{1.5} = \frac{199.5671}{1.5} = 133.0447 \;\; psi$$

i) STEP 9 – Calculate the allowable pressure based on rupture of the crown, P_{ac}.

$$P_{ac} = \frac{2SE}{\frac{L}{t} + 0.5} = \frac{2(18000)(1.0)}{\frac{72.125}{0.5} + 0.5} = 248.7047 \;\; psi$$

j) STEP 10 – Calculate the maximum allowable internal pressure, P_a.

$$P_a = \min\left[P_{ak},\, P_{ac}\right] = \min\left[133.0447,\, 248.7047\right] = 133.0 \; psi$$

k) STEP 11 – If the allowable internal pressure computed from STEP 10 is greater than or equal to the design pressure, then the design is complete. If the allowable internal pressure computed from STEP 10 is less than the design pressure, then increase the head thickness and repeat STEPs 2 through 10.

The MAWP is $133.0\ psi$.

Combined Loadings – torispherical heads subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this calculation, the torispherical head shall be approximated as an equivalent spherical shell with a radius equal to L. In this example problem, the torispherical head is only subject to internal pressure.

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4.3.5 Example E4.3.5 – Elliptical Head

Determine the Maximum Allowable Working Pressure (MAWP) for the proposed seamless 2:1 elliptical head. The Category B joint joining the head to the shell is a Type 1 butt weld and has been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Temperature = $300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength at Design Temperature = $33600\ psi$

Evaluate per paragraph 4.3.7, and paragraph 4.3.6.

Calculate k using the uncorroded inside diameter D and depth of head h.

$$k = \frac{D}{2h} = \frac{90.0}{2(22.5)} = 2.0$$

Verify that the elliptical head diameter to height ratio, k, is within the established limits, permitting the use of the rules of VIII-2, paragraph 4.3.7.

$$1.7 \le \{k = 2\} \le 2.2 \qquad True$$

Determine the variables r and L using the uncorroded inside diameter, D.

$$r = D\left(\frac{0.5}{k} - 0.08\right) = 90.0\left(\frac{0.5}{2.0} - 0.08\right) = 15.3\ in$$

$$L = D(0.44k + 0.02) = 90.0(0.44(2.0) + 0.02) = 81.0\ in$$

Proceed with the design following the steps outlined in paragraph 4.3.6.

a) STEP 1 – Determine, D, assume values for L, r and t (determined from paragraph 4.3.7) and adjust for corrosion.

$$D = 90.0 + 2(Corrosion\ \ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

$$L = 81.0 + Corrosion\ \ Allowance = 81.0 + 0.125 = 81.125\ in$$

$$r = 15.3 + Corrosion\ \ Allowance = 15.3 + 0.125 = 15.425\ in$$

$$t = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

b) STEP 2 – Compute the head L/D, r/D, and L/t ratios and determine if the following equations are satisfied.

$$0.7 \le \left\{ \frac{L}{D} = \frac{81.125}{90.25} = 0.8989 \right\} \le 1.0 \qquad \textit{True}$$

$$\left\{ \frac{r}{D} = \frac{15.425}{90.25} = 0.1709 \right\} \ge 0.06 \qquad \textit{True}$$

$$20 \le \left\{ \frac{L}{t} = \frac{81.125}{1.000} = 81.125 \right\} \le 2000 \qquad \textit{True}$$

c) STEP 3 – Calculate the geometric constants β_{th}, ϕ_{th}, R_{th}.

$$\beta_{th} = \arccos\left[\frac{0.5D - r}{L - r}\right] = \arccos\left[\frac{0.5(90.25) - 15.425}{81.125 - 15.425}\right] = 1.1017 \ rad$$

$$\phi_{th} = \frac{\sqrt{Lt}}{r} = \frac{\sqrt{81.125(1.0)}}{15.425} = 0.5839 \ rad$$

Since $\phi_{th} < \beta_{th}$, calculate R_{th} as follows:

$$R_{th} = \frac{0.5D - r}{\cos[\beta_{th} - \phi_{th}]} + r = \frac{0.5(90.25) - 15.425}{\cos[1.1017 - 0.5839]} + 15.425 = 49.6057 \ in$$

d) STEP 4 – Compute the coefficients C_1 and C_2.

Since $\frac{r}{D} = 0.1709 > 0.08$, calculate C_1 and C_2 as follows:

$$C_1 = 0.692\left(\frac{r}{D}\right) + 0.605 = 0.692(0.1709) + 0.605 = 0.7233$$

$$C_2 = 1.46 - 2.6\left(\frac{r}{D}\right) = 1.46 - 2.6(0.1709) = 1.0157$$

e) STEP 5 – Calculate the value of internal pressure expected to produce elastic buckling of the knuckle, P_{eth}.

$$P_{eth} = \frac{C_1 E_T t^2}{C_2 R_{th}\left(\frac{R_{th}}{2} - r\right)} = \frac{(0.7233)(28.3E+06)(1.0)^2}{1.0157(49.6057)\left(\frac{49.6057}{2} - 15.425\right)} = 43321.6096 \ psi$$

f) STEP 6 – Calculate the value of internal pressure that will result in a maximum stress in the knuckle equal to the material yield strength, P_y.

$$P_y = \frac{C_3 t}{C_2 R_{th}\left(\frac{R_{th}}{2r} - 1\right)}$$

Since the allowable stress at design temperature is governed by time-independent properties, is C_3 the material yield strength at the design temperature, or $C_3 = S_y$.

$$P_y = \frac{33600(1.0)}{1.0157(49.6057)\left(\dfrac{49.6057}{2(15.425)} - 1\right)} = 1096.8927\ psi$$

g) STEP 7 – Calculate the value of internal pressure expected to result in a buckling failure of the knuckle, P_{ck}.

Calculate variable G:

$$G = \frac{P_{eth}}{P_y} = \frac{43321.6096}{1096.8927} = 39.4948$$

Since $G > 1.0$, calculate P_{ck} as follows:

$$P_{ck} = \left(\frac{0.77508G - 0.20354G^2 + 0.019274G^3}{1 + 0.19014G - 0.089534G^2 + 0.0093965G^3}\right)(P_y)$$

$$P_{ck} = \left(\frac{0.77508(39.4948) - 0.20354(39.4948)^2 + 0.019274(39.4948)^3}{1 + 0.19014(39.4948) - 0.089534(39.4948)^2 + 0.0093965(39.4948)^3}\right)(1096.8927)$$

$$P_{ck} = 2206.1634\ psi$$

h) STEP 8 – Calculate the allowable pressure based on a buckling failure of the knuckle, P_{ak}.

$$P_{ak} = \frac{P_{ck}}{1.5} = \frac{2206.1634}{1.5} = 1470.8\ psi$$

i) STEP 9 – Calculate the allowable pressure based on rupture of the crown, P_{ac}.

$$P_{ac} = \frac{2SE}{\dfrac{L}{t} + 0.5} = \frac{2(22400)(1.0)}{\dfrac{81.125}{1.0} + 0.5} = 548.9\ psi$$

j) STEP 10 – Calculate the maximum allowable internal pressure, P_a.

$$P_a = \min[P_{ak}, P_{ac}] = \min[1470.8, 548.9] = 548.9\ psi$$

k) STEP 11 – If the allowable internal pressure computed from STEP 10 is greater than or equal to the design pressure, then the design is complete. If the allowable internal pressure computed from STEP 10 is less than the design pressure, then increase the head thickness and repeat STEPs 2 through 10.

The MAWP is 548.9 psi.

Combined Loadings – ellipsoidal heads subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this calculation, the ellipsoidal head shall be approximated as an equivalent spherical shell with a radius equal to L. In this example problem, the ellipsoidal head is only subject to internal pressure.

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4.3.6 Example E4.3.6 – Combined Loadings and Allowable Stresses

Determine the maximum tensile stress of the proposed cylindrical shell section considering the following design conditions and specified applied loadings. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

<u>Vessel Data:</u>

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psig@300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0
- Axial Force = $-66152.5\ lbs$
- Net Section Bending Moment = $5.08E+06\ in-lbs$
- Torsional Moment = $0.0\ in-lbs$

Adjust variables for corrosion and determine outside dimensions.

$$D = 90.0 + 2(Corrosion\ \ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

$$R = \frac{D}{2} = 45.125\ in$$

$$t = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

$$D_o = 90.0 + 2(Uncorroded\ \ Thickness) = 90.0 + 2(1.125) = 92.25\ in$$

Evaluate per paragraph 4.3.10.

The loads transmitted to the cylindrical shell are given in the Table E4.3.6.3. Note that this table is given in terms of the load parameters shown in Table 4.1.1, and Table 4.1.2 (Table E4.3.6.1 and Table E4.3.6.2 of this example). The load factor, Ω_P, shown in in Table 4.1.2 is used to simulate the maximum anticipated operating pressure acting simultaneously with the occasional loads specified. For this example, problem, $\Omega_P = 0.9$. As shown in Table E4.3.6.2, the acceptance criteria are that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition.

In accordance with paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. By inspection of the results shown in Table E4.3.6.3 and Table E4.3.6.4, Design Load Combination 5 is determined to be a potential governing load combination. The pressure, net section axial force, and bending moment at the location of interest for Design Load Combination 5 are as follows.

$$\Omega_P P + P_s = 0.9P + P_s = 320.4\ \ psi$$

$$F_5 = -66152.5\ lbs$$

$$M_5 = 3048000\ in-lbs$$

Determine applicability of the rules of paragraph 4.3.10 based on satisfaction of the following requirements.

The section of interest is at least $2.5\sqrt{Rt}$ away from any major structural discontinuity.

$$2.5\sqrt{Rt} = 2.5\sqrt{(45.125)(1.0)} = 16.7938\ in$$

Shear force is not applicable.

The shell R/t ratio is greater than 3.0, or:

$$\left\{R/t = \frac{45.125}{1.0} = 45.125\right\} > 3.0 \qquad True$$

a) STEP 1 – Calculate the membrane stress for the cylindrical shell. Note that the maximum bending stress occurs at $\theta = 0.0\ deg$.

$$\sigma_{\theta m} = \frac{PD}{E(D_o - D)} = \frac{320.4(90.25)}{1.0(92.25-90.25)} = 14458.05\ psi$$

$$\sigma_{sm} = \frac{1}{E}\left(\frac{PD^2}{D_o^2 - D^2} + \frac{4F}{\pi\left(D_o^2 - D^2\right)} \pm \frac{32MD_o\cos[\theta]}{\pi\left(D_o^4 - D^4\right)}\right)$$

$$\sigma_{sm} = \frac{1}{1.0}\left(\frac{320.4(90.25)^2}{(92.25)^2-(90.25)^2} + \frac{4(-66152.5)}{\pi\left[(92.25)^2-(90.25)^2\right]} \pm \frac{32(3048000)(92.25)\cos[0.0]}{\pi\left[(92.25)^4-(90.25)^4\right]}\right)$$

$$\sigma_{sm} = \begin{cases} 7149.8028 + (-230.7616) + 471.1299 = 7390.1711\ psi \\ 7149.8028 + (-230.7616) - 471.1299 = 6447.9113\ psi \end{cases}$$

$$\tau = \frac{16M_tD_o}{\pi\left(D_o^4 - D^4\right)} = \frac{16(0.0)(92.25)}{\pi\left[(92.25)^4-(90.25)^4\right]} = 0.0\ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2}\right)$$

$$\sigma_1 = \begin{Bmatrix} 0.5\left(14458.05 + 7390.1711 + \sqrt{\left(14458.05 - 7390.1711\right)^2 + 4\left(0.0\right)^2}\right) = 14458.05\ psi \\ 0.5\left(14458.05 + 6447.9113 + \sqrt{\left(14458.05 - 6447.9113\right)^2 + 4\left(0.0\right)^2}\right) = 14458.05\ psi \end{Bmatrix}$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2}\right)$$

$$\sigma_2 = \begin{Bmatrix} 0.5\left(14458.05 + 7390.1711 - \sqrt{\left(14458.05 - 7390.1711\right)^2 + 4\left(0.0\right)^2}\right) \\ 0.5\left(14458.05 + 6447.9113 - \sqrt{\left(14458.05 - 6447.9113\right)^2 + 4\left(0.0\right)^2}\right) \end{Bmatrix}$$

$$\sigma_2 = \begin{Bmatrix} 7390.1711\ psi \\ 6447.9113\ psi \end{Bmatrix}$$

$$\sigma_3 = \sigma_r = 0.0\ psi \qquad \textit{For stress on the outside surface}$$

c) STEP 3 – At any point on the shell, the following limit shall be satisfied.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]^{0.5} \le S$$

$$\sigma_e = \begin{Bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} \left(14458.05 - 7390.1711\right)^2 + \left(7390.1711 - 0.0\right)^2 \\ +\left(0.0 - 14458.05\right)^2 \end{bmatrix}^{0.5} = 12522.1\ psi \\ \frac{1}{\sqrt{2}}\begin{bmatrix} \left(14458.05 - 6447.9113\right)^2 + \left(6447.9113 - 0.0\right)^2 \\ +\left(0.0 - 14458.05\right)^2 \end{bmatrix}^{0.5} = 12545.4\ psi \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_e = 12522.1\ psi \\ \sigma_e = 12545.4\ psi \end{Bmatrix} \le \left\{S = 22400\ psi\right\} \qquad \textit{True}$$

Since the equivalent stress is less than the acceptance criteria, the shell section is adequately designed.

d) STEP 4 – For cylindrical and conical shells, if the meridional stress, σ_{sm} is compressive, then Equation (4.3.45) shall be satisfied where F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2 with $\lambda = 0.15$.

STEP 4 is not necessary in this example because the meridional stress, σ_{sm}, is tensile.

Table E4.3.6.1 – Design Loads from VIII-2

Table 4.1.1 – Design Loads	
Design Load Parameter	**Description**
P	Internal of External Specified Design Pressure (see paragraph 4.1.5.2.a)
P_s	Static head from liquid or bulk materials (e.g., catalyst)
D	Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following: • Weight of vessel including internals, supports (e.g., skirts, lugs, saddles, and legs), and appurtenances (e.g., platforms, ladders, etc.) • Weight of vessel contents under design and test conditions • Refractory linings, insulation • Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping • Transportation loads (the static forces obtained as equivalent to the dynamic loads experienced during normal operation of a transport vessel [see paragraph 1.2.1.3(b)]
L	• Appurtenance live loading • Effects of fluid flow, steady state or transient • Loads resulting from wave action
E	Earthquake loads [see paragraph 4.1.5.3(b)]
W	Wind loads [see paragraph 4.1.5.3(b)]
S_s	Snow loads
F	Loads due to Deflagration

Table E4.3.6.2 – Design Load Combinations from VIII-2

Table 4.1.2 – Design Load Combinations	
Design Load Combination [Note (1) and (2)]	**General Primary Membrane Allowable Stress [Note (3)]**
(1) $P + P_s + D$	S
(2) $P + P_s + D + L$	S
(3) $P + P_s + D + S_s$	S
(4) $\Omega P + P_s + D + 0.75L + 0.75S_s$	S
(5) $\Omega P + P_s + D + (0.6W \; or \; 0.7E)$	S
(6)$\Omega P + P_s + D + 0.75(0.6W \; or \; 0.7E) + 0.75L + 0.75S_s$	S
(7) $0.6D + (0.6W \; or \; 0.7E)$ (Note (4))	S
(8) $P_s + D + F$	See Annex 4-D
(9) $Other\ load\ combinations\ as\ defined\ in\ the\ UDS$	S

Notes:

1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.
2) See paragraph 4.1.5.3 for additional requirements.
3) S is the allowable stress for the load case combination [see paragraph 4.1.5.3(c)].
4) This load combination addresses an overturning condition for foundation design. It does not apply to design of anchorage (if any) to the foundation. Refer to ASCE/SEI 7, 2.4.1 Exception 2 for an additional reduction to W that may be applicable.

Table E4.3.6.3 – Design Loads (Net-Section Axial Force and Bending Moment) at the Location of Interest

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)	$P = 356.0$
P_s	Static head from liquid or bulk materials (e.g., catalyst)	$P_s = 0.0$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest	$D_F = -66152.5\ lbs$ $D_M = 0.0\ in - lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = 0.0\ lbs$ $L_M = 0.0\ in - lbs$
E	Earthquake loads	$E_F = 0.0\ lbs$ $E_M = 0.0\ in - lbs$
W	Wind Loads	$W_F = 0.0\ lbs$ $W_M = 5.08E + 0.6\ in - lbs$
S_s	Snow Loads	$S_F = 0.0\ lbs$ $S_M = 0.0\ in - lbs$
F	Loads due to Deflagration	$F_F = 0.0\ lbs$ $F_M = 0.0\ in - lbs$

Based on these loads, the shell is required to be designed for the design load combinations shown in Table E4.3.6.4. Note that this table is given in terms of the load combinations shown in Table 4.1.2 (Table E4.3.6.2 of this example).

Table E4.3.6.4 – Design Load Combination at the Location of Interest

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P + P_s + D$	$P + P_s = 356.0\ psi$ $F_1 = -66152.5\ lbs$ $M_1 = 0.0\ in-lbs$	S
2	$P + P_s + D + L$	$P + P_s = 356.0\ psi$ $F_2 = -66152.5\ lbs$ $M_2 = 0.0\ in-lbs$	S
3	$P + P_s + D + S_s$	$P + P_s = 356.0\ psi$ $F_3 = -66152.5\ lbs$ $M_3 = 0.0\ in-lbs$	S
4	$0.9P + P_s + D + 0.75L + 0.75S_s$	$0.9P + P_s = 320.4\ psi$ $F_4 = -66152.5\ lbs$ $M_4 = 0.0\ in-lbs$	S
5	$0.9P + P_s + D + (0.6W\ or\ 0.7E)$	$0.9P + P_s = 320.4\ psi$ $F_5 = -66152.5\ lbs$ $M_5 = 3048000\ in-lbs$	S
6	$(0.9P + P_s + D + 0.75(0.6W\ or\ 0.7E) + 0.75L + 0.75S_s)$	$0.9P + P_s = 320.4\ psi$ $F_6 = -66152.5\ lbs$ $M_6 = 2286000\ in-lbs$	S
7	$0.6D + (0.6W\ or\ 0.7E)$	$F_7 = -39691.5\ lbs$ $M_7 = 3048000\ in-lbs$	S
8	$P_s + D + F$	$P_s = 0.0\ psi$ $F_8 = -66152.5\ lbs$ $M_8 = 0.0\ in-lbs$	See Annex 4-D

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4.3.7 Example E4.3.7 – Conical Transitions Without a Knuckle

Determine if the proposed large and small end cylinder-to-cone transitions are adequately designed considering the following design conditions and applied forces and moments. Evaluate the stresses in the cylinder and cone at both the large and small end junction.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psig@300°F$
- Inside Radius (Large End) = $75.0\ in$
- Thickness (Large End) = $1.8125\ in$
- Cylinder Length (Large End) = $60.0\ in$
- Inside Radius (Small End) = $45.0\ in$
- Thickness (Small End) = $1.125\ in$
- Cylinder Length (Small End) = $48.0\ in$
- Thickness (Conical Section) = $1.9375\ in$
- Length of Conical Section = $78.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle (See Figure E4.3.7) = $21.0375\ deg$
- Axial Force (Large End) = $-99167\ lbs$
- Net Section Bending Moment (Large End) = $5.406E+06\ in-lbs$
- Axial Force (Small End) = $-78104\ lbs$
- Net Section Bending Moment (Small End) = $4.301E+06\ in-lbs$

Adjust variables for corrosion.

$$R_L = 75.0 + Corrosion\ \ Allowance = 75.0 + 0.125 = 75.125\ in$$
$$R_S = 45.0 + Corrosion\ \ Allowance = 45.0 + 0.125 = 45.125\ in$$
$$t_L = 1.8125 - Corrosion\ \ Allowance = 1.8125 - 0.125 = 1.6875\ in$$
$$t_S = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$
$$t_C = 1.9375 - Corrosion\ \ Allowance = 1.9375 - 0.125 = 1.8125\ in$$
$$\alpha = 21.0375\ deg$$

Evaluate per paragraph 4.3.11.

Per paragraph 4.3.11.3, the length of the conical shell, measured parallel to the surface of the cone, shall be equal to or greater than the following:

$$L_C \geq 2.0\sqrt{\frac{R_L t_C}{\cos[\alpha]}} + 1.4\sqrt{\frac{R_S t_C}{\cos[\alpha]}}$$

$$\{L_C = 78.0\ in\} \geq \left\{2.0\sqrt{\frac{75.125(1.8125)}{\cos[21.0375]}} + 1.4\sqrt{\frac{45.125(1.8125)}{\cos[21.0375]}} = 37.2624\ in\right\} \qquad True$$

Evaluate the Large End cylinder-to-cone junction per paragraph 4.3.11.4.

a) STEP 1 – Compute the required thickness of the cylinder at the large end of the cone-to-cylinder junction using paragraph 4.3.3., and select the nominal thickness, t_L (as specified in design conditions).

$$t_L = 1.6875 \; in$$

b) STEP 2 – Determine the cone half-apex angle, α, compute the required thickness of the cone at the large end of the cone-to-cylinder junction using paragraph 4.3.4., and select the nominal thickness, t_C (as specified in design conditions).

$$\alpha = 21.0375 \; deg$$

$$t_C = 1.8125 \; in$$

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with Part 5. In the calculations, if $0 \; deg < \alpha \leq 10 \; deg$, then use $\alpha = 10 \; deg$.

$$20 \leq \left\{ \frac{R_L}{t_L} = \frac{75.125}{1.6875} = 44.5185 \right\} \leq 500 \qquad True$$

$$1 \leq \left\{ \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741 \right\} \leq 2 \qquad True$$

$$\left\{ \alpha = 21.0375 \; deg \right\} \leq \left\{ 60 \; deg \right\} \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force, F_L. Calculate the equivalent line load, X_L, using the specified net section axial force, F_L, and bending moment, M_L.

$$X_L = \frac{F_L}{2\pi R_L} \pm \frac{M_L}{\pi R_L^2} = \left\{ \begin{array}{l} \dfrac{-99167}{2\pi(75.125)} + \dfrac{5406000}{\pi(75.125)^2} = 94.8111 \; \dfrac{lbs}{in} \\ \dfrac{-99167}{2\pi(75.125)} - \dfrac{5406000}{\pi(75.125)^2} = -514.9886 \; \dfrac{lbs}{in} \end{array} \right\}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per Table 4.3.3, and Table 4.3.4, respectively. For calculated values of n other than those presented in Table 4.3.3 and Table 4.3.4, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741$$

$$H = \sqrt{\frac{R_L}{t_L}} = \sqrt{\frac{75.125}{1.6875}} = 6.6722$$

$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in Table 4.3.3 and Table 4.3.4 is required. The results of the interpolation are summarized with the following values for C_i.

Equation Coefficients C_i	VIII-2, Table 4.3.3		VIII-2, Table 4.3.4	
	Pressure Applied Junction Moment Resultant M_{sN}	Pressure Applied Junction Shear Force Resultant Q_N	Equivalent Line Load Junction Moment Resultant M_{sN}	Equivalent Line Load Junction Shear Force Resultant Q_N
1	-3.079751	-1.962370	-5.706141	-4.878520
2	3.662099	2.375540	0.004705	0.006808
3	0.788301	0.582454	0.474988	-0.018569
4	-0.226515	-0.107299	-0.213112	-0.197037
5	-0.080019	-0.103635	2.241065	2.033876
6	0.049314	0.151522	0.000025	-0.000085
7	0.026266	0.010704	0.002759	-0.000109
8	-0.015486	-0.018356	-0.001786	-0.004071
9	0.001773	0.006551	-0.214046	-0.208830
10	-0.007868	-0.021739	0.000065	-0.000781
11	---	---	-0.106223	0.004724

For the applied pressure case both M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\left[\begin{array}{l} C_1 + C_2\ln[H] + C_3\ln[B] + C_4\left(\ln[H]\right)^2 + C_5\left(\ln[B]\right)^2 + \\ C_6\ln[H]\ln[B] + C_7\left(\ln[H]\right)^3 + C_8\left(\ln[B]\right)^3 + \\ C_9\ln[H]\left(\ln[B]\right)^2 + C_{10}\left(\ln[H]\right)^2\ln[B] \end{array}\right]$$

This results in:

$$M_{sN} = -\exp\left[\begin{array}{l} -3.079751 + 3.662099 \cdot \ln[6.6722] + 0.788301 \cdot \ln[0.3846] + \\ (-0.226515)(\ln[6.6722])^2 + (-0.080019)(\ln[0.3846])^2 + \\ 0.049314 \cdot \ln[6.6722] \cdot \ln[0.3846] + \\ 0.026266(\ln[6.6722])^3 + (-0.015486)(\ln[0.3846])^3 + \\ 0.001773 \cdot \ln[6.6722] \cdot (\ln[0.3846])^2 + \\ (-0.007868)(\ln[6.6722])^2 \cdot \ln[0.3846] \end{array}\right] = -10.6148$$

$$Q_N = -\exp\left[\begin{array}{l} -1.962370 + 2.375540 \cdot \ln[6.6722] + 0.582454 \cdot \ln[0.3846] + \\ (-0.107299)(\ln[6.6722])^2 + (-0.103635)(\ln[0.3846])^2 + \\ 0.151522 \cdot \ln[6.6722] \cdot \ln[0.3846] + \\ 0.010704(\ln[6.6722])^3 + (-0.018356)(\ln[0.3846])^3 + \\ 0.006551 \cdot \ln[6.6722] \cdot (\ln[0.3846])^2 + \\ (-0.021739)(\ln[6.6722])^2 \cdot \ln[0.3846] \end{array}\right] = -4.0925$$

For the Equivalent Line Load case, M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\left[\frac{\left(\begin{array}{l} C_1 + C_3 \ln[H^2] + C_5 \ln[\alpha] + C_7 (\ln[H^2])^2 + \\ C_9 (\ln[\alpha])^2 + C_{11} \ln[H^2] \ln[\alpha] \end{array}\right)}{\left(\begin{array}{l} 1 + C_2 \ln[H^2] + C_4 \ln[\alpha] + C_6 (\ln[H^2])^2 + \\ C_8 (\ln[\alpha])^2 + C_{10} \ln[H^2] \ln[\alpha] \end{array}\right)}\right]$$

This results in:

$$M_{sN} = -\exp\left[\frac{\left(\begin{array}{l} -5.706141 + 0.474988 \cdot \ln\left[6.6722^2\right] + \\ 2.241065 \cdot \ln\left[21.0375\right] + 0.002759\left(\ln\left[6.6722^2\right]\right)^2 + \\ (-0.214046)\left(\ln\left[21.0375\right]\right)^2 + \\ (-0.106223)\ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{array}\right)}{\left(\begin{array}{l} 1 + 0.004705 \cdot \ln\left[6.6722^2\right] + (-0.213112)\ln\left[21.0375\right] + \\ 0.000025\left(\ln\left[6.6722^2\right]\right)^2 + (-0.001786)\left(\ln\left[21.0375\right]\right)^2 + \\ 0.000065 \cdot \ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{array}\right)}\right] = -0.4912$$

$$Q_N = -\exp\left[\frac{\left(\begin{array}{l} -4.878520 + (-0.018569)\ln\left[6.6722^2\right] + \\ 2.033876 \cdot \ln\left[21.0375\right] + (-0.000109)\left(\ln\left[6.6722^2\right]\right)^2 + \\ (-0.208830)\left(\ln\left[21.0375\right]\right)^2 + \\ 0.004724 \cdot \ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{array}\right)}{\left(\begin{array}{l} 1 + 0.006808 \cdot \ln\left[6.6722^2\right] + (-0.197037)\ln\left[21.0375\right] + \\ (-0.000085)\left(\ln\left[6.6722^2\right]\right)^2 + (-0.004071)\left(\ln\left[21.0375\right]\right)^2 + \\ (-0.000781)\ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{array}\right)}\right] = -0.1845$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$\begin{array}{lll} \textit{Internal Pressure}: & M_{sN} = -10.6148, & Q_N = -4.0925 \\ \textit{Equivalent Line Load}: & M_{sN} = -0.4912, & Q_N = -0.1845 \end{array}$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in Table 4.3.1 for the Large End Junction.

Evaluate the Cylinder at the Large End:

Stress Resultant Calculations:

$$M_{sP} = Pt_L^2 M_{sN} = 356(1.6875)^2(-10.6148) = -10760.9194 \ \frac{in-lbs}{in}$$

$$M_{sX} = X_L t_L M_{sN} = \begin{cases} 94.8111(1.6875)(-0.4912) = -78.5889 \ \dfrac{in-lbs}{in} \\ -514.9886(1.6875)(-0.4912) = 426.8741 \ \dfrac{in-lbs}{in} \end{cases}$$

$$M_s = M_{sP} + M_{sX} = \begin{cases} -10760.9194 + (-78.5889) = -10839.5083 \ \dfrac{in-lbs}{in} \\ -10760.9194 + 426.8741 = -10334.0453 \ \dfrac{in-lbs}{in} \end{cases}$$

$$Q_P = Pt_L Q_N = 356(1.6875)(-4.0925) = -2458.5694 \ \frac{lbs}{in}$$

$$Q_X = X_L Q_N = \begin{cases} 94.8111(-0.1845) = -17.4926 \ \dfrac{lbs}{in} \\ -514.9886(-0.1845) = 95.0154 \ \dfrac{lbs}{in} \end{cases}$$

$$Q = Q_P + Q_X = \begin{cases} -2458.5694 + (-17.4926) = -2476.0620 \ \dfrac{lbs}{in} \\ -2458.5694 + 95.0154 = -2363.5540 \ \dfrac{lbs}{in} \end{cases}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_L^2 t_L^2}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(75.125)^2(1.6875)^2}\right]^{0.25} = 0.1142 \ in^{-1}$$

$$N_s = \frac{PR_L}{2} + X_L = \begin{cases} \dfrac{356(75.125)}{2} + 94.8111 = 13467.0611 \ \dfrac{lbs}{in} \\ \dfrac{356(75.125)}{2} + (-514.9886) = 12857.2614 \ \dfrac{lbs}{in} \end{cases}$$

$$N_\theta = PR_L + 2\beta_{cy}R_L\left(-M_s\beta_{cy} + Q\right)$$

$$N_\theta = \begin{Bmatrix} 356(75.125)+2(0.1142)(75.125)\left(-(-10839.5083)(0.1142)+(-2476.0620)\right) \\ 356(75.125)+2(0.1142)(75.125)\left(-(-10334.0453)(0.1142)+(-2363.553)\right) \end{Bmatrix}$$

$$N_\theta = \begin{Bmatrix} 5498.9524\ \dfrac{lbs}{in} \\ 6438.9685\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{pc} = 1.0$$

Stress Calculations:

Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_s}{t_L} = \begin{Bmatrix} \dfrac{13467.0611}{1.6875} = 7980.4807\ psi \\ \dfrac{12857.2614}{1.6875} = 7619.1179\ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_s}{t_L^2K_{pc}} = \begin{Bmatrix} \dfrac{6(-10839.5083)}{(1.6875)^2(1.0)} = -22838.7994\ psi \\ \dfrac{6(-10334.0453)}{(1.6875)^2(1.0)} = -21773.7909\ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_L} = \begin{Bmatrix} \dfrac{5498.9524}{1.6875} = 3258.6385\ psi \\ \dfrac{6438.9685}{1.6875} = 3815.6850\ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_L^2K_{pc}} = \begin{Bmatrix} \dfrac{6(0.3)(-10839.5083)}{(1.6875)^2(1.0)} = -6851.6398\ psi \\ \dfrac{6(0.3)(-10334.0453)}{(1.6875)^2(1.0)} = -6532.1373\ psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\left\{\begin{matrix} \sigma_{sm} = 7980.4807\ psi \\ \sigma_{sm} = 7619.1179\ psi \end{matrix}\right\} \leq \left\{1.5S = 1.5(22400) = 33600\ psi\right\} \qquad True$$

$$\left\{\begin{matrix} \sigma_{sm} + \sigma_{sb} = 7980.4807 + (-22838.7994) = \left|-14858.3\ psi\right| \\ \sigma_{sm} - \sigma_{sb} = 7980.4807 - (-22838.7994) = 30819.3\ psi \\ \sigma_{sm} + \sigma_{sb} = 7619.1179 + (-21773.7909) = \left|-14154.7\ psi\right| \\ \sigma_{sm} - \sigma_{sb} = 7619.1179 - (-21773.7909) = 29392.9\ psi \end{matrix}\right\} \leq \left\{S_{PS} = 67200\ psi\right\} \qquad True$$

$$\left\{\begin{matrix} \sigma_{\theta m} = 3258.6385\ psi \\ \sigma_{\theta m} = 3815.6850\ psi \end{matrix}\right\} \leq \left\{1.5S = 1.5(22400) = 33600\ psi\right\} \qquad True$$

$$\left\{\begin{matrix} \sigma_{\theta m} + \sigma_{\theta b} = 3258.6385 + (-6851.6398) = \left|-3593.0\ psi\right| \\ \sigma_{\theta m} - \sigma_{\theta b} = 3258.6385 - (-6851.6398) = 10110.3\ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 3815.6850 + (-6532.1373) = \left|-2716.5\ psi\right| \\ \sigma_{\theta m} - \sigma_{\theta b} = 3815.6850 - (-6532.1373) = 10347.8\ psi \end{matrix}\right\} \leq \left\{S_{PS} = 67200\ psi\right\} \qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cylinder at the cylinder-to-cone junction at the large end is adequately designed.

Evaluate the Cone at the Large End:

Stress Resultant Calculations – as determined above:

$$M_{csP} = M_{sP} = -10760.9194\ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \left\{\begin{matrix} -78.5889\ \dfrac{in-lbs}{in} \\ 426.8741\ \dfrac{in-lbs}{in} \end{matrix}\right\}$$

$$M_{cs} = M_{csP} + M_{csX} = \left\{\begin{matrix} -10760.9194 + (-78.5889) = -10839.5083\ \dfrac{in-lbs}{in} \\ -10760.9194 + 426.8741 = -10334.0453\ \dfrac{in-lbs}{in} \end{matrix}\right\}$$

$$Q_c = Q\cos[\alpha] + N_s\sin[\alpha]$$

$$Q_c = \left\{\begin{matrix} \left((-2476.0620)\cos[21.0375] + (13467.0611)\sin[21.0375]\right) = 2523.3690\ \dfrac{lbs}{in} \\ \left((-2363.5540)\cos[21.0375] + (12857.2614)\sin[21.0375]\right) = 2409.4726\ \dfrac{lbs}{in} \end{matrix}\right\}$$

$$R_C = \frac{R_L}{\cos[\alpha]} = \frac{75.125}{\cos[21.0375]} = 80.4900 \; in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_C^2 t_C^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(80.4900)^2(1.8125)^2}\right]^{0.25} = 0.1064 \; in^{-1}$$

$$N_{cs} = N_s \cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{cases} (13467.0611)\cos[21.0375] - (-2476.0620)\sin[21.0375] = 13458.2772 \; \dfrac{lbs}{in} \\ (12857.2614)\cos[21.0375] - (-2363.5540)\sin[21.0375] = 12848.7353 \; \dfrac{lbs}{in} \end{cases}$$

$$N_{c\theta} = \frac{PR_L}{\cos[\alpha]} + 2\beta_{co}R_C\left(-M_{cs}\beta_{co} - Q_c\right)$$

$$N_{c\theta} = \begin{cases} \dfrac{356(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(-10839.5083)(0.1064) - 2523.3690\right) \\ \dfrac{356(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(-10334.0453)(0.1064) - 2409.4726\right) \end{cases}$$

$$N_{c\theta} = \begin{cases} 5187.9337 \; \dfrac{lbs}{in} \\ 6217.6021 \; \dfrac{lbs}{in} \end{cases}$$

$$K_{cpc} = 1.0$$

Stress Calculations:

Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \begin{Bmatrix} \dfrac{13458.2772}{1.8125} = 7425.2564 \ psi \\ \dfrac{12848.7353}{1.8125} = 7088.9574 \ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(-10839.5083)}{(1.8125)^2(1.0)} = -19797.2470 \ psi \\ \dfrac{6(-10334.0453)}{(1.8125)^2(1.0)} = -18874.0708 \ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \begin{Bmatrix} \dfrac{5187.9337}{1.8125} = 2862.3082 \ psi \\ \dfrac{6217.6021}{1.8125} = 3430.4012 \ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(0.3)(-10839.5083)}{(1.8125)^2(1.0)} = -5939.1741 \ psi \\ \dfrac{6(0.3)(-10334.0453)}{(1.8125)^2(1.0)} = -5662.2213 \ psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\begin{Bmatrix} \sigma_{sm} = 7425.2564 \ psi \\ \sigma_{sm} = 7088.9574 \ psi \end{Bmatrix} \le \left\{1.5S = 1.5(22400) = 33600 \ psi\right\} \qquad True$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = 7425.2564 + (-19797.2470) = \left|-12371.9906 \ psi\right| \\ \sigma_{sm} - \sigma_{sb} = 7425.2564 - (-19797.2470) = 27222.5034 \ psi \\ \sigma_{sm} + \sigma_{sb} = 7088.9574 + (-18874.0708) = \left|-11785.1 \ psi\right| \\ \sigma_{sm} - \sigma_{sb} = 7088.9574 - (-18874.0708) = 25963.0 \ psi \end{Bmatrix} \le \left\{S_{PS} = 67200 \ psi\right\} True$$

$$\begin{Bmatrix} \sigma_{\theta m} = 2862.3082 \ psi \\ \sigma_{\theta m} = 3430.4012 \ psi \end{Bmatrix} \le \left\{1.5S = 1.5(22400) = 33600 \ psi\right\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = 2862.3082 + (-5939.1741) = \left|-3076.8659 \ psi\right| \\ \sigma_{\theta m} - \sigma_{\theta b} = 2862.3082 - (-5939.1741) = 8801.4823 \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 3430.4012 + (-5662.2213) = \left|-2231.8 \ psi\right| \\ \sigma_{\theta m} - \sigma_{\theta b} = 3430.4012 - (-5662.2213) = 9092.6 \ psi \end{Bmatrix} \le \left\{S_{PS} = 67200 \ psi\right\} \quad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cone at the cylinder-to-cone junction at the large end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore, the design is complete.

Evaluate the Small End cylinder-to-cone junction per paragraph 4.3.11.5.

a) STEP 1 – Compute the required thickness of the cylinder at the small end of the cone-to-cylinder junction using paragraph 4.3.3., and select the nominal thickness, t_S, (as specified in design conditions).

b) STEP 2 – Determine the cone half-apex angle, α, compute the required thickness of the cone at the small end of the cone-to-cylinder junction using paragraph 4.3.4., t_C (as specified in design conditions).

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with Part 5. In the calculations, if $0\ deg < \alpha \leq 10\ deg$, then use $\alpha = 10\ deg$.

$$20 \leq \left(\frac{R_S}{t_S} = \frac{45.125}{1.0} = 45.125 \right) \leq 500 \qquad True$$

$$1 \leq \left(\frac{t_C}{t_S} = \frac{1.8125}{1.0} = 1.8125 \right) \leq 2 \qquad True$$

$$\{\alpha = 21.0375\ deg\} \leq \{60\ deg\} \qquad True$$

d) STEP 4 – Calculate the equivalent line load, X_S, given the net section axial force, F_S, and bending moment, M_S, applied at the conical transition.

$$X_S = \frac{F_S}{2\pi R_S} \pm \frac{M_S}{\pi R_S^2} = \begin{Bmatrix} \dfrac{-78104.2}{2\pi(45.125)} + \dfrac{4301000}{\pi(45.125)^2} = 396.8622\ \dfrac{lbs}{in} \\ \dfrac{-78104.2}{2\pi(45.125)} - \dfrac{4301000}{\pi(45.125)^2} = -947.8060\ \dfrac{lbs}{in} \end{Bmatrix}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per Table 4.3.5, and Table 4.3.6, respectively. For calculated values of n other than those presented in Table 4.3.5 and Table 4.3.6, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_S} = \frac{1.8125}{1.000} = 1.8125$$

$$H = \sqrt{\frac{R_S}{t_S}} = \sqrt{\frac{45.125}{1.000}} = 6.7175$$

$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in Table 4.3.5 and Table 4.3.6 is required. The result of the interpolation is summarized with the following values for C_i.

Equation Coefficients C_i	VIII-2, Table 4.3.5		VIII-2, Table 4.3.6	
	Pressure Applied Junction Moment Resultant M_{sN}	Pressure Applied Junction Shear Force Resultant Q_N	Equivalent Line Load Junction Moment Resultant M_{sN}	Equivalent Line Load Junction Shear Force Resultant Q_N
1	-15.144683	0.569891	0.006792	-0.408044
2	3.036812	-0.000027	0.000290	0.021200
3	6.460714	0.008431	-0.000928	-0.325518
4	-0.155909	0.002690	0.121611	-0.003988
5	-1.462862	-0.002884	0.010581	-0.111262
6	-0.369444	0.000000	-0.000011	0.002204
7	0.007742	-0.000005	-0.000008	0.000255
8	0.143191	-0.000117	0.005957	-0.014431
9	0.040944	-0.000087	0.001310	0.000820
10	0.007178	0.000001	0.000186	0.000106
11	---	-0.003778	0.194433	---

For the applied pressure case M_{sN} is calculated using the following equation:

$$M_{sN} = \exp\left[\begin{array}{l} C_1 + C_2\ln\left[H^2\right] + C_3\ln\left[\alpha\right] + C_4\left(\ln\left[H^2\right]\right)^2 + C_5\left(\ln\left[\alpha\right]\right)^2 + \\ C_6\ln\left[H^2\right]\ln\left[\alpha\right] + C_7\left(\ln\left[H^2\right]\right)^3 + C_8\left(\ln\left[\alpha\right]\right)^3 + \\ C_9\ln\left[H^2\right]\left(\ln\left[\alpha\right]\right)^2 + C_{10}\left(\ln\left[H^2\right]\right)^2\ln\left[\alpha\right] \end{array}\right]$$

This results in:

$$M_{sN} = \exp\left[\begin{array}{l} -15.144683 + 3.036812 \cdot \ln\left[6.7175^2\right] + 6.460714 \cdot \ln\left[21.0375\right] + \\ (-0.155909)\left(\ln\left[6.7175^2\right]\right)^2 + (-1.462862)\left(\ln\left[21.0375\right]\right)^2 + \\ (-0.369444)\ln\left[6.7175^2\right] \cdot \ln\left[21.0375\right] + \\ 0.007742\left(\ln\left[6.7175^2\right]\right)^3 + 0.143191\left(\ln\left[21.0375\right]\right)^3 + \\ 0.040944 \cdot \ln\left[6.7175^2\right] \cdot \left(\ln\left[21.0375\right]\right)^2 + \\ 0.007178\left(\ln\left[6.7175^2\right]\right)^2 \cdot \ln\left[21.0375\right] \end{array}\right] = 9.2135$$

For the applied pressure case Q_N is calculated using the following equation,

$$Q_N = \left(\frac{C_1 + C_3 H^2 + C_5 \alpha + C_7 H^4 + C_9 \alpha^2 + C_{11} H^2 \alpha}{1 + C_2 H^2 + C_4 \alpha + C_6 H^4 + C_8 \alpha^2 + C_{10} H^2 \alpha}\right)$$

This results in:

$$Q_N = \left(\frac{\left(\begin{array}{l} 0.569891 + 0.008431(6.7175)^2 + (-0.002884)(21.0375) + \\ (-0.000005)(6.7175)^4 + (-0.000087)(21.0375)^2 + \\ (-0.003778)(6.7175)^2(21.0375) \end{array}\right)}{\left(\begin{array}{l} 1 + (-0.000027)(6.7175)^2 + 0.002690(21.0375) + \\ 0.000000(6.7175)^4 + (-0.000117)(21.0375)^2 + \\ 0.000001(6.7175)^2(21.0375) \end{array}\right)}\right) = -2.7333$$

For the Equivalent Line Load case, M_{sN} is calculated using the following equation,

$$M_{sN} = \left(\frac{C_1 + C_3 H + C_5 B + C_7 H^2 + C_9 B^2 + C_{11} HB}{1 + C_2 H + C_4 B + C_6 H^2 + C_8 B^2 + C_{10} HB}\right)$$

This results in:

$$M_{sN} = \left(\frac{\begin{pmatrix} 0.006792 + (-0.000928)(6.7175) + 0.010581(0.3846) + \\ (-0.000008)(6.7175)^2 + 0.001310(0.3846)^2 + \\ 0.194433(6.7175)(0.3846) \end{pmatrix}}{\begin{pmatrix} 1 + 0.000290(6.7175) + 0.121611(0.3846) + \\ (-0.000011)(6.7175)^2 + 0.005957(0.3846)^2 + \\ 0.000186(6.7175)(0.3846) \end{pmatrix}} \right) = 0.4828$$

For the Equivalent Line Load case, Q_N is calculated using the following equation,

$$Q_N = \begin{pmatrix} C_1 + C_2 \ln[H] + C_3 \ln[B] + C_4 (\ln[H])^2 + C_5 (\ln[B])^2 + C_6 \ln[H] \ln[B] + \\ C_7 (\ln[H])^3 + C_8 (\ln[B])^3 + C_9 \ln[H] (\ln[B])^2 + C_{10} (\ln[H])^2 \ln[B] \end{pmatrix}$$

This results in:

$$Q_N = \begin{pmatrix} -0.408044 + 0.021200 \cdot \ln[6.7175] + (-0.325518) \ln[0.3846] + \\ (-0.003988)(\ln[6.7175])^2 + (-0.111262)(\ln[0.3846])^2 + \\ 0.002204 \cdot \ln[6.7175] \cdot \ln[0.3846] + 0.000255 \cdot (\ln[6.7175])^3 + \\ (-0.014431)(\ln[0.3846])^3 + 0.000820 \cdot \ln[6.7175] \cdot (\ln[0.3846])^2 + \\ 0.000106 (\ln[6.7175])^2 \cdot \ln[0.3846] \end{pmatrix} = -0.1613$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$\begin{aligned} &\textit{Internal Pressure}: \quad && M_{sN} = 9.2135, \quad && Q_N = -2.7333 \\ &\textit{Equivalent Line Load}: \quad && M_{sN} = 0.4828, \quad && Q_N = -0.1613 \end{aligned}$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in Table 4.3.2 for the Small End Junction.

Evaluate the Cylinder at the Small End:

Stress Resultant Calculations:

$$M_{sP} = Pt_S^2 M_{sN} = 356\left(1.000\right)^2\left(9.2135\right) = 3280.0060\ \frac{in-lbs}{in}$$

$$M_{sX} = X_S t_S M_{sN} = \begin{cases} 396.8622\left(1.0000\right)\left(0.4828\right) = -191.6051\ \dfrac{in-lbs}{in} \\ -947.8060\left(1.0000\right)\left(0.4828\right) = -457.6007\ \dfrac{in-lbs}{in} \end{cases}$$

$$M_s = M_{sP} + M_{sX} = \begin{cases} 3280.0060 + \left(191.6051\right) = 3471.6111\ \dfrac{in-lbs}{in} \\ 3280.0060 + \left(-457.6007\right) = 2822.4053\ \dfrac{in-lbs}{in} \end{cases}$$

$$Q_P = Pt_S Q_N = 356\left(1.0000\right)\left(-2.7333\right) = -973.0548\ \frac{lbs}{in}$$

$$Q_X = X_S Q_N = \begin{cases} 396.8622\left(-0.1613\right) = -64.0139\ \dfrac{lbs}{in} \\ -947.8060\left(-0.1613\right) = 152.8811\ \dfrac{lbs}{in} \end{cases}$$

$$Q = Q_P + Q_X = \begin{cases} -973.0548 + \left(-64.0139\right) = -1037.0687\ \dfrac{lbs}{in} \\ -973.0548 + 152.8811 = -820.1737\ \dfrac{lbs}{in} \end{cases}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_S^2 t_S^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{\left(45.1250\right)^2\left(1.000\right)^2}\right]^{0.25} = 0.1914\ in^{-1}$$

$$N_s = \frac{PR_S}{2} + X_S = \begin{cases} \dfrac{356\left(45.125\right)}{2} + 396.8622 = 8429.1122\ \dfrac{lbs}{in} \\ \dfrac{356\left(45.125\right)}{2} + \left(-947.8060\right) = 7084.4440\ \dfrac{lbs}{in} \end{cases}$$

$$N_\theta = PR_S + 2\beta_{cy}R_S\left(-M_s\beta_{cy} - Q\right)$$

$$N_\theta = \begin{Bmatrix} 356(45.125) + 2(0.1914)(45.125)\left(-(3471.6111)(0.1914) - (-1037.0687)\right) \\ 356(45.125) + 2(0.1914)(45.125)\left(-(2822.4053)(0.1914) - (-820.1737)\right) \end{Bmatrix}$$

$$N_\theta = \begin{Bmatrix} 22500.7769 \ \dfrac{lbs}{in} \\ 20900.5790 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{pc} = 1.0$$

Stress Calculations:

Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_s}{t_S} = \begin{Bmatrix} \dfrac{8429.1122}{1.0000} = 8429.1122 \ psi \\ \dfrac{7084.4440}{1.0000} = 7084.4440 \ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(3471.6111)}{(1.0000)^2(1.0)} = 20829.6666 \ psi \\ \dfrac{6(2822.4053)}{(1.0000)^2(1.0)} = 16934.4318 \ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_S} = \begin{Bmatrix} \dfrac{22500.7769}{1.0000} = 22500.7769 \ psi \\ \dfrac{20900.5790}{1.0000} = 20900.5790 \ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(0.3)(3471.6111)}{(1.0000)^2(1.0)} = 6248.8999 \ psi \\ \dfrac{6(0.3)(2822.4053)}{(1.0000)^2(1.0)} = 5080.3295 \ psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\left\{\begin{matrix} \sigma_{sm} = 8429.1122 \ psi \\ \sigma_{sm} = 7084.4440 \ psi \end{matrix}\right\} \le \left\{1.5S = 1.5(22400) = 33600 \ psi\right\} \qquad True$$

$$\left\{\begin{matrix} \sigma_{sm} + \sigma_{sb} = 8429.1122 + (20829.6666) = 29258.8 \ psi \\ \sigma_{sm} - \sigma_{sb} = 8429.1122 - (20829.6666) = |-12400.6 \ psi| \\ \sigma_{sm} + \sigma_{sb} = 7084.4440 + (16934.4318) = 24018.9 \ psi \\ \sigma_{sm} - \sigma_{sb} = 7084.4440 - (16934.4318) = |-9850.0 \ psi| \end{matrix}\right\} \le \left\{S_{PS} = 67200 \ psi\right\} \qquad True$$

$$\left\{\begin{matrix} \sigma_{\theta m} = 22500.7769 \\ \sigma_{\theta m} = 20900.5790 \end{matrix}\right\} \le \left\{1.5S = 1.5(22400) = 33600 \ psi\right\} \qquad True$$

$$\left\{\begin{matrix} \sigma_{\theta m} + \sigma_{\theta b} = 22500.7769 + (6248.8999) = 28749.7 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 22500.7769 - (6248.8999) = 16251.9 \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 20900.5790 + (5080.3295) = 25981.0 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 20900.5790 - (5080.3295) = 15820.2 \ psi \end{matrix}\right\} \le \left\{S_{PS} = 67200 \ psi\right\} \qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cylinder at the cylinder-to-cone junction at the small end is adequately designed.

<u>Evaluate the Cone at the Small End:</u>

Stress Resultant Calculations:

$$M_{csP} = M_{sP} = 3280.0060 \ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \left\{\begin{matrix} 191.6051 \ \frac{in-lbs}{in} \\ -457.6007 \ \frac{in-lbs}{in} \end{matrix}\right\}$$

$$M_{cs} = M_{csP} + M_{csX} = \left\{\begin{matrix} 3280.0060 + 191.6051 = 3471.6111 \ \frac{in-lbs}{in} \\ 3280.0060 + (-457.6007) = 2822.4053 \ \frac{in-lbs}{in} \end{matrix}\right\}$$

$$Q_c = Q\cos[\alpha] + N_s \sin[\alpha]$$

$$Q_c = \left\{\begin{matrix} -1037.0687\cos[21.0375] + 8429.1122\sin[21.0375] = 2057.9298 \ \frac{lbs}{in} \\ -820.1737\cos[21.0375] + 7084.4440\sin[21.0375] = 1777.6603 \ \frac{lbs}{in} \end{matrix}\right\}$$

$$R_c = \frac{R_c}{\cos[\alpha]} = \frac{45.1250}{\cos[21.0375]} = 48.3476\ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_c^{\ 2} t_c^{\ 2}}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(48.3476)^2(1.8125)^2}\right]^{0.25} = 0.1373\ in^{-1}$$

$$N_{cs} = N_s \cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{cases} \left(8429.1122\cos[21.0375] - (-1037.0687)\sin[21.0375]\right) = 8239.5612\ \dfrac{lbs}{in} \\ \left(7084.4440\cos[21.0375] - (-820.1737)\sin[21.0375]\right) = 6906.6602\ \dfrac{lbs}{in} \end{cases}$$

$$N_{c\theta} = \frac{PR_s}{\cos[\alpha]} + 2\beta_{co}R_c\left(-M_{cs}\beta_{co} + Q_c\right)$$

$$N_{c\theta} = \begin{cases} \dfrac{356(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(3471.6111)(0.1373) + 2057.9298\right) \\ \dfrac{356(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(2822.4053)(0.1373) + 1777.6603\right) \end{cases}$$

$$N_{c\theta} = \begin{cases} 38205.1749\ \dfrac{lbs}{in} \\ 35667.6380\ \dfrac{lbs}{in} \end{cases}$$

$$K_{cpc} = 1.0$$

Stress Calculations:

Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \begin{cases} \dfrac{8239.5612}{1.8125} = 4545.9648\ psi \\ \dfrac{6906.6602}{1.8125} = 3810.5711\ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \begin{cases} \dfrac{6(3471.6111)}{(1.8125)^2(1.0)} = 6340.5406\ psi \\ \dfrac{6(2822.4053)}{(1.8125)^2(1.0)} = 5154.8330\ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \begin{cases} \dfrac{38205.1749}{1.8125} = 21078.7172\ psi \\ \dfrac{35667.6380}{1.8125} = 19678.6968\ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(0.3)(3471.6111)}{(1.8125)^2(1.0)} = 1902.1622\ psi \\ \dfrac{6(0.3)(2822.4053)}{(1.8125)^2(1.0)} = 1546.4499\ psi \end{Bmatrix}$$

Check Acceptable Criteria:

$$\begin{Bmatrix} \sigma_{sm} = 4545.9648\ psi \\ \sigma_{sm} = 3810.5711\ psi \end{Bmatrix} \le \{1.5S = 1.5(22400) = 33600\ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = 4545.9648 + (6340.5406) = 10886.5\ psi \\ \sigma_{sm} - \sigma_{sb} = 4545.9648 - (6340.5406) = |-1794.6\ psi| \\ \sigma_{sm} + \sigma_{sb} = 3810.5711 + (5154.8330) = 8965.4\ psi \\ \sigma_{sm} - \sigma_{sb} = 3810.5711 - (5154.8330) = |-1344.3\ psi| \end{Bmatrix} \le \{S_{PS} = 67200\ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} = 21078.7172 \\ \sigma_{\theta m} = 19678.6968 \end{Bmatrix} \le \{1.5S = 1.5(22400) = 33600\ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = 21078.7172 + (1902.1622) = 22980.9\ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 21078.7172 - (1902.1622) = 19176.6\ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 19678.6968 + (1546.4499) = 21225.1\ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 19678.6968 - (1546.4499) = 18132.2\ psi \end{Bmatrix} \le \{S_{PS} = 67200\ psi\} \qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cone at the cylinder-to-cone junction at the small end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore, the design is complete.

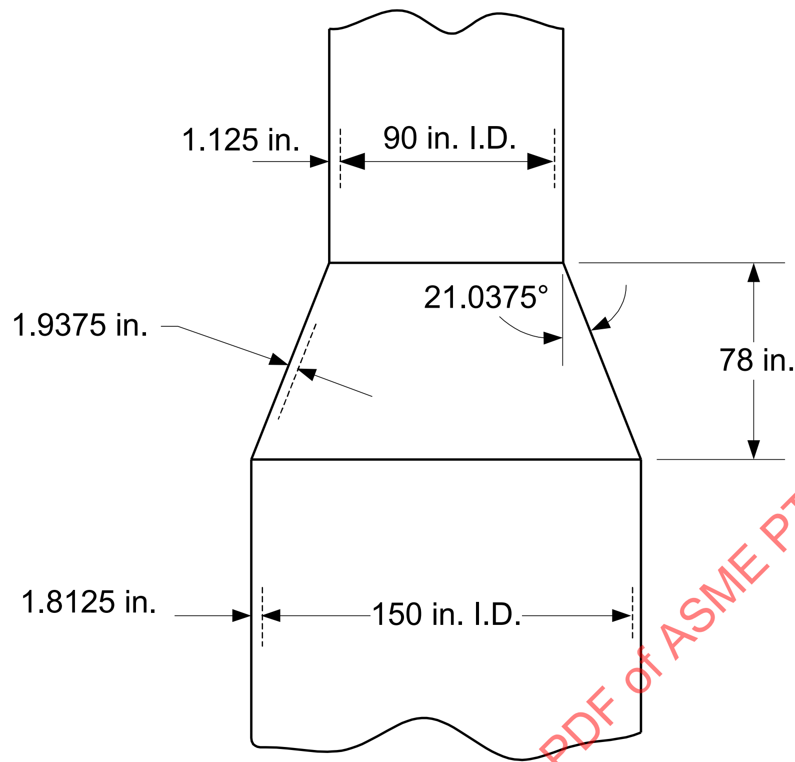

Figure E4.3.7 – Conical Transition

4.3.8 Example E4.3.8 – Conical Transitions with a Knuckle

Determine if the proposed design for the large end of a cylinder-to-cone junction with a knuckle is adequately designed considering the following design conditions and applied forces and moments, see Figure E4.3.8 for details.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $280\ psig@300°F$
- Inside Diameter (Large End) = $120.0\ in$
- Inside Radius (Large End) = $60.0\ in$
- Knuckle Radius = $10.0\ in$
- Large End Thickness = $1.0\ in$
- Cone Thickness = $1.0\ in$
- Knuckle Thickness = $1.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle = $30.0\ deg$
- Axial Force (Large End) = $-10000\ lbs$
- Net Section Bending Moment (Large End) = $2.0E+06\ in-lbs$

Evaluate per paragraph 4.3.12.

a) STEP 1 – Compute the required thickness of the cylinder at the large end of the cone-to-cylinder junction using paragraph 4.3.3., and select the nominal thickness, t_L (as specified in design conditions).

$$t_L = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{120.0}{2}\left(\exp\left[\frac{280.0}{22400.0}\right]-1\right) = 0.7547\ in$$

As specified in the design conditions,

$$t_L = 1.0\ in$$

Since the required thickness is less than the design thickness, the cylinder is adequately designed for internal pressure.

b) STEP 2 – Determine the cone half-apex angle, α, compute the required thickness of the cone at the large end of the cone-to-cylinder junction using paragraph 4.3.4., and select the nominal thickness, t_C.

$$\alpha = 30.0\ deg$$

$$t_C = \frac{D}{2\cos[\alpha]}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{120.0}{2\cos[30.0]}\left(\exp\left[\frac{280.0}{22400.0}\right]-1\right) = 0.8520\ in$$

As specified in the design conditions,

$$t_C = 1.0\ in$$

Since the required thickness is less than the design thickness, the cone is adequately designed for internal pressure.

c) STEP 3 – Proportion the transition geometry by assuming a value for the knuckle radius, r_k, and knuckle thickness, t_k, such that the following equations are satisfied. If all these equations cannot be satisfied, the cylinder-to-cone junction shall be designed in accordance with Part 5.

$$\{t_k = 1.0\ in\} \geq \{t_L = 1.0\ in\} \qquad True$$

$$\{r_k = 10.0\ in\} > \{3t_k = 3.0\ in\} \qquad True$$

$$\left\{\frac{r_k}{R_L} = \frac{10.0}{60.0} = 0.1667\right\} > \{0.03\} \qquad True$$

$$\{\alpha = 30\ deg\} \leq \{60\ deg\} \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition at the location of the knuckle. The thrust load due to pressure shall not be included as part of the axial force, F_L.

$$F_L = -10000\ lbs$$

$$M_L = 2.0E+06\ in-lbs$$

e) STEP 5 – Compute the stresses in the knuckle at the junction using the equations in Table 4.3.7.

Determine if the knuckle is compact or non-compact.

$$\alpha r_k < 2K_m\left(\left\{R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5} + r_k\right\}t_k\right)^{0.5}$$

$$\{0.5236(10.0)\} = \left\{2(0.7)\left(\left\{50.0\left((0.5236)^{-1}\tan[0.5236]\right)^{0.5} + 10\right\}1\right)^{0.5}\right\}$$

$$\{5.2360\ in\} < \{11.0683\ in\} \qquad True$$

where,

$$K_m = 0.7$$

$$\alpha = \frac{30.0}{180}\pi = 0.5236\ rad$$

$$R_k = R_L - r_k = 60.0 - 10.0 = 50.0\ in$$

Therefore, analyze the knuckle junction as a compact knuckle.

Stress Calculations:

Determine the hoop and axial membrane stresses at the knuckle:

$$\sigma_{\theta m} = \frac{PK_m\left(R_L\sqrt{R_L t_L} + L_k\sqrt{L_k t_C}\right) + \alpha\left(PL_{1k}r_k - 0.5P_e L_{1k}^2\right)}{K_m\left(t_L\sqrt{R_L t_L} + t_C\sqrt{L_k t_C}\right) + \alpha t_k r_k}$$

$$\sigma_{sm} = \frac{P_e L_{1k}}{2t_k}$$

where,

$$L_{1k} = R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5} + r_k = 50.0\left((0.5236)^{-1}\tan[0.5236]\right)^{0.5} + 10.0 = 62.5038\ in$$

$$L_k = \frac{R_k}{\cos[\alpha]} + r_k = \frac{50.0}{\cos[0.5236]} + 10.0 = 67.7351\ in$$

$$P_e = P + \frac{F_L}{\pi L_{1k}^2 \cos^2\left[\frac{\alpha}{2}\right]} \pm \frac{2M_L}{\pi L_{1k}^3 \cos^3\left[\frac{\alpha}{2}\right]}$$

$$P_e = \begin{Bmatrix} 280 + \dfrac{-10000.0}{\pi(62.5038)^2\cos^2\left[\dfrac{0.5236}{2}\right]} + \dfrac{2(2.0E+06)}{\pi(62.5038)^3\cos^3\left[\dfrac{0.5236}{2}\right]} \\ 280 + \dfrac{-10000.0}{\pi(62.5038)^2\cos^2\left[\dfrac{0.5236}{2}\right]} - \dfrac{2(2.0E+06)}{\pi(62.5038)^3\cos^3\left[\dfrac{0.5236}{2}\right]} \end{Bmatrix}$$

$$P_e = \begin{Bmatrix} 284.9125\ psi \\ 273.3410\ psi \end{Bmatrix}$$

therefore,

$$\sigma_{\theta m} = \begin{Bmatrix} \dfrac{\begin{pmatrix} 280(0.7)\left(60.0\sqrt{60.0(1.0)} + 67.7351\sqrt{67.7351(1.0)}\right) + \\ 0.5236\left(280(62.5038)(10.0) - 0.5(284.9125)(62.5038)^2\right) \end{pmatrix}}{0.7\left(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}\right) + 0.5236(1.0)(10.0)} = 35.8767\ psi \\ \dfrac{\begin{pmatrix} 280(0.7)\left(60.0\sqrt{60.0(1.0)} + 67.7351\sqrt{67.7351(1.0)}\right) + \\ 0.5236\left(280(62.5038)(10.0) - 0.5(273.3410)(62.5038)^2\right) \end{pmatrix}}{0.7\left(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}\right) + 0.5236(1.0)(10.0)} = 756.6825\ psi \end{Bmatrix}$$

and,

$$\sigma_{sm} = \begin{Bmatrix} \dfrac{P_e L_{1k}}{2t_k} = \dfrac{284.9125(62.5038)}{2(1.0)} = 8904.0570\ psi \\ \dfrac{P_e L_{1k}}{2t_k} = \dfrac{273.3410(62.5038)}{2(1.0)} = 8542.4256\ psi \end{Bmatrix}$$

Check Acceptable Criteria:

$$\begin{Bmatrix} \sigma_{\theta m} = 35.9 \ psi \\ \sigma_{\theta m} = 756.7 \ psi \end{Bmatrix} \leq \{S = 22400 \ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{sm} = 8904.1 \ psi \\ \sigma_{sm} = 8542.4 \ psi \end{Bmatrix} \leq \{S = 22400 \ psi\} \qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ in the knuckle are both tensile, the condition of local buckling need not be considered. Therefore, the knuckle at the cylinder-to-cone junction at the large end is adequately designed.

f) STEP 6 – The stress acceptance criterion in STEP 5 is satisfied for the knuckle. Therefore, the design is complete.

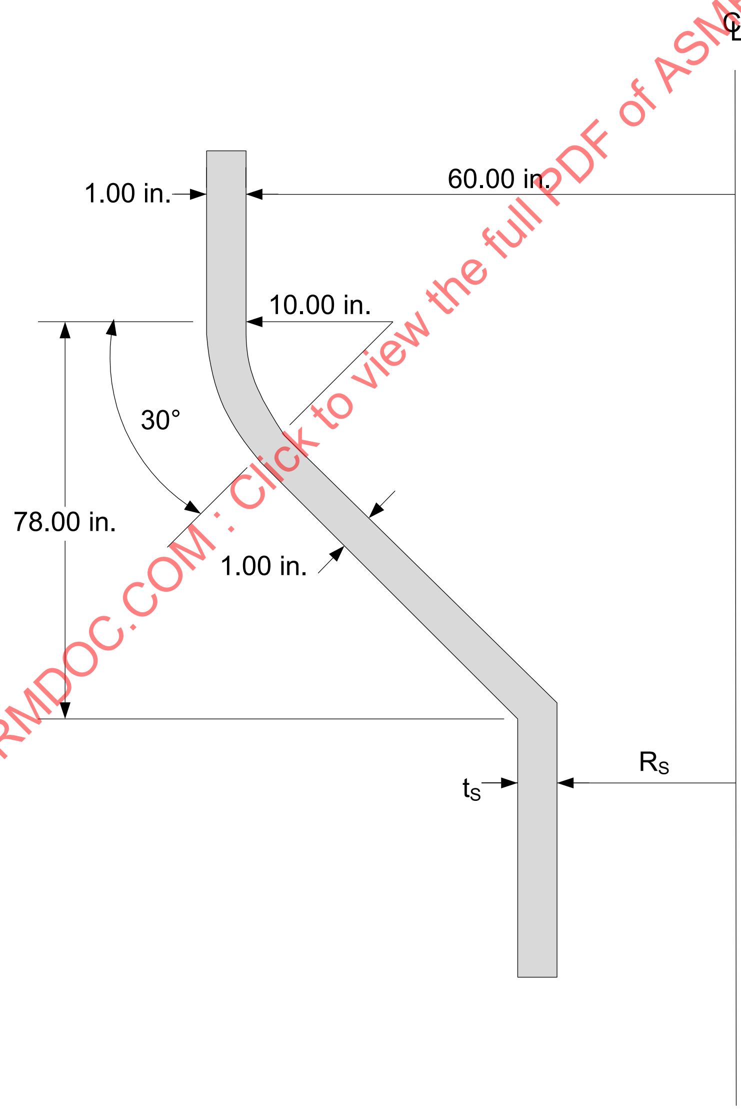

Figure E4.3.8 – Knuckle Detail

4.4 Shells Under External Pressure and Allowable Compressive Stresses

4.4.1 Example E4.4.1 – Cylindrical Shell

Determine the Maximum Allowable External Pressure (MAEP) for a cylindrical shell considering the following design conditions.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Temperature = $300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Unsupported Length = $636.0\ in$
- Mod of Elasticity at Design Temp = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$

Evaluate per paragraph 4.4.5.

a) STEP 1 – Assume an initial thickness, t, and unsupported length, L.

$$t = t - Corrosion\ Allowance = 1.125 - 0.125 = 1.0\ in$$

$$L = 636.0\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{1.6C_h E_y t}{D_o} = \frac{1.6(0.00919142)(28.3E+06)(1.0)}{92.25} = 4511.5189\ psi$$

where,

$$D_o = D + 2(Uncorroded\ Thickness) = 90.0 + 2(1.125) = 92.25\ in$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{\left(\frac{92.25}{2}\right)1.0}} = 93.6459$$

$$2\left(\frac{D_o}{t}\right)^{0.94} = 2\left(\frac{92.25}{1.0}\right)^{0.94} = 140.6366$$

Since $13 < M_x < 2(D_o/t)^{0.94}$, calculate C_h as follows:

$$C_h = 1.12M_x^{-1.058} = 1.12(93.6459)^{-1.058} = 0.00919142$$

c) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

1) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$F_{he} = 4511.5189\ psi$ (as determined in paragraph 4.4.5, STEP 2)

2) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{4511.5189}{28.3E+06} = 0.00015942$$

3) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

$$\frac{F_{ic}}{E_t} = A_e$$

Commentary:

The model used to develop the stress-strain curve is in paragraph 3-D.3. Though outside the scope of this example problem, the user is encouraged to develop the stress-strain curve for the material of interest at the design temperature. Several of the design parameters and supplemental variables used in the development of the stress-strain curve are also used in the determination of the tangent modulus.

Per paragraph 3-D.3: Stress-Strain Curve Model / Tangent Modulus Parameters

- Engineering Ultimate Tensile Stress at Temperature $\sigma_{uts} = 70000\ psi$
- Engineering Yield Stress at Temperature $\sigma_{ys} = 33600\ psi$
- 0.2% Engineering Offset Strain $\varepsilon_{ys} = 0.002$
- Stress-Strain Curve Fitting Parameter, see Table 3-D.1 $\varepsilon_p = 0.00002$
- Modulus of Elasticity at Temperature $E_y = 28.3E+06\ psi$

Engineering yield to engineering tensile ratio,

$$R = \frac{\sigma_{ys}}{\sigma_{uts}} = \frac{33600}{70000} = 0.4800$$

Curve fitting exponent for the stress-strain curve,

$$m_1 = \frac{\left[\ln[R] + (\varepsilon_p - \varepsilon_{ys})\right]}{\ln\left[\frac{\ln[1+\varepsilon_p]}{\ln[1+\varepsilon_{ys}]}\right]} = \frac{\left[\ln[0.48] + (0.00002 - 0.002)\right]}{\ln\left[\frac{\ln[1+0.00002]}{\ln[1+0.002]}\right]} = 0.15984367$$

Curve fitting constant for the elastic region of the stress-strain curve,

$$A_1 = \frac{\sigma_{ys}(1+\varepsilon_{ys})}{\left(\ln[1+\varepsilon_{ys}]\right)^{m_1}} = \frac{33600(1+0.002)}{\left(\ln[1+0.002]\right)^{0.1598}} = 90926.3215$$

Curve fitting exponent for the stress-strain curve, see Table 3-D.1,

$$m_2 = 0.6(1.00 - R) = 0.6(1.00 - 0.4800) = 0.3120$$

Curve fitting constant for the plastic region of the stress-strain curve,

$$A_2 = \frac{\sigma_{uts} \cdot \exp[m_2]}{m_2^{m_2}} = \frac{70000 \cdot \exp[0.3120]}{(0.3120)^{0.3120}} = 137537.6771$$

Material parameter for stress-strain curve model,

$$K = 1.5(R)^{1.5} - 0.5(R)^{2.5} - (R)^{3.5}$$

$$K = 1.5(0.4800)^{1.5} - 0.5(0.4800)^{2.5} - (0.4800)^{3.5} = 0.34239735$$

Stress-Strain curve fitting parameter,

$$H = \frac{2\left(\sigma_t - \left(\sigma_{ys} + K\left(\sigma_{uts} - \sigma_{ys}\right)\right)\right)}{K\left(\sigma_{uts} - \sigma_{ys}\right)} = \frac{2\left(70000 - \left(33600 + 0.3424(70000 - 33600)\right)\right)}{0.3424(70000 - 33600)}$$

$$H = 3.84116670$$

Per paragraph 3-D.5.1: Tangent Modulus Parameters.

- Tangent Modulus, E_t.
- Coefficient used in Tangent Modulus, D_1, D_2, D_3, D_4.

Tangent Modulus:

$$E_t = \left(\frac{1}{E_y} + D_1 + D_2 + D_3 + D_4\right)^{-1}$$

Coefficients:

$$D_1 = \frac{\sigma_t^{\left(\frac{1}{m_1} - 1\right)}}{2m_1 A_1^{\left(\frac{1}{m_1}\right)}}$$

$$D_2 = -\frac{1}{2}\left(\frac{1}{A_1^{\left(\frac{1}{m_1}\right)}}\right) \cdot \left(\sigma_t^{\left(\frac{1}{m_1}\right)}\left(\frac{2}{K\left(\sigma_{uts} - \sigma_{ys}\right)}\right)\left(1 - \tanh^2[H]\right) + \frac{1}{m_1}\sigma_t^{\left(\frac{1}{m_1} - 1\right)}\tanh[H]\right)$$

$$D_3 = \frac{\sigma_t^{\left(\frac{1}{m_2} - 1\right)}}{2m_2 A_2^{\left(\frac{1}{m_2}\right)}}$$

$$D_4 = -\frac{1}{2}\left(\frac{1}{A_2^{\left(\frac{1}{m_2}\right)}}\right) \cdot \left(\sigma_t^{\left(\frac{1}{m_2}\right)}\left(\frac{2}{K\left(\sigma_{uts} - \sigma_{ys}\right)}\right)\left(1 - \tanh^2[H]\right) + \frac{1}{m_2}\sigma_t^{\left(\frac{1}{m_2} - 1\right)}\tanh[H]\right)$$

An example of an iterative solution to determine F_{ic} is shown in Table 4.4.2. When using the procedure of paragraph 3-D.5.1, the value of F_{ic} is substituted for σ_t, which is the value of true stress at which true strain will be evaluated.

The suggested algorithm is an interval-halving approach to "guess" at a value of F_{ic}, and determine the corresponding value of E_t. The relationship, $F_{ic}/E_t = A_e$ is checked and based upon the proximity of the ratio F_{ic}/E_t to the value of A_e, an adjustment is made to the next "guess" of F_{ic}. The iteration continues until the relationship $F_{ic}/E_t = A_e$ is satisfied within a specified tolerance, at which point the iteration process is stopped and the final value of F_{ic} is reported.

Table 4.4.2 – Algorithm for Computation of Predicted Inelastic buckling Stress, F_{ic}.

$$Read\left(F_e, E\right)$$

$$A_e = \frac{F_e}{E}$$

$$F_{icup} = MSTS$$

$$F_{iclow} = 0.0$$

$$TOLA_{diff} = 0.00000001$$

$$A_{diff} = 1.0$$

$$Do\ While\ A_{diff} > TOLA_{diff}$$

$$\quad F_{icg} = 0.5\left(F_{icup} + F_{iclow}\right)$$

$$\quad E_{tg} = \left(\frac{1}{E} + D_1 + D_2 + D_3 + D_4\right)^{-1}$$

$$\quad A_i = \frac{F_{icg}}{E_{tg}}$$

$$\quad A_{diff} = A_i - A_e$$

$$\quad IF\left(A_{diff} < 0.0\right)$$

$$\qquad F_{iclow} = F_{icg}$$

$$\quad ELSE$$

$$\qquad F_{icup} = F_{icg}$$

$$\quad END\ IF$$

$$\quad A_{diff} = \left|A_{diff}\right|$$

$$End\ Do$$

$$F_{ic} = F_{icg}$$

A tabulated summary of the iterative procedure is shown below.

Iteration	A_{diff}	F_{icup}	F_{iclow}	F_{icg}	E_t
1	1.000000000	70000.0000	0.0000	35000.0000	1.6780582E+06
2	0.020698024	35000.0000	0.0000	17500.0000	2.1133306E+07
3	0.000668659	17500.0000	0.0000	8750.0000	2.8052001E+07
4	0.000152503	8750.0000	0.0000	4375.0000	2.8293447E+07
5	0.000004788	8750.0000	4375.0000	6562.5000	2.8244947E+07
6	0.000072925	6562.5000	4375.0000	5468.7500	2.8278853E+07
7	0.000033969	5468.7500	4375.0000	4921.8750	2.8287838E+07
8	0.000014575	4921.8750	4375.0000	4648.4375	2.8290991E+07
9	0.000004890	4648.4375	4375.0000	4511.7188	2.8292298E+07
10	0.000000050	4511.7188	4375.0000	4443.3594	2.8292892E+07
11	0.000002369	4511.7188	4443.3594	4477.5391	2.8292600E+07
12	0.000001159	4511.7188	4477.5391	4494.6289	2.8292450E+07
13	0.000000554	4511.7188	4494.6289	4503.1738	2.8292375E+07
14	0.000000252	4511.7188	4503.1738	4507.4463	2.8292336E+07
15	0.000000101	4511.7188	4507.4463	4509.5825	2.8292317E+07
16	0.000000025	4511.7188	4509.5825	4510.6506	2.8292308E+07
17	0.000000013	4510.6506	4509.5825	**4510.1166**	**2.8292313E+07**
18	**0.000000006**				

$$F_{ic} = 4510.1166 \ psi$$

d) STEP 4 – Calculate the value of design factor, FS, per paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0 \ psi$$

Since $F_{ic} \le 0.55S_y$, calculate FS as follows:

$$FS = 2.0$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{D_o}\right) = 2(2255.0583)\left(\frac{1.0}{92.25}\right) = 48.9 \ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{4510.1166}{2.0} = 2255.0583 \ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness or reduce the unsupported length of the shell (i.e., by the addition of a stiffening rings) and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The allowable external pressure is $P_a = 48.9 \ psi$.

Combined Loadings – cylindrical shells subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the cylindrical shell is only subject to external pressure.

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4.4.2 Example E4.4.2 – Conical Shell

Determine the Maximum Allowable External Pressure (MAEP) for a conical shell considering the following design conditions.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Temperature = $300°F$
- Inside Diameter (Large End) = $150.0\ in$
- Thickness (Large End) = $1.8125\ in$
- Inside Diameter (Small End) = $90.0\ in$
- Thickness (Small End) = $1.125\ in$
- Thickness (Conical Section) = $1.9375\ in$
- Axial Cone Length = $78.0\ in$
- One-Half Apex Angle = $21.0375\ deg$
- Corrosion Allowance = $0.125\ in$
- Mod. of Elasticity at Design Temp. = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$

Evaluate per paragraph 4.4.6. and 4.4.5.

The required thickness of a conical shell subjected to external pressure loading shall be determined using the equations for a cylinder by making the following substitutions:

a) The value of t_c is substituted for t in the equations in paragraph 4.4.5.

$$t_c = t = 1.9375 - Corrosion\ Allowance = 1.9375 - 0.125 = 1.8125\ in$$

b) For offset cones, the cone angle, α, shall satisfy the requirements of paragraph 4.3.4.

The conical shell in this example problem is not of the offset type. Therefore, no additional requirements are necessary.

c) The value of $0.5(D_L + D_S)/\cos[\alpha]$ is substituted for D_o in the equations in paragraph 4.4.5, (concentric cone design with common center line per Figure 4.4.7 Sketch (a)).

$$D_o = \frac{0.5(D_L + D_s)}{\cos[\alpha]} = \frac{0.5\left[\left(150.0+2(1.9375)\right)+\left(90.0+2(1.9375)\right)\right]}{\cos[21.0375]} = 132.7215\ in$$

d) The value of $L_{ce}/\cos[\alpha]$ is substituted for L in the equations in paragraph 4.4.5 where L_{ce} is determined as shown below. For Sketches (a) and (e) in Figure 4.4.7:

$$L_{ce} = L_c$$

$$L = \frac{L_{ce}}{\cos[\alpha]} = \frac{78.0}{\cos[21.0375]} = 83.5703\ in$$

e) Note that the half-apex angle of a conical transition can be computed knowing the shell geometry with the following equations. These equations were developed with the assumption that the conical transition contains a cone section, knuckle, or flare. If the transition does not contain a knuckle or flare, the radii of these components should be set to zero when computing the half-apex angle (see Figure 4.4.7).

$$If\ (R_L - r_k) \geq (R_S + r_f)$$

$$\alpha = \beta + \phi = 0.3672 - 0 = 0.3672\ rad = 21.0375\ deg$$

$$\beta = \arctan\left[\frac{(R_L - r_k) - (R_S + r_f)}{L_c}\right] = \arctan\left[\frac{(75.0 - 0) - (45.0 + 0)}{78.0}\right] = 0.3672\ rad$$

$$\phi = \arcsin\left[\frac{(r_f + r_k)\cos[\beta]}{L_c}\right] = \arcsin\left[\frac{(0.0 + 0.0)\cos[0.3672]}{78.0}\right] = 0.0\ rad$$

Proceed with the design following the steps outlined in paragraph 4.4.5.

a) STEP 1 – Assume an initial thickness, t, and unsupported length, L (see VIII-2, Figures 4.4.1 and 4.4.2).

$$t = 1.8125\ in$$

$$L = 83.5703\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{1.6C_h E_y t}{D_o} = \frac{1.6(0.1306633)(28.3E+06)(1.8125)}{132.7215} = 80796.7762\ psi$$

where,

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{83.5703}{\sqrt{\left(\frac{132.7215}{2.0}\right)1.8125}} = 7.6200$$

$$2\left(\frac{D_o}{t}\right)^{0.94} = 2\left(\frac{132.7215}{1.8125}\right)^{0.94} = 113.1914$$

Since $1.5 < M_x < 13$, calculate C_h as follows:

$$C_h = \frac{0.92}{(M_x - 0.579)} = \frac{0.92}{(7.6200 - 0.579)} = 0.1306633$$

c) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

1) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$F_{he} = 80796.7762\ psi$ (as determined in paragraph 4.4.5, STEP 2)

2) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{80796.7762}{28.3E+06} = 0.00285501$$

3) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

A tabulated summary of the iterative procedure is shown below.

Iteration	A_{diff}	F_{icup}	F_{iclow}	F_{icg}	E_t
1	1.000000000	70000.0000	0.0000	35000.0000	1.6780582E+06
2	0.018002431	35000.0000	0.0000	17500.0000	2.1133306E+07
3	0.002026933	35000.0000	17500.0000	26250.0000	7.1896405E+06
4	0.000796077	26250.0000	17500.0000	21875.0000	1.3450426E+07
5	0.001228667	26250.0000	21875.0000	24062.5000	9.9795641E+06
6	0.000443832	26250.0000	24062.5000	25156.2500	8.4945724E+06
7	0.000106440	25156.2500	24062.5000	24609.3750	9.2145856E+06
8	0.000184312	25156.2500	24609.3750	24882.8125	8.8489245E+06
9	0.000043051	25156.2500	24882.8125	25019.5313	8.6703335E+06
10	0.000030638	25019.5313	24882.8125	24951.1719	8.7592753E+06
11	0.000006467	25019.5313	24951.1719	24985.3516	8.7147160E+06
12	0.000012020	24985.3516	24951.1719	24968.2617	8.7369735E+06
13	0.000002760	24968.2617	24951.1719	24959.7168	8.7481189E+06
14	0.000001857	24968.2617	24959.7168	24963.9893	8.7425448E+06
15	0.000000450	24963.9893	24959.7168	24961.8530	8.7453315E+06
16	0.000000704	24963.9893	24961.8530	24962.9211	8.7439381E+06
17	0.000000127	24963.9893	24962.9211	24963.4552	8.7432415E+06
18	0.000000162	24963.4552	24962.9211	24963.1882	8.7435898E+06
19	0.000000018	24963.1882	24962.9211	24963.0547	8.7435898E+06
20	0.000000055	24963.1882	24963.0547	24963.1214	8.7436769E+06
21	0.000000018	24963.1882	24963.1214	**24963.1548**	**8.7436333E+06**
22	**0.000000000**				

$$F_{ic} = 24963.1548\ psi$$

d) STEP 4 – Calculate the value of design factor, FS, per VIII-2, paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0\ psi$$

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{24963.1548}{33600.0}\right) = 1.8565$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{D_o}\right) = 2(13446.3533)\left(\frac{1.8125}{132.7215}\right) = 367.3\ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{24963.1548}{1.8565} = 13446.3533\ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness or reduce the unsupported length of the shell (i.e., by the addition of a stiffening rings) and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 367.3\ psi$.

Combined Loadings – conical shells subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the conical shell is only subject to external pressure.

4.4.3 Example E4.4.3 – Spherical Shell and Hemispherical Head

Determine the Maximum Allowable External Pressure (MAEP) for a hemispherical head considering the following design conditions.

Vessel Data:

- Material = $SA-542,\ Type\ D,\ Class\ 4a$
- Design Temperature = $350°F$
- Inside Diameter = $149.0\ in$
- Thickness = $2.8125\ in$
- Corrosion Allowance = $0.0\ in$
- Modulus of Elasticity at Design Temperature = $29.1E+06\ psi$
- Yield Strength = $58000\ psi$

Evaluate per paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the spherical shell.

$$t = 2.8125\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075E_y\left(\frac{t}{R_o}\right) = 0.075(29.1E+06)\left(\frac{2.8125}{77.3125}\right) = 79395.7154\ psi$$

c) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

1) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$$F_{he} = 79395.7154\ psi$$ (as determined in paragraph 4.4.7, STEP 2)

2) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{79395.7154}{29.1E+06} = 0.00272838$$

3) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

A tabulated summary of the iterative procedure is shown below.

Iteration	A_{diff}	F_{icup}	F_{iclow}	F_{icg}	E_t
1	1.000000000	85000.0000	0.0000	42500.0000	2.0983586E+07
2	0.000702983	85000.0000	42500.0000	63750.0000	6.1671934E+05
3	0.100641174	63750.0000	42500.0000	53125.0000	5.1280086E+06
4	0.007631397	53125.0000	42500.0000	47812.5000	1.2010241E+07
5	0.001252603	47812.5000	42500.0000	45156.2500	1.6580360E+07
6	0.000004897	47812.5000	45156.2500	46484.3750	1.4259656E+07
7	0.000531477	46484.3750	45156.2500	45820.3125	1.5417952E+07
8	0.000243505	45820.3125	45156.2500	45488.2813	1.5999552E+07
9	0.000114722	45488.2813	45156.2500	45322.2656	1.6290170E+07
10	0.000053810	45322.2656	45156.2500	45239.2578	1.6435333E+07
11	0.000024186	45239.2578	45156.2500	45197.7539	1.6507865E+07
12	0.000009578	45197.7539	45156.2500	45177.0020	1.6544117E+07
13	0.000002324	45177.0020	45156.2500	45166.6260	1.6562240E+00
14	0.000001291	45177.0020	45166.6260	45171.8140	1.6553179E+07
15	0.000000515	45171.8140	45166.6260	45169.2200	1.6557709E+07
16	0.000000388	45171.8140	45169.2200	45170.5170	1.6555444E+07
17	0.000000064	45170.5170	45169.2200	45169.8685	1.6556577E+07
18	0.000000162	45170.5170	45169.8685	45170.1927	1.6556011E+07
19	0.000000049	45170.5170	45170.1927	**45170.3548**	**1.6555727E+07**
20	**0.000000007**				

$$F_{ic} = 45170.3548 \ psi$$

d) STEP 4 – Calculate the value of design margin, FS, per VIII-2, paragraph 4.4.2.

$$0.55S_y = 0.55(58000.0) = 31900.0 \ psi$$

Since $0.55S_y < F_{ic} < S_y$, calculate the FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{45170.3548}{58000.0}\right) = 1.8299$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(24684.6)\left(\frac{2.8125}{77.3125}\right) = 1796.0 \ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{45170.3548}{1.8299} = 24684.6 \ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 1796.0 \, psi$.

Combined Loadings – spherical shells and hemispherical heads subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the hemispherical head is only subject to external pressure.

4.4.4 Example E4.4.4 – Torispherical Head

Determine the Maximum Allowable External Pressure (MAEP) for a torispherical head considering the following design conditions.

Vessel Data:

- Material = $SA-387,\ Grade\ 11,\ Class\ 1$
- Design Temperature = $650°F$
- Inside Diameter = $72.0\ in$
- Crown Radius = $72.0\ in$
- Knuckle Radius = $4.375\ in$
- Thickness = $0.625\ in$
- Corrosion Allowance = $0.125\ in$
- Modulus of Elasticity at Design Temperature = $26.55E+06\ psi$
- Yield Strength at Design Temperature = $26900\ psi$

Evaluate per paragraph 4.4.8 and 4.4.7.

The required thickness of a torispherical head subjected to external pressure loading shall be determined using the equations for a spherical shell in paragraph 4.4.7 by substituting the outside crown radius for R_o.

$$R_o = 72.0 + 0.625 = 72.625\ in$$

Restrictions on Torispherical Head Geometry – the restriction of paragraph 4.3.6 shall apply. See paragraph 4.3.6.1.b and STEP 2 of E4.3.4.

Torispherical heads With Different Dome and Knuckle Thickness – heads with this configuration shall be designed in accordance with Part 5. In this example problem, the dome and knuckle thickness are the same.

Proceed with the design following the steps outlined in paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the torispherical head.

$$t = 0.625 - Corrosion\ Allowance = 0.625 - 0.125 = 0.500\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075E_y\left(\frac{t}{R_o}\right) = 0.075(26.55E+06)\left(\frac{0.500}{72.625}\right) = 13709.1222\ psi$$

c) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

1) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$$F_{he} = 13709.1222 \ psi$$ (as determined in paragraph 4.4.7, STEP 2)

2) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{13709.1222}{26.55E+06} = 0.00051635$$

3) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

A tabulated summary of the iterative procedure is shown below.

Iteration	A_{diff}	F_{icup}	F_{iclow}	F_{icg}	E_t
1	1.000000000	60000.0000	0.0000	30000.0000	9.9279405E+05
2	0.029701396	30000.0000	0.0000	15000.0000	1.5506005E+07
3	0.000451016	15000.0000	0.0000	7500.0000	2.5857734E+07
4	0.000226303	15000.0000	7500.0000	11250.0000	2.2459426E+07
5	0.000015448	15000.0000	11250.0000	13125.0000	1.9267796E+07
6	0.000164837	13125.0000	11250.0000	12187.5000	2.0970842E+07
7	0.000064813	12187.5000	11250.0000	11718.7500	2.1745287E+07
8	0.000022559	11718.7500	11250.0000	11484.3750	2.2110214E+07
9	0.000003064	11484.3750	11250.0000	11367.1875	2.2286818E+07
10	0.000006310	11484.3750	11367.1875	11425.7813	2.2199012E+07
11	0.000001653	11484.3750	11425.7813	11455.0781	2.2154736E+07
12	0.000000698	11455.0781	11425.7813	11440.4297	2.2176905E+07
13	0.000000480	11455.0781	11440.4297	11447.7539	2.2165828E+07
14	0.000000108	11447.7539	11440.4297	11444.0918	2.2171369E+07
15	0.000000186	11447.7539	11444.0918	11445.9229	2.2168599E+07
16	0.000000039	11447.7539	11445.9229	11446.8384	2.2167214E+07
17	0.000000035	11446.8384	11445.9229	**11446.3806**	**2.2167906E+07**
18	**0.000000002**				

$$F_{ic} = 11446.3806 \ psi$$

d) STEP 4 – Calculate the value of design margin, FS, per paragraph 4.4.2.

$$0.55S_y = 0.55(26900.0) = 14795.0 \ psi$$

Since $F_{ic} \le 0.55S_y$, calculate the FS as follows:

$$FS = 2.0$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(5723.1903)\left(\frac{0.500}{72.625}\right) = 78.8 \ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{11446.3806}{2.0} = 5723.1903 \ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 78.8\,psi$.

Combined Loadings – torispherical heads subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the torispherical head is only subject to external pressure.

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4.4.5 Example E4.4.5 – Elliptical Head

Determine the maximum allowable external pressure (MAEP) for a 2:1 elliptical head considering the following design conditions.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Temperature = $300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$

Evaluate per paragraph 4.4.9 and 4.4.7.

The required thickness of an elliptical head subjected to external pressure loading shall be determined using the equations for a spherical shell in paragraph 4.4.7 by substituting $K_o D_o$ for R_o where K_o is given by the following equation.

$$K_o = 0.25346 + 0.13995\left(\frac{D_o}{2h_o}\right) + 0.12238\left(\frac{D_o}{2h_o}\right)^2 - 0.015297\left(\frac{D_o}{2h_o}\right)^3$$

$$K_o = \left(\begin{array}{l} 0.25346 + 0.13995\left(\dfrac{92.25}{2(23.0625)}\right) + 0.12238\left(\dfrac{92.25}{2(23.0625)}\right)^2 - \\ 0.015297\left(\dfrac{92.25}{2(23.0625)}\right)^3 \end{array}\right) = 0.900504$$

$$D_o = 90.0 + 2(1.125) = 92.25\ in$$

$$h_o = \left(\frac{D_o}{4}\right) = \frac{92.25}{4} = 23.0625\ in$$

therefore,

$$R_o = K_o D_o = 0.9005040(92.25) = 83.0715\ in$$

Proceed with the design following the steps outlined in paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the elliptical head.

$$t = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075E_y\left(\frac{t}{R_o}\right) = 0.075(28.3E+06)\left(\frac{1.0}{83.0715}\right) = 25550.2790\ \ psi$$

c) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic} see paragraph 4.4.2.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

1) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$$F_{he} = 25550.2790 \ psi$$ (as determined in paragraph 4.4.7, STEP 2)

2) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{25550.2790}{28.3E+06} = 0.00090284$$

3) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

A tabulated summary of the iterative procedure is shown below.

Iteration	**A_{diff}**	**F_{icup}**	**F_{iclow}**	**F_{icg}**	**E_t**
1	1.000000000	70000.0000	0.0000	35000.0000	1.6780582E+06
2	0.019954604	35000.0000	0.0000	17500.0000	1.6780582E+07
3	0.000074760	35000.0000	17500.0000	26250.0000	7.1896405E+06
4	0.002748250	26250.0000	17500.0000	21875.0000	1.3450426E+07
5	0.000723506	21875.0000	17500.0000	19687.5000	1.7344726E+07
6	0.000232234	19687.5000	17500.0000	18593.7500	1.9291520E+07
7	0.000060993	18593.7500	17500.0000	18046.8750	2.0230363E+07
8	0.000010768	18593.7500	18046.8750	18320.3125	1.9764850E+07
9	0.000024077	18320.3125	18046.8750	18183.5938	1.9998658E+07
10	0.000006404	18183.5938	18046.8750	18115.2344	2.0114783E+07
11	0.000002244	18183.5938	18115.2344	18149.4141	2.0056787E+07
12	0.000002065	18149.4141	18115.2344	18132.3242	2.0085802E+07
13	0.000000093	18149.4141	18132.3242	18140.8691	2.0071299E+07
14	0.000000985	18140.8691	18132.3242	18136.5967	2.0078551E+07
15	0.000000445	18136.5967	18132.3242	18134.4604	2.0082177E+07
16	0.000000176	18134.4604	18132.3242	18133.3923	2.0083989E+07
17	0.000000041	18133.3923	18132.3242	18132.8583	2.0084896E+07
18	0.000000026	18133.3923	18132.8583	**18133.1253**	**2.0084442E+07**
19	**0.000000008**				

$$F_{ic} = 18133.1253 \ psi$$

d) STEP 4 – Calculate the value of design margin, FS, per paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0 \ psi$$

Since $F_{ic} \leq 0.55S_y$, calculate the FS as follows:

$$FS = 2.0$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(9066.5627)\left(\frac{1.0}{83.0715}\right) = 218.3 \ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{18133.1253}{2.0} = 9066.5627 \ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 218.3\,psi$.

Combined Loadings – ellipsoidal heads subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the ellipsoidal head is only subject to external pressure.

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4.4.6 Example E4.4.6 – Combined Loadings and Allowable Compressive Stresses

Determine the allowable compressive stresses of the proposed cylindrical shell section considering the following design conditions and specified applied loadings. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $-14.7\ psig\ @\ 300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Unsupported Length = $636.0\ in$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$
- Applied Axial Force = $-66152.5\ lbs$
- Applied Net Section Bending Moment = $5.08E+06\ in-lbs$
- Applied Shear Force = $18762.6\ lbs$

Adjust variables for corrosion and determine outside dimensions.

$$D = 90.0 + 2(Corrosion\ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

$$R = \frac{D}{2} = \frac{90.25}{2} = 45.125\ in$$

$$t = 1.125 - Corosion\ Allowance = 1.125 - 0.125 = 1.0\ in$$

$$D_o = 90.0 + 2(Uncorroded\ Thickness) = 90.0 + 2(1.125) = 92.25\ in$$

$$R_o = \frac{D}{2} = \frac{92.25}{2} = 46.125\ in$$

Evaluate per paragraph 4.4.12.2.

The loads transmitted to the cylindrical shell are given in the Table E4.4.6.3. Note that this table is given in terms of the load parameters shown in Table 4.1.1, and Table 4.1.2. (Table E4.4.6.1 and Table E4.4.6.2 of this example). As shown in Table E4.4.6.2, the acceptance criteria are that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition.

In accordance with paragraph 4.4.12.2, the following procedure shall be used to determine the allowable compressive stresses for cylindrical shells that are based on loading conditions. By inspection of the results shown in Table E4.4.6.3 and Table E4.4.6.4, Load Case 5 is determined to be a potential governing load case. The pressure, net section axial force, bending moment, and radial shear force at the location of interest for Load Case 5 are as follows. Note, the load factor, Ω_p, shown in Table 4.1.2 is used to simulate the maximum anticipated operating pressure acting simultaneously with the occasional loads specified. For this example, problem, $\Omega_p = 1.0$.

$$\Omega P + P_s = (1.0)P + P_s = -14.7\ psi$$
$$F_5 = -66152.5\ lbs$$
$$M_5 = 3048000\ in-lbs$$
$$V_5 = 11257.6\ lbs$$

Common parameters used in each of the loading conditions are given in paragraph 4.4.12.2.k.

Per paragraph 4.4.12.2.k:

$$A = \frac{\pi\left(D_o^2 - D_i^2\right)}{4} = \frac{\pi\left(92.25^2 - 90.25^2\right)}{4} = 286.6703\ in^2$$

$$S = \frac{\pi\left(D_o^4 - D_i^4\right)}{32D_o} = \frac{\pi\left(92.25^4 - 90.25^4\right)}{32(92.25)} = 6469.5531\ in^3$$

$$f_h = \frac{PD_o}{2t} = \frac{14.7(92.25)}{2(1.0)} = 678.0375\ psi$$

$$f_b = \frac{M}{S} = \frac{3.048E+06}{6469.5531} = 471.1299\ psi$$

$$f_a = \frac{F}{A} = \frac{66152.5}{286.6703} = 230.7616\ psi$$

$$f_v = \frac{V\sin[\phi]}{A} = \frac{11257.6\sin[90.0]}{286.6703} = 39.2702\ psi$$

Note: ϕ is defined as the angle measured around the circumference from the direction of the applied shear force to the point under consideration. For this example, problem, $\phi = 90\ deg$ to maximize the shear force.

$$r_g = 0.25\sqrt{D_o^2 + D_i^2} = 0.25\sqrt{\left(92.25^2 + 90.25^2\right)} = 32.2637\ in$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{(46.125)1.0}} = 93.6459$$

The value of the slenderness factor for column buckling, λ_c is calculated in paragraph 4.4.12.2.b.

Per paragraph 4.4.12.2:

a) External Pressure Acting Alone, (paragraph 4.4.12.2.a) – the allowable hoop compressive membrane stress of a cylinder subject to external pressure acting alone, F_{ha}, is computed using the equations in VIII-2, paragraph 4.4.5.

From Example E4.4.1,

$$F_{ha} = 2255.0583\ psi$$

b) Axial Compressive Stress Acting Alone, (paragraph 4.4.12.2.b) – the allowable axial compressive membrane stress of a cylinder subject to an axial compressive load acting alone, F_{xa}, is computed using the following equations:

For $\lambda_c \le 0.15$ (Local Buckling):

1) STEP 1 – Calculate the predicted elastic buckling stress, F_{xe}.

$$F_{xe} = \frac{C_x E_y t}{D_o} = \frac{0.8499(28.3E+06)(1.0)}{92.25} = 260728.1301\ psi$$

where,

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{46.125(1.0)}} = 93.6459$$

Since $D_o/t < 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389+\frac{D_o}{t}\right)},\ 0.9\right] = \min\left[\frac{409(1.0)}{389+\frac{92.25}{1.0}},\ 0.9\right] = 0.8499$$

Since $M_x \ge 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

2) STEP 2 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{xe}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic} , is determined using the following procedure.

i) STEP 2.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$$F_{xe} = 260728.1301\ psi$$ (as determined in STEP 1 above)

ii) STEP 2.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{xe}}{E} = \frac{260728.1301}{28.3E+06} = 0.00921301$$

iii) STEP 2.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 30967.6147\ psi$$

3) STEP 3 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{30967.6147}{33600}\right) = 1.7241$$

4) STEP 4 – Calculate the allowable axial compressive membrane stress as follows:

$$F_{xa} = \frac{F_{ic}}{FS} = \frac{30967.6147}{1.7241} = 17961.6117 \ psi$$

For $\lambda_c > 0.15$ and $K_u L_u / r_g < 200$, (Column Buckling).

With F_{xa} calculated, determine the value of λ_c from paragraph 4.4.12.2.k. For a cylinder with end conditions with one end free and the other end fixed, $K_u = 2.1$.

$$\left\{\frac{K_u L_u}{r_g} = \frac{2.1(636.0)}{32.2637} = 41.3964\right\} < \{200\} \qquad True$$

$$\lambda_c = \frac{K_u L_u}{\pi r_g}\left(\frac{F_{xa} \cdot FS}{E_y}\right)^{0.5} = \frac{2.1(636.0)}{\pi(32.2637)}\left(\frac{17961.6117(1.7241)}{28.3E+06}\right)^{0.5} = 0.4359$$

$$F_{ca} = F_{xa}\left[1 - 0.74(\lambda_c - 0.15)\right]^{0.3}$$

$$F_{ca} = 17961.6117\left[1 - 0.74(0.4359 - 0.15)\right]^{0.3} = 16725.3381 \ psi$$

c) <u>Compressive Bending Stress, (paragraph 4.4.12.2.c)</u> – the allowable axial compressive membrane stress of a cylindrical shell subject to a bending moment acting across the full circular cross section, F_{ba}, is computed using the procedure in paragraph 4.4.12.2.b. For this example, problem, since $0.15 < \{\lambda_c = 0.4359\} \le 1.2$, $F_{ba} = F_{ca}$.

As shown, $F_{ba} = F_{ca} = 16725.3381 \ psi$

d) <u>Shear Stress, (paragraph 4.4.12.2.d)</u> – the allowable shear stress of a cylindrical shell, F_{va}, is computed using the following equations:

1) STEP 1 – Calculate the predicted elastic buckling stress, F_{ve}.

$$F_{ve} = \alpha_v C_v E_y\left(\frac{t}{D_o}\right) = 0.8(0.1542)(28.3E+06)\left(\frac{1.0}{92.25}\right) = 37843.7724 \ psi$$

where,

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{46.125(1.0)}} = 93.6459$$

$$4.347\left(\frac{D_o}{t}\right) = 4.347(92.25) = 401.0108$$

Since $26 \le \{M_x = 93.6459\} < \{4.347(D_o/t)\}$, calculate C_v as follows:

$$C_v = \frac{1.492}{M_x^{0.5}} = \frac{1.492}{(93.6459)^{0.5}} = 0.1542$$

Since $\{D_o/t = 92.25\} \le 500$, calculate α_v as follows:

$$\alpha_v = 0.8$$

2) STEP 2 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{ve}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 2.1 – Calculate the predicted elastic buckling stress, F_{ve}.

$F_{ve} = 37843.7724\ psi$ (as determined in STEP 1 above)

ii) STEP 2.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{ve}}{E} = \frac{37843.7724}{28.3E+06} = 0.00133724$$

iii) STEP 2.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 20714.3593\ psi$$

3) STEP 3 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{20714.3593}{33600}\right) = 1.9502$$

4) STEP 4 – Calculate the allowable compressive shear stress as follows:

$$F_{va} = \frac{F_{ic}}{FS} = \frac{20714.3593}{1.9502} = 10621.7984\ psi$$

e) Axial Compressive Stress and Hoop Compression, (paragraph 4.4.12.2.e) – the allowable compressive stress for the combination of uniform axial compression and hoop compression, F_{xha}, is computed using the following equations:

1) For $\lambda_c = 0.15$, F_{xha} is computed using the following equation with F_{ha} and F_{xa} evaluated using the equations in paragraphs 4.4.12.2.a and 4.4.12.2.b.1, respectively.

$$F_{xha} = \left[\left(\frac{1}{F_{xa}^2}\right) - \left(\frac{C_1}{C_2 F_{xa} F_{ha}}\right) + \left(\frac{1}{C_2^2 F_{ha}^2}\right)\right]^{-0.5}$$

$$F_{xha} = \left[\begin{array}{l}\left(\frac{1}{(17961.6117)^2}\right) - \\ \left(\frac{0.0559}{0.8241(17961.6117)(2255.0583)}\right) + \\ \left(\frac{1}{(0.8241)^2 (2255.0583)^2}\right)\end{array}\right]^{-0.5} = 1853.8375\ psi$$

where,

$$C_1 = \frac{(F_{xa} \cdot FS + F_{ha} \cdot FS)}{S_y} - 1.0 = \frac{17961.6117(1.7241) + 2255.0583(2.0)}{33600} - 1.0$$

$$C_1 = 0.0559$$

$$C_2 = \frac{f_x}{f_h} = \frac{558.7957}{678.0375} = 0.8241$$

$$f_x = f_a + f_q = 230.7616 + 328.0341 = 558.7957\ psi$$

Note: this step is not required since $\lambda_c > 0.15$.

2) For $0.15 < \lambda_c \leq 1.2$, F_{xha}, is computed from the following equation with $F_{ah1} = F_{xha}$ evaluated using the equations in paragraph 4.4.12.2.e.1, and F_{ah2} using the following procedure. The value of F_{ca} used in the calculation for F_{ah2} is evaluated using the equation in paragraph 4.4.12.2.b.2 with $F_{xa} = F_{xha}$ as determined in paragraph 4.4.12.2.e.1. As noted, the load on the end of a cylinder due to external pressure does not contribute to column buckling and therefore F_{ah1} is compared with f_a rather than f_x. The stress due to the pressure load does, however, lower the effective yield stress and the quantity in $(1 - f_q/S_y)$ accounts for this reduction.

$$F_{xha} = \min\left[F_{ah1}, F_{ah2}\right] = \min\left[1853.8375, 1709.3873\right] = 1709.3873\ psi$$

$$F_{ah1} = F_{xha} = 1853.8375\ psi$$

$$F_{ah2} = F_{ca}\left(1 - \frac{f_q}{S_y}\right) = 1726.2404\left(1 - \frac{328.0341}{33600}\right) = 1709.3873\ psi$$

where,

$$F_{ca} = F_{xa}\left[1 - 0.74(\lambda_c - 0.15)\right]^{0.3}$$

$$F_{ca} = 1853.8375\left[1 - 0.74(0.4359 - 0.15)\right]^{0.3} = 1726.2404\ psi$$

3) For $\lambda_c \leq 0.15$, the allowable hoop compressive membrane stress, F_{hxa}, is given by the following equation.

$$F_{hxa} = \frac{F_{xha}}{C_2} = \frac{1853.8375}{0.8241} = 2249.5298\ psi$$

Note: this step is not required since $\lambda_c > 0.15$.

4) For $\lambda_c \geq 1.2$, the rules of paragraph 4.4.12.2.e do not apply.

f) Compressive Bending Stress and Hoop Compression, (paragraph 4.4.12.2.f) – the allowable compressive stress for the combination of axial compression due to a bending moment and hoop compression, F_{bha}, is computed using the following equations.

1) An iterative solution procedure is utilized to solve these equations for C_3 with F_{ha} and F_{ba} evaluated using the equations in paragraphs 4.4.12.2.a and 4.4.12.2.c, respectively. For this example, problem, since $0.15 < \{\lambda_c = 0.4359\} \leq 1.2$, $F_{ba} = F_{bc}$.

$$F_{bha} = C_3 C_4 F_{ba} = (0.9926)(0.0937)(16725.3381) = 1555.5672\ psi$$

where,

$$C_4 = \left(\frac{f_b}{f_h}\right)\left(\frac{F_{ha}}{F_{ba}}\right) = \left(\frac{471.1299}{678.0375}\right)\left(\frac{2255.0583}{16725.3381}\right) = 0.0937$$

$$C_3^2\left(C_4^2 + 0.6C_4\right) + C_3^{2n} - 1 = 0$$

$$n = 5 - \frac{4F_{ha} \cdot FS}{S_y} = 5 - \frac{4(2255.0583)(2.0)}{33600} = 4.4631$$

A tabulated summary of the iterative procedure to solve for C_3 is shown below.

Iteration	**A_{diff}**	**$C3_{up}$**	**$C3_{low}$**	**$C3_g$**	**$Equation_g$**
1	1.000000000	1.000000000	0.000000000	0.500000000	-9.81697352E-01
2	0.981697352	1.000000000	0.500000000	0.750000000	-8.86747671E-01
3	0.886747671	1.000000000	0.750000000	0.875000000	-6.46606859E-01
4	0.646606859	1.000000000	0.875000000	0.937500000	-3.80785061E-01
5	0.380785061	1.000000000	0.937500000	0.968750000	-1.85787513E-01
6	0.185787513	1.000000000	0.968750000	0.984375000	-6.81664050E-02
7	0.068166405	1.000000000	0.984375000	0.992187500	-3.63864700E-03
8	0.003638647	1.000000000	0.992187500	0.996093750	3.01482310E-02
9	0.030148231	0.996093750	0.992187500	0.994140625	1.31249790E-02
10	0.013124979	0.994140625	0.992187500	0.993164063	4.71093300E-03
11	0.004710933	0.993164063	0.992187500	0.992675781	5.28112000E-04
12	0.000528112	0.992675781	0.992187500	0.992431641	-1.55727200E-03
13	0.001557272	0.992675781	0.992431641	0.992553711	-5.15082000E-04
14	0.000515082	0.992675781	0.992553711	0.992614746	6.38970000E-06
15	0.000006390	0.992614746	0.992553711	0.992584229	-2.54377000E-04
16	0.000254377	0.992614746	0.992584229	0.992599487	-1.24002000E-04
17	0.000124002	0.992614746	0.992599487	0.992607117	-5.88079000E-05
18	0.000058808	0.992614746	0.992607117	0.992610931	-2.62096000E-05
19	0.000026210	0.992614746	0.992610931	0.992612839	-9.91008000E-06
20	0.000009910	0.992614746	0.992612839	0.992613792	-1.76022000E-06
21	0.000001760	0.992614746	0.992613792	0.992614269	2.31473000E-06
22	0.000002315	0.992614269	0.992613792	**0.992614031**	**2.77253000E-07**
23	**0.000000277**	0.992614031	0.992613792		

$$C_3 = 0.9926$$

2) The allowable hoop compressive membrane stress, F_{hba}, is given by the following equation.

$$F_{hba} = F_{bha}\left(\frac{f_h}{f_b}\right) = 1555.5672\left(\frac{678.0375}{471.1299}\right) = 2238.7305\ psi$$

g) <u>Shear Stress and Hoop Compression, (paragraph 4.4.12.2.g)</u> – the allowable compressive stress for the combination of shear, F_{vha}, and hoop compression is computed using the following equations.

Note: This load combination is only applicable for shear stress and hoop compression, in the absence of axial compressive stress and compressive bending stress. It is shown in this example problem for informational purposes only. The effect of shear is accounted for in the interaction equations of paragraphs 4.4.12.2.h and 4.4.12.2.i through the variable K_s.

1) The allowable shear stress is given by the following equation with F_{ha} and F_{va} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.and 4.4.12.2.d, respectively.

$$F_{vha} = \left[\left(\frac{F_{va}^2}{2C_5F_{ha}}\right)^2 + F_{va}^2\right]^{0.5} - \frac{F_{va}^2}{2C_5F_{ha}}$$

$$F_{vha} = \left[\left(\frac{(10621.7984)^2}{2(0.0579)(2255.0583)}\right)^2 + (10621.7984)^2\right]^{0.5} - \left[\frac{(10621.7984)^2}{2(0.0579)(2255.0583)}\right]$$

$$F_{vha} = 130.5482\ psi$$

where,

$$C_5 = \frac{f_v}{f_h} = \frac{39.2702}{678.0375} = 0.0579$$

2) The allowable hoop compressive membrane stress, F_{hva}, is given by the following equation.

$$F_{hva} = \frac{F_{vha}}{C_5} = \frac{130.5482}{0.0579} = 2254.7185\ psi$$

h) <u>Axial Compressive Stress, Compressive Bending Stress, Shear Stress, and Hoop Compression, (paragraph 4.4.12.2.h)</u> – the allowable compressive stress for the combination of uniform axial compression, axial compression due to a bending moment, and shear in the presence of hoop compression is computed using the following interaction equations.

1) The shear coefficient is determined using the following equation with F_{va} from paragraph 4.4.12.2.d.

$$K_s = 1.0 - \left(\frac{f_v}{F_{va}}\right)^2 = 1.0 - \left(\frac{39.2702}{10621.7984}\right)^2 = 0.9999$$

2) For $\lambda_c \leq 0.15$, the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{xha} and F_{bha} evaluated using the equations in paragraphs 4.4.12.2.e.1 and 4.4.12.2.f.1, respectively.

$$\left(\frac{f_a}{K_s F_{xha}}\right)^{1.7} + \left(\frac{f_b}{K_s F_{bha}}\right) \leq 1.0$$

$$\left\{\left(\frac{230.7616}{0.9999(1853.8375)}\right)^{1.7} + \left(\frac{471.1299}{0.9999(1555.5672)}\right) = 0.3319\right\} \leq 1.0 \qquad True$$

Note: this step is not required since $\lambda_c > 0.15$.

3) For $0.15 < \lambda_c \leq 1.2$, the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{xha} and F_{bha} evaluated using the equations in paragraphs 4.4.12.2.e.2 and 4.4.12.2.f.1, respectively.

$$\frac{f_a}{K_s F_{xha}} = \frac{230.7616}{0.9999(1709.3873)} = 0.1350$$

Since $f_a/K_s F_{xha} < 0.2$, the following equation shall be used:

$$\left(\frac{f_a}{2K_s F_{xha}}\right) + \left(\frac{\Delta f_b}{K_s F_{bha}}\right) \leq 1.0$$

$$\left\{\left(\frac{230.7616}{2(0.9999)(1709.3873)}\right) + \left(\frac{1.0024(471.1299)}{0.9999(1555.5672)}\right) = 0.3711\right\} \leq \{1.0\} \qquad True$$

where,

$$\Delta = \frac{C_m}{1 - \left(\frac{f_a \cdot FS}{F_e}\right)} = \frac{1.0}{1 - \left(\frac{230.7616(1.7241)}{162990.2785}\right)} = 1.0024$$

$$F_e = \frac{\pi^2 E_y}{\left(\frac{K_u L_u}{r_g}\right)^2} = \frac{\pi^2 (28.3E+06)}{\left(\frac{2.1(636.0)}{32.2637}\right)^2} = 162990.2785 \ psi$$

Note: $C_m = 1.0$ for unbraced skirt supported vessels, see paragraph 4.4.15.

i) Axial Compressive Stress, Compressive Bending Stress, and Shear Stress, (paragraph 4.4.12.2.i) – the allowable compressive stress for the combination of uniform axial compression, axial compression due to a bending moment, and shear in the absence of hoop compression is computed using the following interaction equations.

1) The shear coefficient is determined using the equation in paragraph 4.4.12.2.h.1 with F_{va} from paragraph 4.4.12.2.d.

$$K_s = 1.0 - \left(\frac{f_v}{F_{va}}\right)^2 = 1.0 - \left(\frac{39.2702}{10621.7984}\right)^2 = 0.9999$$

2) For $\lambda_c \leq 0.15$ the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{xa} and F_{ba} evaluated using the equations in paragraphs 4.4.12.2.b.1 and 4.4.12.2.c, respectively.

$$\left(\frac{f_a}{K_s F_{xa}}\right)^{1.7} + \left(\frac{f_b}{K_s F_{ba}}\right) \leq 1.0$$

$$\left\{\left(\frac{230.7616}{0.9999(17961.6117)}\right)^{1.7} + \left(\frac{471.1299}{0.9999(16725.3381)}\right) = 0.0288\right\} \leq 1.0 \qquad True$$

Note: this step is not required since $\lambda_c > 0.15$.

3) For $0.15 < \lambda_c \leq 1.2$ the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{ca} and F_{ba} evaluated using the equations in paragraphs 4.4.12.2.b.2 and 4.4.12.2.c, respectively. The coefficient Δ is evaluated using the equations in paragraph 4.4.12.2.h.3.

$$\frac{f_a}{K_s F_{ca}} = \frac{230.7616}{0.9999(16725.3381)} = 0.0138$$

Since $f_a/K_s F_{ca} < 0.2$, the following equation shall be used:

$$\left(\frac{f_a}{2K_s F_{ca}}\right) + \left(\frac{\Delta f_b}{K_s F_{ba}}\right) \leq 1.0$$

$$\left\{\left(\frac{230.7616}{2(0.9999)(16725.3381)}\right) + \left(\frac{1.0024(471.1299)}{0.9999(16725.3381)}\right) = 0.0351\right\} \leq 1.0 \quad True$$

j) (Paragraph 4.4.12.2.j) – The maximum deviation, e, may exceed the value e_x given in VIII-2, paragraph 4.4.4.2 if the maximum axial stress is less than F_{xa} for shells designed for axial compression only, or less than F_{xha} for shells designed for combinations of axial compression and external pressure. The change in buckling stress, F'_{xe}, is given by Equation (4.4.114). The reduced allowable buckling stress, $F_{xa(reduced)}$, is determined using Equation (4.4.115) where e is the new maximum deviation, F_{xa} is determined using Equation 4.4.61, and FS_{xa} is the value of the stress reduction factor used to determine F_{xa}.

<u>**From paragraph 4.4.4.1:**</u>

$$e = \min[e_c,\ 2t] = \min\left[1.3007,\ \{2(1.0) = 2.0\}\right] = 1.3007\ in$$

Where, e_c is valid for the following range:

$$\{0.1t = 0.1(1.0) = 0.1\ in\} \leq e_c \leq \{0.0282R_o = 0.0282(46.125) = 1.3007\ in\}$$

$$e_c = 0.0165t\left(\frac{L}{\sqrt{R_o t}} + 3.25\right)^{1.069} = 0.0165(1.0)\left(\frac{636.0}{\sqrt{46.125(1.0)}} + 3.25\right)^{1.069} = 2.1920\ in$$

<u>**From paragraph 4.4.4.2:**</u>

$$e_x = 0.002R_m = 0.002(45.625) = 0.0913\ in$$

where,

$$R_m = \frac{(D_o + D_i)}{4} = \frac{(92.25 + 90.25)}{4} = 45.625 \ in$$

For axial compression only,

$$\{f_a + f_b\} \le F_{xa}$$

$$\{230.7616 + 471.1299 = 701.8915 \ psi\} \le 16725.3381 \ psi \qquad True$$

For axial compression and external pressure,

$$\{f_a + f_b + f_q\} \le F_{xha}$$

$$\{230.7616 + 471.1299 + 328.0341 = 1029.9256 \ psi\} \le 1709.3873 \ psi \qquad True$$

Since, both criteria are satisfied,

$$F_{xe}^{'} = \left(0.944 - \left|0.286 \log\left[\frac{0.0005e}{e_x}\right]\right|\right)\left(\frac{E_y t}{R}\right)$$

$$F_{xe}^{'} = \left(0.944 - \left|0.286 \log\left[\frac{0.0005(1.3007)}{(0.0913)}\right]\right|\right)\left(\frac{(28.3E+06)(1.0)}{46.125}\right) = 202388.986 \ psi$$

Therefore, the reduced allowable bucking stress is determined as follows.

$$F_{xa(reduced)} = \frac{F_{xa} \cdot FS_{xa} - F_{xe}^{'}}{FS_{xa}}$$

$$F_{xa(reduced)} = \frac{17961.6117(1.7241) - (202388.986)}{1.7241} = -99426.5827 \ psi$$

<u>A summary of the allowable compressive stresses are as follows:</u>

Paragraph 4.4.12.2.a, External Pressure Acting Alone

$$F_{ha} = 2255.0583 \ psi$$

Paragraph 4.4.12.2.b, Axial Compressive Stress Acting Alone

$$F_{xa} = 17961.6117 \ psi$$

$$F_{ca} = 16725.3381 \ psi$$

Paragraph 4.4.12.2.c, Compressive Bending Stress

$$F_{ba} = 16725.3381 \ psi$$

Paragraph 4.4.12.2.d, Shear Stress

$$F_{va} = 10621.7984 \ psi$$

Paragraph 4.4.12.2.e, Axial Compressive Stress and Hoop Compression

$$F_{xha} = 1709.3873 \ psi$$

Paragraph 4.4.12.2.f, Compressive Bending Stress and Hoop Compression

$$F_{bha} = 1555.5672 \ psi$$

$$F_{hba} = 2238.7305 \ psi$$

Paragraph 4.4.12.2.g, Shear Stress and Hoop Compression

$$F_{vha} = 130.5482 \ psi$$

$$F_{hva} = 2254.7185 \ psi$$

Table E4.4.6.1 – Design Loads from VIII-2

Table 4.1.1 – Design Loads	
Design Load Parameter	**Description**
P	Internal of External Specified Design Pressure (see paragraph 4.1.5.2.a)
P_s	Static head from liquid or bulk materials (e.g., catalyst)
D	Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following: • Weight of vessel including internals, supports (e.g., skirts, lugs, saddles, and legs), and appurtenances (e.g., platforms, ladders, etc.) • Weight of vessel contents under design and test conditions • Refractory linings, insulation • Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping • Transportation loads (the static forces obtained as equivalent to the dynamic loads experienced during normal operation of a transport vessel [see paragraph 1.2.1.3(b)]
L	• Appurtenance live loading • Effects of fluid flow, steady state or transient • Loads resulting from wave action
E	Earthquake loads [see paragraph 4.1.5.3(b)]
W	Wind loads [see paragraph 4.1.5.3(b)]
S_s	Snow loads
F	Loads due to Deflagration

Table E4.4.6.2 – Design Load Combinations from VIII-2

Table 4.1.2 – Design Load Combinations	
Design Load Combination [Note (1) and (2)]	**General Primary Membrane Allowable Stress [Note (3)]**
(1) $P+P_s+D$	S
(2) $P+P_s+D+L$	S
(3) $P+P_s+D+S_s$	S
(4) $\Omega P+P_s+D+0.75L+0.75S_s$	S
(5) $\Omega P+P_s+D+(0.6W\ or\ 0.7E)$	S
(6) $\Omega P+P_s+D+0.75(0.6W\ or\ 0.7E)+0.75L+0.75S_s$	S
(7) $0.6D+(0.6W\ or\ 0.7E)$ [Note(4)]	S
(8) P_s+D+F	See Annex 4-D
(9) Other load combinations as defined in the UDS	S

Notes:

1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.
2) See 4.1.5.3 for additional requirements.
3) S is the allowable stress for the load case combination [see paragraph 4.1.5.3(c)].
4) This load combination addresses an overturning condition for foundation design. It does not apply to design of anchorage (if any) to the foundation. Refer to ASCE/SEI 7, 2.4.1 Exception 2 for an additional reduction to W that may be applicable.

Table E4.4.6.3 – Design Loads (Net-Section Axial Force and Bending Moment) at the Location of Interest

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)	$P = -14.7\ psi$
P_s	Static head from liquid or bulk materials (e.g., catalyst)	$P_s = 0.0\ psi$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest	$D_F = -66152.5\ lbs$ $D_M = 0.0\ in-lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = 0.0\ lbs$ $L_M = 0.0\ in-lbs$
E	Earthquake loads	$E_F = 0.0\ lbs$ $E_M = 0.0\ in-lbs$
W	Wind Loads	$W_F = 0.0\ lbs$ $W_M = 5.08E+06\ in-lbs$ $W_V = 18762.6\ lbs$
S_s	Snow Loads	$S_F = 0.0\ lbs$ $S_M = 0.0\ in-lbs$
F	Loads due to Deflagration	$F_F = 0.0\ lbs$ $F_M = 0.0\ in-lbs$

Based on these loads, the shell is required to be designed for the load case combinations shown in Table E4.4.6.4. Note that this table is given in terms of the load combinations shown in Table 4.1.2 (Table E4.4.6.2 of this example).

Table E4.4.6.4 – Load Case Combination at the Location of Interest

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P+P_s+D$	$P+P_s=-14.7\ psi$ $F_1=-66152.5\ lbs$ $M_1=0.0\ in-lbs$	S
2	$P+P_s+D+L$	$P+P_s=-14.7\ psi$ $F_2=-66152.5\ lbs$ $M_2=0.0\ in-lbs$	S
3	$P+P_s+D+S_s$	$P+P_s=-14.7\ psi$ $F_3=-66152.5\ lbs$ $M_3=0.0\ in-lbs$	S
4	$\Omega P+P_s+D+0.75L+0.75S_s$	$\Omega P+P_s=-14.7\ psi$ $F_4=-66152.5\ lbs$ $M_4=0.0\ in-lbs$	S
5	$\Omega P+P_s+D+(0.6W\ or\ 0.7E)$	$\Omega P+P_s=-14.7\ psi$ $F_5=-66152.5\ lbs$ $M_5=3048000\ in-lbs$ $V_5=11257.6\ lbs$	S
6	$\Omega P+P_s+D+0.75(0.6W\ or\ 0.7E)+$ $0.75L+0.75S_s$	$\Omega P+P_s=-14.7\ psi$ $F_6=-66152.5\ lbs$ $M_6=2286000\ in-lbs$ $V_6=8443.0\ lbs$	S
7	$0.6D+(0.6W\ or\ 0.7E)$	$F_6=-39691.5\ lbs$ $M_6=3048000\ in-lbs$	S
8	P_s+D+F	$P_s=0.0\ psi$ $F_8=-66152.5\ lbs$ $M_8=0.0\ in-lbs$	See Annex 4-D

4.4.7 Example E4.4.7 – Conical Transitions without a Knuckle

Determine if the proposed large and small end cylinder-to-cone transitions are adequately designed considering the following design conditions and applied forces and moments.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $-14.7\ psig@300°F$
- Inside Radius (Large End) = $75.0\ in$
- Thickness (Large End) = $1.8125\ in$
- Inside Radius (Small End) = $45.0\ in$
- Thickness (Small End) = $1.125\ in$
- Thickness (Conical Section) = $1.9375\ in$
- Length of Conical Section = $78.0\ in$
- Unsupported Length of Large Cylinder = $732.0\ in$
- Unsupported Length of Small Cylinder = $636.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Yield Strength = $33600\ psi$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle (See E4.3.2) = $21.0375\ deg$
- Axial Force (Large End) = $-99167\ lbs$
- Net Section Bending Moment (Large End) = $5.406E+06\ in-lbs$
- Axial Force (Small End) = $-78104\ lbs$
- Net Section Bending Moment (Small End) = $4.301E+06\ in-lbs$

Adjust variables for corrosion and determine outside dimensions.

$$t_L = 1.8125 - Corrosion\ Allowance = 1.8125 - 0.125 = 1.6875\ in$$
$$t_S = 1.125 - Corrosion\ Allowance = 1.125 - 0.125 = 1.0\ in$$
$$t_C = 1.9375 - Corrosion\ Allowance = 1.9375 - 0.125 = 1.8125\ in$$

$$R_L = 75.0 + Uncorroded\ Thickness = 75.0 + 1.9375 = 76.8125\ in$$
$$R_S = 45.0 + Uncorroded\ Thickness = 45.0 + 1.9375 = 46.125\ in$$

$$D_L = 2R_L = 2(76.8125) = 153.625\ in$$
$$D_S = 2R_S = 2(46.125) = 92.25\ in$$

Evaluate per paragraphs 4.4.13 and 4.3.11.

The design rules in paragraph 4.3.11 shall be satisfied. In these calculations, a negative value of pressure shall be used in all applicable equations.

Per paragraph 4.3.11.3, the length of the conical shell, measured parallel to the surface of the cone, shall be equal to or greater than the following:

$$\left\{L_C = 78.0\right\} \geq \left\{ \begin{array}{l} 2.0\sqrt{\dfrac{R_L t_C}{\cos[\alpha]}} + 1.4\sqrt{\dfrac{R_S t_C}{\cos[\alpha]}} \\ 2.0\sqrt{\dfrac{75.125(1.8125)}{\cos[21.0375]}} + 1.4\sqrt{\dfrac{45.125(1.8125)}{\cos[21.0375]}} = 37.2624\ in \end{array} \right\} \quad True$$

Evaluate the Large End cylinder-to-cone junction per VIII-2, paragraph 4.3.11.4.

a) STEP 1 – Compute the large end cylinder thickness, t_L, using paragraph 4.3.3., (as specified in design conditions)

$$t_L = 1.6875\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the large end using paragraph 4.3.4., (as specified in design conditions)

$$\alpha = 21.0375\ deg$$

$$t_C = 1.8125\ in$$

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with Part 5. In the calculations if $0\ deg < \alpha \leq 10\ deg$, then use $\alpha = 10\ deg$.

$$20 \leq \left\{\frac{R_L}{t_L} = \frac{75.125}{1.6875} = 44.5185\right\} \leq 500 \qquad True$$

$$1 \leq \left\{\frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741\right\} \leq 2 \qquad True$$

$$\left\{\alpha = 21.0375\ deg\right\} \leq 60\ deg \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force, F_L. Calculate the equivalent line load, X_L, using the specified net section axial force, F_L, and bending moment, M_L.

$$X_L = \frac{F_L}{2\pi R_L} \pm \frac{M_L}{\pi R_L^2} = \left\{ \begin{array}{l} \dfrac{-99167}{2\pi(75.125)} + \dfrac{5.406E+06}{\pi(75.125)^2} = 94.8111\ \dfrac{lbs}{in} \\ \dfrac{-99167}{2\pi(75.125)} - \dfrac{5.406E+06}{\pi(75.125)^2} = -514.9886\ \dfrac{lbs}{in} \end{array} \right\}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per Table 4.3.3 and VIII-2, Table 4.3.4, respectively. For calculated values of n other than those presented in Table 4.3.3 and Table 4.3.4, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741$$

$$H = \sqrt{\frac{R_L}{t_L}} = \sqrt{\frac{75.125}{1.6875}} = 6.6722$$

$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in Table 4.3.3 and Table 4.3.4 is required. The results of the interpolation are summarized with the following values for C_i (see paragraph 4.3.11.4 and STEP 5 of E4.3.7).

For the applied pressure case both M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\left[\begin{array}{l} C_1 + C_2\ln[H] + C_3\ln[B] + C_4(\ln[H])^2 + C_5(\ln[B])^2 + \\ C_6\ln[H]\ln[B] + C_7(\ln[H])^3 + C_8(\ln[B])^3 + \\ C_9\ln[H](\ln[B])^2 + C_{10}(\ln[H])^2\ln[B] \end{array}\right]$$

This results in the following (see paragraph 4.3.11.4 and STEP 5 of E4.3.7):

$$M_{sN} = -10.6148$$
$$Q_N = -4.0925$$

For the Equivalent Line Load case, M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\left[\frac{\left(\begin{array}{l} C_1 + C_3\ln[H^2] + C_5\ln[\alpha] + C_7(\ln[H^2])^2 + \\ C_9(\ln[\alpha])^2 + C_{11}\ln[H^2]\ln[\alpha] \end{array}\right)}{\left(\begin{array}{l} 1 + C_2\ln[H^2] + C_4\ln[\alpha] + C_6(\ln[H^2])^2 + \\ C_8(\ln[\alpha])^2 + C_{10}\ln[H^2]\ln[\alpha] \end{array}\right)}\right]$$

This results in the following (see paragraph 4.3.11.4 and STEP 5 of E4.3.7):

$$M_{sN} = -0.4912$$
$$Q_N = -0.1845$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$Internal\ Pressure: \quad M_{sN} = -10.6148, \quad Q_N = -4.0925$$
$$Equivalent\ Line\ Load: \quad M_{sN} = -0.4912, \quad Q_N = -0.1845$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in Table 4.3.1 for the Large End Junction.

Evaluate the Cylinder at the Large End:

Stress Resultant Calculations:

$$M_{sP} = Pt_L^2 M_{sN} = -14.7(1.6875)^2(-10.6148) = 444.3413 \ \frac{in-lbs}{in}$$

$$M_{sX} = X_L t_L M_{sN} = \begin{Bmatrix} 94.8111(1.6875)(-0.4912) = -78.5889 \ \frac{in-lbs}{in} \\ -514.9886(1.6875)(-0.4912) = 426.8741 \ \frac{in-lbs}{in} \end{Bmatrix}$$

$$M_s = M_{sP} + M_{sX} = \begin{Bmatrix} 444.3413 + (-78.5889) = 365.7524 \ \frac{in-lbs}{in} \\ 444.3413 + 426.8741 = 871.2154 \ \frac{in-lbs}{in} \end{Bmatrix}$$

$$Q_P = Pt_L Q_N = -14.7(1.6875)(-4.0925) = 101.5196 \ \frac{lbs}{in}$$

$$Q_X = X_L Q_N = \begin{Bmatrix} 94.8111(-0.1845) = -17.4926 \ \frac{lbs}{in} \\ -514.9886(-0.1845) = 95.0154 \ \frac{lbs}{in} \end{Bmatrix}$$

$$Q = Q_P + Q_X = \begin{Bmatrix} 101.5196 + (-17.4926) = 84.0270 \ \frac{lbs}{in} \\ 101.5196 + 95.0154 = 196.5350 \ \frac{lbs}{in} \end{Bmatrix}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_L^2 t_L^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(75.125)^2(1.6875)^2}\right]^{0.25} = 0.1142 \ in^{-1}$$

$$N_s = \frac{PR_L}{2} + X_L = \begin{Bmatrix} \frac{-14.7(75.125)}{2} + 94.8111 = -457.3577 \ \frac{lbs}{in} \\ \frac{-14.7(75.125)}{2} + (-514.9886) = -1067.1574 \ \frac{lbs}{in} \end{Bmatrix}$$

$$N_\theta = PR_L + 2\beta_{cy} R_L \left(-M_s \beta_{cy} + Q\right)$$

$$N_\theta = \begin{Bmatrix} -14.7(75.125) + 2(0.1142)(75.125)(-(365.7524)(0.1142) + 84.0270) \\ -14.7(75.125) + 2(0.1142)(75.125)(-(871.2154)(0.1142) + 196.5350) \end{Bmatrix}$$

$$N_\theta = \begin{Bmatrix} -379.2502 \ \frac{lbs}{in} \\ 560.7660 \ \frac{lbs}{in} \end{Bmatrix}$$

$$K_{pc} = 1.0$$

Stress Calculations:

Determine the axial and hoop membrane and bending stresses.

$$\sigma_{sm} = \frac{N_s}{t_L} = \begin{Bmatrix} \dfrac{-457.3577}{1.6875} = -271.0268 \; psi \\ \dfrac{-1067.1574}{1.6875} = -632.3895 \; psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_s}{t_L^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(365.7524)}{(1.6875)^2(1.0)} = 770.6388 \; psi \\ \dfrac{6(871.2154)}{(1.6875)^2(1.0)} = 1835.6472 \; psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_L} = \begin{Bmatrix} \dfrac{-379.2502}{1.6875} = -224.7409 \; psi \\ \dfrac{560.7660}{1.6875} = 332.3058 \; psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_L^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(0.3)(365.7524)}{(1.6875)^2(1.0)} = 231.1916 \; psi \\ \dfrac{6(0.3)(871.2154)}{(1.6875)^2(1.0)} = 550.6942 \; psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\begin{Bmatrix} \sigma_{sm} = -271.0268 \; psi \\ \sigma_{sm} = -632.3895 \; psi \end{Bmatrix} \le \{1.5S, \text{ not applicable due to compressive stress}\}$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = -271.0268 + 770.6388 = 499.6 \; psi \\ \sigma_{sm} - \sigma_{sb} = -271.0268 - 770.6388 = |-1041.7| \; psi \\ \sigma_{sm} + \sigma_{sb} = -632.3895 + 1835.6472 = 1203.3 \; psi \\ \sigma_{sm} - \sigma_{sb} = -632.3895 - 1835.6472 = |-2468.0| \; psi \end{Bmatrix} \le \{S_{PS} = 67200 \; psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} = -224.7409 \\ \sigma_{\theta m} = 332.3058 \end{Bmatrix} \le \begin{Bmatrix} 1.5S, \text{ not applicable due to compressive stress} \\ 1.5S = 1.5(22400) = 33600 \; psi \end{Bmatrix} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = -224.7409 + 231.1916 = 6.5 \; psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -224.7409 - 231.1916 = |-455.9| \; psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 332.3058 + 550.6942 = 883.0 \; psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 332.3058 - 550.6942 = |-218.4| \; psi \end{Bmatrix} \le \{S_{PS} = 67200 \; psi\} \qquad True$$

Since the hoop membrane stress, $\sigma_{\theta m}$ and the axial membrane stress, σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m} \le F_{ha}$$
$$\sigma_{sm} \le F_{xa}$$

F_{ha} is evaluated using paragraph 4.4.5.1 but substituting F_{he} with the following equation.

$$F_{he} = \frac{0.4 E_y t}{D_o}$$

F_{xa} is evaluated using paragraph 4.4.12.2.b with $\lambda = 0.15$.

In accordance with paragraph 4.4.5.1, the value of F_{ha} is calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

$$t = 1.6875\ in$$
$$L \rightarrow Not\ required,\ as\ the\ equation\ for\ F_{he}\ is\ independent\ of\ L$$

2) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{0.4 E_y t}{D_o} = \frac{0.4(28.3E+06)(1.6875)}{153.625} = 124344.9959\ psi$$

3) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$$F_{he} = 124344.9959\ psi$$ (as determined in STEP 2 above)

ii) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{124344.9959}{28.3E+06} = 0.00439382$$

iii) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 27206.0299\ psi$$

4) STEP 4 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55 S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{27206.0299}{33600}\right) = 1.8070$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{27206.0299}{1.8070} = 15055.9103 \ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ to the allowable hoop compressive membrane stress, F_{ha} per following criteria.

$$\{\sigma_{\theta m} = 224.7 \ psi\} \le \{F_{ha} = 15055.9 \ psi\} \qquad True$$

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

In accordance with paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

1) STEP 1 – Calculate the predicted elastic buckling stress, F_{xe}.

$$F_{xe} = \frac{C_x E_y t}{D_o} = \frac{0.8520(28.3E+06)(1.6875)}{153.625} = 264854.8413 \ psi$$

where,

$$\frac{D_o}{t} = \frac{153.625}{1.6875} = 91.0370$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{732.0}{\sqrt{76.8125(1.6875)}} = 64.2944$$

Since $D_o/t < 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \frac{D_o}{t}\right)}, \ 0.9\right] = \min\left[\frac{409(1.0)}{389 + \frac{153.625}{1.6875}}, \ 0.9\right] = 0.8520$$

Since $M_x \ge 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

2) STEP 2 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{xe}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 2.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$F_{xe} = 264854.8413 \ psi$ (as determined in STEP 1 above)

ii) STEP 2.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{xe}}{E} = \frac{264854.8413}{28.3E+06} = 0.00935883$$

iii) STEP 2.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 31046.7970 \ psi$$

3) STEP 3 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{31046.7970}{33600}\right) = 1.7223$$

4) STEP 4 – Calculate the allowable axial compressive membrane stress as follows:

$$F_{xa} = \frac{F_{ic}}{FS} = \frac{31046.7970}{1.7223} = 18026.3584 \ psi$$

5) STEP 5 – Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria.

$$\{\sigma_{sm} = 632.4 \ psi\} \leq \{F_{xa} = 18026.4 \ psi\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

Evaluate the Cone at the Large End:

Stress Resultant Calculations, as determined above.

$$M_{csP} = M_{sP} = 444.3413 \ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \begin{Bmatrix} -78.5889 \ \frac{in-lbs}{in} \\ 426.8741 \ \frac{in-lbs}{in} \end{Bmatrix}$$

$$M_{cs} = M_{csP} + M_{csX} = \begin{Bmatrix} 444.3413 + (-78.5889) = 365.7524 \ \frac{in-lbs}{in} \\ 444.3413 + 426.8741 = 871.2154 \ \frac{in-lbs}{in} \end{Bmatrix}$$

$$Q_c = Q\cos[\alpha] + N_s \sin[\alpha]$$

$$Q_c = \begin{Bmatrix} 84.0270(\cos[21.0375]) + (-457.3577)\sin[21.0375] = -85.7555 \ \frac{lbs}{in} \\ 196.5350(\cos[21.0375]) + (-1067.1574)\sin[21.0375] = -199.6519 \ \frac{lbs}{in} \end{Bmatrix}$$

$$R_C = \frac{R_L}{\cos[\alpha]} = \frac{75.125}{\cos[21.0375]} = 80.4900\ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_C^2 t_C^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(80.4900)^2(1.8125)^2}\right]^{0.25} = 0.1064\ in^{-1}$$

$$N_{cs} = N_s \cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{Bmatrix} -457.3577(\cos[21.0375]) - 84.0270\sin[21.0375] = -457.0368\ \dfrac{lbs}{in} \\ -1067.1574(\cos[21.0375]) - 196.5350\sin[21.0375] = -1066.5786\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_{c\theta} = \frac{PR_L}{\cos[\alpha]} + 2\beta_{co}R_C\left(-M_{cs}\beta_{co} - Q_c\right)$$

$$N_{c\theta} = \begin{Bmatrix} \dfrac{-14.7(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(365.7524)(0.1064) - (-85.7555)\right) \\ \dfrac{-14.7(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(871.2154)(0.1064) - (-199.6519)\right) \end{Bmatrix}$$

$$N_{c\theta} = \begin{Bmatrix} -380.9244\ \dfrac{lbs}{in} \\ 648.7441\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{cpc} = 1.0$$

Stress Calculations:

Determine the axial and hoop membrane and bending stresses.

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \begin{Bmatrix} \dfrac{-457.0368}{1.8125} = -252.1582\ psi \\ \dfrac{-1066.5786}{1.8125} = -588.4572\ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(365.7524)}{(1.8125)^2(1.0)} = 668.0091\ psi \\ \dfrac{6(871.2154)}{(1.8125)^2(1.0)} = 1591.1853\ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \begin{Bmatrix} \dfrac{-380.9244}{1.8125} = -210.1652\ psi \\ \dfrac{648.7441}{1.8125} = 357.9278\ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(0.3)(365.7524)}{(1.8125)^2(1.0)} = 200.4027 \ psi \\ \dfrac{6(0.3)(871.2154)}{(1.8125)^2(1.0)} = 477.3556 \ psi \end{Bmatrix}$$

f) STEP 6 – Check Acceptance Criteria:

$$\begin{Bmatrix} \sigma_{sm} = -252.1582 \ psi \\ \sigma_{sm} = -588.4572 \ psi \end{Bmatrix} \le \{1.5S, \ not \ applicable \ due \ to \ compressive \ stress\}$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = -252.1582 + 668.0091 = 415.6 \ psi \\ \sigma_{sm} - \sigma_{sb} = -252.1582 - 668.0091 = |-920.2| \ psi \\ \sigma_{sm} + \sigma_{sb} = -588.4572 + 1591.1853 = 1002.7 \ psi \\ \sigma_{sm} - \sigma_{sb} = -588.4572 - 1591.1853 = |-2179.6| \ psi \end{Bmatrix} \le \{S_{PS} = 67200 \ psi\} \quad True$$

$$\begin{Bmatrix} \sigma_{\theta m} = -210.1652 \\ \sigma_{\theta m} = 357.9278 \end{Bmatrix} \le \begin{Bmatrix} 1.5S, \ not \ applicable \ due \ to \ compressive \ stress \\ 1.5S = 1.5(22400) = 33600 \ psi \end{Bmatrix} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = -210.1652 + 200.4027 = |-9.7| \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -210.1652 - 200.4027 = |-410.6| \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 357.9278 + 477.3556 = 835.3 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 357.9278 - 477.3556 = |-119.4| \ psi \end{Bmatrix} \le \{S_{PS} = 67200 \ psi\} \qquad True$$

Since the hoop membrane stress, $\sigma_{\theta m}$ and the axial membrane stress, σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m} \le F_{ha}$$
$$\sigma_{sm} \le F_{xa}$$

F_{ha} is evaluated using paragraph 4.4.5.1 but substituting F_{he} with the following equation.

$$F_{he} = \frac{0.4 E_y t}{D_o}$$

where,

$$t = t_c = 1.8125 \ in$$

$$D_o = \frac{0.5(D_{cL} + D_{cs})}{\cos[\alpha]} = \frac{0.5(153.875 + 93.875)}{\cos[21.0375]} = 132.7215 \ in$$

F_{xa} is evaluated using paragraph 4.4.12.2.b with $\lambda = 0.15$ and the following substitutions.

$$t = t_c = 1.8125 \ in$$

$$D_o = 132.7215 \ in$$

$$L = \frac{L_{ce}}{\cos[\alpha]} = \frac{L_c}{\cos[\alpha]} = \frac{78.0}{\cos[21.0375]} = 83.5703\ in$$

Using the procedure shown above for the large end cylindrical shell and the above noted substitutions, the allowable compressive hoop membrane and axial membrane stresses, F_{ha} and F_{xa}, respectively, are as follows.

$$F_{ha} = 15888.9\ psi$$
$$F_{xa} = 19150.9\ psi$$

Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ and axial compressive membrane stress, σ_{sm}, to the allowable hoop compressive membrane stress, F_{ha} and axial compressive membrane stress, F_{xa} per following criteria.

$$\{\sigma_{\theta m} = 210.2\ psi\} \le \{F_{ha} = 15888.9\ psi\} \qquad True$$
$$\{\sigma_{sm} = 588.5\ psi\} \le \{F_{xa} = 19150.9\ psi\} \qquad True$$

Therefore, local buckling due to hoop and axial compressive membrane stress is not a concern.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore, the design is complete.

Evaluate the Small End cylinder-to-cone junction per paragraph 4.3.11.5.

a) STEP 1 – Compute the small end cylinder thickness, t_S, using paragraph 4.3.3., (as specified in design conditions).

$$t_S = 1.0\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the small end using paragraph 4.3.4., (as specified in design conditions)

$$\alpha = 21.0375\ deg$$
$$t_C = 1.8125\ in$$

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with Part 5. In the calculations, if $0\ deg < \alpha \le 10\ deg$, then use $\alpha = 10\ deg$.

$$20 \le \left\{\frac{R_S}{t_S} = \frac{45.125}{1.0} = 45.125\right\} \le 500 \qquad True$$

$$1 \le \left(\frac{t_C}{t_S} = \frac{1.8125}{1.0} = 1.8125\right) \le 2 \qquad True$$

$$\{\alpha = 21.0375\ deg\} \le 60\ deg \qquad True$$

d) STEP 4 – Determine the net section axial force, F_S, and bending moment, M_S, applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force, F_S. Calculate the equivalent line load, X_S, using the specified net section axial force, F_S, and bending moment, M_S.

$$X_S = \frac{F_S}{2\pi R_S} \pm \frac{M_S}{\pi R_S^2} = \begin{Bmatrix} \dfrac{-78104}{2\pi(45.125)} + \dfrac{4.301E+06}{\pi(45.125)^2} = 396.8629\ \dfrac{lbs}{in} \\ \dfrac{-78104}{2\pi(45.125)} - \dfrac{4.301E+06}{\pi(45.125)^2} = -947.8053\ \dfrac{lbs}{in} \end{Bmatrix}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per Table 4.3.5 and Table 4.3.6, respectively. For calculated values of n other than those presented in Table 4.3.5 and Table 4.3.6, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_S} = \frac{1.8125}{1.0} = 1.8125$$

$$H = \sqrt{\frac{R_S}{t_S}} = \sqrt{\frac{45.125}{1.0}} = 6.7175$$

$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in Table 4.3.5 and Table 4.3.6 is required. The results of the interpolation are summarized with the following values for C_i (see paragraph 4.3.11.5 and STEP 5 of E4.3.7).

For the applied pressure case M_{sN} is calculated using the following equation:

$$M_{sN} = \exp\begin{bmatrix} C_1 + C_2\ln\left[H^2\right] + C_3\ln[\alpha] + C_4\left(\ln\left[H^2\right]\right)^2 + C_5\left(\ln[\alpha]\right)^2 + \\ C_6\ln\left[H^2\right]\ln[\alpha] + C_7\left(\ln\left[H^2\right]\right)^3 + C_8\left(\ln[\alpha]\right)^3 + \\ C_9\ln\left[H^2\right]\left(\ln[\alpha]\right)^2 + C_{10}\left(\ln\left[H^2\right]\right)^2\ln[\alpha] \end{bmatrix}$$

This results in the following (see paragraph 4.3.11.5 and STEP 5 of E4.3.7).

$$M_{sN} = 9.2135$$

For the applied pressure case Q_N is calculated using the following equation:

$$Q_N = \left(\frac{C_1 + C_3H^2 + C_5\alpha + C_7H^4 + C_9\alpha^2 + C_{11}H^2\alpha}{1 + C_2H^2 + C_4\alpha + C_6H^4 + C_8\alpha^2 + C_{10}H^2\alpha}\right)$$

This results in the following (see paragraph 4.3.11.5 and STEP 5 of E4.3.7).

$$Q_N = -2.7333$$

For the Equivalent Line Load case, M_{sN} is calculated using the following equation:

$$M_{sN} = \left(\frac{C_1 + C_3H + C_5B + C_7H^2 + C_9B^2 + C_{11}HB}{1 + C_2H + C_4B + C_6H^2 + C_8B^2 + C_{10}HB}\right)$$

This results in the following (see paragraph 4.3.11.5 and STEP 5 of E4.3.7).

$$M_{sN} = 0.4828$$

For the Equivalent Line Load case, Q_N is calculated using the following equation:

$$Q_N = \left(\begin{array}{l} C_1 + C_2 \ln[H] + C_3 \ln[B] + C_4 (\ln[H])^2 + C_5 (\ln[B])^2 + C_6 \ln[H]\ln[B] + \\ C_7 (\ln[H])^3 + C_8 (\ln[B])^3 + C_9 \ln[H](\ln[B])^2 + C_{10} (\ln[H])^2 \ln[B] \end{array} \right)$$

This results in the following (see paragraph 4.3.11.5 and STEP 5 of E4.3.7).

$$Q_N = -0.1613$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$\begin{array}{lll} \textit{Internal Pressure}: & M_{sN} = 9.2135, & Q_N = -2.7333 \\ \textit{Equivalent Line Load}: & M_{sN} = 0.4828, & Q_N = -0.1613 \end{array}$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in Table 4.3.2. for the Small End Junction.

<u>Evaluate the Cylinder at the Small End.</u>

Stress Resultant Calculations.

$$M_{sP} = Pt_S^2 M_{sN} = -14.7(1.0)^2(9.2135) = -135.4385 \ \frac{in-lbs}{in}$$

$$M_{sX} = X_S t_S M_{sN} = \begin{Bmatrix} 396.8629(1.0)(0.4828) = 191.6054 \ \dfrac{in-lbs}{in} \\ -947.8053(1.0)(0.4828) = -457.6004 \ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$M_s = M_{sP} + M_{sX} = \begin{Bmatrix} -135.4385 + (191.6054) = 56.1669 \ \dfrac{in-lbs}{in} \\ -135.4385 + (-457.6004) = -593.0389 \ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$Q_P = Pt_S Q_N = -14.7(1.0)(-2.7333) = 40.1795 \ \frac{lbs}{in}$$

$$Q_X = X_S Q_N = \begin{Bmatrix} 396.8629(-0.1613) = -64.0140 \ \dfrac{lbs}{in} \\ -947.8053(-0.1613) = 152.8810 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$Q = Q_P + Q_X = \begin{Bmatrix} 40.1795 + (-64.0140) = -23.8345 \ \dfrac{lbs}{in} \\ 40.1795 + 152.8810 = 193.0605 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_S^2 t_S^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(45.1250)^2(1.000)^2}\right]^{0.25} = 0.1914 \ in^{-1}$$

$$N_s = \frac{PR_S}{2} + X_S = \begin{Bmatrix} \dfrac{-14.7(45.125)}{2} + 396.8629 = 65.1942 \ \dfrac{lbs}{in} \\ \dfrac{-14.7(45.125)}{2} + (-947.8053) = -1279.4741 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_\theta = PR_S + 2\beta_{cy} R_S \left(-M_s \beta_{cy} - Q\right)$$

$$N_\theta = \begin{Bmatrix} -14.7(45.125) + 2(0.1914)(45.125)(-(56.1669)(0.1914) - (-23.8345)) \\ -14.7(45.125) + 2(0.1914)(45.125)(-(-593.0389)(0.1914) - 193.0605) \end{Bmatrix}$$

$$N_\theta = \begin{Bmatrix} -437.3238 \ \dfrac{lbs}{in} \\ -2037.5216 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{pc} = 1.0$$

Stress Calculations:

Determine the axial and hoop membrane and bending stresses:

$$\sigma_{sm} = \frac{N_s}{t_S} = \begin{Bmatrix} \dfrac{65.1942}{1.0} = 65.1942 \ psi \\ \dfrac{-1279.4741}{1.0} = -1279.4741 \ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(56.1669)}{(1.0)^2(1.0)} = 337.0014 \ psi \\ \dfrac{6(-593.0389)}{(1.0)^2(1.0)} = -3558.2334 \ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_S} = \begin{Bmatrix} \dfrac{-437.3238}{1.0} = -437.3238 \ psi \\ \dfrac{-2037.5216}{1.0} = -2037.5216 \ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(0.3)(56.1669)}{(1.0)^2(1.0)} = 101.1004 \; psi \\ \dfrac{6(0.3)(-593.0389)}{(1.0)^2(1.0)} = -1067.4700 \; psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\begin{Bmatrix} \sigma_{sm} = 65.1942 \; psi \\ \sigma_{sm} = -1279.4741 \; psi \end{Bmatrix} \le \begin{Bmatrix} 1.5S = 1.5(22400) = 33600 \; psi \\ 1.5S, \; not \; applicable \; due \; to \; compressive \; stress \end{Bmatrix} \qquad True$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = 65.1942 + 337.0014 = 402.2 \; psi \\ \sigma_{sm} - \sigma_{sb} = 65.1942 - 337.0014 = |-271.8| \; psi \\ \sigma_{sm} + \sigma_{sb} = -1279.4741 + (-3558.2334) = |-4837.7| \; psi \\ \sigma_{sm} - \sigma_{sb} = -1279.4741 - (-3558.2334) = 2278.8 \; psi \end{Bmatrix} \le \{S_{PS} = 67200 \; psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} = -437.3238 \\ \sigma_{\theta m} = -2037.5216 \end{Bmatrix} \le \{1.5S, \; not \; applicable \; due \; to \; compressive \; stress\}$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = -437.3238 + 101.1004 = |-336.2| \; psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -437.3238 - 101.1004 = |-538.4| \; psi \\ \sigma_{\theta m} + \sigma_{\theta b} = -2037.5216 + (-1067.4700) = |-3105.0| \; psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -2037.5216 - (-1067.4700) = |-970.1| \; psi \end{Bmatrix} \le \{S_{PS} = 67200 \; psi\} \qquad True$$

Since the hoop membrane stress $\sigma_{\theta m}$ and axial membrane stress σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m} \le F_{ha}$$

$$\sigma_{sm} \le F_{xa}$$

F_{ha} is evaluated using paragraph 4.4.5.1 but substituting F_{he} with the following equation.

$$F_{he} = \frac{0.4E_y t}{D_o}$$

F_{xa} is evaluated using paragraph 4.4.12.2.b with $\lambda = 0.15$.

In accordance with paragraph 4.4.5.1, the value of F_{ha} is calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

$$t = 1.0 \; in$$

$$L \rightarrow Not \; required, \; as \; the \; equation \; for \; F_{he} \; is \; independent \; of \; L$$

2) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{0.4E_y t}{D_o} = \frac{0.4(28.3E+06)(1.0)}{92.25} = 122710.0271 \; psi$$

3) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$F_{he} = 122710.0271\ psi$ (as determined in STEP 2 above)

ii) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{122710.0271}{28.3E+06} = 0.00433604$$

iii) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 27137.9709\ psi$$

4) STEP 4 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{27137.9709}{33600}\right) = 1.8085$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{27137.9709}{1.8085} = 15005.8\ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ to the allowable hoop compressive membrane stress, F_{ha} per following criteria.

$$\{\sigma_{\theta m} = 2037.5\ psi\} \le \{F_{ha} = 15005.8\ psi\} \qquad True$$

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

In accordance with paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

1) STEP 1 – Calculate the predicted elastic buckling stress, F_{xe}.

$$F_{xe} = \frac{C_x E_y t}{D_o} = \frac{0.8499(28.3E+06)(1.0)}{92.25} = 260728.1301\ psi$$

where,

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{46.125(1.0)}} = 93.6459$$

Since $D_o/t < 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \frac{D_o}{t}\right)},\ 0.9\right] = \min\left[\frac{409(1.0)}{389 + \frac{92.25}{1.0}},\ 0.9\right] = 0.8499$$

Since $M_x \geq 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

2) STEP 2 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{xe}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 2.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$F_{xe} = 260728.1301\ psi$ (as determined in STEP 1 above)

ii) STEP 2.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{xe}}{E} = \frac{260728.1301}{28.3E+06} = 0.00921301$$

iii) STEP 2.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 30967.6147\ psi$$

3) STEP 3 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{30967.6147}{33600}\right) = 1.7241$$

4) STEP 4 – Calculate the allowable axial compressive membrane stress as follows:

$$F_{xa} = \frac{F_{ic}}{FS} = \frac{30967.6147}{1.7241} = 17961.6117\ psi$$

5) STEP 5 – Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria.

$$\left\{\sigma_{sm} = 632.4 \ psi\right\} \le \left\{F_{xa} = 17961.6 \ psi\right\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

Evaluate the Cone at the Small End.

Stress Resultant Calculations as determined above.

$$M_{csP} = M_{sP} = -135.4385 \ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \begin{Bmatrix} 191.6054 \ \frac{in-lbs}{in} \\ -457.6004 \ \frac{in-lbs}{in} \end{Bmatrix}$$

$$M_{cs} = M_{csP} + M_{csX} = \begin{Bmatrix} -135.4385 + 191.6054 = 56.1669 \ \frac{in-lbs}{in} \\ -135.4385 + (-457.6004) = -593.0389 \ \frac{in-lbs}{in} \end{Bmatrix}$$

$$Q_c = Q\cos[\alpha] + N_s\sin[\alpha]$$

$$Q_c = \begin{Bmatrix} (-23.8345)\cos[21.0375] + 65.1942\sin[21.0375] = 1.1575 \ \frac{lbs}{in} \\ 193.0605\cos[21.0375] + (-1279.4741)\sin[21.0375] = -279.1120 \ \frac{lbs}{in} \end{Bmatrix}$$

$$R_c = \frac{R_s}{\cos[\alpha]} = \frac{45.1250}{\cos[21.0375]} = 48.3476 \ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_c^{\,2} t_c^{\,2}}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(48.3476)^2(1.8125)^2}\right]^{0.25} = 0.1373 \ in^{-1}$$

$$N_{cs} = N_s\cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{Bmatrix} 65.1942\cos[21.0375] - (-23.8345)\sin[21.0375] = 69.4048 \ \frac{lbs}{in} \\ (-1279.4741)\cos[21.0375] - 193.0605\sin[21.0375] = -1263.4963 \ \frac{lbs}{in} \end{Bmatrix}$$

$$N_{c\theta}=\frac{PR_s}{\cos[\alpha]}+2\beta_{co}R_c\left(-M_{cs}\beta_{co}+Q_c\right)$$

$$N_{c\theta}=\begin{Bmatrix}\dfrac{-14.7(45.125)}{\cos[21.0375]}+2(0.1373)(48.3476)\left(-(56.1669)(0.1373)+1.1575\right)\\ \dfrac{-14.7(45.125)}{\cos[21.0375]}+2(0.1373)(48.3476)\left(-(-593.0389)(0.1373)+(-279.1120)\right)\end{Bmatrix}$$

$$N_{c\theta}=\begin{Bmatrix}-797.7248\dfrac{lbs}{in}\\ -3335.2619\dfrac{lbs}{in}\end{Bmatrix}$$

$$K_{cpc}=1.0$$

Stress Calculations:

Determine the axial and hoop membrane and bending stresses:

$$\sigma_{sm}=\frac{N_{cs}}{t_C}=\begin{Bmatrix}\dfrac{69.4048}{1.8125}=38.2923\ psi\\ \dfrac{-1263.4963}{1.8125}=-697.1014\ psi\end{Bmatrix}$$

$$\sigma_{sb}=\frac{6M_{cs}}{t_C^2K_{cpc}}=\begin{Bmatrix}\dfrac{6(56.1669)}{(1.8125)^2(1.0)}=102.5831\ psi\\ \dfrac{6(-593.0389)}{(1.8125)^2(1.0)}=-1083.1246\ psi\end{Bmatrix}$$

$$\sigma_{\theta m}=\frac{N_{c\theta}}{t_C}=\begin{Bmatrix}\dfrac{-797.7248}{1.8125}=-440.1240\ psi\\ \dfrac{-3335.2619}{1.8125}=-1840.1445\ psi\end{Bmatrix}$$

$$\sigma_{\theta b}=\frac{6\nu M_{cs}}{t_C^2K_{cpc}}=\begin{Bmatrix}\dfrac{6(0.3)(56.1669)}{(1.8125)^2(1.0)}=30.7749\ psi\\ \dfrac{6(0.3)(-593.0389)}{(1.8125)^2(1.0)}=-324.9374\ psi\end{Bmatrix}$$

Check Acceptable Criteria:

$$\left\{\begin{array}{l}\sigma_{sm}=38.2923\ \ psi\\ \sigma_{sm}=-697.1014\ \ psi\end{array}\right\}\leq\left\{\begin{array}{l}1.5S=1.5(22400)=33600\ \ psi\\ 1.5S,\ not\ applicable\ due\ to\ compressive\ stress\end{array}\right\}\qquad True$$

$$\left\{\begin{array}{l}\sigma_{sm}+\sigma_{sb}=38.2923+102.5831=140.9\ \ psi\\ \sigma_{sm}-\sigma_{sb}=38.2923-102.5831=\left|-64.3\right|\ \ psi\\ \sigma_{sm}+\sigma_{sb}=-697.1014+(-1083.1246)=\left|-1780.2\right|\ \ psi\\ \sigma_{sm}-\sigma_{sb}=-697.1014-(-1083.1246)=386.0\ \ psi\end{array}\right\}\leq\left\{S_{PS}=67200\ \ psi\right\}\qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m}=-440.1240\\ \sigma_{\theta m}=-1840.1445\end{array}\right\}\leq\left\{1.5S,\ not\ applicable\ due\ to\ compressive\ stress\right\}$$

$$\left\{\begin{array}{l}\sigma_{\theta m}+\sigma_{\theta b}=-440.1240+30.7749=\left|-409.3\right|\ \ psi\\ \sigma_{\theta m}-\sigma_{\theta b}=-440.1240-30.7749=\left|-470.9\right|\ psi\\ \sigma_{\theta m}+\sigma_{\theta b}=-1840.1445+(-324.9374)=\left|-2164.9\right|\ \ psi\\ \sigma_{\theta m}-\sigma_{\theta b}=-1840.1445-(-324.9374)=\left|-1515.1\right|\ psi\end{array}\right\}\leq\left\{S_{PS}=67200\ \ psi\right\}\qquad True$$

Since the hoop membrane stress, $\sigma_{\theta m}$ and the axial membrane stress, σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m}\leq F_{ha}$$
$$\sigma_{sm}\leq F_{xa}$$

F_{ha} is evaluated using VIII-2, paragraph 4.4.5.1, but substituting F_{he} with the following equation.

$$F_{he}=\frac{0.4E_y t}{D_o}$$

where,

$$t=t_c=1.8125\ in$$

$$D_o=\frac{0.5(D_{cL}+D_{cs})}{\cos[\alpha]}=\frac{0.5(153.875+93.875)}{\cos[21.0375]}=132.7215\ in$$

F_{xa} is evaluated using paragraph 4.4.12.2.b with $\lambda=0.15$ and the following substitutions.

$$t=t_c=1.8125\ in$$
$$D_o=132.7215\ in$$
$$L=\frac{L_{ce}}{\cos[\alpha]}=\frac{L_c}{\cos[\alpha]}=\frac{78.0}{\cos[21.0375]}=83.5703\ in$$

Using the procedure shown above for the large end cylindrical shell and the above noted substitutions, the allowable compressive hoop membrane and axial membrane stresses, F_{ha} and F_{xa}, respectively, are as follows.

$$F_{ha} = 15888.9 \ psi$$
$$F_{xa} = 19150.9 \ psi$$

Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ and axial compressive membrane stress, σ_{sm}, to the allowable hoop compressive membrane stress, F_{ha} and axial compressive membrane stress, F_{xa} per following criteria.

$$\{\sigma_{\theta m} = 697.1 \ psi\} \le \{F_{ha} = 15888.9 \ psi\} \qquad True$$
$$\{\sigma_{sm} = 1840.1 \ psi\} \le \{F_{xa} = 19150.9 \ psi\} \qquad True$$

Therefore, local buckling due to hoop and axial compressive membrane stress is not a concern.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore, the design is complete.

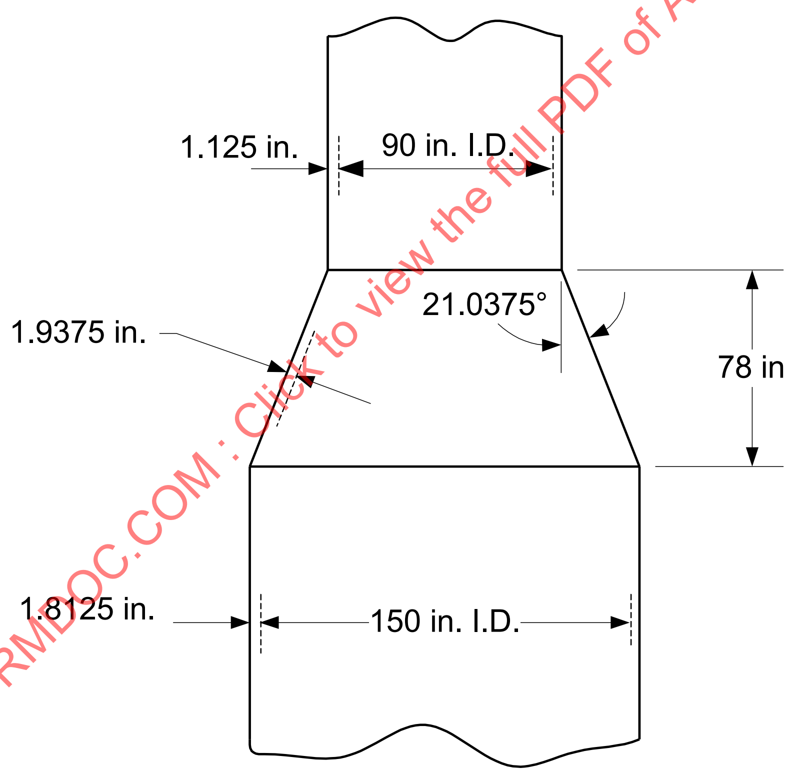

Figure E4.4.7 – Conical Transition

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4.4.8 Example E4.4.8 – Conical Transitions with a Knuckle

Determine if the proposed design for the large end of a cylinder-to-cone junction with a knuckle is adequately designed considering the following design conditions and applied forces and moments.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $-14.7\ psig@300°F$
- Inside Diameter (Large End) = $120.0\ in$
- Large End Thickness = $1.0\ in$
- Inside Diameter (Small End) = $33.0\ in$
- Small End Thickness = $1.0\ in$
- Knuckle Radius = $10.0\ in$
- Cone Thickness = $1.0\ in$
- Knuckle Thickness = $1.0\ in$
- Length of Conical Section = $73.0\ in$
- Unsupported Length of Large Cylinder = $240.0\ in$
- Unsupported Length of Small Cylinder = $360.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $22400\ psi$
- Yield Strength = $33600\ psi$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle = $30.0\ deg$
- Axial Force (Large End) = $-10000\ lbs$
- Net Section Bending Moment (Large End) = $2.0E+06\ in-lbs$

Evaluate per paragraphs 4.4.14 and 4.3.12.

The design rules in paragraph 4.3.12 shall be satisfied. In these calculations, a negative value of pressure shall be used in all applicable equations.

a) STEP 1 – Compute the large end cylinder thickness, t_L, using paragraph 4.4.5, (as specified in design conditions)

$$t_L = 1.0\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the large end using paragraph 4.4.5, (as specified in design conditions).

$$\alpha = 30\ deg$$

$$t_C = 1.0\ in$$

c) STEP 3 – Proportion the transition geometry by assuming a value for the knuckle radius, r_k, and knuckle thickness, t_k, such that the following equations are satisfied. If all these equations cannot be satisfied, the cylinder-to-cone junction shall be designed in accordance with Part 5.

$$\{t_k = 1.0\ in\} \geq \{t_L = 1.0\ in\} \qquad True$$

$$\{r_k = 10.0\ in\} > \{3t_k = 3.0\ in\} \qquad True$$

$$\left\{\frac{r_k}{R_L} = \frac{10.0}{60.0} = 0.1667\right\} > \{0.03\} \qquad True$$

$$\{\alpha = 30\ deg\} \leq \{60\ deg\} \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition at the location of the knuckle. The thrust load due to pressure shall not be included as part of the axial force, F_L.

$$F_L = -10000\ lbs$$
$$M_L = 2.0E+06\ in-lbs$$

e) STEP 5 – Compute the stresses in the knuckle at the junction using the equations in Table 4.3.7.

Determine if the knuckle is considered to be compact or non-compact.

$$\alpha r_k < 2K_m\left(\left\{R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5} + r_k\right\}t_k\right)^{0.5}$$

$$\{0.5236(10.0)\} < \left\{2(0.7)\left(\left\{50.0\left((0.5236)^{-1}\cdot\tan[0.5236]\right)^{0.5} + 10\right\}1\right)^{0.5}\right\}$$

$$\{5.2360\ in\} < \{11.0683\ in\} \qquad True$$

where,

$$K_m = 0.7$$

$$\alpha = \frac{30.0}{180}\pi = 0.5236\ rad$$

$$R_k = R_L - r_k = 60.0 - 10.0 = 50.0\ in$$

Therefore, analyze the knuckle junction as a compact knuckle.

Stress Calculations:

Determine the hoop and axial membrane stresses at the knuckle:

$$\sigma_{\theta m} = \frac{PK_m\left(R_L\sqrt{R_L t_L} + L_k\sqrt{L_k t_C}\right) + \alpha\left(PL_{1k}r_k - 0.5P_e L_{1k}^2\right)}{K_m\left(t_L\sqrt{R_L t_L} + t_C\sqrt{L_k t_C}\right)\alpha t_k r_k}$$

$$\sigma_{sm} = \frac{P_e L_{1k}}{2t_k}$$

where,

$$L_{1k} = R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5} + r_k = 50.0\left((0.5236)^{-1}\tan[0.5236]\right)^{0.5} + 10.0 = 62.5038\ in$$

$$L_k = \frac{R_k}{\cos[\alpha]} + r_k = \frac{50.0}{\cos[0.5236]} + 10.0 = 67.7351\ in$$

$$P_e = P + \frac{F_L}{\pi L_{1k}^2 \cos^2\left[\dfrac{\alpha}{2}\right]} \pm \frac{2M_L}{\pi L_{1k}^3 \cos^3\left[\dfrac{\alpha}{2}\right]}$$

$$P_e = \begin{Bmatrix} -14.7 + \dfrac{-10000.0}{\pi(62.5038)^2 \cdot \cos^2\left[\dfrac{0.5236}{2}\right]} + \dfrac{2(2.0E+06)}{\pi(62.5038)^3 \cdot \cos^3\left[\dfrac{0.5236}{2}\right]} \\ -14.7 + \dfrac{-10000.0}{\pi(62.5038)^2 \cdot \cos^2\left[\dfrac{0.5236}{2}\right]} - \dfrac{2(2.0E+06)}{\pi(62.5038)^3 \cdot \cos^3\left[\dfrac{0.5236}{2}\right]} \end{Bmatrix}$$

$$P_e = \begin{Bmatrix} -9.7875\ psi \\ -21.3590\ psi \end{Bmatrix}$$

therefore,

$$\sigma_{\theta m} = \begin{Bmatrix} \dfrac{\begin{pmatrix} (-14.7)(0.7)\left(60.0\sqrt{60.0(1.0)} + 67.7351\sqrt{67.7351(1.0)}\right) + \\ 0.5236\left((-14.7)(62.5038)(10.0) - 0.5(-9.7875)(62.5038)^2\right) \end{pmatrix}}{0.7\left(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}\right) + 0.5236(1.0)(10.0)} = -323.9558\ psi \\ \dfrac{\begin{pmatrix} (-14.7)(0.7)\left(60.0\sqrt{60.0(1.0)} + 67.7351\sqrt{67.7351(1.0)}\right) + \\ 0.5236\left((-14.7)(62.5038)(10.0) - 0.5(-21.3590)(62.5038)^2\right) \end{pmatrix}}{0.7\left(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}\right) + 0.5236(1.0)(10.0)} = 396.8501\ psi \end{Bmatrix}$$

and,

$$\sigma_{sm} = \begin{Bmatrix} \dfrac{P_e L_{1k}}{2t_k} = \dfrac{-9.7875(62.5038)}{2(1.0)} = -305.8780\ psi \\ \dfrac{P_e L_{1k}}{2t_k} = \dfrac{-21.3590(62.5038)}{2(1.0)} = -667.5093\ psi \end{Bmatrix}$$

Check Acceptable Criteria:

$$\begin{Bmatrix} \sigma_{\theta m} = -324.0\ psi \\ \sigma_{\theta m} = 396.9\ psi \end{Bmatrix} \le \begin{Bmatrix} S,\ not\ applicable\ due\ to\ compressive\ stress \\ 20000 \qquad True \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{sm} = -305.9\ psi \\ \sigma_{sm} = -667.5\ psi \end{Bmatrix} \le \left\{S,\ not\ applicable\ due\ to\ compressive\ stress\right\}$$

Since the hoop membrane stress $\sigma_{\theta m}$ and axial membrane stress σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m} \le F_{ha}$$

$$\sigma_{sm} \le F_{xa}$$

F_{ha} is evaluated using VIII-2, paragraph 4.4.5.1, but substituting F_{he} with the following equation.

$$F_{he} = \frac{0.4 E_y t}{D_o}$$

F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2.b with $\lambda = 0.15$.

In accordance with VIII-2, paragraph 4.4.5.1, the value of F_{ha} is calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

$$t = 1.0\ in$$

$$L \rightarrow Not\ required,\ as\ the\ equation\ for\ F_{he}\ is\ independent\ of\ L$$

2) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{0.4 E_y t}{D_o} = \frac{0.4\left(28.3E+06\right)\left(1.0\right)}{122.0} = 92786.8853\ psi$$

3) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$$F_{he} = 92786.8853\ psi \quad \text{(as determined in STEP 2 above)}$$

ii) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{92786.8853}{28.3E+06} = 0.00327869$$

iii) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.2 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 25689.9738\ psi$$

4) STEP 4 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{25689.9738}{33600}\right) = 1.8404$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{25689.9738}{1.8404} = 13958.9077 \ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ to the allowable hoop compressive membrane stress, F_{ha} per following criteria.

$$\left\{\sigma_{\theta m} = 324.0 \ psi\right\} \leq \left\{F_{ha} = 13958.9 \ psi\right\} \qquad True$$

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

In accordance with paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

1) STEP 1 – Calculate the predicted elastic buckling stress, F_{xe}.

$$F_{xe} = \frac{C_x E_y t}{D_o} = \frac{0.8004(28.3E+06)(1.0)}{122.0} = 185666.5574 \ psi$$

where,

$$\frac{D_o}{t} = \frac{122.0}{1.0} = 122.0$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{240.0}{\sqrt{61.0(1.0)}} = 30.7289$$

Since $D_o/t < 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \frac{D_o}{t}\right)}, 0.9\right] = \min\left[\frac{409(1.0)}{\left(389 + \frac{122.0}{1.0}\right)}, \ 0.9\right] = 0.8004$$

Since $M_x \geq 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

2) STEP 2 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{xe}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 2.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$F_{xe} = 185666.5574 \ psi$ (as determined in STEP 1 above)

ii) STEP 2.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{xe}}{E} = \frac{185666.5574}{28.3E+06} = 0.00656066$$

iii) STEP 2.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 29252.6889\ psi$$

3) STEP 3 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{29252.6889}{33600}\right) = 1.7619$$

4) STEP 4 – Calculate the allowable axial compressive membrane stress as follows:

$$F_{xa} = \frac{F_{ic}}{FS} = \frac{29252.6889}{1.7619} = 16602.9224\ psi$$

5) STEP 5 – Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria.

$$\{\sigma_{sm} = 667.5\ psi\} \le \{F_{xa} = 16602.9\ psi\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

f) STEP 6 – The stress acceptance criterion in STEP 5 is satisfied. Therefore, the design is complete.

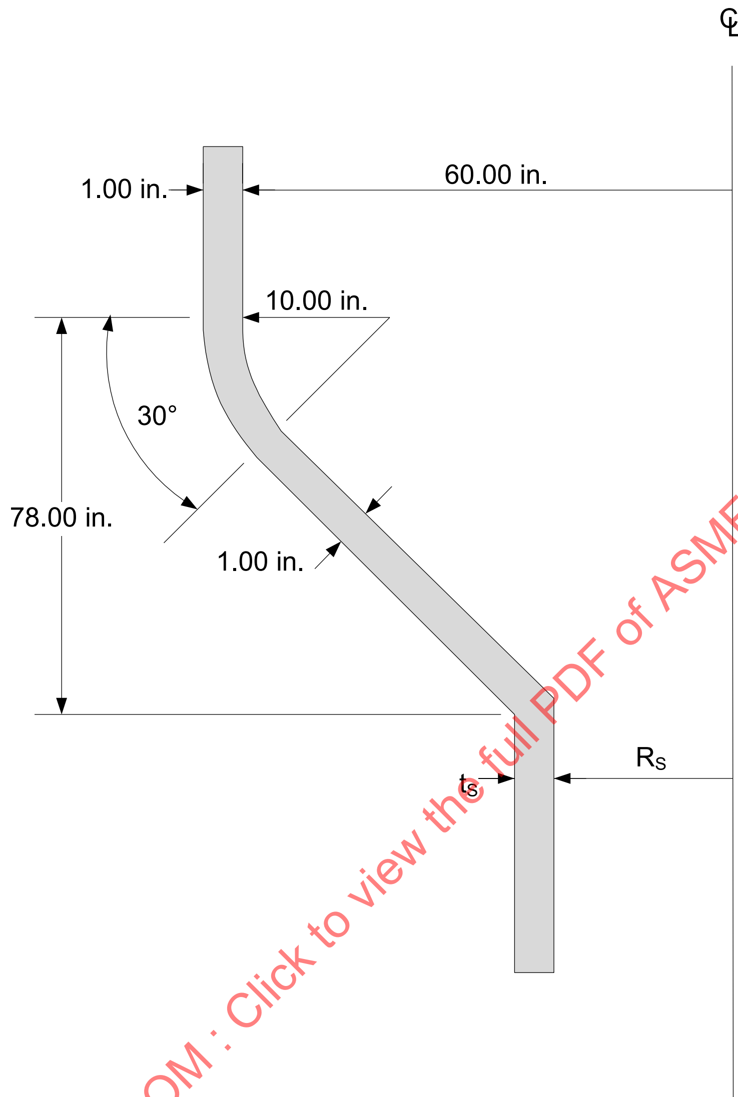

Figure E4.4.8 – Knuckle Detail

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4.5 Shells Openings in Shells and Heads

4.5.1 Example E4.5.1 – Radial Nozzle in Cylindrical Shell

Design an integral nozzle in a cylindrical shell based on the vessel and nozzle data below. The parameters used in this design procedure are shown in Figure E4.5.1.

Vessel and Nozzle Data:

- Design Conditions = $356\ psig@300°F$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0
- Shell Material = $SA-516,\ Grade\ 70,\ Normalized$
- Shell Allowable Stress = $22400\ psi$
- Yield Strength = $33600\ psi$
- Nozzle Material = $SA-105$
- Nozzle Allowable Stress = $21200\ psi$
- Shell Inside Diameter = $150.0\ in$
- Shell Thickness = $1.8125\ in$
- Nozzle Outside Diameter = $19.0\ in$
- Nozzle Hub Outside Diameter = $25.5\ in$
- Nozzle Hub Height = $7.1875\ in$
- Nozzle Thickness = $4.75\ in$
- External Nozzle Projection = $14.1875\ in$
- Internal Nozzle Projection = $0.0 in$

The nozzle is inserted through the shell, i.e., set–in type nozzle, see Figure 4.5.13.

Establish the corroded dimensions.

Shell:

$$D_i = 150.0 + 2(Corrosion\ Allowance) = 150.0 + 2(0.125) = 150.25\ in$$
$$t = 1.8125 - Corrosion\ Allowance = 1.8125 - 0.125 = 1.6875\ in$$

Nozzle:

$$t_n = 4.75 - Corrosion\ Allowance = 4.75 - 0.125 = 4.625\ in$$
$$R_n = \frac{D - 2(t_n)}{2} = \frac{25.5 - 2(4.625)}{2} = 8.125\ in$$

Evaluate per paragraph 4.5.5, The procedure to design a radial nozzle in a cylindrical shell subject to pressure loading is shown below.

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in paragraph 4.5.13 would need to be checked.

a) STEP 1 – Determine the effective radius of the shell as follows.

$$R_{eff} = 0.5D_i = 0.5(150.25) = 75.125\ in$$

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall.

For set–in integrally reinforced nozzles,

$$L_R = \min\left[\sqrt{R_{eff}t},\ 2R_n\right]$$

$$L_R = \min\left[\sqrt{(75.125)(1.6875)},\ \ 2(8.125)\right] = \min[11.2594, 16.25] = 11.2594\ in$$

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface.

For set–in nozzles,

$$L_{H1} = \min[1.5t,\ t_e] + \sqrt{R_n t_n} = \min\left[1.5(1.6875), 0.0\right] + \sqrt{8.125(4.625)} = 6.1301\ in$$

$$L_{H2} = L_{pr1} = 14.1875\,in$$

$$L_{H3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5\ in$$

$$L_H = \min\left[L_{H1},\ L_{H2},\ L_{H3}\right] + t = \min[6.1301,\ 15.875,\ 13.5] + 1.6875 = 7.8176\ in$$

d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable.

$$L_{I1} = \sqrt{R_n t_n} = \sqrt{8.125(4.625)} = 6.1301\ in$$

$$L_{I2} = L_{pr2} = 0.0\ in$$

$$L_{I3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5\ in$$

$$L_I = \min\left[L_{I1},\ L_{I2},\ L_{I3}\right] = \min[6.1301,\ 0.0,\ 13.5] = 0.0\ in$$

e) STEP 5 – Determine the total available area near the nozzle opening (see Figure 4.5.2). Do not include any area that falls outside of the limits defined by L_H, L_R, and L_I. For variable thickness nozzles, see Figure 4.5.13 for metal area definitions for A_2.

For set–in nozzles,

$$A_T = A_1 + f_{rn}(A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp}A_5$$

$$A_1 = (tL_R)\cdot\max\left[\left(\frac{\lambda}{5}\right)^{0.85},\ 1.0\right] = 1.6875(11.2594)\cdot\max\left[\left(\frac{1.3037}{5}\right)^{0.85},\ 1.0\right]$$

$$A_1 = 19.0002\ in^2$$

$$\lambda = \min\left[\left\{\frac{(2R_n + t_n)}{\sqrt{(D_i + t_{eff})t_{eff}}}\right\},\ 12.0\right] = \min\left[\frac{2(8.125) + 4.625}{\sqrt{150.25 + 1.6875}(1.6875)},\ 12.0\right] = 1.3037$$

$$t_{eff} = t + \left(\frac{A_5 f_{rp}}{L_R}\right) = 1.6875 + \left(\frac{0.0(1.0)}{11.2594}\right) = 1.6875\ in$$

$$f_{rp} = \min\left[\frac{S_p}{S}, 1\right] = 1.0$$

$$f_{rn} = \min\left[\frac{S_n}{S}, 1\right] = \frac{21200}{22400} = 0.9464$$

Since $\{L_H = 7.8176\ in\} \le \{L_{x3} = L_{pr3} + t = 7.1875 + 1.6875 = 8.875\ in\}$, calculate A_2 as follows, see Figure 4.5.13,

$$A_2 = t_n L_H = 4.625(7.8176) = 36.1564\ in^2$$

$$A_3 = t_n L_I = 0.0\ in^2$$

$$A_{41} = 0.5L_{41}^2 = 0.5(0.375)^2 = 0.0703\ in^2$$

$$A_{42} = 0.5L_{42}^2 = 0.0\ in^2$$

$$A_{43} = 0.5L_{43}^2 = 0.0\ in^2$$

$$A_5 = \min\left[A_{5a}, A_{5b}\right]$$

$$A_{5a} = Wt_e = 0.0\ in^2$$

$$A_{5b} = L_R t_e = 0.0\ in^2$$

$$A_5 = 0.0\ in^2$$

$$A_T = 19.0002 + 0.9464(36.1564 + 0.0) + 0.0703 + 0.0 + 0.0 + 0.0 = 53.2889\ in^2$$

f) STEP 6 – Determine the applicable forces.

For set–in nozzles,

$$f_N = PR_{xn}L_H = 356(10.2644)(7.8176) = 28566.4985\ lbs$$

$$R_{xn} = \frac{t_n}{\ln\left[\dfrac{R_n + t_n}{R_n}\right]} = \frac{4.625}{\ln\left[\dfrac{8.125 + 4.625}{8.125}\right]} = 10.2644\ in$$

$$f_S = PR_{xs}(L_R + t_n) = 356(75.9656)(11.2594 + 4.625) = 429573.7997\ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[\dfrac{R_{eff} + t_{eff}}{R_{eff}}\right]} = \frac{1.6875}{\ln\left[\dfrac{75.125 + 1.6875}{75.125}\right]} = 75.9656\ in$$

$$f_Y = PR_{xs}R_{nc} = 356(75.9656)(8.125) = 219730.4980\ lbs$$

Note: For radial nozzles, $R_{nc} = R_n$.

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection.

$$\sigma_{avg} = \frac{(f_N + f_S + f_Y)}{A_T} = \frac{28566.4985 + 429573.7997 + 219730.4980}{53.2889} = 12720.6753\ psi$$

$$\sigma_{circ} = \frac{PR_{xs}}{t_{eff}} = \frac{356(75.9656)}{1.6875} = 16025.9281\ psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection.

$$P_L = \max\left[\{2\sigma_{avg} - \sigma_{circ}\},\ \sigma_{circ}\right]$$

$$P_L = \max\left[\{2(12720.6753) - 16025.9281\},\ 16025.9281\right] = 16025.9281\ psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy Equation 4.5.56. If the nozzle is subjected to internal pressure, then the allowable stress, S_{allow}, is given by Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by Equation 4.5.58.

$$\{P_L = 16025.9281\ psi\} \le \{S_{allow} = 1.5SE = 1.5(22400)(1.0) = 33600\ psi\}$$

j) STEP 10 – Determine the maximum allowable working pressure at the nozzle intersection.

$$P_{\max 1} = \frac{S_{allow}}{\dfrac{2A_p}{A_T} - \dfrac{R_{xs}}{t_{eff}}} = \frac{33600}{\left(\dfrac{2(1904.1315)}{53.2889}\right) - \left(\dfrac{75.9656}{1.6875}\right)} = 1270.4262\ psi$$

$$A_p = \frac{f_N + f_S + f_Y}{P} = \frac{28566.4985 + 429573.7997 + 219730.4980}{356.0} = 1904.1315\ in^2$$

$$P_{\max 2} = S\left(\frac{t}{R_{xs}}\right) = 22400\left(\frac{1.6875}{75.9656}\right) = 497.5936\ psi$$

$$P_{\max} = \min\left[P_{\max 1},\ P_{\max 2}\right] = \min\left[1270.4262,\ 497.5936\right] = 497.6\ psi$$

The nozzle is acceptable because $P_{\max} = 497.6\ psi$ is greater than the specified design pressure of $356\ psig$.

Weld Strength Analysis

The procedure to evaluate attachment welds of nozzles in a cylindrical, conical, or spherical shell or formed head subject to pressure loading per paragraph 4.5.14.2 is shown below.

a) STEP 1 – Determine the discontinuity force factor,

For set-in nozzles:

$$k_y = \frac{R_{nc} + t_n}{R_{nc}} = \frac{8.125 + 4.625}{8.125} = 1.5692$$

Note, for radial nozzles, $R_{nc} = R_n$.

b) STEP 2 – Calculate weld length resisting continuity force,

Weld length of nozzle to shell weld, for radial nozzles:

$$L_\tau = \frac{\pi}{2}(R_n + t_n) = \frac{\pi}{2}(8.125 + 4.625) = 20.0277 \; in$$

Weld length of pad to shell weld, for radial nozzles:

$$L_{\tau p} = \frac{\pi}{2}(R_n + t_n + W) \qquad \textit{Not Applicable}$$

c) STEP 3 – Compute the weld throat dimensions, as applicable

$$L_{41T} = 0.7071 L_{41} = 0.7071(0.375) = 0.2652 \; in$$
$$L_{42T} = 0.0 \; in$$
$$L_{43T} = 0.0 \; in$$

d) STEP 4 – Determine if the weld sizes are acceptable. If the nozzle is integrally reinforced, and the computed shear stress in the weld given by Equation (4.5.182) satisfies Equation (4.5.183), then the design is complete. If the shear stress in the weld does not satisfy Equation (4.5.183), increase the weld size, and return to Step 3.

$$\tau = \frac{f_{welds}}{L_\tau \left(0.49 L_{41T} + 0.6 t_{w1} + 0.49 L_{43T}\right)}$$
$$\tau = \frac{45450.9764}{20.0277\left(0.49(0.2652) + 0.6(1.6875) + 0.49(0.0)\right)} = 1986.4411 \; psi$$

Where,

$$f_{welds} = \min\left[f_Y k_y, \; 1.5 S_n \left(A_2 + A_3\right), \; \frac{\pi}{4} P R_n^2 k_y^2\right]$$

$$f_{welds} = \min\left[\begin{array}{l} \left\{219730.4980(1.5692) = 344801.0975\right\} \\ \left\{1.5(21200)(36.1564 + 0) = 1149773.52\right\} \\ \left\{\frac{\pi}{4}(356)(8.125)^2(1.5692)^2 = 45450.9764\right\} \end{array}\right] = 45450.9764 \; lbs$$

$$\left\{\tau = 1986.4 \; psi\right\} \le \left\{S = 22400 \; psi\right\}$$

The weld strength is acceptable.

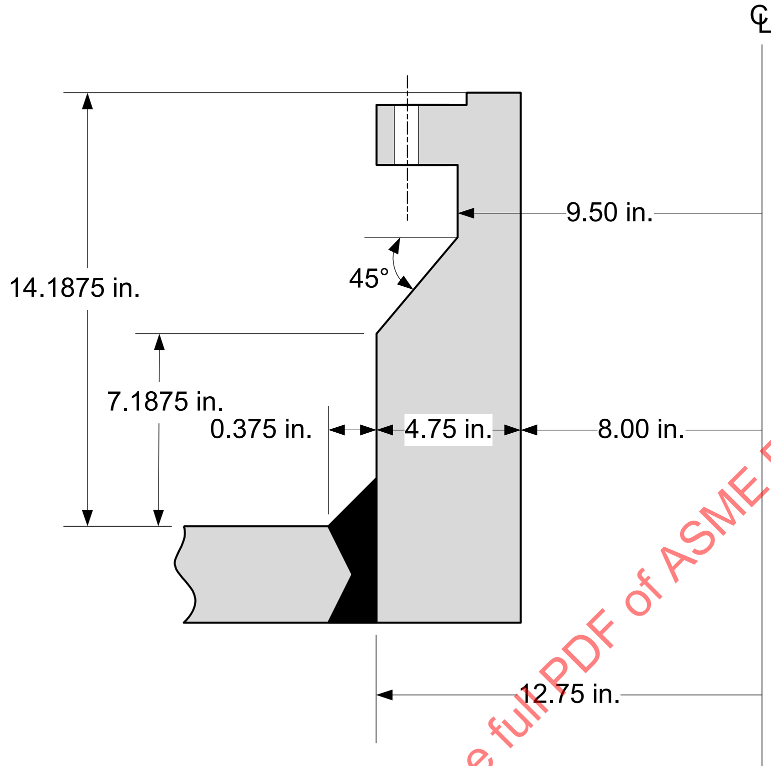

Figure E4.5.1 – Nozzle Detail

4.5.2 Example E4.5.2 – Hillside Nozzle in Cylindrical Shell

Design an integral hillside nozzle in a cylindrical shell based on the vessel and nozzle data below. The parameters used in this design procedure are shown in Figure E4.5.2.

Vessel and Nozzle Data:

- Design Conditions = $356\ psig@300°F$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0
- Shell Material = $SA-516,\ Grade\ 70,\ Normalized$
- Shell Allowable Stress = $22400\ psi$
- Shell Yield Strength = $33600\ psi$
- Nozzle Material = $SA-105$
- Nozzle Allowable Stress = $21200\ psi$
- Shell Inside Diameter = $150.0\ in$
- Shell Thickness = $1.8125\ in$
- Nozzle Outside Diameter = $11.56\ in$
- Nozzle Thickness = $1.97\ in$
- External Nozzle Projection = $19.0610\ in$
- Internal Nozzle Projection = $0.0\ in$
- Nozzle Offset = $34.875\ in$

The nozzle is inserted through the shell, i.e., set–in type nozzle, see Figure 4.5.4.

Establish the corroded dimensions.

Shell:

$$D_i = 150.0 + 2(Corrosion\ Allowance) = 150.0 + 2(0.125) = 150.25\ in$$

$$t = 1.8125 - Corrosion\ Allowance = 1.8125 - 0.125 = 1.6875\ in$$

Nozzle:

$$t_n = 1.97 - Corrosion\ Allowance = 1.97 - 0.125 = 1.845\ in$$

$$R_n = \frac{D - 2(t_n)}{2} = \frac{11.56 - 2(1.845)}{2} = 3.935\ in$$

Evaluate per paragraph 4.5.5 and paragraph 4.5.6. For a hillside nozzle in a cylindrical shell (see Figure 4.5.4), the design procedure in paragraph 4.5.5 shall be used with the following substitutions from paragraph 4.5.6.

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in paragraph 4.5.13 would need to be checked.

$$R_{nc} = \max\left[\left(\frac{R_{ncl}}{2}\right),\ R_n\right]$$

where,

$$R_{ncl} = R_{eff}\left(\theta_1 - \theta_2\right)$$

$$\theta_1 = \cos^{-1}\left[\frac{D_X}{R_{eff}}\right] = \cos^{-1}\left[\frac{34.875}{75.125}\right] = 62.3398\ deg = 1.0880\ rad$$

$$\theta_2 = \cos^{-1}\left[\frac{D_X + R_n}{R_{eff}}\right] = \cos^{-1}\left[\frac{34.875 + 3.935}{75.125}\right] = 58.8952\ deg = 1.0279\ rad$$

$$R_{ncl} = 75.125\left(1.0880 - 1.0279\right) = 4.5150\ in$$

$$R_{nc} = \max\left[\left(\frac{4.5150}{2}\right),\ 3.935\right] = 3.935\ in$$

The procedure in paragraph 4.5.5 is shown below.

a) STEP 1 – Determine the effective radius of the shell as follows:

$$R_{eff} = 0.5D_i = 0.5\left(150.25\right) = 75.125\ in$$

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall:

For integrally reinforced nozzles:

$$L_R = \min\left[\sqrt{R_{eff}t},\ 2R_n\right] = \min\left[\sqrt{75.125\left(1.6875\right)},\ 2\left(3.935\right)\right] = 7.87\ in$$

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface.

For set–in nozzles,

$$L_{H1} = \min\left[1.5t, t_e\right] + \sqrt{R_n t_n} = \min\left[1.5 \times 1.6875, 0\right] + \sqrt{3.935\left(1.845\right)} = 2.6945\ in$$

$$L_{H2} = L_{pr1} = 19.0610\ in$$

$$L_{H3} = 8\left(t + t_e\right) = 8\left(1.6875 + 0.0\right) = 13.5\ in$$

$$L_H = \min\left[L_{H1},\ L_{H2},\ L_{H3}\right] + t = 4.3820\ in$$

d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable:

$$L_{I1} = \sqrt{R_n t_n} = \sqrt{3.935\left(1.845\right)} = 2.6945$$

$$L_{I2} = L_{pr2} = 0.0\ in$$

$$L_{I3} = 8\left(t + t_e\right) = 8\left(1.6875 + 0.0\right) = 13.5\ in$$

$$L_I = \min\left[L_{I1},\ L_{I2},\ L_{I3}\right] = 0.0\ in$$

e) STEP 5 – Determine the total available area near the nozzle opening (see VIII-2, Figures 4.5.1 and 4.5.2). Do not include any area that falls outside of the limits defined by L_H, L_R, and L_I.

For set–in nozzles,

$$A_T = A_1 + f_{rn}\left(A_2 + A_3\right) + A_{41} + A_{42} + A_{43} + f_{rp}A_5$$

$$A_1 = \left(tL_R\right)\cdot \max\left[\left(\frac{\lambda}{5}\right)^{0.85}, 1.0\right] = 1.6875\left(7.87\right)\cdot \max\left[\left(\frac{0.6067}{5}\right)^{0.85}, 1.0\right] = 13.2806$$

$$\lambda = \min\left[\left\{\frac{\left(2R_n + t_n\right)}{\sqrt{\left(D_i + t_{eff}\right)t_{eff}}}\right\}, 12.0\right] = \min\left[\left\{\frac{2\left(3.935\right)+1.845}{\sqrt{\left(150.25+1.6875\right)\left(1.6875\right)}}\right\}, 12.0\right] = 0.6067$$

$$t_{eff} = t + \left(\frac{A_5 f_{rp}}{L_R}\right) = 1.6875 + \left(\frac{0.0\left(1.0\right)}{7.87}\right) = 1.6875\ in$$

$$f_{rn} = \frac{S_n}{S} = \frac{21200}{22400} = 0.9464$$

$$f_{rp} = \frac{S_p}{S} = 1.0$$

Since $\{t_n = 1.845\ in\} = \{t_{n2} = 1.845\ in\}$, calculate A_2 as follows:

$$A_2 = t_n L_H = 1.845\left(4.3820\right) = 8.0848\ in^2$$

$$A_3 = t_n L_I = 1.845\left(0.0\right) = 0.0\ in^2$$

$$A_{41} = 0.5L_{41}^2 = 0.5\left(0.375\right) = 0.0703\ in^2$$

$$A_{42} = 0.5L_{42}^2 = 0.0\ in^2$$

$$A_{43} = 0.5L_{43}^2 = 0.0\ in^2$$

$$A_5 = \min\left[A_{5a},\ A_{5b}\right]$$

$$A_{5a} = Wt_e = 0.0\ in^2$$

$$A_{5b} = L_R t_e = 0.0\ in^2$$

$$A_5 = 0.0\ in^2$$

$$A_T = 13.2806 + 0.9464\left(8.0848 + 0.0\right) + 0.0 + 0.0703 + 0.0 + 0.0 = 21.0024\ in^2$$

f) STEP 6 – Determine the applicable forces:

For set–in nozzles,

$$f_N = PR_{xn}L_H = 356\left(4.7985\right)\left(4.3820\right) = 7485.6216\ lbs$$

$$R_{xn} = \frac{t_n}{\ln\left[\dfrac{R_n + t_n}{R_n}\right]} = \frac{1.845}{\ln\left[\dfrac{3.935+1.845}{3.935}\right]} = 4.7985 \ in$$

$$f_S = PR_{xs}\left(L_R + t_n\right) = 356(75.9656)(7.87+1.845) = 262730.0662 \ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[\dfrac{R_{eff} + t_{eff}}{R_{eff}}\right]} = \frac{1.6875}{\ln\left[\dfrac{75.125+1.6875}{75.125}\right]} = 75.9656 \ in$$

$$f_Y = PR_{xs}R_{nc} = 356(75.9656)(3.935) = 106417.1704 \ lbs$$

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection:

$$\sigma_{avg} = \frac{\left(f_N + f_S + f_Y\right)}{A_T} = \frac{(7485.6216 + 262730.0662 + 106417.1704)}{21.0024} = 17932.8485 \ psi$$

$$\sigma_{circ} = \frac{PR_{xs}}{t_{eff}} = \frac{356(75.9656)}{1.6875} = 16025.9281 \ psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection:

$$P_L = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\}, \ \sigma_{circ}\right]$$

$$P_L = \max\left[\left\{2(17932.8485) - 16025.9281\right\}, 16025.9281\right] = 19839.7689 \ psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy Equation 4.5.56. If the nozzle is subjected to internal pressure, then the allowable stress, S_{allow}, is given by Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by Equation 4.5.58.

$$\left\{P_L = 19839.7689 \ psi\right\} \le \left\{S_{allow} = 1.5SE = 1.5(22400)(1.0) = 33600 \ psi\right\}$$

j) STEP 10 – Determine the maximum allowable working pressure of the nozzle:

$$P_{\max 1} = \frac{S_{allow}}{\dfrac{2A_p}{A_T} - \dfrac{R_{xs}}{t_{eff}}} = \frac{33600}{\dfrac{2(1057.9575)}{21.0024} - \dfrac{75.9656}{1.6875}} = 602.9102 \ psi$$

$$A_p = \frac{\left(f_N + f_S + f_Y\right)}{A_T} = \frac{(7485.6216 + 262730.0662 + 106417.1704)}{356} = 1057.9575 \ in^2$$

$$P_{\max 2} = S\left(\frac{t}{R_{xs}}\right) = 20000\left(\frac{1.6875}{75.9656}\right) = 444.28 \ psi$$

$$P_{\max} = \min\left[P_{\max 1}, \ P_{\max 2}\right] = \min\left[602.9102, \ 444.28\right] = 444.3 \ psi$$

The nozzle is acceptable because $P_{\max} = 444.3 \ psi$ is greater than the specified design pressure of $356 \ psig$.

Weld Strength Analysis.

The procedure to evaluate attachment welds of nozzles in a cylindrical, conical, or spherical shell or formed head subject to pressure loading per paragraph 4.5.14 is shown below.

a) STEP 1 – Determine the discontinuity force factor,

For set–in Nozzles:

$$k_y = \frac{R_{nc} + t_n}{R_{nc}} = \frac{3.935 + 1.845}{3.935} = 1.4689$$

b) STEP 2 – Calculate weld length resisting continuity force,

Weld length of nozzle to shell weld, for non–radial nozzles:

$$L_\tau = \frac{\pi}{2}\sqrt{\frac{\left(R_{nc} + t_n\right)^2 + \left(R_n + t_n\right)^2}{2}} = \frac{\pi}{2}\sqrt{\frac{\left(3.935 + 1.845\right)^2 + \left(3.935 + 1.845\right)^2}{2}} = 9.0792 \ in$$

Weld length of pad to shell weld, for non–radial nozzles:

$$L_{\tau p} = \frac{\pi}{2}\sqrt{\frac{\left(R_{nc} + t_n + W\right)^2 + \left(R_n + t_n + W\right)^2}{2}} \qquad Not \ Applicable$$

c) STEP 3 – Compute the weld throat dimensions, as applicable.

$$L_{41T} = 0.7071 L_{41} = 0.7071(0.375) = 0.2652 \ in$$
$$L_{42T} = 0.0 \ in$$
$$L_{43T} = 0.0 \ in$$

d) STEP 4 – Determine if the weld sizes are acceptable. If the nozzle is integrally reinforced, and the computed shear stress in the weld given by Equation (4.5.182) satisfies Equation (4.5.183), then the design is complete. If the shear stress in the weld does not satisfy Equation (4.5.183), increase the weld size, and return to Step 3.

$$\tau = \frac{f_{welds}}{L_\tau\left(0.49 L_{41T} + 0.6 t_{w1} + 0.49 L_{43T}\right)}$$

$$\tau = \frac{9341.4397}{9.0792\left(0.49(0.2652) + 0.6(1.6875) + 0.49(0.0)\right)} = 900.5955 \ psi$$

Where,

$$f_{welds} = \min\left[f_Y k_y,\ 1.5 S_n\left(A_2 + A_3\right),\ \frac{\pi}{4} P R_n^2 k_y^2\right]$$

$$f_{welds} = \min\begin{bmatrix} \{106417.1704(1.4689) = 156316.1816\} \\ \{1.5(21200)(8.0848 + 0) = 257096.64\} \\ \left\{\frac{\pi}{4}(356)(3.935)^2(1.4689)^2 = 9341.4397\right\} \end{bmatrix} = 9341.4397 \ lbs$$

$$\{\tau = 900.6 \ psi\} \le \{S = 22400 \ psi\}$$

The weld strength is acceptable.

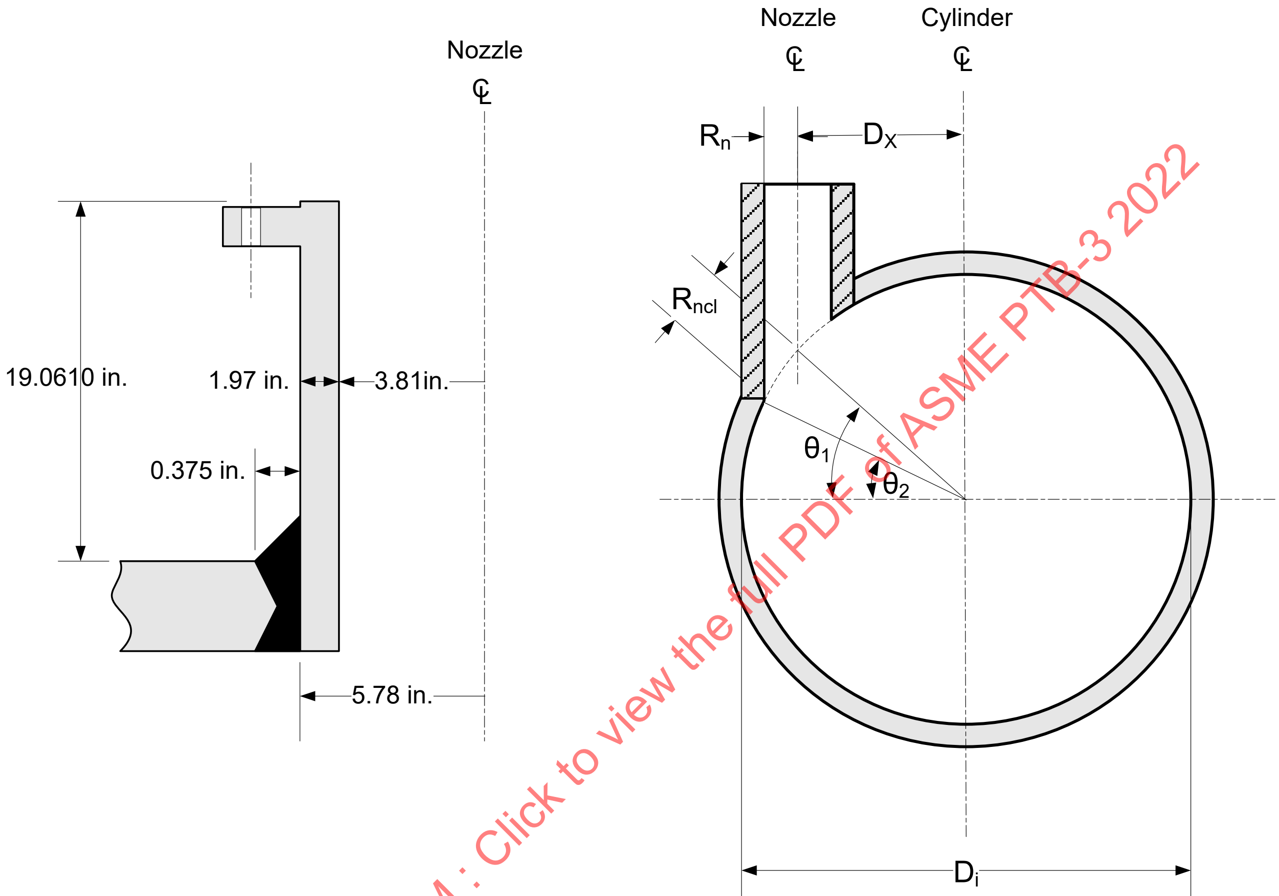

Figure E4.5.2 – Nozzle Detail

4.5.3 Example E4.5.3 – Radial Nozzle in Ellipsoidal Head

Design an integral radial nozzle centrally located in a 2:1 ellipsoidal head based on the vessel and nozzle data below. The parameters used in this design procedure are shown in Figure E4.5.3.

Vessel and Nozzle Data:

- Design Conditions = $356\ psig@300°F$
- Vessel and Nozzle Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0
- Head Material = $SA-516,\ Grade\ 70,\ Norm.$
- Head Allowable Stress = $22400\ psi$
- Head Yield Strength = $33600\ psi$
- Nozzle Material = $SA-105$
- Nozzle Allowable Stress = $21200\ psi$
- Head Inside Diameter = $90.0\ in$
- Height of the Elliptical Head, (2:1) = $22.5\ in$
- Head Thickness = $1.0\ in$
- Nozzle Outside Diameter = $15.94\ in$
- Nozzle Thickness = $2.28\ in$
- External Nozzle Projection = $13.5\ in$
- Nozzle Internal Projection = $0.0\ in$

The nozzle is inserted centrally through the head, i.e., set–in type nozzle, see Figure 4.5.3.

Establish the corroded dimensions.

Ellipsoidal Head:

$$D_i = 90.0 + 2\left(Corrosion\ \ Allowance\right) = 90.0 + 2\left(0.125\right) = 90.25\ in$$

$$R = \frac{D_i}{2} = \frac{90.25}{2} = 45.125\ in$$

$$t = 1.0 - Corrosion\ \ Allowance = 1.0 - 0.125 = 0.875\ in$$

$$h = 22.5 + Corrosion\ \ Allowance = 22.5 + 0.125 = 22.625\ in$$

Nozzle:

$$t_n = 2.28 - Corrosion\ \ Allowance = 2.28 - 0.125 = 2.155\ in$$

$$R_n = \frac{D_n - 2\left(t_n\right)}{2} = \frac{15.94 - 2\left(2.155\right)}{2} = 5.815\ in$$

Evaluate per paragraph 4.5.10, The procedure to design a radial nozzle in an ellipsoidal head subject to pressure loading is shown below.

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in paragraph 4.5.13 would need to be checked.

a) STEP 1 – Determine the effective radius of the ellipsoidal head as follows.

$$R_{eff}=\frac{0.9D_i}{6}\left[2+\left(\frac{D_i}{2h}\right)^2\right]=\frac{0.9(90.25)}{6}\left[2+\left(\frac{90.25}{2(22.625)}\right)^2\right]=80.9262\ in$$

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall.

For integrally reinforced set–in nozzles in ellipsoidal heads,

$$L_R=\min\left[\sqrt{R_{eff}t},\ 2R_n\right]=\min\left[\sqrt{80.9262(0.875)},\ 2(5.8150)\right]=8.4149\ in$$

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in VIII-2, paragraph 4.5.13 would need to be checked.

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface. See VIII-2, Figures 4.5.9 and 4.5.10.

For set–in nozzles in ellipsoidal heads,

$$L_H=\min\left[t+t_e+F_p\sqrt{R_nt_n},\ L_{pr1}+t\right]$$

$$X_o=\min\left[D_R+(R_n+t_n)\cdot\cos[\theta],\ \frac{D_i}{2}\right]$$

$$X_o=\min\left[0.0+(5.8150+2.1550)\cdot\cos[0.0],\ \frac{90.25}{2}\right]=7.97\ in$$

where,

$$\theta=\arctan\left[\left(\frac{h}{R}\right)\cdot\left(\frac{D_R}{\sqrt{R^2-D_R^2}}\right)\right]=\arctan\left[\left(\frac{22.625}{45.125}\right)\cdot\left(\frac{0.0}{\sqrt{45.125^2-0.0^2}}\right)\right]=0.0\ rad$$

Since $\{X_o = 7.97\ in\} \le \{0.35D_i = 0.35(90.25) = 31.5875\ in\}$, calculate F_p as follows:

$$F_p=C_n=\min\left[\left(\frac{t+t_e}{t_n}\right)^{0.35},\ 1.0\right]=\min\left[\left(\frac{0.875+0.0}{2.1550}\right)^{0.35},\ 1.0\right]=0.7295$$

therefore,

$$L_H=\min\left[0.875+0.0+(0.7295)\sqrt{5.8150(2.1550)},\ 13.5+0.875\right]=3.4574\ in$$

d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable.

$$L_{pr2}=0.0\ in$$

$$L_I=\min\left[F_p\sqrt{R_nt_n},L_{pr2}\right]=0.0\ in$$

e) STEP 5 – Determine the total available area near the nozzle opening (see Figures 4.5.1 and 4.5.2) where f_{rn} and f_{rp} are given by Equations (4.5.21) and (4.5.22) respectively. Do not include any area that falls outside of the limits defined by L_H, L_R, and L_I.

For set–in nozzles:

$$A_T = A_1 + f_{rn}\left(A_2 + A_3\right) + A_{41} + A_{42} + A_{43} + f_{rp}A_5$$

$$A_1 = tL_R = 0.875\left(8.4149\right) = 7.3630\ in^2$$

Since $\{t_n = 2.1550\ in\} = \{t_{n2} = 2.1550\ in\}$, calculate A_2 as follows:

$$A_2 = t_n L_H = 2.1550\left(3.4574\right) = 7.4507\ in^2$$

$$A_3 = t_n L_I = 0.0\ in^2$$

$$A_{41} = 0.5L_{41}^2 = 0.5\left(0.375\right)^2 = 0.0703\ in^2$$

$$A_{42} = 0.5L_{42}^2 = 0.0\ in^2$$

$$A_{43} = 0.5L_{43}^2 = 0.0\ in^2$$

$$t_e = 0.0\ in$$

$$A_{5a} = Wt_e = 0.0\ in$$

$$A_{5b} = \left(L_R - t_n\right)t_e = 0.0\ in$$

$$A_5 = \min\left[A_{5a},\ A_{5b}\right] = 0.0\ in$$

$$f_{rn} = \frac{S_n}{S} = \frac{21200}{22400} = 0.9464$$

$$f_{rp} = \frac{S_p}{S} = 1.0$$

$$A_T = 7.363 + 0.9464\left(7.4507 + 0.0\right) + 0.0703 + 0.0 + 0.0 + 0.0\left(0.0\right) = 14.4846\ in^2$$

f) STEP 6 – Determine the applicable forces.

For set–in nozzles,

$$f_N = PR_{xn}L_H = 356\left(6.8360\right)\left(3.4572\right) = 8413.4972\ lbs$$

$$R_{xn} = \frac{t_n}{\ln\left[\dfrac{R_n + t_n}{R_n}\right]} = \frac{2.1550}{\ln\left[\dfrac{5.8150 + 2.1550}{5.8150}\right]} = 6.8360\ in$$

$$f_S = \frac{PR_{xs}\left(L_R + t_n\right)}{2} = \frac{356\left(81.3629\right)\left(8.4149 + 2.1550\right)}{2} = 153079.5936\ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[\dfrac{R_{eff} + t_{eff}}{R_{eff}}\right]} = \frac{0.875}{\ln\left[\dfrac{80.9262 + 0.875}{80.9262}\right]} = 81.3629 \; in$$

$$t_{eff} = t + \left(\frac{A_5 f_{rp}}{L_R}\right) = 0.875 + \left(\frac{0.0}{8.4149}\right) = 0.875 \; in$$

$$f_Y = \frac{P R_{xs} R_{nc}}{2} = \frac{356(81.3629)(5.8150)}{2} = 84216.2969 \; lbs$$

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection.

$$\sigma_{avg} = \frac{(f_N + f_S + f_Y)}{A_T} = \frac{8413.4972 + 153079.5936 + 84216.2969}{14.4846} = 16963.4914 \; psi$$

$$\sigma_{circ} = \frac{P R_{xs}}{2 t_{eff}} = \frac{356(81.3629)}{2(0.875)} = 16551.5385 \; psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection.

$$P_L = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\}, \, \sigma_{circ}\right]$$

$$P_L = \max\left[\left\{2(16963.4914) - 16551.5385\right\}, \, 16551.5385\right] = 17375.4443 \; psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy VIII-2, Equation 4.5.146. If the nozzle is subjected to internal pressure, then the allowable stress, S_{allow}, is given by Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by Equation 4.5.58.

$$\left\{P_L = 16551.5385\right\} \le \left\{S_{allow} = 1.5SE = 1.5(22400)(1.0) = 33600 \; psi\right\}$$

j) STEP 10 – Determine the maximum allowable working pressure of the nozzle.

$$P_{\max 1} = \frac{S_{allow}}{\left(\dfrac{2A_p}{A_T}\right) - \left(\dfrac{R_{xs}}{2t_{eff}}\right)} = \frac{33600}{\left(\dfrac{2(690.1949)}{14.884}\right) - \left(\dfrac{81.3629}{2(0.875)}\right)} = 688.4198 \; psi$$

$$A_p = \frac{(f_N + f_S + f_Y)}{P}$$

$$A_p = \frac{8413.4972 + 153079.5936 + 84216.2969}{356} = 690.1949 \; in^2$$

$$P_{\max 2} = 2S\left(\frac{t}{R_{xs}}\right) = 2(22400)\left(\frac{0.875}{81.3629}\right) = 481.7921 \; psi$$

$$P_{\max} = \min\left[P_{\max 1}, \, P_{\max 2}\right] = \min\left[648.647, \, 481.7921\right] = 481.7921 \; psi$$

The nozzle is acceptable because $P_{\max} = 481.8 \; psi$ is greater than the specified design pressure of $356 \; psig$.

Weld Strength Analysis

The procedure to evaluate attachment welds of nozzles in a cylindrical, conical, or spherical shell or formed head subject to pressure loading per paragraph 4.5.14.2 is shown below.

a) STEP 1 – Determine the discontinuity force factor,

For set–in Nozzles:

$$k_y = \frac{R_{nc} + t_n}{R_{nc}} = \frac{5.8150 + 2.155}{5.8150} = 1.3706$$

Note, for radial nozzles, $R_{nc} = R_n$.

b) STEP 2 – Calculate weld length resisting continuity force,

Weld length of nozzle to shell weld, for radial nozzles:

$$L_\tau = \frac{\pi}{2}\left(R_n + t_n\right) = \frac{\pi}{2}\left(5.8150 + 2.155\right) = 12.5192\ in$$

Weld length of pad to shell weld, for radial nozzles:

$$L_{\tau p} = \frac{\pi}{2}\left(R_n + t_n + W\right) \qquad Not\ \ Applicable$$

c) STEP 3 – Compute the weld throat dimensions, as applicable.

$$L_{41T} = 0.7071 L_{41} = 0.7071\left(0.375\right) = 0.2652\ in$$

$$L_{42T} = 0.7071 L_{42} = 0.7071\left(0.0\right) = 0.0\ in$$

$$L_{43T} = 0.7071 L_{43} = 0.7071\left(0.0\right) = 0.0\ in$$

e) STEP 4 – Determine if the weld sizes are acceptable. If the nozzle is integrally reinforced, and the computed shear stress in the weld given by Equation (4.5.182) satisfies Equation (4.5.183), then the design is complete. If the shear stress in the weld does not satisfy Equation (4.5.183), increase the weld size, and return to Step 3. For nozzles on heads, A_2 and A_3 are to be calculated using $F_p = 1.0$, when computing f_{welds} using Equation (4.5.184).

From STEP 3 of paragraph 4.5.10, re–calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface using $F_p = 1.0$.

$$L_H = \min\left[t + t_e + F_p\sqrt{R_n t_n},\ L_{pr1} + t\right]$$

$$L_H = \min\left[0.875 + 0.0 + \left(1.0\right)\sqrt{5.8150\left(2.1550\right)},\ 13.5 + 0.875\right] = 4.4150\ in$$

Re–calculate the values of A_2 using the new value of L_H.

$$A_2 = t_n L_H = 2.1550\left(4.4150\right) = 9.5143\ in^2$$

From STEP 4 of paragraph 4.5.10, re–calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface using $F_p = 1.0$. Since the nozzle does not have an internal projection, the value of $L_I = 0.0\ in$; therefore, $A_3 = 0.0\ in^2$.

$$\tau = \frac{f_{welds}}{L_\tau\left(0.49L_{41T} + 0.6t_{w1} + 0.49L_{43T}\right)}$$

$$\tau = \frac{17760.7284}{12.5192\left(0.49(0.2652) + 0.6(0.875) + 0.49(0.0)\right)} = 2166.0944\ psi$$

Where,

$$f_{welds} = \min\left[f_Y k_y,\ 1.5S_n\left(A_2 + A_3\right),\ \frac{\pi}{4}PR_n^2k_y^2\right]$$

$$f_{welds} = \min\left[\begin{array}{l}\{219730.4980(1.3706) = 301162.6206\}, \\ \{1.5(21200)(9.5143 + 0) = 302554.74\}, \\ \left\{\frac{\pi}{4}(356)(5.8150)^2(1.3706)^2 = 17760.7284\right\}\end{array}\right] = 17760.7284\ lbs$$

$$\{\tau = 2166.1\ psi\} \le \{S = 22400\ psi\}$$

The weld strength is acceptable.

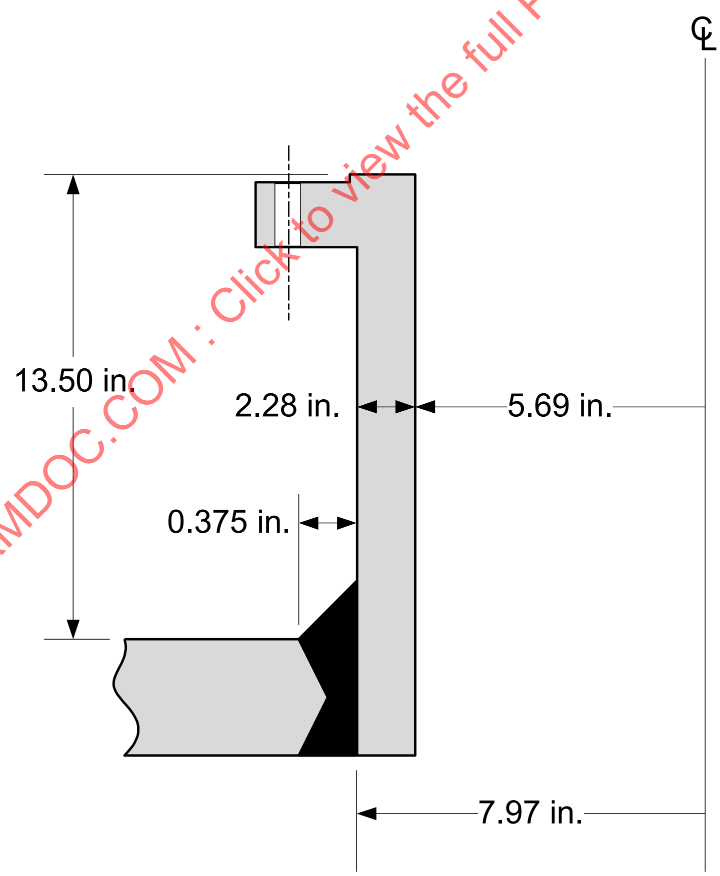

Figure E4.5.3 – Nozzle Details

4.5.4 Example E4.5.4 – Piping Load Evaluation of ASME B16.5 Nozzle Flange

Evaluate the following ASME B16.5 Class 150 Weld Neck Flange attached to a vessel nozzle considering the anticipated applied axial force and bending moment determined from a pipe flexibility analysis.

Nozzle/Flange and Loading Data:

- Design Conditions = $200\ psi\ @\ 300°F$
- Flange Material = $SA-105$
- Bolt Material = $SA-193,\ Grade\ B7$
- Nominal Pipe Size = $NPS\ 10$
- ASME B16.5 Weld Neck Flange = $Class\ 150$
- ASME B16.5 Pressure/Temperature Rating = $230\ psig\ @\ 300°F$
- Gasket Type = $Class\ 150\ SWG$
- Gasket Outside/Inside Diameter = $12.5\ in\ /\ 11.31\ in$
- External Tensile Axial Force = $5000\ lbs$
- External Moment = $120000.0\ in-lbs$

Reference Standards:

ASME B16.5 *Pipe Flanges and Flanged Fittings NPS ½ Through NPS 24*

Table II-2-1.1 Pressure-Temperature Ratings for Group 1.1 Materials

ASME B16.20 *Metallic Gaskets for Pipe Flanges – Ring Joint, Spiral-Wound, and Jacketed*

Table I-4 Dimensions for Spiral-Wound Gaskets Used with ASME B16.5 Flanges

Evaluate per paragraph 4.16.12.

External loads (forces and moments) may be evaluated for flanged joints with welding neck flanges chosen in accordance with ASME B16.5 and ASME B16.47 using the following requirements.

1) The vessel design pressure (corrected for the static pressure acting on the flange) at the design temperature cannot exceed the pressure-temperature rating of the flange.

2) The actual assembly bolt load (see paragraph 4.16.11) shall comply with ASME PCC-1, Nonmandatory Appendix O.

3) The bolt material shall have an allowable stress equal to or greater than $SA-193, Grade\ B7, Class\ 2$ at the specified bolt size and temperature.

4) The combination of vessel design pressure (corrected for the static pressure acting on the flange) with external moment and external axial force shall satisfy the following equation (the units of the variables in this equation shall be consistent with the pressure rating).

$$\left\{\begin{array}{l} 16M_E + 4F_E G \le \pi G^3\left[\left(P_R - P_D\right) + F_M P_R\right] \\ 16(120000) + 4(5000)(11.905) \le \pi(11.905)^3\left[(230-200)+1.2(230)\right] \\ 2158100\ in-lbs \le 1622032.3\ in-lbs \end{array}\right\} \qquad False$$

where,

$$F_E = 5000\ lbs$$

$$F_M = 1.2 \qquad (per\ Table\ 4.16.12,\ See\ Figure\ E4.5.7)$$

$$G = \frac{Gasket\ OD - Gasket\ ID}{2} = \frac{12.5-11.31}{2} = 11.905\ in$$

$$M_E = 120000\ in-lbs$$

$$P_D = 230\ psig$$

$$P_R = 200\ psig$$

Since the above expression is not satisfied, the Class 150 flange is not adequate for the proposed combination of applied loads and design pressure. The designer may consider the following options.

1) Reduce the applied loads on the nozzle flange via modifications to the support layout for the piping system,

2) Use a Class 300 flange, pending satisfaction of the above expression.

Table 4.16.12
Moment Factor, F_M

Standard	Size Range	Flange Pressure Rating Class 150	300	600	900	1500	2500
ASME B16.5	≤NPS 12	1.2	0.5	0.5	0.5	0.5	0.5
	>NPS 12 and ≤NPS 24	1.2	0.5	0.5	0.3	0.3	...
ASME B16.47							
Series A	All	0.6	0.1	0.1	0.1	...	...
Series B	<NPS 48	[Note (1)]	[Note (1)]	0.13	0.13	...	...
	≥NPS 48	0.1	[Note (2)]	...	...	...	...

GENERAL NOTES:
(a) The combinations of size ranges and flange pressure classes for which this Table gives no moment factor value are outside the scope of this Table.
(b) The designer should consider reducing the moment factor if the loading is primarily sustained in nature and the bolted flange joint operates at a temperature where gasket creep/relaxation will be significant.

NOTES:
(1) F_M = 0.1 + (48 – NPS)/56.
(2) F_M = 0.1 except NPS 60, Class 300, in which case F_M = 0.03.

Figure E4.5.4 – Table 4.16.12

4.6 Flat Heads

4.6.1 Example E4.6.1 – Flat Unstayed Circular Heads Attached by Bolts

Determine the required thickness for a heat exchanger blind flange.

Blind Flange Data:

- Material = $SA-105$
- Design Conditions = $135\ psig@650°F$
- Flange Bolt-Up Temperature = $100°F$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $17800\ psi$
- Allowable Stress at Flange Bolt-Up Temp. = $24000\ psi$
- Weld Joint Efficiency = 1.0
- Mating flange information and gasket details are provided in Example Problem E4.16.1.

Evaluate the blind flange in accordance with paragraph 4.6.2.

The minimum required thickness of a flat unstayed circular head, cover, or blind flange that is attached with bolting that results in an edge moment (see Table 4.6.1, Detail 7), shall be calculated by the equations shown below. The operating and gasket seating bolt loads, W_o and W_g, and the moment arm of this load, h_G, in these equations shall be computed based on the flange geometry and gasket material as described in paragraph 4.16.

a) STEP 1 – Calculate the gasket moment arm, h_G, and the diameter of the gasket load reaction, d in accordance with paragraph 4.16, as demonstrated in Example Problem E4.16.1.

   See paragraph 4.16.7, Flange Design Procedure, STEP 6: $h_G = 0.875\ in$

   See paragraph 4.16.6, Gasket Reaction Diameter, STEP 3: $d = G = 29.5\ in$

b) STEP 2 – Calculate the operating and gasket seating bolt loads, W_o and W_g, in accordance with paragraph 4.16, as demonstrated in Example Problem E4.16.1.

   See paragraph 4.16.6, Design Bolt Loads, STEP 4: $W_o = 111282.7\ lbs$

   See paragraph 4.16.6, Design Bolt Loads, STEP 5: $W_g = 237626.3\ lbs$

c) STEP 3 – Identify the appropriate attachment factor, C, from Table 4.6.1, Detail 7.

$$C = 0.3$$

d) STEP 4 – The required thickness of the blind flange is the maximum of the thickness required for the operating and gasket seating conditions.

$$t = \max\left[t_o,\ t_g\right] = \max\left[1.6522,\ 0.8720\right] = 1.6522\ in$$

   Where the required thickness in the operating condition is in accordance with Equation (4.6.3).

$$t_o = d\sqrt{\left(\frac{CP}{SE}\right) + \left(\frac{1.9Wh_G}{SEd^3}\right)} + CA$$

$$t_o = (29.5)\sqrt{\left(\frac{0.3(135)}{17800(1.0)}\right) + \left(\frac{1.9(111282.7)(0.875)}{17800(1.0)(29.5)^3}\right)} + 0.125 = 1.6522\ in$$

And the required thickness in the gasket seating condition is in accordance with Equation (4.6.4).

$$t_g = d\sqrt{\frac{1.9Wh_G}{SEd^3}} + CA$$

$$t_g = (29.5)\sqrt{\frac{1.9(237626.3)(0.875)}{24000(1.0)(29.5)^3}} + 0.125 = 0.8720\ in$$

4.6.2 Example E4.6.2 – Flat Un-stayed Non-Circular Heads Attached by Welding

Determine the required thickness for an air-cooled heat exchanger end plate. The end plate is welded to the air-cooled heat exchanger box with a full penetration Category C, Type 7 corner joint.

End Plate Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $400\ psig@500°F$
- Short Span Length = $7.125\ in$
- Long Span Length = $9.25\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20600\ psi$
- Weld Joint Efficiency = 1.0

Evaluate the welded end plate in accordance with paragraph 4.6.3.

The minimum required thickness of a flat unstayed non-circular head or cover that is not attached with bolting that results in an edge moment shall be calculated by the following equations.

a) STEP 1 – Determine the short and long span dimensions of the non-circular plate, d and D, respectively (in the corroded state) as demonstrated in Example Problem E4.12.1.

$$d = H = 7.125 + 2(0.125) = 7.375\ in$$

$$D = h = 9.25 + 2(0.125) = 9.500\ in$$

Note, the variables d and D used in paragraph 4.6.3 are denoted as H and h, respectively, in paragraph 4.12.

b) STEP 2 – Calculate the Z factor in accordance with Equation (4.6.6).

$$Z = \min\left[2.5, \left(3.4 - \left(\frac{2.4d}{D}\right)\right)\right] = \min\left[2.5, \left(3.4 - \left(\frac{2.4(7.375)}{9.5}\right)\right)\right] = 1.5368\ in$$

c) STEP 3 – The appropriate attachment factor, C, is taken from paragraph 4.12.2.6. For end closures of non-circular vessels constructed of flat plate, the design rules of paragraph 4.6 shall be used except that 0.20 shall be used for the value of C in all the calculations.

$$C = 0.20$$

d) STEP 4 – Calculate the required thickness using Equation (4.6.5).

$$t = d\sqrt{\frac{ZCP}{SE}} + CA = 7.375\sqrt{\frac{1.5368(0.20)(400)}{20600(1.0)}} + 0.125 = 0.6947\ in$$

The required thickness is $0.6947\ in$.

[Intentionally Left Blank]

4.6.3 Example E4.6.3 – Integral Flat Head with a Centrally Located Opening

Determine if the stresses in the integral flat head with a centrally located opening are within acceptable limits, considering the following design conditions. The head, shell and opening detail is shown in Figure E4.6.3.

Flat Head Data:

- Material = $SA-240,\ Type\ 304$
- Design Conditions = $100\ psig@400°F$
- Outside diameter of flat head and shell, A = $72.0\ in$
- Inside diameter of shell, B_s = $70.0\ in$
- Diameter of central opening, B_n = $40.0\ in$
- Thickness of the flat head, t = $3.0\ in$
- Thickness of nozzle above the transition, g_{0n} = $0.5625\ in$
- Thickness of nozzle at the flat head, g_{1n} = $1.125\ in$
- Length of nozzle transition, h_n = $2.0\ in$
- Thickness of shell below transition, g_{0s} = $1.0\ in$
- Thickness of shell at head, g_{1s} = $2.0\ in$
- Length of shell transition, h_s = $3.0\ in$
- Allowable stress = $18600\ psi$

Evaluate the integral flat head with a single, circular, centrally located opening in accordance with paragraph 4.6.4.

a) STEP 1 – Determine the design pressure and temperature of the flat head opening.

See the specified data above.

b) STEP 2 – Determine the geometry of the flat head opening.

See Figure E4.6.3 and the specified data above.

c) STEP 3 – Calculate the operating moment, M_o, using the following equation.

$$M_o = 0.785B_n^2P\left(R+\frac{g_{1n}}{2}\right)+0.785\left(B_s^2-B_n^2\right)P\left(\frac{R+g_{1n}}{2}\right)$$

$$M_o = 0.785(40.0)^2(100.0)\left(13.875+\frac{1.125}{2}\right)+0.785\left(70.0^2-40.0^2\right)(100.0)\left(\frac{13.875+1.125}{2}\right)$$

$$M_o = 3756225.0\ in-lbs$$

where,

$$R=\frac{B_s-B_n}{2}-g_{1n}=\frac{70.0-40.0}{2}-1.125=13.875\ in$$

d) STEP 4 – Calculate F, V, and f based on B_n, g_{1n}, g_{0n}, and h_n using the equations in Table 4.16.4 and Table 4.16.5, designating the resulting values as F_n, V_n, and f_n.

Table 4.16.4:

$$h_{on} = \sqrt{B_n g_{0n}} = \sqrt{(40.0)(0.5625)} = 4.75\ in$$

$$X_{gn} = \frac{g_{1n}}{g_{0n}} = \frac{1.125}{0.5625} = 2.0$$

$$X_{hn} = \frac{h_n}{h_{on}} = \frac{2.0}{4.75} = 0.4211$$

Table 4.16.5, with $X_{gn} = 2.0$ and $X_{hn} = 0.4211$.

$$F_n = \begin{pmatrix} 0.897697 - 0.297012\ln\left[X_{gn}\right] + 9.5257\left(10^{-3}\right)\ln\left[X_{hn}\right] + \\ 0.123586\left(\ln\left[X_{gn}\right]\right)^2 + 0.0358580\left(\ln\left[X_{hn}\right]\right)^2 - \\ 0.194422\left(\ln\left[X_{gn}\right]\right)\left(\ln\left[X_{hn}\right]\right) - 0.0181259\left(\ln\left[X_{gn}\right]\right)^3 + \\ 0.0129360\left(\ln\left[X_{hn}\right]\right)^3 - 0.0377693\left(\ln\left[X_{gn}\right]\right)\left(\ln\left[X_{hn}\right]\right)^2 + \\ 0.0273791\left(\ln\left[X_{gn}\right]\right)^2\left(\ln\left[X_{hn}\right]\right) \end{pmatrix}$$

$$F_n = \begin{pmatrix} 0.897697 - 0.297012\ln[2.0] + 9.5257\left(10^{-3}\right)\ln[0.4211] + \\ 0.123586\left(\ln[2.0]\right)^2 + 0.0358580\left(\ln[0.4211]\right)^2 - \\ 0.194422\left(\ln[2.0]\right)\left(\ln[0.4211]\right) - 0.0181259\left(\ln[2.0]\right)^3 + \\ 0.0129360\left(\ln[0.4211]\right)^3 - 0.0377693\left(\ln[2.0]\right)\left(\ln[0.4211]\right)^2 + \\ 0.0273791\left(\ln[2.0]\right)^2\left(\ln[0.4211]\right) \end{pmatrix}$$

$$F_n = 0.8410$$

$$For\ 0.1 \le X_{hn} \le 0.5,$$

$$V_n = \begin{pmatrix} 0.0.500244 - \dfrac{0.227914}{X_{gn}} - 1.87071X_{hn} + \dfrac{0.344410}{X_{gn}^2} + 2.49189X_{hn}^2 + \\ 0.873446\left(\dfrac{X_{hn}}{X_{gn}}\right) + \dfrac{0.189953}{X_{gn}^3} - 1.06082X_{hn}^3 - 1.49970\left(\dfrac{X_{hn}^2}{X_{gn}}\right) + 0.719413\left(\dfrac{X_{hn}}{X_{gn}^2}\right) \end{pmatrix}$$

$$V_n = \begin{pmatrix} 0.0.500244 - \dfrac{0.227914}{2.0} - 1.87071(0.4211) + \dfrac{0.344410}{(2.0)^2} + 2.49189(0.4211)^2 + \\ 0.873446\left(\dfrac{0.4211}{2.0}\right) + \dfrac{0.189953}{(2.0)^3} - 1.06082(0.4211)^3 - 1.49970\left(\dfrac{(0.4211)^2}{2.0}\right) + \\ 0.719413\left(\dfrac{0.4211}{(2.0)^2}\right) \end{pmatrix}$$

$$V_n = 0.2534$$

$$f_n = \max\left[1.0, \left(\frac{\begin{pmatrix} 0.0927779 - 0.0336633X_g + 0.964176X_g^2 + \\ 0.0566286X_h + 0.347074X_h^2 - 4.18699X_h^3 \end{pmatrix}}{\begin{pmatrix} 1 - 5.96093\left(10^{-3}\right)X_g + 1.62904X_h + \\ 3.49329X_h^2 + 1.39052X_h^3 \end{pmatrix}}\right)\right]$$

$$f_n = \max\left[1.0, \left(\frac{\begin{pmatrix} 0.0927779 - 0.0336633(2.20) + 0.964176(2.20)^2 + \\ 0.0566286(0.7419) + 0.347074(0.7419)^2 - 4.18699(0.7419)^3 \end{pmatrix}}{\begin{pmatrix} 1 - 5.96093\left(10^{-3}\right)(2.20) + 1.62904(0.7419) + \\ 3.49329(2.20)^2 + 1.39052(0.7419)^3 \end{pmatrix}}\right)\right]$$

$$f_n = 1.5248$$

e) STEP 5 – Calculate F, V, and f based on B_s, g_{1s}, g_{0s}, and h_s using the equations in Table 4.16.4 and Table 4.16.5 and designating the resulting values as F_s, V_s, and f_s.

Table 4.16.4:

$$h_{os} = \sqrt{B_s g_{0s}} = \sqrt{(70.0)(1.0)} = 8.3666\ in$$

$$X_{gs} = \frac{g_{1s}}{g_{0s}} = \frac{2.0}{1.0} = 2.0$$

$$X_{hs} = \frac{h_s}{h_{os}} = \frac{3.0}{8.3666} = 0.3586$$

Similarly, from Table 4.16.5 with $X_{gs} = 2.0$ and $X_{hs} = 0.3586$.

$$F_s = 0.8564$$
$$V_s = 0.2772$$
$$f_s = 1.8001$$

f) STEP 6 – Calculate Y, T, U, Z, L, e, and d based on $K = A/B_n$ using the equations/direct interpretation from VIII-1 Figure 2-7.1.

$$K = \frac{A}{B_n} = \frac{72.0}{40.0} = 1.8$$

$$Y = \frac{1}{K-1}\left[0.66845 + 5.71690\left(\frac{K^2 \log_{10} K}{K^2 - 1}\right)\right]$$

$$Y = \frac{1}{(1.8)-1}\left[0.66845 + 5.71690\left(\frac{(1.8)^2 \log_{10}[1.8]}{(1.8)^2 - 1}\right)\right] = 3.4742$$

$$T = \frac{K^2\left(1+8.55246\log_{10}K\right)-1}{\left(1.04720+1.9448K^2\right)\left(K-1\right)} = \frac{(1.8)^2\left(1+8.55246\log_{10}[1.8]\right)-1}{\left(1.04720+1.9448(1.8)^2\right)(1.8-1)} = 1.5801$$

$$U = \frac{K^2\left(1+8.55246\log_{10}K\right)-1}{1.36136\left(K^2-1\right)\left(K-1\right)} = \frac{(1.8)^2\left(1+8.55246\log_{10}[1.8]\right)-1}{1.36136\left((1.8)^2-1\right)\left((1.8)-1\right)} = 3.8076$$

$$Z = \frac{\left(K^2+1\right)}{\left(K^2-1\right)} = \frac{\left((1.8)^2+1\right)}{\left((1.8)^2-1\right)} = 1.8929$$

$$d = \frac{Ug_{on}^2 h_{on}}{V_n} = \frac{(3.8076)(0.5625)^2(4.75)}{0.2534} = 22.5831\ in^3$$

$$e = \frac{F_n}{h_{on}} = \frac{0.8410}{4.75} = 0.1771\ in^{-1}$$

$$L = \frac{te+1}{T} + \frac{t^3}{d} = \frac{(3)(0.1771)+1}{1.5801} + \frac{(3)^3}{22.5831} = 2.1647$$

g) STEP 7 – Calculate the quantity $(E\theta)^*$ for an opening with an integrally attached nozzle using the following equation.

$$(E\theta)^* = \frac{0.91\left(\frac{g_{1n}}{g_{0n}}\right)^2 (B_n+g_{0n})V_n}{f_n\sqrt{(B_n g_{0n})}} \cdot S_H$$

$$(E\theta)^* = \frac{0.91\left(\frac{1.125}{0.5625}\right)^2 (40.0+0.5625)(0.2534)}{(1.5248)\sqrt{(40.0)(0.5625)}} \cdot (52263.9) = 270352.5\ psi$$

where, S_H is evaluated from Table 4.6.2.

$$S_H = \frac{f_n M_o}{L g_{1n}^2 B_n} = \frac{1.5248(3756225.0)}{2.1647(1.125)^2(40.0)} = 52263.9\ psi$$

h) STEP 8 – Calculate the quantity M_H using the following equation.

$$M_H = \frac{(E\theta)^*}{\frac{1.74V_s\sqrt{B_s g_{0s}}}{g_{0s}^3(B_s+g_{0s})} + \frac{(E\theta)^*}{M_o}\left(1+\frac{F_s t}{\sqrt{B_s g_{0s}}}\right)}$$

$$M_H = \frac{270352.5}{\frac{1.74(0.2772)\sqrt{(70.0)(1.0)}}{(1.0)^3(70.0+1.0)} + \frac{270352.5}{3756225}\left(1+\frac{(0.8564)(3.0)}{\sqrt{(70.0)(1.0)}}\right)} = 1793295.2\ in-lb$$

i) STEP 9 – Calculate the quantity X_1 using the following equation.

$$X_1 = \frac{M_o - M_H\left(1 + \frac{F_s t}{\sqrt{B_s g_{0s}}}\right)}{M_o} = \frac{3756225.0 - 1793295.2\left(1 + \frac{(0.8564)(3.0)}{\sqrt{(70.0)(1.0)}}\right)}{3756225.0} = 0.3770$$

j) STEP 10 – Calculate the stresses at the shell-to-flat-head junction and opening-to-flat-head junction using Table 4.6.2.

Head/Shell Junction Stresses:

Longitudinal hub stress in shell:

$$S_{HS} = \frac{1.1 f_s X_1 (E\theta)^* \left(\sqrt{B_s g_{0s}}\right)}{\left(\frac{g_{1s}}{g_{0s}}\right)^2 B_s V_s} = \frac{1.1(1.8001)(0.3727)(270352.5)\left(\sqrt{(70.0)(1.0)}\right)}{\left(\frac{2.0}{1.0}\right)^2 (70.0)(0.2772)} = 21659.9\ psi$$

Radial stress at outside diameter:

$$S_{RS} = \frac{1.91 M_H\left(1 + \frac{F_s t}{\sqrt{B_s g_{0s}}}\right)}{B_s t^2} + \frac{0.64 F_s M_H}{B_s \sqrt{B_s g_{0s}}\, t}$$

$$S_{RS} = \left(\frac{1.91(1793295.2)\left(1 + \frac{(0.8564)(3.0)}{\sqrt{(70.0)(1.0)}}\right)}{(70.0)(3.0)^2} + \frac{0.64(0.8564)(1793295.2)}{(70.0)\sqrt{(70.0)(1.0)}(3.0)}\right) = 7665.8\ psi$$

Tangential stress at outside diameter:

$$S_{TS} = \frac{X_1 (E\theta)^* t}{B_s} - \frac{0.57 M_H\left(1 + \frac{F_s t}{\sqrt{B_s g_{0s}}}\right)}{B_s t^2} + \frac{0.64 Z F_s M_H}{B_s \sqrt{B_s g_{0s}}\, t}$$

$$S_{TS} = \left(\begin{array}{l} \frac{(0.3770)(270352.5)(3.0)}{70.0} - \frac{0.57(1793295.2)\left(1 + \frac{(0.8564)(3.0)}{\sqrt{(70.0)(1.0)}}\right)}{(70.0)(3.0)^2} + \\ \frac{0.64(1.8929)(0.8564)(1793295.2)}{(70.0)\sqrt{(70.0)(1.0)}(3.0)} \end{array}\right) = 3306.3\ psi$$

Opening/Head Junction Stresses:

Longitudinal hub stress in central opening:

$$S_{HO} = X_1 S_H = (0.3770)(52263.9) = 19703.5\ psi$$

Radial stress at central opening:

$$S_{RO} = X_1 S_R = (0.3770)(8226.0) = 3101.2 \ psi$$

Where, S_R is evaluated from Table 4.6.2.

$$S_R = \frac{(1.33te+1)M_o}{Lt^2 B_n} = \frac{(1.33(3.0)(0.1771)+1)(3756225.0)}{(2.1647)(3.0)^2(40.0)} = 8226.0 \ psi$$

Tangential stress at diameter of central opening:

$$S_{TO} = X_1 S_T + \frac{0.64 Z_1 F_s M_H}{B_s \sqrt{B_s g_{0s}}\, t} = \left(\begin{array}{c} (0.3770)(20678.7) + \\ \dfrac{0.64(2.8929)(0.8564)(1793295.2)}{(70.0)\sqrt{(70.0)(1.0)}(3.0)} \end{array} \right) = 9414.2 \ psi$$

Where, Z_1 and S_T are evaluated from Table 4.6.2.

$$Z_1 = \frac{2K^2}{K^2 - 1} = \frac{2(1.8)^2}{\left((1.8)^2 - 1\right)} = 2.8929$$

$$S_T = \frac{YM_o}{t^2 B_n} - ZS_R = \frac{(3.4742)(3756225.0)}{(3.0)^2(40.0)} - (1.8929)(8226.0) = 20678.7 \ psi$$

k) STEP 11 – Check the flange stress acceptance criteria in Table 4.6.3. If the stress criteria are satisfied, then the design is complete. If the stress criteria are not satisfied, then re-proportion the flat head and/or opening dimensions and go to STEP 3.

Head/Shell Junction Stresses:

$$\{S_{HS} = 21659.9 \ psi\} \le \{1.5S_f = 1.5(18600) = 27900 \ psi\} \qquad True$$

$$\{S_{RS} = 7665.8 \ psi\} \le \{S_f = 18600 \ psi\} \qquad True$$

$$\{S_{TS} = 3306.3 \ psi\} \le \{S_f = 18600 \ psi\} \qquad True$$

$$\left\{ \frac{(S_{HS} + S_{RS})}{2} = \frac{(21659.9 + 7665.8)}{2} = 14662.9 \ psi \le S_f = 18600 \ psi \right\} \qquad True$$

$$\left\{ \frac{(S_{HS} + S_{TS})}{2} = \frac{(21659.9 + 3306.3)}{2} = 12483.1 \ psi \le S_f = 18600 \ psi \right\} \qquad True$$

Opening/Head Junction Stresses:

$$\{S_{HO} = 19703.5 \ psi \le 1.5S_f = 1.5(18600) = 27900 \ psi\} \qquad True$$

$$\{S_{RO} = 3101.2 \ psi\} \le \{S_f = 18600 \ psi\} \qquad True$$

$$\{S_{TO} = 9414.2 \ psi\} \le \{S_f = 18600 \ psi\} \qquad True$$

$$\left\{\frac{(S_{HO}+S_{RO})}{2}=\frac{(19703.5+3101.2)}{2}=11402.4\ psi \le S_f=18600\ psi\right\} \qquad True$$

$$\left\{\frac{(S_{HO}+S_{TO})}{2}=\frac{(19703.5+9414.2)}{2}=14558.9\ psi \le S_f=18600\ psi\right\} \qquad True$$

Stress acceptance criteria are satisfied, the design is complete.

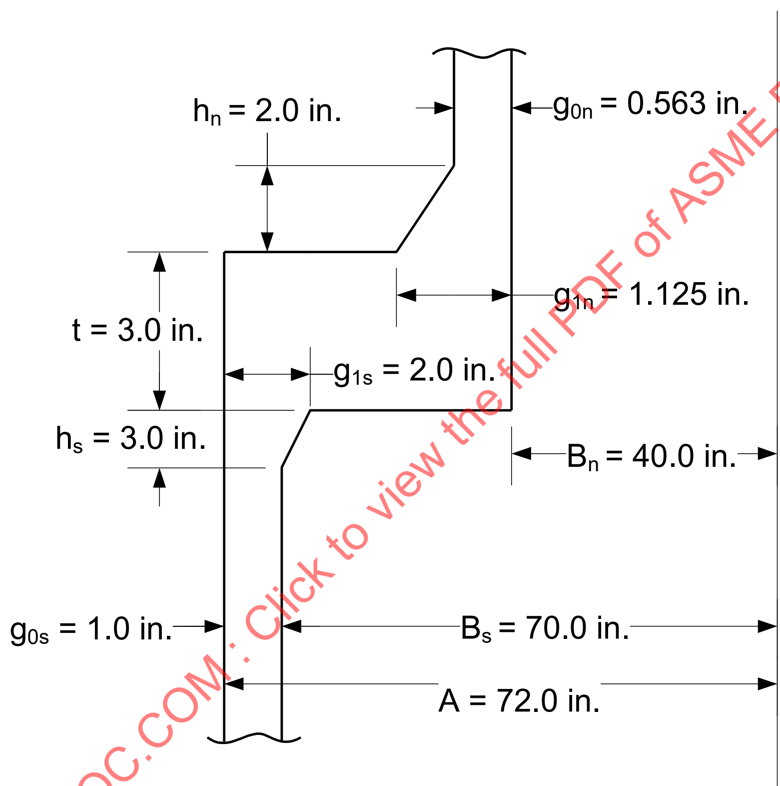

Figure E4.6.3 – Head, Shell, and Nozzle Geometry

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4.7 Spherically Dished Bolted Covers

4.7.1 Example E4.7.1 – Thickness Calculation for a Type D Head

Determine if the proposed Type D spherically dished bolted cover, used in a heat exchanger application, is adequately designed considering the following design conditions. The spherically dished head is seamless. See Figure E4.7.1 for details.

Tubeside Data:

- Design Conditions = $213\ psig\ @\ 400°F$
- Corrosion Allowance (CAT) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Shellside Data:

- Design Conditions = $305\ psig\ @\ 250°F$
- Corrosion Allowance (CAS) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Flange Data:

- Material = $SA-105$
- Allowable Stress at Ambient Temperature = $24000\ psi$
- Allowable Stress at Tubeside Design Temperature = $20500\ psi$
- Allowable Stress at Shellside Design Temperature = $21600\ psi$

Head Data:

- Material = $SA-515,\ Grade\ 60$
- Allowable Stress at Ambient Temperature = $21300\ psi$
- Allowable Stress at Tubeside Design Temperature = $18200\ psi$
- Yield Stress at Tubeside Design Temperature = $27300\ psi$
- Modulus of Elasticity at Tubeside Design Temp. = $27.9E+06\ psi$
- Allowable Stress at Shellside Design Temperature = $19200\ psi$
- Yield Stress at Shellside Design Temperature = $28800\ psi$
- Modulus of Elasticity at Shellside Design Temp. = $28.55E+06\ psi$

Bolt Data:

- Material = $SA-193,\ Grade\ B7$
- Diameter = $0.75\ in^2$
- Cross-Sectional Root Area = $0.302\ in^2$
- Number of Bolts = 20
- Allowable Stress at Ambient Temperature = $25000\ psi$

- Allowable Stress at Tubeside Design Temperature = $25000\ psi$
- Allowable Stress at Shellside Design Temperature = $25000\ psi$

Gasket Data:

- Material = Solid Flat Metal (Iron/Soft Steel)
- Gasket Factor = 5.5
- Gasket Seating Factor = $18000\ psi$
- Inside Diameter = $16.1875\ in$
- Outside Diameter = $17.0625\ in$

Commentary:

In accordance with paragraph 4.1.8.1, a combination unit is a pressure vessel that consists of more than one independent or dependent pressure chamber, operating at the same or different pressures and temperatures. The parts separating each pressure chamber are the common elements. Each element, including the common elements, shall be designed for at least the most severe condition of coincident pressure and temperature expected in normal operation. The common elements under consideration in this example are that of the head and flange that make-up the floating head. While this example will separately evaluate tubeside and shellside pressures for each common element, the design temperature of the tubeside will conservatively be applied to both evaluations.

Per paragraph 4.7.1.3, the calculations are performed using dimensions in the corroded condition and the uncorroded condition, and the more severe case shall control. This example only evaluates the spherically dished bolted cover in the corroded condition.

Per paragraph 4.7.5.1, the thickness of the head for a Type D Head Configuration see Figure 4.7.4 shall be determined by the following equations.

a) Internal pressure (pressure on the concave side) – the head thickness shall be determined using Equation (4.7.2).

$$t=\left(\frac{5PL}{6S}\right)=\frac{5(213)(16.125)}{6(18200)}=0.1573\ in$$

where,

$$L=L+CAS=16.0+0.125=16.125\ in$$

This calculated thickness is increased for corrosion allowance on both the shell and tube side.

$$t=t+CAS+CAT$$
$$t=0.1573+0.125+0.125=0.4073\ in$$

b) External pressure (pressure on the convex side) – the head thickness shall be determined in accordance with the rules of paragraph 4.4.

Per paragraph 4.4.7.1, the required thickness of a spherical or hemispherical head subjected to external pressure loading shall be determined using the following procedure.

1) STEP 1 – Assume an initial thickness, t, for the spherical shell. The specified head thickness shall consider corrosion from tubeside and shellside, resulting in the following.

$$t = t - CAS - CAT = 0.875 - 0.125 - 0.125 = 0.625\ in$$

2) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075E_y\left(\frac{t}{R_o}\right) = 0.075(27.9E+06)\left(\frac{0.625}{16.75}\right) = 78078.3582\ psi$$

where,

$$R_o = L + t = 16.125 + 0.625 = 16.75\ in$$

3) STEP 3 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{he}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 3.1 – Calculate the predicted elastic buckling stress due to external pressure, F_{he}.

$$F_{he} = 78078.3582\ psi \quad \text{(as determined in paragraph 4.4.7, STEP 2)}$$

ii) STEP 3.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{he}}{E} = \frac{78078.3582}{27.9E+06} = 0.00279851$$

iii) STEP 3.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE

$$F_{ic} = 20252.1229\ psi$$

4) STEP 4 – Calculate the value of design margin, FS, per VIII-2, paragraph 4.4.2.

$$0.55S_y = 0.55(27300.0) = 15015.0\ psi$$

Since $0.55S_y < F_{ic} < S_y$, calculate the FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{20252.1229}{27300.0}\right) = 1.8573$$

5) STEP 5 – Calculate the allowable external working pressure P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(10904.1)\left(\frac{0.625}{16.75}\right) = 813.7\ psi$$

where,

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{20252.1229}{1.8573} = 10904.1\ psi$$

6) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2.

Since $\{P_a = 813.7\, psi\} > \{P = 305\, psi\}$, the specified head thickness is acceptable for external pressure.

The flange thickness of the head for a Type D Head Configuration is determined per paragraph 4.7.5.2. To compute the required flange thickness, the flange operating, and gasket seating moments are determined using the flange design procedure from paragraphs 4.16.6 and 4.16.7.

Paragraph 4.16.6: Design Bolt Loads. The procedure to determine the bolt loads for the operating and gasket seating conditions is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

$$Tubeside\ Conditions: P = 213\ psig\ at\ 400°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 4.16.1.

$$m = 5.5$$
$$y = 18000\ psi$$

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5(GOD - GID) = 0.5(17.0625 - 16.1875) = 0.4375\ in$$

From Table 4.16.3, Facing Sketch Detail 2, Column I,

$$b_o = \frac{w + N}{4} = \frac{(0.125 + 0.4375)}{4} = 0.1406\ in$$

where,

$$w = raised\ nubbin\ width = 0.125\ in$$

for $b_o \le 0.25\ in$,

$$b = b_o = 0.1406\ in$$

$$G = mean\ diameter\ of\ the\ gasket\ contact\ face$$
$$G = 0.5(17.0625 + 16.1875) = 16.625\ in$$

d) STEP 4 – Determine the design bolt load for the operating condition.

$$W_o = H + H_p = 0.785G^2P + 2b\pi GmP\ \ for\ non-self-energized\ gaskets$$
$$W_o = 0.785(16.625)^2(213) + 2(0.1406)(\pi)(16.625)(5.5)(213) = 63419.5\ lbs$$

e) STEP 5 – Determine the design bolt load for the gasket seating condition.

$$W_g = \frac{(A_m + A_b)}{2} S_{bg} = \frac{(5.2872 + 6.04)}{2}(25000) = 141590.0\ lbs$$

where, the parameter A_b is the actual cross-sectional area of the bolts that is selected such that $A_b \ge A_m$.

$$A_b = (Number\ of\ bolts)(Root\ area\ of\ one\ bolt) = 20(0.302) = 6.04\ in^2$$

$$A_m = \max\left[\left(\frac{W_o + F_A + \frac{4M_E}{G}}{S_{bo}}\right), \left(\frac{W_{gs}}{S_{bg}}\right)\right] = \max\left[\left(\frac{63419.5 + 0.0 + 0.0}{25000}\right), \left(\frac{132181.1}{25000}\right)\right]$$

$$A_m = \max[2.5368,\ 5.2872] = 5.2872\ in^2$$

and,

$$W_{gs} = \pi bGy \qquad for\ non-self-energized\ gaskets$$

$$W_{gs} = \pi(0.1406)(16.625)(18000) = 132181.1\ lbs$$

and, $F_A = 0$ and $M_E = 0$ since there are no externally applied net-section forces and bending moments.

Paragraph 4.16.7: Flange Design Procedure. The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint, $F_A = 0$ and $M_E = 0$.

$$Tubeside\ Conditions: P = 213\ psig\ at\ 400°F$$

$$Shellside\ Conditions: P = 305\ psig\ at\ 400°F$$

b) STEP 2 – Determine the design bolt loads for operating condition W_o, and the gasket seating condition W_g, and the corresponding actual bolt load area A_b, from paragraph 4.16.6.

$$W_o = 63419.5\ lbs$$

$$W_g = 141590.0\ lbs$$

$$A_b = 6.04\ in^2$$

c) STEP 3 – Determine an initial flange geometry (see Figure E4.7.1), in addition to the information required to determine the bolt load, the following geometric parameters are required.

1) Flange bore

$$B = B + 2(CAT) = 16.25 + 2(0.125) = 16.50\ in$$

2) Bolt circle diameter

$$C = 18.125\ in$$

3) Outside diameter of the flange

$$A = A - 2(CAS) = 19.625 - 2(0.125) = 19.375\ in$$

4) Flange thickness, (see Figure E4.7.1)

$$T = T - 2(CAT) = 2.3125 - 2(0.125) = 2.0625\ in$$

5) Thickness of the hub at the large end

$$Not\ \ Applicable$$

6) Thickness of the hub at the small end

$$Not\ \ Applicable$$

7) Hub length

$$Not\ \ Applicable$$

d) STEP 4 – Determine the flange stress factors using the equations in Table 4.16.4 and 4.16.5.

$$Not\ \ Applicable$$

e) STEP 5 – Determine the flange forces.

Tubeside Conditions:

$$H_D = 0.785B^2P = (0.785)(16.5)^2(213) = 45521.6\ lbs$$

$$H = 0.785G^2P = (0.785)(16.625)^2(213) = 46213.9\ lbs$$

$$H_T = H - H_D = 46213.9 - 45521.6 = 692.3\ lbs$$

$$H_G = W_o - H = 63419.5 - 46213.9 = 17205.6\ lbs$$

Shellside Conditions:

$$H_D = 0.785B^2P = (0.785)(16.5)^2(305) = 65183.5\ lbs$$

$$H = 0.785G^2P = (0.785)(16.625)^2(305) = 66174.8\ lbs$$

$$H_T = H - H_D = 66174.8 - 65183.5 = 991.3\ lbs$$

$$H_G = Not\ \ Applicable$$

f) STEP 6 – Determine the flange moment for the operating condition. When specified by the user or his designated agent, the maximum bolt spacing, B_{smax}, and the bolt spacing correction factor, B_{SC}, shall be applied in calculating the flange moment for internal pressure using the equations in Table 4.16.11. The flange moment M_o for the operating condition and flange moment M_g for the gasket seating condition without correction for bolt spacing $B_{SC} = 1$ is used for the calculation of the rigidity index in STEP 10. In these equations the moment arm, h_D, is computed using Equation (4.7.21), and h_T, and h_G are determined from Table 4.16.6.

For internal pressure (Tubeside Conditions):

$$M_o = abs\left[\left(\left(H_D h_D + H_T h_T + H_G h_G\right) B_{SC} + M_{oe}\right) F_s\right]$$

$$M_o = abs\left[\left(\left(45521.6(0.8125) + 692.3(0.7813) + 17205.6(0.75)\right)\cdot 1.0 + 0.0\right)\cdot 1.0\right]$$

$$M_o = 50431.4\ in - lbs$$

For external pressure (Shellside Conditions):

$$M_o = abs\left[\left(\left(H_D\left(h_D - h_G\right) + H_T\left(h_T - h_G\right)\right) + M_{oe}\right)F_s\right]$$

$$M_o = abs\left[\left(\left(65183.5\left(0.8125 - 0.75\right) + 991.3\left(0.7813 - 0.75\right)\right) + 0.0\right)\cdot 1.0\right]$$

$$M_o = 4105.0 \ in - lbs$$

Where the moment arm, h_D, is computed using Equation (4.7.21),

$$h_D = 0.5\left(C - B\right) = 0.5\left(18.125 - 16.50\right) = 0.8125 \ in$$

From Table 4.16.6 for loose type flanges,

$$h_G = \frac{C - G}{2} = \frac{18.125 - 16.625}{2} = 0.75 \ in$$

$$h_T = \frac{h_D + h_G}{2} = \frac{0.8125 + 0.75}{2} = 0.7813 \ in$$

g) STEP 7 – Determine the flange moment for the gasket seating condition using Equations (4.16.17) or (4.16.18).

For internal pressure (Tubeside Conditions):

$$M_g = \frac{W_g\left(C - G\right)B_{SC}F_S}{2} = \frac{\left(141590.0\right)\left(18.125 - 16.625\right)\left(1.0\right)\left(1.0\right)}{2} = 106192.5 \ in - lbs$$

For external pressure (Shellside Conditions):

$$M_g = W_g h_G F_s = \left(141590.0\right)\left(0.75\right)\left(1.0\right) = 106192.5 \ in - lbs$$

Per paragraph 4.7.5.2 – the flange thickness of the head for a Type D Head Configuration shall be determined by the following equations. When determining the flange design moment for the design condition, M_o, using paragraph 4.16, the following modifications must be made. An additional moment term, M_r, computed using Equation (4.7.22) shall be added to M_o as defined in paragraph 4.16. Note that this term may be positive or negative depending on the location of the head–to–flange ring intersection with relation to the flange ring centroid. Since the head–to–flange ring intersection is above the flange centroid, the sign of the M_r value is negative.

$$T = \max\left[T_g,\ T_o\right] = \max\left[T_g,\ \max\left[T_{o(tubeside)},\ T_{o(shellside)}\right]\right]$$

where,

$$T_g = \sqrt{\frac{M_g}{S_{fg}B}\cdot\left(\frac{A + B}{A - B}\right)} + CAS + CAT$$

$$T_o = Q + \sqrt{Q^2 + \frac{M_o}{S_{fo}B}\cdot\left(\frac{A + B}{A - B}\right)} + CAS + CAT$$

$$Q = \frac{|P| B\sqrt{4L^2 - B^2}}{8S_{fo}(A - B)}$$

a) STEP 1 – Calculate the additional moment, M_r, using Equation (4.7.22).

$$M_r = \left(0.785B^2 P \cot[\beta_1]\right) h_r$$

where,

$$\beta_1 = \arcsin\left[\frac{B}{2L+t}\right] = \arcsin\left[\frac{(16.5)}{2(16.125)+(0.625)}\right] = \begin{Bmatrix} 0.5258\ rad \\ 30.1259\ deg \end{Bmatrix}$$

and,

$$L = 16.0 + CAT = 16.0 + 0.125 = 16.125\ in$$
$$t = t - CAT - CAS = 0.875 - 0.125 - 0.125 = 0.625\ in$$

Commentary:

Paragraph 4.7.5.2 does not include guidance as to a method to calculate the lever arm, h_r of force H_r about the centroid of the flange ring. The procedure shown in Annex E4.7.1 provides one method to calculate h_r; however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

Referencing Annex E4.7.1,

$$h_r = 0.2654\ in$$

For internal pressure (Tubeside Conditions),

$$M_r = \left(0.785B^2 P \cot[\beta_1]\right) h_r$$
$$M_r = \left((0.785)(16.5)^2 (213)\cot[30.1259]\right)(0.2654) = 20819.9\ in-lbs$$

For external pressure (Shellside Conditions),

$$M_r = \left(0.785B^2 P \cot[\beta_1]\right) h_r$$
$$M_r = \left((0.785)(16.5)^2 (305)\cot[30.1259]\right)(0.2654) = 29812.5\ in-lbs$$

b) STEP 2 – Calculate the modified flange moment for the design condition, M_o, using paragraph 4.16 including the additional moment, M_r.

For internal pressure (Tubeside Conditions),

$$M_{o(tubeside)} = M_o - M_r = 50431.4 - 20819.9 = 29611.5\ in-lbs$$

For external pressure (Shellside Conditions),

$$M_{o(shellside)} = M_o - M_r = 4105.0 - 29812.5 = -25707.5\ in-lbs$$

c) STEP 3 – Calculate the flange thickness for the gasket seating condition, T_g.

$$T_g = \sqrt{\frac{M_g}{S_{fg}B}\cdot\left(\frac{A+B}{A-B}\right)} + CAS + CAT$$

$$T_g = \sqrt{\left(\frac{106192.5}{(24000)(16.5)}\right)\cdot\left(\frac{19.375+16.5}{19.375-16.5}\right)} + 0.125 + 0.125 = 2.0793\ in$$

d) STEP 4 – Calculate the flange thickness for the operating conditions, $T_{o(tubeside)}$ and $T_{o(shellside)}$.

For internal pressure (Tubeside Conditions),

$$T_o = Q + \sqrt{Q^2 + \frac{M_o}{S_{fo}B}\cdot\left(\frac{A+B}{A-B}\right)} + CAS + CAT$$

$$T_o = 0.2065 + \sqrt{(0.2065)^2 + \left(\frac{29611.5}{(20500)(16.5)}\right)\cdot\left(\frac{(19.375+16.5)}{(19.375-16.5)}\right)} + 0.125 + 0.125 = 1.5219\ in$$

where,

$$Q = \frac{|P|B\sqrt{4L^2 - B^2}}{8S_{fo}(A-B)} = \frac{|213|(16.5)\sqrt{4(16.125)^2 - (16.5)^2}}{8(20500)(19.375-16.5)} = 0.2065$$

For external pressure (Shellside Conditions),

$$T_o = Q + \sqrt{Q^2 + \frac{M_o}{S_{fo}B}\cdot\left(\frac{A+B}{A-B}\right)} + CAS + CAT$$

$$T_o = 0.2958 + \sqrt{(0.2958)^2 + \left(\frac{|25707.5|}{(20500)(16.5)}\right)\cdot\left(\frac{(19.375+16.5)}{(19.375-16.5)}\right)} + 0.125 + 0.125 = 1.5636\ in$$

where,

$$Q = \frac{|P|B\sqrt{4L^2 - B^2}}{8S_{fo}(A-B)} = \frac{|305|(16.5)\sqrt{4(16.125)^2 - (16.5)^2}}{8(20500)(19.375-16.5)} = 0.2958$$

e) STEP 5 – Determine the required flange thickness using the thicknesses determined in STEP 3 and STEP 4.

$$T = \max\left[T_g,\ T_o\right] = \max\left[T_g,\ \max\left[T_{o(tubeside)},\ T_{o(shellside)}\right]\right]$$

$$T = \max\left[2.0793,\ \max\left[1.5219,\ 1.5636\right]\right] = 2.0793\ in$$

The specified head thickness, $\{t = 0.875\ in\} > \{t_{req} = 0.4073\ in\}$ for internal pressure (tubeside conditions) and the external pressure (shellside conditions) calculations verified the maximum allowable external pressure, $\{MAEP = 866.4\ psi\} > \{P_{shellside} = 305\ psi\}$.

The specified flange thickness, $\{T = 2.3125\ in\} > \{T_{req} = 2.0793\ in\}$ for design internal pressure (tubeside conditions), external pressure (shellside conditions), and gasket seating conditions. Therefore, the proposed type D spherically dished bolted cover is adequately designed.

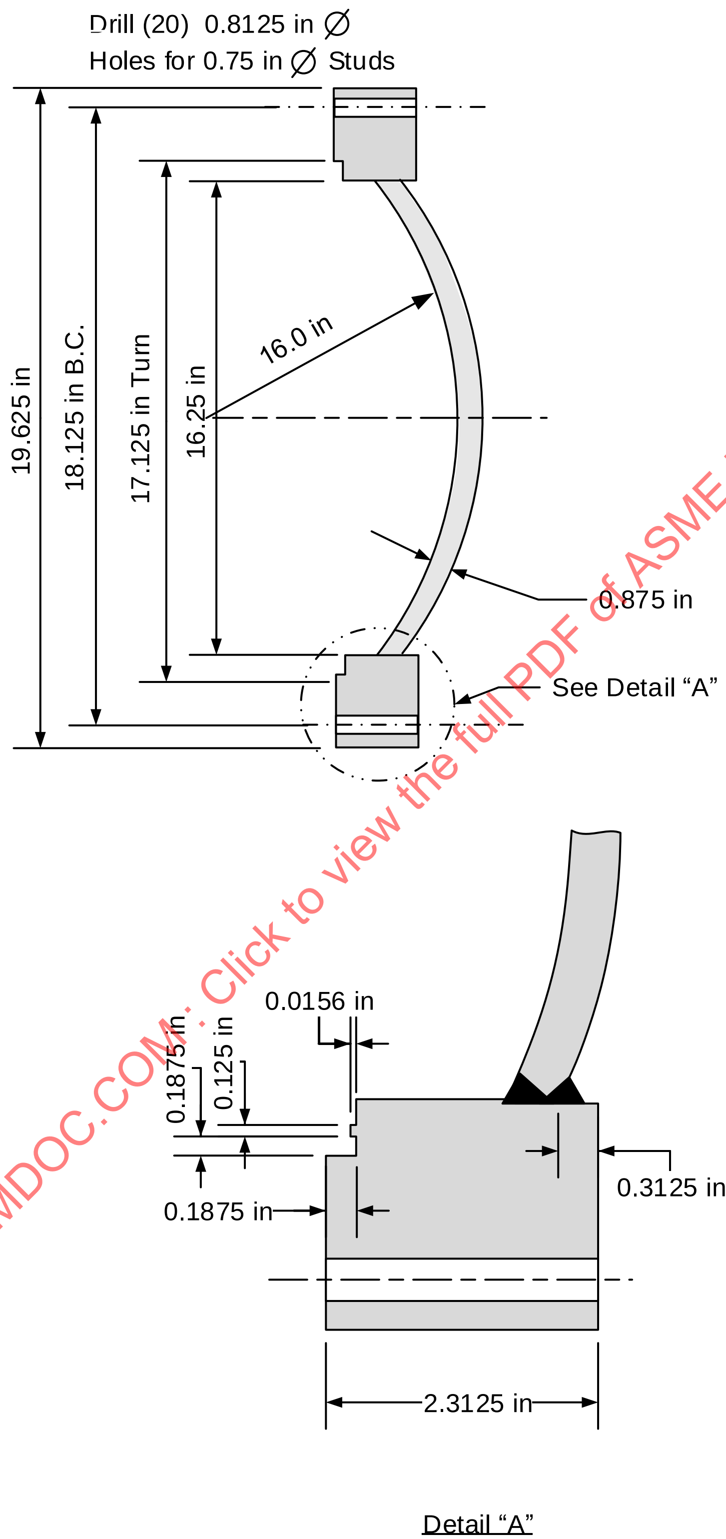

Figure E4.7.1 – Floating Head Geometry

Annex E4.7.1

ASME VIII-2 does not provide explicit guidance for computing the lever arm, h_r. This Annex provides one possible method; however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

Assumptions used in the development of the procedure include the following.

1) Shellside and tubeside corrosion allowance is applied to the dished head.
2) Tubeside corrosion allowance is only applied to the flange ring inside radius.
3) Shellside corrosion allowance is applied to the outer surfaces of the flange ring.
4) The geometry of the dished head to flange ring is assumed to be rectilinear.
5) The projected thickness of the dished head and associated dimensions are based off the angle β_1.
6) The location of the dished head to flange ring attachment point is established from the measured value from the outside depth of the flange ring to the outside base of the head. This variable is referenced as $DFHEAD$, (distance from head).
7) The ring flange is rectangular in shape, i.e., the portion of the flange that is removed by machining for the gasket surface is not considered.

The lever arm h_r measured from the centerline of the projected dished head thickness on the flange ring to the flange ring centroid is determined geometrically considering the above established assumptions. The variable $DFHEAD$ along with the inside radius and thickness of the dished head, L and t, respectively, and the flange ring radius, R set the initial location of the head centerline with the flange ring. The angle β_1 formed by the tangent to the centerline of the dished head at its point of intersection with the flange ring and a line perpendicular to the axis of the dished head is then established. From this point, the dished head and flange ring are subject to the applicable tubeside and shellside corrosion allowances resulting in the final corroded geometry from which the corroded lever arm is determined.

Refer to Figure AE4.7.1.

a) STEP 1 – Establish the variables used in the calculations.

Flange (uncorroded):

$$Thickness: \quad T = 2.3125\ in$$

$$Inside\ Diameter: \quad B = 16.25\ in$$

$$Inside\ Radius: \quad R = \frac{B}{2} = \frac{16.25}{2} = 8.125\ in$$

$$Head\ \ Location: \quad DFHEAD = 0.3125\ in$$

Spherically Dished Head (uncorroded):

$$Thickness: \quad t = 0.875\ in$$

$$Inside\ Radius: \quad L = 16.0\ in$$

Shellside and Tubeside Corrosion Allowance:

$$Shellside: \qquad CAS = 0.125\ in$$

$$Tubeside: \qquad CAT = 0.125\ in$$

b) STEP 2 – Establish the corroded dimensions of the input variables.

$$t_c = t - CAS - CAT = 0.875 - 0.125 - 0.125 = 0.625\ \ in$$

$$L_{mc} = L + CAT + \frac{t_c}{2} = 16.0 + 0.125 + \frac{0.625}{2} = 16.4375\ in$$

$$T = T - 2(CAS) = 2.3125 - 2(0.125) = 2.0625\ in$$

$$R_c = R + CAT = 8.125 + 0.125 = 8.25\ in$$

$$DFHEAD_c = DFHEAD - CAS = 0.3125 - 0.125 = 0.1875\ in$$

c) STEP 3 – Calculate the angle β_1 formed by the corroded mean radius of the dished head at the intersection with the flange ring and the corresponding corroded radius of the flange ring, measured to the axis of the dished head assembly.

$$\beta_1 = \arcsin\left[\frac{R_c}{L_{mc}}\right] = \arcsin\left[\frac{8.25}{16.4375}\right] = \begin{Bmatrix} 0.5258\ rad \\ 30.1259\ deg \end{Bmatrix}$$

d) STEP 4 – Calculate the axial adjustment of $DFHEAD$ due to the applied tubeside corrosion allowance on the flange inside radius. See Figure AE4.7.1.

$$X_{DFHEAD} = CAT \cdot \tan\left[\beta_1\right] = 0.125 \cdot \tan\left[30.1259\right] = 0.0725\ in$$

e) STEP 5 – Calculate the projected shellside corrosion allowance of the dished head on the flange ring, X_{cas}.

$$X_{cas} = \frac{CAS}{\cos\left[\beta_1\right]} = \frac{0.125}{\cos\left[30.1259\right]} = 0.1445\ in$$

f) STEP 6 – Calculate the projected corroded dished head thickness on the flange ring, X_c.

$$X_c = \frac{t_c}{\cos\left[\beta_1\right]} = \frac{0.625}{\cos\left[30.1259\right]} = 0.7226\ in$$

g) STEP 7 – Calculate the moment arm based on the corroded dimensions, h_r.

$$h_r = 0.5T_c - \left[DFHEAD_c + X_{DFHEAD} + X_{cas}\right] - 0.5X_c$$

$$h_r = 0.5(2.0625) - \left[0.1875 + 0.0725 + 0.1445\right] - 0.5(0.7227) = 0.2654\ in$$

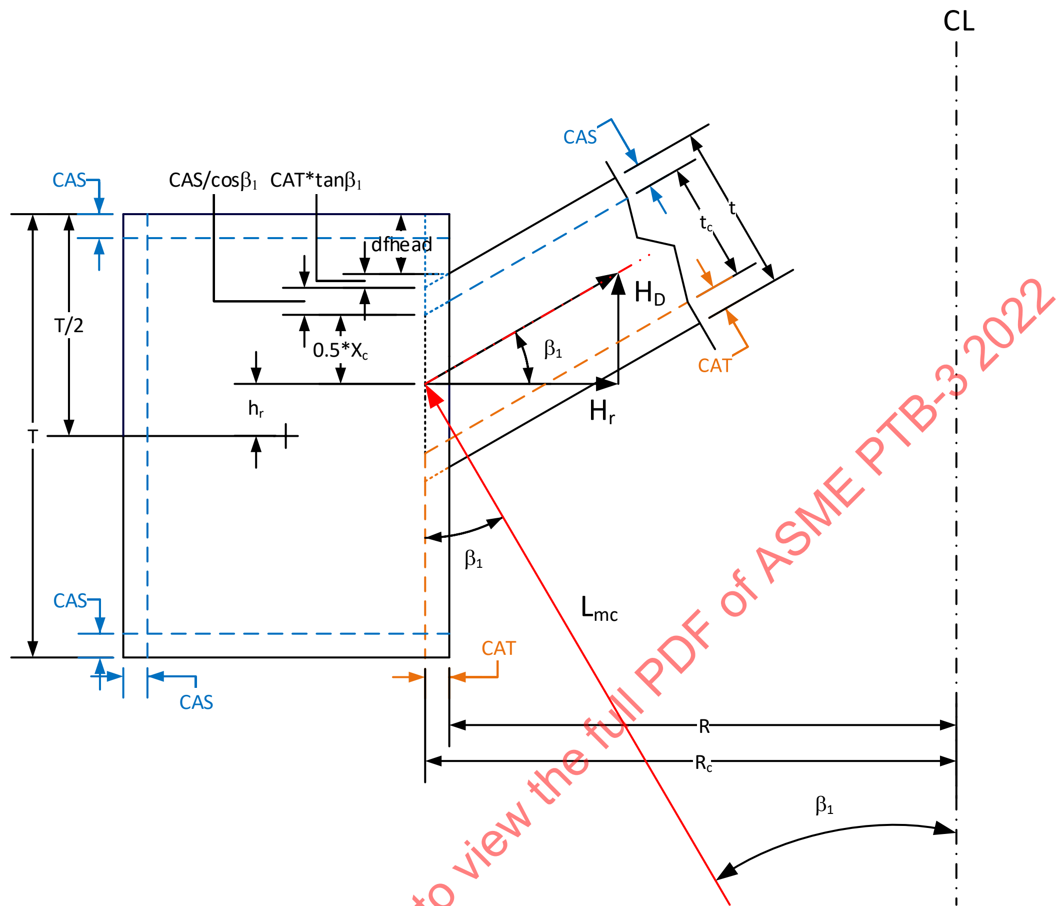

Figure AE4.7.1 – Lever Arm of Floating Head Geometry

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4.7.2 Example E4.7.2 – Thickness Calculation for a Type D Head Using the Alternative Rule in VIII-2, Paragraph 4.7.5.3

Mandatory Appendix 1-6(h) indicates that the equations for the bolted heads with a dished cover are approximate in that they do not consider continuity between the flange ring and the dished head. A more exact method of analysis which takes the continuity of the flange and head into account may be used if it meets the requirements of U-2(g). The alternate design method provided in VIII-2; paragraph 4.7.5.3 satisfies this requirement.

Determine if the proposed Type D spherically dished bolted cover is adequately designed, considering the following design conditions. The spherically dished head is seamless. Evaluate using the alternative procedure in VIII-2, paragraph 4.7.5.3.

Tubeside Data:

- Design Conditions = $213\ psig\ @\ 400°F$
- Corrosion Allowance (CAT) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Shellside Data:

- Design Conditions = $305\ psig\ @\ 250°F$
- Corrosion Allowance (CAS) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Flange Data:

- Material = $SA-105$
- Allowable Stress at Ambient Temperature = $24000\ psi$
- Allowable Stress at Tubeside Design Temperature = $20500\ psi$
- Allowable Stress at Shellside Design Temperature = $21600\ psi$

Head Data:

- Material = $SA-515,\ Grade\ 60$
- Allowable Stress at Ambient Temperature = $21300\ psi$
- Allowable Stress at Tubeside Design Temperature = $18200\ psi$
- Yield Stress at Tubeside Design Temperature = $27300\ psi$
- Modulus of Elasticity at Tubeside Design Temp. = $27.9E+06\ psi$
- Allowable Stress at Shellside Design Temperature = $19200\ psi$
- Yield Stress at Shellside Design Temperature = $28800\ psi$
- Modulus of Elasticity at Shellside Design Temp. = $28.55E+06\ psi$

Bolt Data:

- Material = $SA-193,\ Grade\ B7$
- Diameter = $0.75\ in^2$

- Cross-Sectional Root Area = $0.302\ in^2$
- Number of Bolts = 20
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Allowable Stress at Tubeside Design Temperature = $25000\ psi$
- Allowable Stress at Shellside Design Temperature = $25000\ psi$

Gasket Data:

- Material = Solid Flat Metal (Iron/Soft Steel)
- Gasket Factor = 5.5
- Gasket Seating Factor = $18000\ psi$
- Inside Diameter = $16.1875\ in$
- Outside Diameter = $17.0625\ in$

Commentary:

In accordance with paragraph 4.1.8.1, a combination unit is a pressure vessel that consists of more than one independent or dependent pressure chamber, operating at the same or different pressures and temperatures. The parts separating each pressure chamber are the common elements. Each element, including the common elements, shall be designed for at least the most severe condition of coincident pressure and temperature expected in normal operation. The common elements under consideration in this example are that of the head and flange that make-up the floating head. While this example will separately evaluate tubeside and shellside pressures for each common element, the design temperature of the tubeside will conservatively be applied to both evaluations.

Per paragraph 4.7.5.3, the following procedure can be used to determine the required head and flange thickness of a Type D head. This procedure accounts for the continuity between the flange ring and the head and represents a more accurate method of analysis.

a) STEP 1 – Determine the design pressure and temperature of the flange joint. When evaluating external pressure, a negative value of the pressure is used in all equations of this procedure.

$$Tubeside\ Conditions: P = 213\ psig\ at\ 400°F$$
$$Shellside\ Conditions: P = 305\ psig\ at\ 400°F$$

b) STEP 2 – Determine an initial Type D head configuration geometry (see Figure E4.7.1). The following geometry parameters are required.

1) Flange bore.

$$B = B_{nom} + 2(CAT) = 16.25 + 2(0.125) = 16.50\ in$$

2) Bolt circle diameter.

$$C = 18.125\ in$$

3) Outside diameter of the flange.

$$A = A - 2(CAS) = 19.625 - 2(0.125) = 19.375\ in$$

4) Flange thickness, (see Figure E4.7.1).

$$T = T - 2(CAS) = 2.3125 - 2(0.125) = 2.0625\ in$$

5) Mean head radius, (see Figure 4.7.5).

$$R = \frac{(L + t_{nom} - CAS) + (L + CAT)}{2}$$

$$R = \frac{(16.0 + 0.875 - 0.125) + (16.0 + 0.125)}{2} = 16.4375\ in$$

6) Head thickness.

$$t = t_{nom} - CAT - CAS = 0.875 - 0.125 - 0.125 = 0.625\ in$$

7) Initial inside depth of flange to the base of the head, (see Figure AE4.7.2).

$$q_o = q_{nom} - CAS = 1.0 - 0.125 = 0.875\ in$$

Commentary:

Although the procedure shown in paragraph 4.7.5.3 provides an equation to calculate the lever arm e of shell discontinuity force V about the centroid of the flange ring, the original development of the equation did not lend itself well to the consideration of corrosion allowance for the calculation of the variable q. The procedure shown in Annex E4.7.2 provides one method of determining these adjustments to q; however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

Referencing Annex E4.7.2,

$$q = 0.9470\ in$$

c) STEP 3 – Select a gasket configuration and determine the location of the gasket reaction, G, and the design bolt loads for the gasket seating, W_g, and operating conditions, W_o, using the rules of paragraph 4.16. Computations for the following parameters are shown in Example Problem E4.7.1.

$$G = 16.625\ in$$

$$W_g = 141590.0\ lbs$$

$$W_o = 63419.5\ lbs$$

$$W_{oe} = 0.785G^2 P + 2b\pi GmP$$

$$W_{oe} = 0.785(16.625)^2(-305) + 2(0.1406)\pi(16.625)(5.5)(-305) = -90799.4\ lbs$$

d) STEP 4 – Determine the geometry parameters.

$$h_1 = \frac{(C - G)}{2} = \frac{(18.125 - 16.625)}{2} = 0.75\ in$$

$$h_2 = \frac{(G - B)}{2} = \frac{(16.625 - 16.5)}{2} = 0.0625\ in$$

$$d = \frac{(A-B)}{2} = \frac{(19.375-16.5)}{2} = 1.4375\ in$$

$$n = \frac{T}{t} = \frac{2.0625}{0.625} = 3.3$$

$$K = \frac{A}{B} = \frac{19.375}{16.5} = 1.1742$$

$$\phi = \arcsin\left[\frac{B}{2R}\right] = \arcsin\left[\frac{16.5}{2(16.4375)}\right] = 30.1259\ deg$$

$$e = q - \frac{1}{2}\left[T - \frac{t}{\cos[\phi]}\right] = 0.9470 - \frac{1}{2}\left[2.0625 - \frac{0.625}{\cos[30.1259]}\right] = 0.2770\ in$$

$$k_1 = 1 - \left(\frac{1-2v}{2\lambda}\right)\cot[\phi] = 1 - \left[\frac{1-2(0.3)}{2(6.5920)}\right]\cot[30.1259] = 0.9477$$

$$k_2 = 1 - \left(\frac{1+2v}{2\lambda}\right)\cot[\phi] = 1 - \left[\frac{1+2(0.3)}{2(6.5920)}\right]\cot[30.1259] = 0.7909$$

where,

$$v = 0.3$$

$$\lambda = \left[3\left(1-v^2\right)\left(\frac{R}{t}\right)^2\right]^{0.25} = \left\{3\left(1-0.3^2\right)\left(\frac{16.4375}{0.625}\right)^2\right\}^{0.25} = 6.5920$$

e) STEP 5 – Determine the shell discontinuity geometry factors.

$$C_1 = \frac{0.275n^3t \cdot \ln[K]}{k_1} - e = \left(\frac{0.275(3.3)^3(0.625)\cdot\ln[1.1742]}{0.9477}\right) - (0.2770) = 0.7696$$

$$C_2 = \frac{1.1\lambda n^3 t \ln[K]}{Bk_1} + 1 = \left(\frac{1.1(6.5920)(3.3)^3(0.625)\cdot\ln[1.1742]}{(16.5)(0.9477)}\right) + 1 = 2.6726$$

$$C_4 = \frac{\lambda \sin[\phi]}{2}\left(k_2 + \frac{1}{k_1}\right) + \frac{B}{4nd} + \frac{1.65e}{tk_1}$$

$$C_4 = \left[\begin{array}{l} \dfrac{(6.5920)\sin[30.1259]}{2}\left(0.7907 + \dfrac{1}{0.9477}\right) + \\ \dfrac{16.5}{4(3.3)(1.4375)} + \dfrac{1.65(0.2770)}{(0.625)(0.9477)} \end{array}\right] = 4.6951$$

$$C_5 = \frac{1.65}{tk_1}\left(1 + \frac{4\lambda e}{B}\right) = \left(\frac{1.65}{(0.625)(0.9477)}\right)\left(1 + \frac{4(6.5920)(0.2770)}{(16.5)}\right) = 4.0188$$

f) STEP 6 – Determine the shell discontinuity load factors for the operating and gasket seating conditions.

Operating Condition – Tubeside:

$$C_{3o} = \frac{\pi B^2 P}{4}\left[e\cot[\phi] + \frac{2q(T-q)}{B} - h_2\right] - W_o h_1$$

$$C_{3o} = \left(\frac{\pi(16.5)^2(213)}{4}\right)\left[\begin{array}{l}(0.2770)\cot[30.1259] + \\ \dfrac{2(0.9470)(2.0625-0.9470)}{16.5} - 0.0625\end{array}\right] - 63419.5(0.75)$$

$$C_{3o} = -22838.5 \; in-lbs$$

$$C_{6o} = \frac{\pi B^2 P}{4}\left(\frac{4q - B\cot[\phi]}{4nd} - \frac{0.35}{\sin[\phi]}\right)$$

$$C_{6o} = \frac{\pi(16.5)^2(213)}{4}\left(\frac{4(0.9470)-(16.5)\cot[30.1259]}{4(3.3)(1.4375)} - \frac{0.35}{\sin[30.1259]}\right)$$

$$C_{6o} = -90917.8 \; lbs$$

Operating Condition – Shellside:

$$C_{3o} = \frac{\pi B^2 P}{4}\left[e\cot[\phi] + \frac{2q(T-q)}{B} - h_2\right] - W_o h_1$$

$$C_{3o} = \left(\frac{\pi(16.5)^2(-305)}{4}\right)\left[\begin{array}{l}(0.2770)\cot[30.1259] + \\ \dfrac{2(0.9470)(2.0625-0.9470)}{16.5} - 0.0625\end{array}\right] - (-90799.4)(0.75)$$

$$C_{3o} = 32693.6 \; in-lbs$$

$$C_{6o} = \frac{\pi B^2 P}{4}\left(\frac{4q - B\cot[\phi]}{4nd} - \frac{0.35}{\sin[\phi]}\right)$$

$$C_{6o} = \frac{\pi(16.5)^2(-305)}{4}\left(\frac{4(0.9470)-(16.5)\cot[30.1259]}{4(3.3)(1.4375)} - \frac{0.35}{\sin[30.1259]}\right)$$

$$C_{6o} = 130187.4 \; lbs$$

Gasket Seating Condition:

$$C_{3g} = -W_g h_1 = -(141590.0)(0.75) = -106192.5 \; in-lbs$$

$$C_{6g} = 0.0$$

g) STEP 7 – Determine the shell discontinuity force and moment for the operating and gasket condition.

Operating Condition – Tubeside:

$$V_{do} = \frac{C_2 C_{6o} - C_{3o} C_5}{C_2 C_4 - C_1 C_5}$$

$$V_{do} = \frac{(2.6726(-90917.8)) - (-22838.5(4.0188))}{(2.6726(4.6951)) - (0.7696(4.0188))} = -15991.5 \; lbs$$

$$M_{do} = \frac{C_1 C_{6o} - C_{3o} C_4}{C_2 C_4 - C_1 C_5}$$

$$M_{do} = \frac{(0.7696(-90917.8)) - (-22838.3(4.6951))}{(2.6726(4.6951)) - (0.7696(4.0188))} = 3940.5 \; lbs$$

Operating Condition – Shellside:

$$V_{do} = \frac{C_2 C_{6o} - C_{3o} C_5}{C_2 C_4 - C_1 C_5}$$

$$V_{do} = \frac{(2.6726(130187.4)) - (32693.6(4.0188))}{(2.6726(4.6951)) - (0.7696(4.0188))} = 22902.6 \; lbs$$

$$M_{do} = \frac{C_1 C_{6o} - C_{3o} C_4}{C_2 C_4 - C_1 C_5}$$

$$M_{do} = \frac{(0.7696(130187.4)) - (32693.6(4.6951))}{(2.6726(4.6951)) - (0.7696(4.0188))} = -5637.9 \; lbs$$

Gasket Seating Condition:

$$V_{dg} = \frac{C_2 C_{6g} - C_{3g} C_5}{C_2 C_4 - C_1 C_5} = \frac{(2.6726(0.0)) - (-106192.5(4.0188))}{(2.6726(4.6951)) - (0.7696(4.0188))} = 45135.4 \; lbs$$

$$M_{dg} = \frac{C_1 C_{6g} - C_{3g} C_4}{C_2 C_4 - C_1 C_5} = \frac{(0.7696(0.0)) - (-106192.5(4.6951))}{(2.6726(4.6951)) - (0.7696(4.0188))} = 52730.9 \; lbs$$

h) STEP 8 – Calculate the stresses in the head and at the head to flange junction using Table 4.7.1 and check the stress criteria for both the operating and gasket conditions.

Calculated Stresses – Operating Conditions – Tubeside (omitting shellside pressure):

$$S_{hm} = \frac{PR}{2t} + P_e = \frac{213(16.4375)}{2(0.625)} + 0.0 = 2801.0 \;\; psi$$

$$S_{hl} = \frac{PR}{2t} + \frac{V_{do}\cos[\phi]}{\pi Bt} + P_e$$

$$S_{hl} = \frac{213(16.4375)}{2(0.625)} + \frac{(-15996.5)\cos[30.1259]}{\pi(16.5)(0.625)} + 0.0 = 2373.9 \;\; psi$$

$$S_{hb} = \frac{6M_{do}}{\pi Bt^2} = \frac{6(3940.5)}{\pi(16.5)(0.625)^2} = 1167.6 \ psi$$

$$S_{hlbi} = S_{hl} - S_{hb} = 2373.9 - 1166.6 = 1207.3 \ psi$$

$$S_{hlbo} = S_{hl} + S_{hb} = 2373.9 + 1166.6 = 3540.5 \ psi$$

$$S_{fm} = \frac{1}{\pi BT}\left(\frac{\pi B^2 P}{4}\left(\frac{4q}{B} - \cot[\phi]\right) - V_{do}\right)\left(\frac{K^2+1}{K^2-1}\right) + P_e$$

$$S_{fm} = \left(\frac{1}{\pi(16.5)(2.0625)}\right)\left(\begin{array}{l}\left(\dfrac{\pi(16.5)^2(213)}{4}\right) \\ \left(\dfrac{4(0.9470)}{(16.5)} - \cot[30.1259]\right) - \\ (-15996.5)\end{array}\right)\left(\frac{(1.1742)^2+1}{(1.1742)^2-1}\right) + 0.0$$

$$S_{fm} = -3056.8 \ psi$$

$$S_{fb} = \frac{0.525n}{Btk_1}\left(V_{do} - \frac{4M_{do}\lambda}{B}\right)$$

$$S_{fb} = \frac{0.525(3.3)}{(16.5)(0.625)(0.9477)}\left((-15996.5) - \frac{4(3940.5)(6.5920)}{(16.5)}\right) = -3952.0 \ psi$$

$$S_{fmbi} = S_{fm} + S_{fb} = -3056.8 + (-3952.0) = -7008.8 \ psi$$

$$S_{fmbo} = S_{fm} - S_{fb} = -3056.8 - (-3952.0) = 895.2 \ psi$$

Acceptance Criteria – Operating Conditions – Tubeside:

$\{S_{hm} = 2801.0 \ psi\} \le \{S_{ho} = 18200 \ psi\}$ *True*

$\{S_{hl} = 2373.9 \ psi\} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi\}$ *True*

$\{S_{hlbi} = 1207.3 \ psi\} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi\}$ *True*

$\{S_{hlbo} = 3540.5 \ psi\} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi\}$ *True*

$\{S_{fm} = -3056.8 \ psi\} \le \{S_{fo} = 20500 \ psi\}$ *True*

$\{S_{fmbi} = -7008.8 \ psi\} \le \{1.5S_{fo} = 1.5(20500) = 30750 \ psi\}$ *True*

$\{S_{fmbo} = 895.2 \ psi\} \le \{1.5S_{fo} = 1.5(20500) = 30750 \ psi\}$ *True*

Calculated Stresses – Operating Conditions – Shellside (omitting tubeside pressure):

$$S_{hm} = \frac{PR}{2t} + P_e = \frac{(-305)(16.4375)}{2(0.625)} + (-305) = -4315.8 \ psi$$

$$S_{hl} = \frac{PR}{2t} + \frac{V_{do}\cos[\phi]}{\pi Bt} + P_e$$

$$S_{hl} = \frac{(-305)(16.4375)}{2(0.625)} + \frac{(22902.6)\cos[30.1259]}{\pi(16.5)(0.625)} + (-305) = -3704.3 \ psi$$

$$S_{hb} = \frac{6M_{do}}{\pi Bt^2} = \frac{6(-5637.9)}{\pi(16.5)(0.625)^2} = -1670.6 \ psi$$

$$S_{hlbi} = S_{hl} - S_{hb} = -3704.3 - (-1670.6) = -2033.7 \ psi$$

$$S_{hlbo} = S_{hl} + S_{hb} = -3704.3 + (-1670.6) = -5374.9 \ psi$$

$$S_{fm} = \frac{1}{\pi BT}\left(\frac{\pi B^2 P}{4}\left(\frac{4q}{B} - \cot[\phi]\right) - V_{do}\right)\left(\frac{K^2+1}{K^2-1}\right) + P_e$$

$$S_{fm} = \left(\frac{1}{\pi(16.5)(2.0625)}\right)\left(\begin{array}{l}\left(\dfrac{\pi(16.5)^2(-305)}{4}\right)\\ \left(\dfrac{4(0.9470)}{(16.5)} - \cot[30.1259]\right) - \\ (22902.6)\end{array}\right)\left(\frac{(1.1742)^2+1}{(1.1742)^2-1}\right) + (-305)$$

$$S_{fm} = 4072.3 \ psi$$

$$S_{fb} = \frac{0.525n}{Btk_1}\left(V_{do} - \frac{4M_{do}\lambda}{B}\right)$$

$$S_{fb} = \frac{0.525(3.3)}{(16.5)(0.625)(0.9477)}\left(22902.6 - \frac{4(-5637.9)(6.5920)}{(16.5)}\right) = 5657.1 \ psi$$

$$S_{fmbi} = S_{fm} + S_{fb} = 4072.3 + 5657.1 = 9729.4 \ psi$$

$$S_{fmbo} = S_{fm} - S_{fb} = 4072.3 - 5657.1 = -1584.8 \ psi$$

Acceptance Criteria – Operating Conditions – Shellside:

$$\{S_{hm} = -4315.8 \ psi\} \le \{S_{ho} = 18200 \ psi\} \qquad True$$

$$\{S_{hl} = -3704.3 \ psi\} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi\} \qquad True$$

$$\{S_{hlbi} = -2033.7 \ psi\} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi\} \qquad True$$

$$\{S_{hlbo} = -5374.9 \ psi\} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi\} \qquad True$$

$$\{S_{fm} = 4072.3\ psi\} \le \{S_{fo} = 20500\ psi\} \qquad True$$

$$\{S_{fmbi} = 9729.4\ psi\} \le \{1.5S_{fo} = 1.5(20500) = 30750\ psi\} \qquad True$$

$$\{S_{fmbo} = -1584.8\ psi\} \le \{1.5S_{fo} = 1.5(20500) = 30750\ psi\} \qquad True$$

Calculated Stresses – Gasket Seating Conditions:

$$S_{hm} = 0.0$$

$$S_{hl} = \frac{V_{dg}\cos[\phi]}{\pi Bt} = \frac{(45135.4)\cos[30.1259]}{\pi(16.5)(0.625)} = 1205.0\ psi$$

$$S_{hb} = \frac{6M_{dg}}{\pi Bt^2} = \frac{6(52730.9)}{\pi(16.5)(0.625)^2} = 15625.1\ psi$$

$$S_{hlbi} = S_{hl} - S_{hb} = 1205.0 - 15625.1 = -14420.1\ psi$$

$$S_{hlbo} = S_{hl} + S_{hb} = 1205.0 + 15625.1 = 16830.1\ psi$$

$$S_{fm} = \frac{1}{\pi BT}\left(-V_{dg}\right)\left(\frac{K^2+1}{K^2-1}\right)$$

$$S_{fm} = \left(\frac{1}{\pi(16.5)(2.0625)}\right)\left(-(45135.4)\right)\left(\frac{(1.1742)^2+1}{(1.1742)^2-1}\right) = -2651.5\ psi$$

$$S_{fb} = \frac{0.525n}{Btk_1}\left(V_{dg} - \frac{4M_{dg}\lambda}{B}\right)$$

$$S_{fb} = \frac{0.525(3.3)}{(16.5)(0.625)(0.9477)}\left(45135.4 - \frac{4(52730.9)(6.5920)}{(16.5)}\right) = -6938.7\ psi$$

$$S_{fmbi} = S_{fm} + S_{fb} = -2651.5 + (-6938.7) = -9590.2\ psi$$

$$S_{fmbo} = S_{fm} - S_{fb} = -2651.5 - (-6938.7) = 4287.2\ psi$$

Acceptance Criteria – Gasket Seating Conditions:

$$\{S_{hm} = 0.0\ psi\} \le \{S_{hg} = 21300\ psi\} \qquad True$$

$$\{S_{hl} = 1205.0\ psi\} \le \{1.5S_{hg} = 1.5(21300) = 31950\ psi\} \qquad True$$

$$\{S_{hlbi} = -14420.1\ psi\} \le \{1.5S_{hg} = 1.5(21300) = 31950\ psi\} \qquad True$$

$$\{S_{hlbo} = 16830.1\ psi\} \le \{1.5S_{hg} = 1.5(21300) = 31950\ psi\} \qquad True$$

$$\{S_{fm} = -2651.5\ psi\} \le \{S_{fg} = 24000\ psi\} \qquad True$$

$$\{S_{fmbi} = -9590.2\ psi\} \le \{1.5S_{fg} = 1.5(24000) = 36000\ psi\} \qquad True$$

$$\left\{ S_{fmbo} = 4287.2 \ \ psi \right\} \le \left\{ 1.5 S_{fg} = 1.5\left(24000\right) = 36000 \ \ psi \right\} \qquad True$$

Since the calculated stresses in both the head and flange ring are shown to be within the acceptance criteria, for both internal pressure (tubeside conditions), external pressure (shellside conditions), and gasket seating conditions, the proposed Type D spherically dished bolted cover is adequately designed.

Annex E4.7.2

Although the procedure shown in VIII-2, paragraph 4.7.5.3 provides an equation to calculate the lever arm e of shell discontinuity force V about the centroid of the flange ring, the original development of the equation did not lend itself well to the consideration of corrosion allowance. The variable q is defined as the inside depth of the flange to the base of the head, see VIII-2, Figure 4.7.5, to establish the location of the dished head to flange ring attachment point. However, when the tubeside corrosion allowance is applied to the inside diameter of the dished head and flange ring, the value of q must change accordingly. Additionally, the value of q must be adjusted to account for the shellside corrosion allowance acting on the flange ring. This Annex provides one method of determining these adjustments to q; however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

Assumptions used in the development of the procedure include the following.

1) Shellside and tubeside corrosion allowance is applied to the dished head.
2) Tubeside corrosion allowance is only applied to the flange ring inside radius.
3) Shellside corrosion allowance is applied to the outer surfaces of the flange ring.
4) The geometry of the dished head to flange ring is assumed to be rectilinear.
5) The projected thickness of the dished head and associated dimensions are based off the angle ϕ.
6) The location of the dished head to flange ring attachment point is established from the measured value from the inside depth of the flange ring to the inside base of the head. This variable is referenced as q.
7) The ring flange is rectangular in shape, i.e., the portion of the flange that is removed by machining for the gasket surface is not considered.

The lever arm e measured from the centerline of the projected dished head thickness on the flange ring to the flange ring centroid is determined geometrically considering the above established assumptions. The variable q along with the inside radius and thickness of the dished head, R_{nom} and t_{nom}, respectively, and the flange ring diameter, B set the initial location of the head centerline with the flange ring. The angle ϕ formed by the tangent to the centerline of the dished head at its point of intersection with the flange ring and a line perpendicular to the axis of the dished head is then established. From this point, the dished head and flange ring are subject to the applicable tubeside and shellside corrosion allowances resulting in the final corroded geometry from which the corroded lever arm is determined.

Refer to Figure AE4.7.2.

a) STEP 1 – Establish the variables used in the calculations.

Flange (uncorroded):

$$Thickness: \qquad T_{nom} = 2.3125\ in$$

$$Inside\ Diameter: \qquad B_{nom} = 16.25\ in$$

$$Head\ \ Location: \qquad q_o = 1.0\ in$$

Spherically Dished Head (uncorroded):

$$Thickness: \quad t_{nom} = 0.875 \ in$$
$$Inside\ Radius: \quad L = 16.0 \ in$$

Shellside and Tubeside Corrosion Allowance:

$$Shellside: \quad CAS = 0.125 \ in$$
$$Tubeside: \quad CAT = 0.125 \ in$$

b) STEP 2 – Establish the corroded dimensions of the input variables, as previously defined.

$$t = 0.625 \ in$$
$$R = 16.4375 \ in$$
$$T = 2.0625 \ in$$
$$B = 16.5 \ in$$
$$q_o = 0.875 \ in$$

c) STEP 3 – Calculate the angle ϕ formed by the corroded mean radius of the dished head at the intersection with the flange ring and the corresponding corroded radius of the flange ring, measured to the axis of the dished head assembly.

$$\phi = \arcsin\left[\frac{B}{2R}\right] = \arcsin\left[\frac{16.5}{2(16.4375)}\right] = \begin{Bmatrix} 0.5258 \ rad \\ 30.1259 \ deg \end{Bmatrix}$$

d) STEP 4 – Calculate the projected tubeside corrosion allowance of the dished head on the flange ring, X_{cat}.

$$X_{cat} = \frac{CAT}{\cos[\phi]} = \frac{0.125}{\cos[30.1259]} = 0.1445 \ in$$

e) STEP 5 – Calculate the axial adjustment of X_{adj} due to the applied tubeside corrosion allowance on the flange inside radius.

$$X_{adj} = CAT \cdot \tan[\phi] = 0.125 \cdot \tan[30.1259] = 0.0725 \ in$$

f) STEP 6 – Calculate the adjusted value of q based on the corroded dimensions.

$$q = q_o + X_{cat} - X_{adj} = 0.875 + 0.1445 - 0.0725 = 0.9470 \ in$$

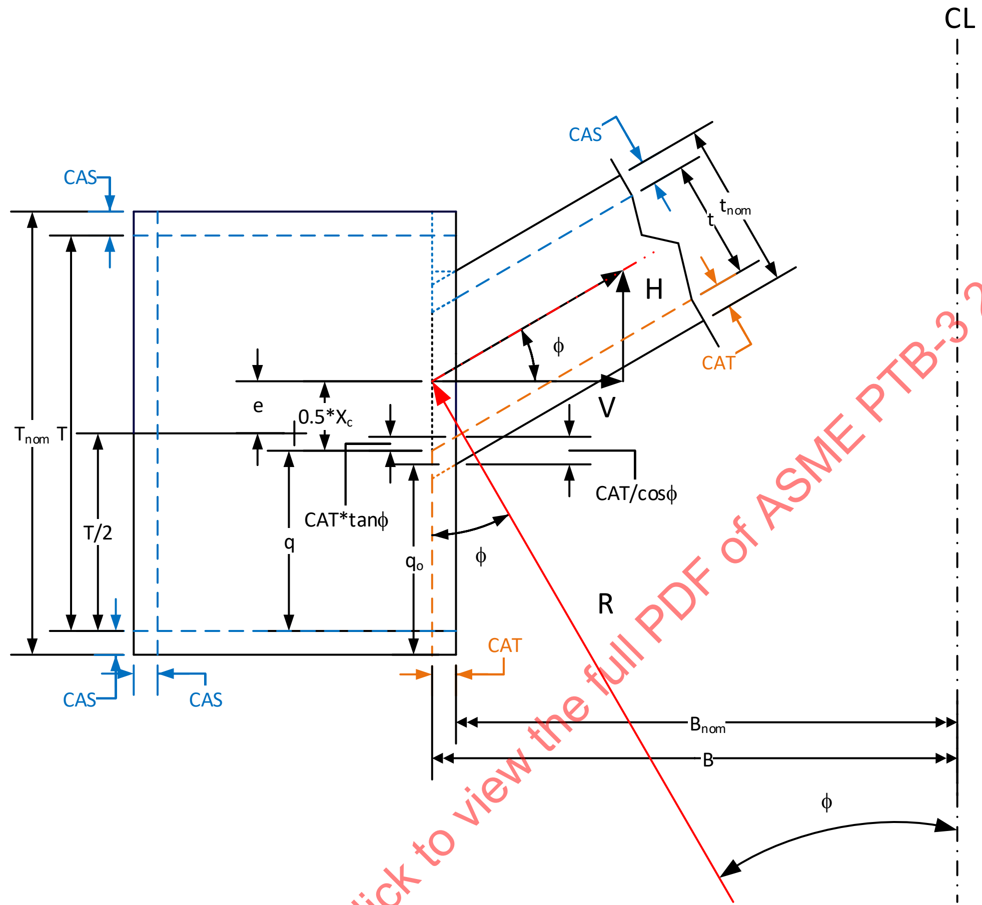

Figure AE4.7.2 – "q" Floating Head Geometry

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4.8 Quick-Actuating (Quick Opening) Closures

4.8.1 Example E4.8.1 – Review of Requirements for Quick-Actuating Closures

An engineer is tasked with developing a design specification for an air filter vessel to be equipped with a quick-actuating closure that is to be constructed in accordance with paragraph 4.8. As part of developing the design specification, the following items need to be considered.

a) The Manufacturer's Design Report shall be certified by a Certifying Engineer in accordance with Annex 2B. See paragraphs 2.3.3.1 and 2.3.3.2. In addition, the competency requirements and qualification requirements as outlined in Annex 2J shall be verified.

b) Scope

Specific calculation methods are not provided in paragraph 4.8. However, both general and specific design requirements are provided.

c) General Design Requirements

Quick-actuating closures shall be designed such that:

1) The locking elements will be engaged prior to or upon application of the pressure and will not disengaged until the pressure is released.

2) The failure of a single locking component while the vessel is pressurized will not:

i) Cause or allow the closure to be opened or leaked; or

ii) Result in the failure of any other locking component or holding element; or

iii) Increase the stress in any other locking component or holding element by more than 50% above the allowable stress of the component.

3) All locking components can be verified to be fully engaged by visual observation or other means prior to application of pressure to the vessel.

4) The use of multilink component, such as a chain, is not permitted.

5) When installed:

i) It may be determined by visual external observation that the holding elements are in satisfactory condition.

ii) All vessels shall be provided with a pressure-indicating device visible from the operating area and suitable for detecting pressure at the closure.

d) Specific Design Requirements

1) Quick-actuating closures that are held in position by positive locking devices and that are fully released by partial rotation or limited movement of the closure itself or the locking mechanism and any closure that is other than manually operated shall be so designed that when the vessel is installed the following conditions are met:

i) The closure and its holding elements are fully engaged in their intended operating position before pressure can be applied in the vessel.

ii) Pressure tending to force the closure open or discharge the vessel contents clear of the vessel shall be released before the closure can be fully opened for access.

iii) In the event compliance with the above conditions is not inherent in the design of the closure and its holding elements, provisions shall be made so that devices to accomplish this can be added when the vessel is installed.

2) The design rules of paragraph 4.16 may not be applicable, see paragraph 4.16.1.4.

3) The designer shall consider the effects of cyclic loading, other loadings (see paragraph 4.1.5.3) and mechanical wear on the holding and locking components.

4) Any device or devices that will provide the safeguards broadly described above within these specific design requirements will meet the intent of these rules.

e) Alternative Designs for Manually Operated Closures

1) Quick-actuating closures that are held in position by a locking mechanism designed for manual operation shall be designed such that if an attempt is made to open the closure when the vessel is under pressure, the closure will leak prior to full disengagement of the locking components and release of the closure. Any leakage shall be directed away from the normal position of the operator.

2) Manually operated closures need not satisfy specific design requirements found in (c)(1) above, but such closures shall be equipped with an audible or visible warning device that will warn the operator if pressure is applied to the vessel before the holding elements and locking components are fully engaged in their intended position or if an attempt is made to disengage the locking mechanism before the pressure within the vessel is released.

f) Supplementary Requirements for Quick-Actuating (Quick-Opening) Closures

Annex 4B provides additional design information for the Manufacturer and provides installation, operational, and maintenance requirements for the Owner.

4.9 Braced and Stayed Surfaces

4.9.1 Example E4.9.1 – Braced and Stayed Surfaces

Determine the required thickness for a flat plate with welded staybolts considering the following design condition. Verify that the welded staybolts are adequately designed. See Figure E4.9.1.

Vessel Data:

- Plate Material = $SA-516,\ Grade\ 70$
- Design Conditions = $100\ psig@300°F$
- Staybolt Material = $SA-675,\ Grade\ 70$
- Staybolt Diameter = $1.5\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress Plate Material = $22400\ psi$
- Allowable Stress Staybolt Material = $20600\ psi$
- Staybolt Pattern = $Equilateral\ Triangle$
- Staybolt Pitch = $p_s = p_{horizontal} = p_{diagonal} = 15.0\ in$

Using the procedure in paragraph 4.9, calculate the required thickness of the flat plate, the load carried by each staybolt, and the required diameter of the staybolt.

Paragraph 4.9.2, the minimum required thickness for braced and stayed flat plates and those parts that, by these rules, require staying as flat plates or staybolts of uniform diameter symmetrically spaced, shall be calculated by the following equation.

Assume, $C = 2.2$ from Table 4.9.1 with the Welded Staybolt Construction per Figure 4.9.1 Detail (c).

$$t = p_s\sqrt{\frac{P}{SC}} = 15.0\sqrt{\frac{100.0}{22400(2.2)}} = 0.6757\ in$$

Paragraph 4.9.3, the required area of a staybolt or stay as its minimum cross section, usually located at the root of the thread, exclusive of any corrosion allowance, shall be obtained by dividing the load on the staybolt computed in accordance with paragraph 4.9.3.2 by the allowable tensile stress value for the staybolt material, multiplying the result by 1.10.

The area supported by a staybolt or stay shall be computed based on the full pitch dimensions, with a deduction for the area occupied by the stay. The load carried by a stay is the product of the area supported by the stay and the maximum allowable working pressure.

a) The area of the flat plate supported by the staybolt, A_p, is calculated as follows.

$$A_p = \left(p_{horizontal} \cdot p_{diagonal} \cdot \cos[\theta]\right) - A_{sb} = 15.0\left(15.0 \cdot \cos[30]\right) - 1.7671 = 193.0886\ in^2$$

Where,

$$\theta = 30\ deg, \quad See\ Figure\ E4.9.1$$

$$A_{sb} = \frac{\pi}{4}(1.5)^2 = 1.7671\ in^2$$

a) The load carried by the staybolt, L_{sb}, is calculated as follows.

$$L_{sb} = A_p \cdot P = 193.0886(100.0) = 19308.9\ lbs$$

b) The required area of the staybolt, A_{rsb}, is calculated as follows.

$$A_{rsb} = 1.10\left(\frac{L_{sb}}{S_{sb}}\right) = 1.10\left(\frac{19308.9}{20000}\right) = 1.0620\ in^2$$

Since $\{A_{sb} = 1.7671\ in^2\} \le \{A_{rsb} = 1.0311\ in^2\}$ the staybolt is adequately designed.

Paragraph 4.9.4.1, welded-in staybolts may be used provided the following requirements are satisfied.

c) The configuration is in accordance with the typical arrangements shown in Figure 4.9.1.

$$Construction\ per\ Figure\ 4.9.1(c) \qquad satisfied$$

d) The required thickness of the plate shall not exceed 38 mm (1.5 in).

$$t \le 1.5\ in \qquad t = 0.6757\ in \qquad satisfied$$

e) The maximum pitch shall not exceed 15 times the diameter of the staybolt.

$$p_s \le 15(d_{sb}) \quad 15.0 \le 15(1.5) = 22.5\ in \qquad satisfied$$

f) The size of the attachment welds is not less than that shown in Figure 4.9.1.

$$Full\ Penetration\ Weld\ \ per\ Figure\ 4.9.1(c) \qquad satisfied$$

g) The allowable load on the welds shall not exceed the product of the weld area (based on the weld dimension parallel to the staybolt), the allowable tensile stress of the material being welded, and a weld joint factor of 60%.

$$\{L_{sb} = 19308.9\ lbs\} \le \{L_a = 39356.2\ lbs\} \qquad satisfied$$

Where,

$$L_a = E(t \cdot \pi d_{sb})S_{sb} = 0.6\left(0.6757 \cdot \left(\pi(1.5)\right)\right)20600 = 39356.2\ lbs$$

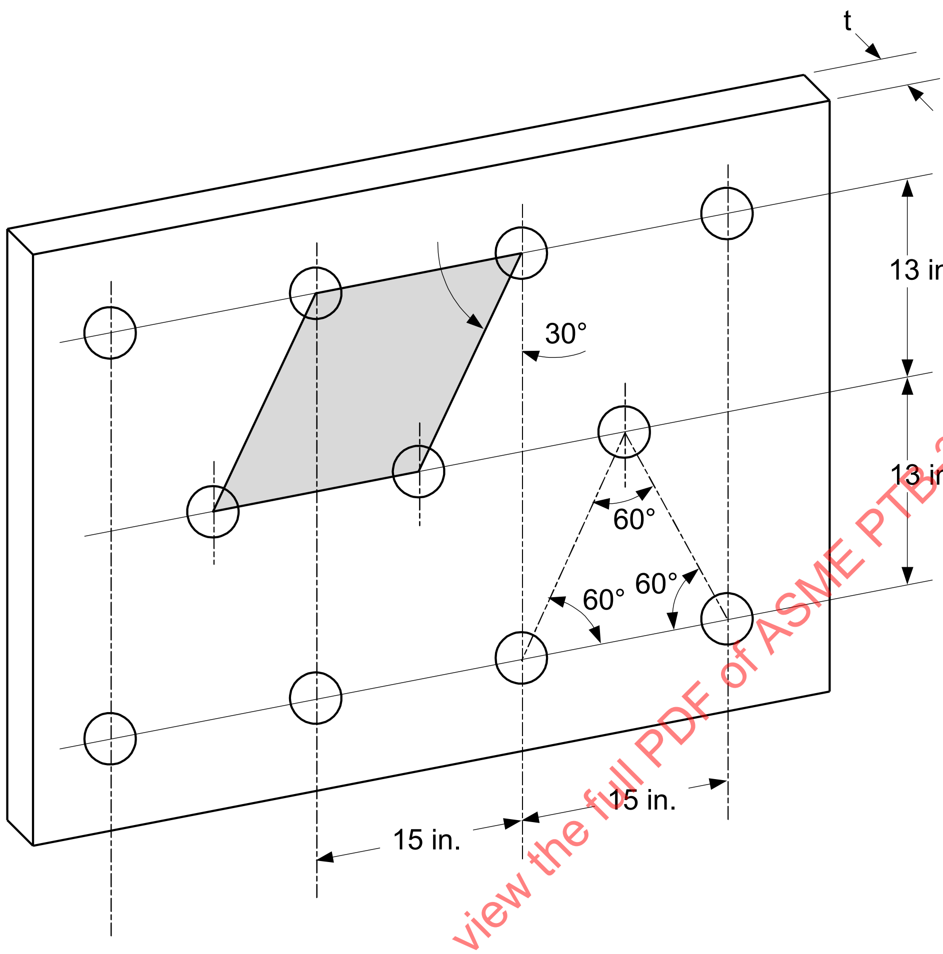

Figure E4.9.1 – Stayed Plate Detail

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4.10 Ligaments

4.10.1 Example E4.10.1 – Ligaments

Determine the ligament efficiency and corresponding efficiency to be used in the design equations of paragraph 4.3 for a group of tube holes in a cylindrical shell as shown in Figure E4.10.1.

Using the procedure in paragraph 4.10, calculate the ligament efficiency for the group of tube holes. As shown in Figure E4.10.1, three ligaments are produced; longitudinal, circumferential, and diagonal.

Paragraph 4.10.2.1.d, when a cylindrical shell is drilled for holes so as to form diagonal ligaments, as shown in Figure E4.10.1, the efficiency of these ligaments shall be determined by paragraph 4.10, Figures 4.10.5 or 4.10.6. Figure 4.10.5 is used when either or both longitudinal and circumferential ligaments exist with diagonal ligaments. The procedure to determine the ligament efficiency is provided in paragraph 4.10.2.1(d)(1).

a) STEP 1 – Compute the value of p^*/p_1.

$$Diagonal\ Pitch,\ p^* = 3.75\ in$$

$$Unit\ Length\ of\ Ligament,\ p_1 = 4.5\ in$$

$$\frac{p^*}{p_1} = \frac{3.75}{4.5} = 0.8333$$

b) STEP 2 – Compute the efficiency of the longitudinal ligament in accordance with Figure 4.10.5, Note (d).

$$E_{long} = 100\left(\frac{p_1 - d}{p_1}\right) = 100\left(\frac{4.5 - 2.25}{4.5}\right) = 50\%$$

where,

$$Diameter\ of\ Tube\ Holes,\ d = 2.25\ in$$

c) STEP 3 – Compute the diagonal efficiency in accordance with Figure 4.10.5, Note (b).

$$E_{diag} = \frac{J + 0.25 - \left(1 - 0.01 \cdot E_{long}\right)\sqrt{0.75 + J}}{0.00375 + 0.005J}$$

$$E_{diag} = \frac{0.6944 + 0.25 - \left(1 - 0.01(50)\right)\sqrt{(0.75 + 0.6944)}}{0.00375 + 0.005(0.6944)} = 47.56\%$$

where,

$$J = \left(\frac{p'}{p_1}\right)^2 = \left(\frac{3.75}{4.5}\right)^2 = 0.6944$$

Alternatively, STEP 3 can be replaced with the following procedure.

STEP 3 (Alternate) – Enter Figure 4.10.5 at the vertical line corresponding to the value of the longitudinal efficiency, E_{long}, and follow this line vertically to the point where it intersects the diagonal line representing the ratio of the value of p^*/p_1. Then, project this point horizontally to the left, and read the diagonal efficiency of the ligament on the scale at the edge of the diagram.

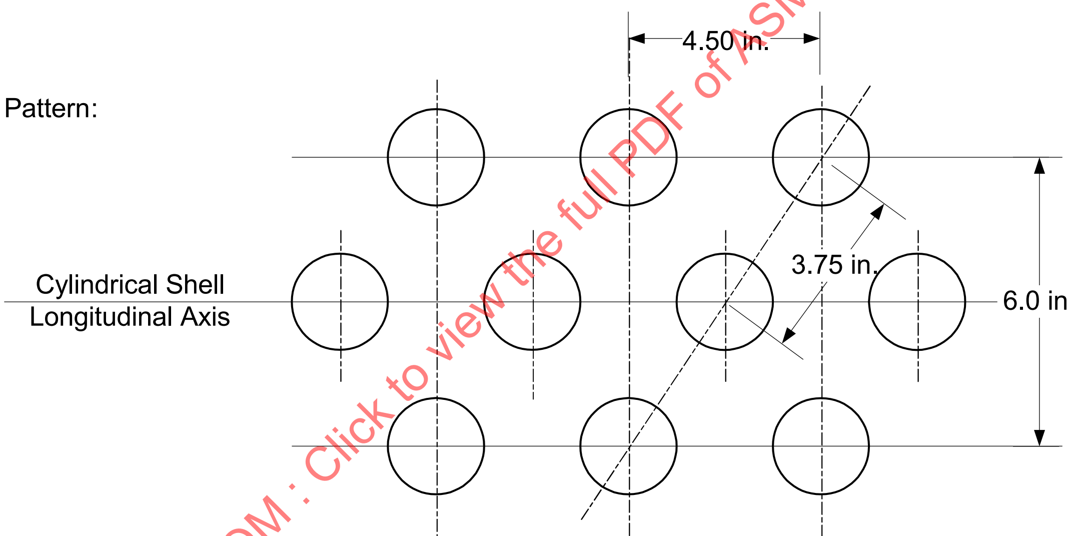
$$E_{diag} \approx 47.5\%$$

d) STEP 4 – The minimum shell thickness and the maximum allowable working pressure shall be based on the ligament that has the lower efficiency.

$$E = \min\left[E_{long}, E_{diag}\right] = \min\left[50\%, 47.5\%\right] = 47.5\%$$

In accordance with paragraph 4.10.3, when ligaments occur in cylindrical shells made from welded pipe or tubes and their calculated efficiency is less than 85% (longitudinal) or 50% (circumferential), the efficiency to be used in paragraph 4.3 to determine the minimum required thickness is the calculated ligament efficiency. In this case, the appropriate stress value in tension may be multiplied by the factor 1.18.

- Installation Pattern:



- All Finished Hole Diameters are 2.25 in.

Figure E4.10.1 – Installation Pattern

4.11 Jacketed Vessels

4.11.1 Example E4.11.1 – Jacketed Vessel

Design a jacketed vessel to be installed on the outside diameter of a section of a tower in accordance with Figure 4.11.1, Type 1. The jacket closure will be made using closure members per Table 4.11.1, Detail 4, Figure (b).

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $350\ psig@300°F$
- Vessel ID = $90.0\ in$
- Nominal Thickness = $1.125\ in$
- Allowable Stress = $22400\ psi$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0

Jacket Data:

- Jacket Type = $Figure\ 4.11.1,\ Type\ 1$
- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $150\ psig@400°F$
- Allowable Stress = $21600\ psi$
- Yield Stress at Design Temperature = $32500\ psi$
- Minimum Ultimate Tensile Strength = $70000\ psi$
- Jacket ID = $96.0\ in$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0

Establish the corroded dimensions.

$$t_s = \begin{Bmatrix} t_s - Vessel\ Corrosion\ Allowance - Jacket\ Corrosion\ Allowance \\ 1.125 - 0.125 - 0.125 = 0.8750\ in \end{Bmatrix}$$

$$OD\ of\ Inner\ Shell = \begin{Bmatrix} Vessel\ ID + 2\left(t_s - Jacket\ Corrosion\ Allowance\right) \\ 90 + 2\left(1.125 - 0.125\right) = 92.0\ in \end{Bmatrix}$$

$$R_s = \frac{OD\ of\ Inner\ Shell}{2} = 46.0\ in$$

$$t_j = t_j - Jacket\ Corrosion\ Allowance = 0.5 - 0.125 = 0.375\ in$$

$$R_j = R_j + Jacket\ Corrosion\ Allowance = 48.0 + 0.125 = 48.125\ in$$

$$ID\ of\ Jacket = 2\left(48.125\right) = 96.25\ in$$

Evaluate the partial jacket per Paragraph 4.11:

a) Paragraph 4.11.2.1, determine required thickness of the jacket using equation (4.3.1).

$$t_{rj} = R_j\left(\exp\left[\frac{P_j}{S_j E}\right]-1\right) = 48.125\left(\exp\left[\frac{150}{21600(1.0)}\right]-1\right) = 0.3354\ in$$

$$t_{rj} = t_{rj} + Jacket\ \ Corrosion\ \ Allowance = 0.3354 + 0.125 = 0.4604\ in$$

Use jacket plates with a wall thickness of $t_j = 0500\ in.$

b) Paragraph 4.11.3: Design of Closure Member of Jacket to Vessel

1) Paragraph 4.11.3.1, the design of jacket closure members shall be in accordance with Table 4.11.1 and the additional requirements of paragraph 4.11.3.

2) Paragraph 4.11.3.2, radial welds in closure members shall be butt-welded joints through the full thickness of the member.

3) Paragraph 4.11.3.3, partial penetration and fillet welds are permitted when both of the following requirements are satisfied.

i) The material of construction satisfies the following equation,

$$\left\{\frac{S_{yT}}{S_u} = \frac{32500}{70000} = 0.464\right\} \le 0.625 \qquad\qquad True$$

ii) The component is not in cyclic service.

c) Determine thickness of jacket closures. Use closure detail in Table 4.11.1, Detail 4, Figure (b).

$$t_{rc} = \max\left[2t_{rj}, 0.707\sqrt{\frac{P_j}{S_c}}\right]$$

$$t_{rc} = \max\left[\left\{2(0.3354) = 0.6708\right\}, \left\{0.707\sqrt{\frac{150}{21600}} = 0.0589\right\}\right] = 0.6708\ in$$

$$t_{rc} = t_{rc} + Jacket\ \ Corrosion\ \ Allowance = 0.6708 + 0.125 = 0.7958\ in$$

Use end closure plates with a wall thickness of $t_c = 0.8125\ in.$

d) Determine the fillet weld sizes for the closure to shell weld per Table 4.11.1, Detail 4, Figure (b).

$$t_s = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

$$t_c = 0.8125 - Corrosion\ \ Allowance = 0.8125 - 0.125 = 0.6875\ in$$

$$t_j = 0.500 - Corrosion\ \ Allowance = 0.500 - 0.125 = 0.375\ in$$

Closure to shell weld:

$$Y \ge \min\left[0.75t_c,\ 0.75t_s\right] = \min\left[\left\{0.75(0.6875) = 0.5156\right\},\ \left\{0.75(1.0) = 0.75\right\}\right] = 0.75\ in$$

Jacket to closure weld:

$$Z \ge \left\{t_j = 0.375\ in\right\}$$

4.11.2 Example E4.11.2 – Half-Pipe Jacket

Design a half-pipe jacket for a section of a tower in accordance with Paragraph 4.11.6, using the information shown below.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $350\ psig@300°F$
- Vessel ID = $90.0\ in$
- Nominal Thickness = $1.125\ in$
- Allowable Stress = $22400\ psi$
- Corrosion Allowance = $0.125\ in$
- Applied Axial Force = $-78104.2\ lbs$
- Applied Net Section Bending Moment = $4.301E+06\ in-lbs$

Half-Pipe Jacket Data:

- Material = $SA-106,\ Grade\ B$
- Design Conditions = $150\ psig@400°F$
- Jacket ID = $NPS\ 4\ (STD\ WT) \rightarrow 0.237\ in$
- Allowable Stress = $19900\ psi$
- Yield Stress at Design Temperature = $29900\ psi$
- Minimum Ultimate Tensile Strength = $60000\ psi$
- Weld Joint Efficiency = 1.0
- Corrosion Allowance = $0.0\ in$

Establish the corroded dimensions.

Vessel:

$$D_0 = 90.0 + 2t_s = 90.0 + 2(1.125) = 92.25\ in$$
$$t_s = t_s - Corrosion\ Allowance = 1.125 - 0.125 = 1.0\ in$$
$$D = D + 2(Corrosion\ \ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

Half-Pipe Jacket:

$$d_j = 4.5 - 2t_j = 4.5 - 2(0.237) = 4.026\ in$$
$$r_j = \frac{d_j}{2} = \frac{4.026}{2} = 2.013\ in$$

Evaluate half-pipe jacket per paragraph 4.11.6.

a) Paragraph 4.11.6.1, the rules in this section are applicable for the design of half-pipe jackets constructed of NPS 2, NPS 3, or NPS 4 pipes and subject to internal pressure loading.

$$Half-Pipe\ Jacket \rightarrow NPS\ 4 \qquad True$$

b) Paragraph 4.11.6.2:

1) The fillet weld attaching the half-pipe jacket to the vessel shall have a throat thickness not less than the smaller of the jacket or shell thickness.

$$throat_f = 0.707 \cdot leg = 0.707(0.375) = 0.265\ in$$

$$\left\{throat_f = 0.265\ in\right\} \geq \left\{\min\left[0.237,\ (1.125-0.125)\right] in = 0.237\ in\right\} \qquad True$$

2) The requirements of paragraph 4.11.3.3 shall be satisfied. Paragraph 4.11.3.3, partial penetration and fillet welds are permitted when both of the following requirements are satisfied.

i) The material of construction satisfies the following equation, $SA-106, Grade\ B$.

$$\left\{\frac{S_{yT}}{S_u} = \frac{29900}{60000} = 0.498\right\} \leq 0.625 \qquad True$$

ii) The component is not in cyclic service.

c) Paragraph 4.11.6.3:

1) Calculate the minimum required thickness for the $NPS\ 4\ STD\ WT$ half-pipe jacket.

$$t_{rp} = \frac{P_1 r_j}{0.85S_j - 0.6P_j} = \frac{150(2.0130)}{0.85(19900) - 0.6(150)} = 0.0179\ in$$

Since $\{t_j = 0.237\ in\} \geq \{t_{rj} = 0.0179\ in\}$, the thickness of $NPS\ 4\ STD\ WT$ pipe is acceptable.

2) The additional condition must be satisfied, as shown in paragraph 4.11.6.4, where $P_j \leq P_{jpm}$.

d) Paragraph 4.11.6.4, the maximum permissible pressure in the half-pipe jacket, P_{jpm}, shall be determined using the following equation.

$$P_{jpm} = \frac{F_p}{K_p}$$

where,

$$F_p = \min\left[\left(1.5S - S^*\right),\ 1.5S\right]$$

$$K_p = C_1 + C_2\frac{D^{0.5}}{C_{ul}} + C_3\frac{D}{C_{ul}} + C_4\frac{D^{1.5}}{C_{ul}} + C_5\frac{D^2}{C_{ul}} + C_6\frac{D^{2.5}}{C_{ul}} + C_7\frac{D^3}{C_{ul}} + C_8\frac{D^{3.5}}{C_{ul}} + C_9\frac{D^4}{C_{ul}} + C_{10}\frac{D^{4.5}}{C_{ul}}$$

$$C_{ul} = 1.0 \rightarrow US\ Customary\ Units$$

The coefficients C_1, C_2, C_n are provided in Table 4.11.3.

To compute P_{jpm}, the parameter S^*, defined as the actual longitudinal stress in the shell, must be computed. This stress may be computed using the equations in paragraph 4.3.10.2. However, since this is a thin shell, the thin-wall equations for a cylindrical shell will be used. If the combination of axial forces and pressure result in a negative value of S^*, then $S^* = 0$.

$$S^* = \ Pressure\ Stress + Axial\ Stress \pm Bending\ Stress$$

$$S^* = \frac{PD}{4t_s} + \frac{F}{A} \pm \frac{Mc}{I}$$

$$S^* = \begin{cases} \dfrac{350(90.25)}{4(1.0)} + \dfrac{-78104.2}{286.6703} + \dfrac{4.301E+06\left(\dfrac{92.25}{2}\right)}{298408.1359} = 8289.2283\ psi \\ \dfrac{350(90.25)}{4(1.0)} + \dfrac{-78104.2}{286.6703} - \dfrac{4.301E+06\left(\dfrac{92.25}{2}\right)}{298408.1359} = 6959.6156\ psi \end{cases}$$

where,

$$I = \frac{\pi}{64}\left(D_o^4 - D^4\right) = \frac{\pi}{64}\left((92.25)^4 - (90.25)^4\right) = 298408.1359\ in^4$$

$$A = \frac{\pi}{4}\left(D_o^2 - D^2\right) = \frac{\pi}{4}\left((92.25)^2 - (90.25)^2\right) = 286.6703\ in^2$$

therefore,

$$F_p = \min\left[\left(1.5S - S^*\right),\ 1.5S\right]$$

$$F_p = \min\left[\left(1.5(22400) - 8289.2283\right), 1.5(22400)\right] = 25310.7717\ psi$$

The coefficients for the equation K_p are obtained from, Table 4.11.3 for NPS 4 and shell nominal thickness of 1.0 in.

$$C_1 = -2.5016604E+02 \qquad C_2 = 1.7178270E+02 \qquad C_3 = -4.6844914E+01$$

$$C_4 = 6.6874346E+00 \qquad C_5 = -5.2507555E-01 \qquad C_6 = 2.1526948E-02$$

$$C_7 = -3.6091550E-04 \qquad C_8 = C_9 = C_{10} = 0.0$$

with a vessel diameter, $D = 90.25\ in$, the value of K_p is calculated as,

$$K_p = C_1 + C_2\frac{D^{0.5}}{C_{ul}} + C_3\frac{D}{C_{ul}} + C_4\frac{D^{1.5}}{C_{ul}} + C_5\frac{D^2}{C_{ul}} + C_6\frac{D^{2.5}}{C_{ul}} + C_7\frac{D^3}{C_{ul}} + C_8\frac{D^{3.5}}{C_{ul}} + C_9\frac{D^4}{C_{ul}} + C_{10}\frac{D^{4.5}}{C_{ul}}$$

$$K_p = 11.2903$$

Therefore, the maximum permissible pressure in the half-pipe is calculated as,

$$\left\{P_{jpm} = \frac{F_p}{K_p} = \frac{25310.7717}{11.2903} = 2241.8\ psi\right\} \geq \left\{P_j = 150\ psi\right\}$$

Therefore, the half-pipe jacket design is acceptable.

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4.12 NonCircular Vessels

4.12.1 Example E4.12.1 – Unreinforced Vessel of Rectangular Cross Section, Type 1

Using the data shown below, design a Type 1 non-circular pressure vessel per paragraph 4.12.7.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $400\ psig@500°F$
- Inside Length (Short Side) = $7.125\ in$
- Inside Length (Long Side) = $9.25\ in$
- Overall Vessel Length = $40.0\ in$
- Thickness (Short Side) = $1.0\ in$
- Thickness (Long Side) = $1.0\ in$
- Thickness (End Plate) = $0.75\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20600\ psi$
- Weld Joint Efficiency (Corner Joint) = 1.0
- Tube Outside Diameter = $1.0000\ in$
- Tube Pitch = $2.3910\ in$

Adjust variables for corrosion.

$$h = 9.25 + 2(Corrosion\ \ Allowance) = 9.25 + 2(0.125) = 9.50\ in$$
$$H = 7.125 + 2(Corrosion\ \ Allowance) = 7.125 + 2(0.125) = 7.375\ in$$
$$t_1 = 1.0 - Corrosion\ \ Allowance = 1.0 - 0.125 = 0.875\ in$$
$$t_2 = 1.0 - Corrosion\ \ Allowance = 1.0 - 0.125 = 0.875\ in$$
$$t_5 = 0.75 - Corrosion\ \ Allowance = 0.75 - 0.125 = 0.625\ in$$

Evaluate per paragraph 4.12.

Paragraph 4.12.2, General Design Requirements:

Paragraph 4.12.2.7 – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. These equations are conservatively applicable to vessels of aspect ratio less than four. Vessels with aspect ratios less than four may be designed in accordance with VIII-2, paragraph 4.12.5 and may be designed in accordance with the provisions of Part 5.

$$Aspect\ \ Ratio = \frac{L_v}{h} = \frac{40.0}{9.5} = 4.21 \qquad\qquad Satisfied$$

Paragraph 4.12.2.9 – The openings in this noncircular vessel meet the requirements of paragraph 4.5.2.

Paragraphs 4.12.3, 4.12.4 and 4.12.5 – These paragraphs are not applicable to this design.

Paragraph 4.12.6, Weld Joint Factors and Ligament Efficiency:

Paragraph 4.12.6.1 – The non-circular vessel is constructed with corner joints typical of paragraph 4.2. Therefore, the weld joint efficiencies E_m and E_b are set to 1.0 at stress calculation locations in the corners of the vessel. Since there are no welds or hole pattern in the short side plates of the vessel, the weld joint efficiencies E_m and E_b are set to 1.0 for these stress calculation locations. For the stress calculation locations on the long side plates that do not contain welded joints, but do contain a hole pattern, the weld joint efficiencies E_m and E_b are set equal to the ligament efficiencies e_m and e_b, respectively.

Paragraph 4.12.6.3 – It is assumed that the holes drilled in the long side plates (tube sheet and plug sheet) are of uniform diameter. Therefore, e_m and e_b shall be the same value and calculated in accordance with paragraph 4.10.

$$e_m = e_b = \frac{p-d}{p} = \frac{2.3910-1.0}{2.3910} = 0.5818$$

Paragraph 4.12.7, Design Procedure:

a) STEP 1 – The design pressure and temperature are listed in the information given above.

b) STEP 2 – The vessel to be designed is a Type 1 vessel, see Figure 4.12.1.

c) STEP 3 – The vessel configuration and wall thicknesses of the pressure containing plates are listed in the information given above.

d) STEP 4 – Determine the location of the neutral axis from the inside and outside surfaces. Since the section under evaluation does not have stiffeners, but has uniform diameter holes, then $c_i = c_o = t/2$ where t is the thickness of the plate.

$$c_i = c_o = \frac{t_1}{2} = \frac{t_2}{2} = \frac{0.875}{2} = 0.4375\ in$$

e) STEP 5 – Determine the weld joint factor and ligaments efficiencies as applicable, see paragraph 4.12.6, and determine the factors E_m and E_b.

f) STEP 6 – Complete the stress calculation for the selected noncircular vessel Type, see Table 4.12.1, and check the acceptance criteria.

For non-circular vessel Type 1, the applicable table for stress calculations is Table 4.12.2 and the corresponding details are shown in Figure 4.12.1

<u>Equation Constants:</u>

$$b = 1.0\ in$$

$$J_{2s} = 1.0$$
$$J_{3s} = 1.0$$

$$J_{2l} = 1.0$$
$$J_{3l} = 1.0$$

$$I_1 = \frac{bt_1^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558\ in^4$$

$$I_2 = \frac{bt_2^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558\ in^4$$

$$\alpha = \frac{H}{h} = \frac{7.375}{9.5} = 0.7763$$

$$K = \frac{I_2}{I_1}\alpha = \left(\frac{0.0558}{0.0558}\right)0.7763 = 0.7763$$

Nomenclature for Stress Results:

S_m^s membrane stress, short side.

S_{bi}^{sC}, S_{bo}^{sC} bending stress, short side at point C on the inside and outside surfaces, respectively.

S_{bi}^{sB}, S_{bo}^{sB} bending stress, short side at point B on the inside and outside surfaces, respectively.

S_m^l membrane stress, long side.

S_{bi}^{lA}, S_{bo}^{lA} bending stress, long side at point A on the inside and outside surfaces, respectively.

S_{bi}^{lB}, S_{bo}^{lB} bending stress, long side at point B on the inside and outside surfaces, respectively.

Membrane and Bending Stresses – Critical Locations of Maximum Stress:

Membrane Stress on the short side plate:

$$S_m^s = \frac{Ph}{2(t_1)E_m} = \frac{400(9.5)}{2(0.875)(1.0)} = 2171.4\ psi$$

Bending Stress at Location C on the short side plate:

$$S_{bi}^{sC} = -S_{bo}^{sC}\left(\frac{c_i}{c_o}\right) = \frac{PbJ_{2s}c_i}{12I_1E_b}\left[-1.5H^2 + h^2\left(\frac{1+\alpha^2 K}{1+K}\right)\right]$$

$$S_{bi}^{sC} = \frac{400(1.0)(1.0)(0.4375)}{12(0.0558)(1.0)}\left[-1.5(7.375)^2 + (9.5)^2\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right] = -1831.7\ psi$$

$$S_{bo}^{sC} = -S_{bi}^{sC}\left(\frac{c_o}{c_i}\right) = -(-1831.7)\left(\frac{0.4375}{0.4375}\right) = 1831.7\ psi$$

Bending Stress at Location B on the short side plate:

$$S_{bi}^{sB} = -S_{bo}^{sB}\left(\frac{c_i}{c_o}\right) = \frac{Pbh^2J_{3s}c_i}{12I_1E_b}\left[\frac{1+\alpha^2 K}{1+K}\right]$$

$$S_{bi}^{sB} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(0.0558)(1.0)}\left[\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right] = 19490.8\ psi$$

$$S_{bo}^{sB} = -S_{bi}^{sB}\left(\frac{c_o}{c_i}\right) = -19490.8\left(\frac{0.4375}{0.4375}\right) = -19490.8\ psi$$

Membrane Stress on the long side plate:

$$S_m^l = \frac{PH}{2t_2E_m} = \frac{400(7.375)}{2(0.875)(0.5818)} = 2897.4 \ psi$$

Bending Stress at Location A on the long side plate:

$$S_{bi}^{lA} = -S_{bo}^{lA}\left(\frac{c_i}{c_o}\right) = \frac{Pbh^2J_{2l}c_i}{12I_2E_b}\left[-1.5+\left(\frac{1+\alpha^2K}{1+K}\right)\right]$$

$$S_{bi}^{lA} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(1.0)(0.0558)(0.5818)}\left[-1.5+\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right] = -27310.9 \ psi$$

$$S_{bo}^{lA} = -S_{bi}^{lA}\left(\frac{c_o}{c_i}\right) = -(-27310.9)\left(\frac{0.4375}{0.4375}\right) = 27310.9 \ psi$$

Bending Stress at Location B on the long side plate:

$$S_{bi}^{lB} = -S_{bo}^{lB}\left(\frac{c_i}{c_o}\right) = \frac{Pbh^2J_{3l}c_i}{12I_2E_b}\left[\frac{1+\alpha^2K}{1+K}\right]$$

$$S_{bi}^{lB} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(0.0558)(1.0)}\left[\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right] = 19490.8 \ psi$$

$$S_{bo}^{lB} = -S_{bi}^{lB}\left(\frac{c_o}{c_i}\right) = -19490.8\left(\frac{0.4375}{0.4375}\right) = -19490.8 \ psi$$

Acceptance Criteria – Critical Locations of Maximum Stress:

$$\left\{S_m^s = 2171.4 \ psi\right\} \le \left\{S = 20600 \ psi\right\} \qquad True$$

$$\begin{Bmatrix} S_m^s + S_{bi}^{sC} = 2171.4 + (-1831.7) = 339.7 \ psi \\ S_m^s + S_{bo}^{sC} = 2171.4 + 1831.7 = 4003.1 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m^s + S_{bi}^{sC} = 339.7 \ psi \\ S_m^s + S_{bo}^{sC} = 4003.1 \ psi \end{Bmatrix} \le \left\{1.5S = 1.5(20600) = 30900 \ psi\right\} \qquad True$$

$$\begin{Bmatrix} S_m^s + S_{bi}^{sB} = 2171.4 + 19490.8 = 21662.2 \ psi \\ S_m^s + S_{bo}^{sB} = 2171.4 + (-19490.8) = -17319.4 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m^s + S_{bi}^{sB} = 21662.2 \ psi \\ S_m^s + S_{bo}^{sB} = -17319.4 \ psi \end{Bmatrix} \le \left\{1.5S = 1.5(20600) = 30900 \ psi\right\} \qquad True$$

$$\left\{S_m^l = 2897.4 \ psi\right\} \le \left\{S = 20600 \ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lA} = 2897.4 + (-27310.9) = -24413.5 \ psi \\ S_m^l + S_{bo}^{lA} = 2897.4 + 27310.9 = 30208.3 \ psi \end{array}\right\}$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lA} = -24413.5 \ psi \\ S_m^l + S_{bo}^{lA} = 30208.3 \ psi \end{array}\right\} \le \left\{1.5S = 1.5(20600) = 30900 \ psi\right\} \qquad \{True\}$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lB} = 2897.4 + 19490.8 = 22388.2 \ psi \\ S_m^l + S_{bo}^{lB} = 2897.4 + (-19490.8) = -16593.4 \ psi \end{array}\right\}$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lB} = 22388.2 \ psi \\ S_m^l + S_{bo}^{lB} = -16593.4 \ psi \end{array}\right\} \le \left\{1.5S = 1.5(20600) = 30900 \ psi\right\} \qquad True$$

Since the acceptance criteria are satisfied, the design is complete. The vessel satisfies the requirements as designed and no further iterations are necessary.

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4.12.2 Example E4.12.2 – Reinforced Vessel of Rectangular Cross Section, Type 4

Using the data shown below, design a Type 4 non-circular pressure vessel per paragraph 4.12.7. The stiffeners are attached with continuous fillets welds on both sides of the member (Category E, Type 10) and satisfy the requirements of paragraph 4.2, Figure 4.2.2.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $50\,psig\,@\,200°F$
- Inside Length (Short Side) = $30.0\,in$
- Inside Length (Long Side) = $60.0\ in$
- Overall Vessel Length = $240.0\ in$
- Unstiffened Span Length (pitch) = $12.0\ in$
- Thickness (Short Side) = $0.4375\ in$
- Thickness (Long Side) = $0.4375\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $23200\ psi$
- Weld Joint Efficiency = 1.0
- Yield Stress at Design Temperature = $34800\ psi$
- Modulus of Elasticity at Design Temperature = $28.8E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$

Stiffener Data (W4x13 I-Beam):

- Material = $SA-36$
- Allowable Stress = $22000\ psi$
- Stiffener Yield Stress at Design Temperature = $33000\,psi$
- Modulus of Elasticity at Design Temperature = $28.8E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$
- Stiffener Cross Sectional Area = $3.83\ in^2$
- Stiffener Moment of Inertia = $11.3\ in^4$
- Stiffener Height = $4.125\ in$
- Stiffener Centerline Distance (Short Side) = $35.0\ in$
- Stiffener Centerline Distance (Long Side) = $65.0\ in$

Required variables.

$$h = 60.0\ in$$

$$H = 30.0\ in$$

$$t_1 = 0.4375\ in$$

$$t_2 = 0.4375\ in$$

Evaluate per paragraph 4.12.

Paragraph 4.12.2, General Design Requirements.

Paragraph 4.12.2.3.c – For a vessel with reinforcement, when the reinforcing member does not have the same allowable stress as the vessel, the total stress shall be determined at the inside and outside surfaces of each component of the composite section. The total stresses at the inside and outside surfaces shall be compared to the allowable stress.

i) For locations of stress below the neutral axis, the bending equation used to compute the stress shall be that considered acting on the inside surface.

ii) For locations of stress above the neutral axis, the bending equation used to compute the stress shall be that considered acting on the outside surface.

Paragraph 4.12.2.7 – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. These equations are conservatively applicable to vessels of aspect ratio less than four. Vessels with aspect ratios less than four may be designed in accordance with VIII-2, paragraph 4.12.5 and may be designed in accordance with the provisions of Part 5.

$$Aspect\ \ Ratio = \frac{L_v}{h} = \frac{240.0}{60.0} = 4.0$$

Paragraph 4.12.2.9 – There are no specified openings for this example problem.

Paragraph 4.12.3, Requirements for Vessels with Reinforcement.

Paragraph 4.12.3.1 – Design rules are provided for Type 4 configurations where the welded-on reinforcement members are in a plane perpendicular to the long axis of the vessel. All reinforcement members attached to two opposite plates shall have the same moment of inertia.

Paragraph 4.12.3.5 – Reinforcing members shall be placed on the outside of the vessel and shall be attached to the plates of the vessel by welding on each side of the reinforcing member. For continuous reinforcement, the welding may be continuous or intermittent.

Paragraph 4.12.3.6 – The maximum distance between reinforcing members is computed paragraph 4.12.3 and are covered in STEP 3 of the Design Procedure in paragraph 4.12.7.

Paragraphs 4.12.4 and 4.12.5.

These paragraphs are not applicable to this design.

Paragraph 4.12.6, Weld Joint Factors and Ligament Efficiency.

Paragraph 4.12.6.1 – The non-circular vessel is constructed with corner joints typical of paragraph 4.2. Therefore, the weld joint efficiencies E_m and E_b are set to 1.0 at stress calculation locations in the corners of the vessel. Since there are no welds or hole pattern in either the short side plates or long side plates of the vessel, the weld joint efficiencies E_m and E_b are set to 1.0 for these stress calculation locations.

Paragraph 4.12.7, Design Procedure.

a) STEP 1 – The design pressure and temperature are listed in the information given above.

b) STEP 2 – The vessel to be designed is a Type 4 vessel, see Figure 4.12.4.

c) STEP 3 – The vessel configuration and wall thicknesses of the pressure containing plates are listed in the information given above. The vessel has stiffeners; therefore, calculate the maximum spacing and size of the stiffeners per paragraph 4.12.3.

Paragraph 4.12.3.6.a – The maximum distance between reinforcing member centerlines is given by Equation (4.12.1). In the equations for calculating stresses for reinforced noncircular vessels, the value of p shall be the sum of one-half the distances to the next reinforcing member on each side.

For the short side plate, where $\{H = 30.0\ in\} \geq \{p = 12.0\ in\}$,

$$p_1 = t_1\sqrt{\frac{SJ_1}{P}} = 0.4375\sqrt{\frac{(23200)2.2931}{50}} = 14.2708\ in$$

where,

$$J_1 = -0.26667 + \frac{24.222}{(\beta_{1\max})} - \frac{99.478}{(\beta_{1\max})^2} + \frac{194.59}{(\beta_{1\max})^3} - \frac{169.99}{(\beta_{1\max})^4} + \frac{55.822}{(\beta_{1\max})^5}$$

$$J_1 = -0.26667 + \frac{24.222}{(2.1967)} - \frac{99.478}{(2.1967)^2} + \frac{194.59}{(2.1967)^3} - \frac{169.99}{(2.1967)^4} + \frac{55.822}{(2.1967)^5}$$

$$J_1 = 2.2931$$

$$\beta_{1\max} = \min\left[\max\left[\beta_1, \frac{1}{\beta_1}\right], 4.0\right]$$

$$\beta_{1\max} = \min\left[\max\left[2.1967, \frac{1}{2.1967}\right], 4.0\right] = 2.1967$$

$$\beta_1 = \frac{H}{p_{b1}} = \frac{30.0000}{13.6567} = 2.1967 \qquad (for\ rectangular\ vessels)$$

$$p_{b1} = t_1\sqrt{\frac{2.1S}{P}} = t_1\sqrt{\frac{2.1S}{P}} = 0.4375\sqrt{\frac{2.1(23200)}{50}} = 13.6567\ in$$

For the long side plate, where $\{h = 60.0\ in\} \geq \{p = 12.0\ in\}$,

$$p_2 = t_2\sqrt{\frac{SJ_2}{P}} = 0.4375\sqrt{\frac{(23200)2.0000}{50}} = 13.3276\ in$$

where,

$$J_2 = -0.26667 + \frac{24.222}{(\beta_{1\max})} - \frac{99.478}{(\beta_{1\max})^2} + \frac{194.59}{(\beta_{1\max})^3} - \frac{169.99}{(\beta_{1\max})^4} + \frac{55.822}{(\beta_{1\max})^5}$$

$$J_2 = -0.26667 + \frac{24.222}{(4.0)} - \frac{99.478}{(4.0)^2} + \frac{194.59}{(4.0)^3} - \frac{169.99}{(4.0)^4} + \frac{55.822}{(4.0)^5} = 2.0000$$

$$\beta_{2\max} = \min\left[\max\left[\beta_2, \frac{1}{\beta_2}\right], 4.0\right]$$

$$\beta_{2\max} = \min\left[\max\left[4.3934, \frac{1}{4.3934}\right], 4.0\right] = 4.0$$

$$\beta_2 = \frac{h}{p_{b2}} = \frac{60.0000}{13.6567} = 4.3934 \qquad (for\ rectangular\ vessels)$$

$$p_{b2} = t_2\sqrt{\frac{2.1S}{P}} = t_2\sqrt{\frac{2.1S}{P}} = 0.4375\sqrt{\frac{2.1(23200)}{50}} = 13.6567\ in$$

therefore,

$$p = \min\left[p_1,\ p_2\right] = \min\left[14.2708,\ 13.3276\right] = 13.3276\ in$$

Since $\{p_{design} = 12.0\ in\} < \{p_{allow} = 13.3276\ in\}$, the design is acceptable.

Paragraph 4.12.3.6.b – The allowable effective widths of shell plate, w_1 and w_2 shall not be greater than the value given by Equation (4.12.16) or Equation (4.12.17), nor greater than the actual value of p if this value is less than that computed in paragraph 4.12.3.6.a. One half of w shall be considered effective on each side of the reinforcing member centerline, but the effective widths shall not overlap. The effective width shall not be greater than the actual width available.

$$w_1 = \min\left[p,\ \min\left[w_{\max},\ p_1\right]\right] = \min\left[12.0, \min\left[13.7843,\ 14.2708\right]\right] = 12.0\ in$$

$$w_2 = \min\left[p,\ \min\left[w_{\max},\ p_2\right]\right] = \min\left[12.0, \min\left[13.7843,\ 13.3276\right]\right] = 12.0\ in$$

where,

$$w_{\max} = \frac{t_1\Delta}{\sqrt{S_y}}\left(\frac{E_y}{E_{ya}}\right) = \frac{t_2\Delta}{\sqrt{S_y}}\left(\frac{E_y}{E_{ya}}\right) = \frac{0.4375(6000)}{\sqrt{34800}}\left(\frac{28.8E+06}{29.4E+06}\right) = 13.7843\ in$$

$$\Delta = 6000\ \sqrt{psi} \qquad From\ Table\ 4.12.14$$

Paragraph 4.12.3.6.c – At locations, other than in the corner regions where the shell plate is in tension, the effective moments of inertia, I_{11} and I_{21}, of the composite section (reinforcement and shell plate acting together) shall be computed based on the values of w_1 and w_2 computed in paragraph 4.12.3.6.b.

NOTE – A composite structure may include the use of two or more different materials, each carrying a part of the load. Unless all the various materials used have the same Modulus of Elasticity, the evaluation of the composite section will need to consider the ratio of the moduli. Although the material specifications for the shell plate and stiffeners are different, their Moduli of Elasticity are the same; therefore, no adjustment to the procedure to calculate the composite section moment of inertia is required.

Calculate the short side stiffener/plate composite section neutral axis as follows, see Figure E4.12.2.

$$\bar{y} = \frac{A_{stif}\left(t_1 + \frac{h_s}{2}\right) + A_{plate}\left(\frac{t_1}{2}\right)}{\left(A_{stif} + A_{plate}\right)}$$

$$\bar{y} = \frac{3.83\left(0.4375 + \frac{4.125}{2}\right) + 0.4375(12.0)\left(\frac{0.4375}{2}\right)}{\left(3.83 + 0.4375(12.0)\right)} = 1.1810\ in$$

Calculate the short side composite section moment of inertia, I_{11}, using parallel axis theorem.

$$I_{11} = I_{stif} + A_{stif}\left(t_1 + \frac{h_s}{2} - \bar{y}\right)^2 + \frac{w_1(t_1)^3}{12} + w_1(t_1)\left(\bar{y} - \frac{t_1}{2}\right)^2$$

$$I_{11} = \left\{ \begin{array}{l} 11.3 + 3.83\left(0.4375 + \frac{4.125}{2} - 1.1810\right)^2 + \\ \frac{12.0(0.4375)^3}{12} + 12.0(0.4375)\left(1.1810 - \frac{0.4375}{2}\right)^2 \end{array} \right\} = 22.9081\ in^4$$

Since the stiffener is continuous around the vessel with a consistent net section, the plate thicknesses of the short side and long side are equal, $t_1 = t_2$, the pitch of stiffeners are equal, $w_1 = w_2$, it follows that $\bar{y}$ for the short side and long side plates are equal and $I_{11} = I_{21}$.

d) STEP 4 – Determine the location of the neutral axis from the inside and outside surfaces. If the section under evaluation has stiffeners, then c_i and c_o are determined from the cross section of the combined plate and stiffener section using strength of materials concepts.

For the short side plate,

$$c_i = \bar{y} = 1.1810\ in$$

$$c_o = t_1 + h_s - \bar{y} = 0.4375 + 4.125 - 1.1810 = 3.3815\ in$$

For the long side plate,

$$c_i = \bar{y} = 1.1810\ in$$

$$c_o = t_2 + h_s - \bar{y} = 0.4375 + 4.125 - 1.1810 = 3.3815\ in$$

The reinforcing member does not have the same allowable stress as the vessel; therefore, the stress at the interface of the components of the composite section shall be determined. Since the interface between components is oriented below the composite section neutral axis, the bending equation used to compute the stress shall be that considered acting on the inside surface. The distance between the composite section neutral axis and the interface of the components is calculated as follows.

For the short side and long side plates, respectively,

$$c_{i(interface)} = \bar{y} - t_1 = 1.1810 - 0.4375 = 0.7435\ in$$

$$c_{i(interface)} = \bar{y} - t_2 = 1.1810 - 0.4375 = 0.7435\ in$$

e) STEP 5 – Determine the weld joint factor and ligaments efficiencies, as applicable, see paragraph 4.12.6, and determine the factors E_m and E_b.

$$E_m = E_b = 1.0$$

f) STEP 6 – Complete the stress calculation for the selected noncircular vessel Type, see Table 4.12.1, and check the acceptance criteria.

For non-circular vessel Type 4, the applicable table for stress calculations is Table 4.12.5 and the corresponding details are shown in Figure 4.12.4.

Equation Constants:

$$\alpha_1 = \frac{H_1}{h_1} = \frac{H + 2(t_1) + h_s}{h + 2(t_2) + h_s} = \frac{30 + 2(0.4375) + 4.125}{60 + 2(0.4375) + 4.125} = 0.5385$$

$$k = \frac{I_{21}}{I_{11}}\alpha_1 = \left(\frac{22.9081}{22.9081}\right)0.5385 = 0.5385$$

Nomenclature for Stress Results:

S_m^s membrane stress, short side.

S_{bi}^{sC}, S_{bo}^{sC} bending stress, short side at point C on the inside and outside surfaces, respectively.

S_{bi}^{sB}, S_{bo}^{sB} bending stress, short side at point B on the inside and outside surfaces, respectively.

S_m^l membrane stress, long side.

S_{bi}^{lA}, S_{bo}^{lA} bending stress, long side at point A on the inside and outside surfaces, respectively.

S_{bi}^{lB}, S_{bo}^{lB} bending stress, long side at point B on the inside and outside surfaces, respectively.

Membrane and Bending Stresses – Critical Locations of Maximum Stress:

Membrane Stress on the short side plate:

$$S_m^s = \frac{Php}{2(A_1 + t_1 p)E_m} = \frac{50(60.0)12.0}{2(3.83 + 0.4375(12.0))1.0} = 1982.4\ psi$$

Bending Stress at Location C on the short side plate:

$$S_{bi}^{sC} = -S_{bo}^{sC}\left(\frac{c_i}{c_o}\right) = \frac{Ppc_i}{24I_{11}E_b}\left[-3H^2 + 2h^2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{sC} = \frac{50(12.0)1.1810}{24(22.9081)1.0}\left[-3(30.0)^2 + 2(60.0)^2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bi}^{sC} = 3493.6\ psi$$

$$S_{bo}^{sC} = -S_{bi}^{sC}\left(\frac{c_o}{c_i}\right) = -3493.6\left(\frac{3.3815}{1.1810}\right) = -10003.1\ psi$$

Bending Stress at Location B on the short side plate:

$$S_{bi}^{sB} = -S_{bo}^{sB}\left(\frac{c_i}{c_o}\right) = \frac{Ph^2 pc_i}{12I_{11}E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{sB} = \frac{50(60.0)^2(12.0)(1.1810)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right]$$

$$S_{bi}^{sB} = 6973.5 \ psi$$

$$S_{bo}^{sB} = -S_{bi}^{sB}\left(\frac{c_o}{c_i}\right) = -6973.5\left(\frac{3.3815}{1.1810}\right) = -19966.9 \ psi$$

Membrane Stress on the long side plate:

$$S_m^l = \frac{PHp}{2(A_2+t_2 p)E_m} = \frac{50(30.0)(12.0)}{2(3.83+0.4375(12.0))1.0} = 991.2 \ psi$$

Bending Stress at Location A on the long side plate:

$$S_{bi}^{lA} = -S_{bo}^{lA}\left(\frac{c_i}{c_o}\right) = \frac{Ph^2 pc_i}{24I_{21}E_b}\left[-3+2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{lA} = \frac{50(60.0)^2(12.0)(1.1810)}{24(22.9081)1.0}\left[-3+2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bi}^{lA} = -6946.0 \ psi$$

$$S_{bo}^{lA} = -S_{bi}^{lA}\left(\frac{c_o}{c_i}\right) = -(-6946.0)\left(\frac{3.3815}{1.1810}\right) = 19888.2 \ psi$$

Bending Stress at Location B on the long side plate:

$$S_{bi}^{lB} = -S_{bo}^{lB}\left(\frac{c_i}{c_o}\right) = \frac{Ph^2 pc_i}{12I_{21}E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{lB} = \frac{50(60.0)^2(12.0)(1.1810)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right]$$

$$S_{bi}^{lB} = 6973.5 \ psi$$

$$S_{bo}^{lB} = -S_{bi}^{lB}\left(\frac{c_o}{c_i}\right) = -6973.5\left(\frac{3.3815}{1.1810}\right) = -19966.9 \ psi$$

Calculate the bending stresses at the interface of the shell plate and stiffener at the Critical Locations of Maximum Stress.

Bending Stress at Location C on the short side plate:

$$S_{bi}^{sC} = \frac{Ppc_i}{24 I_{11} E_b}\left[-3H^2 + 2h^2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{sC} = \frac{50(12.0)(0.7435)}{24(22.9081)1.0}\left[-3(30.0)^2 + 2(60.0)^2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bi}^{sC} = 2199.4\ psi$$

Bending Stress at Location B on the short side plate:

$$S_{bi}^{sB} = \frac{Ph^2 pc_i}{12 I_{11} E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{sB} = \frac{50(60.0)^2(12.0)(0.7435)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] = 4390.2\ psi$$

Bending Stress at Location A on the long side plate:

$$S_{bi}^{lA} = \frac{Ph^2 pc_i}{24 I_{21} E_b}\left[-3 + 2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{lA} = \frac{50(60.0)^2(12.0)(0.7435)}{24(22.9081)1.0}\left[-3 + 2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right] = -4372.9\ psi$$

Bending Stress at Location B on the long side plate:

$$S_{bi}^{lB} = \frac{Ph^2 pc_i}{12 I_{21} E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{lB} = \frac{50(60.0)^2(12.0)(0.7435)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] = 4390.2\ psi$$

Acceptance Criteria – Critical Locations of Maximum Stress: The stiffener allowable stress, S_{stif}, is used for the membrane stress and membrane plus bending stress for the outside fiber stress acceptance criteria, while the plate allowable stress, S, is used for the membrane plus bending stress for inside fiber allowable stress criteria.

$$\left\{S_m^s = 1982.4\ psi\right\} \le \left\{S = 22000\ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_m^s + S_{bi}^{sC} = 1982.4 + 3493.5 = 5476.0\ psi \\ S_m^s + S_{bo}^{sC} = 1982.4 + (-10003.1) = -8020.7\ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(23200) = 34800\ psi \\ 1.5S = 1.5(22000) = 33000\ psi \end{array}\right\} \left\{\begin{array}{l} True \\ True \end{array}\right\}$$

$$\left\{\begin{array}{l} S_m^s + S_{bi}^{sB} = 1982.4 + 6973.5 = 8955.9\ psi \\ S_m^s + S_{bo}^{sB} = 1982.4 + (-19966.9) = -17984.5\ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(23200) = 34800\ psi \\ 1.5S = 1.5(22000) = 33000\ psi \end{array}\right\} \left\{\begin{array}{l} True \\ True \end{array}\right\}$$

$$\left\{S_m^l = 991.2\ psi\right\} \le \left\{S = 22000\ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lA} = 991.2 + (-6946.0) = -5954.8\ psi \\ S_m^l + S_{bo}^{lA} = 991.2 + 19888.2 = 20879.4\ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(23200) = 34800\ psi \\ 1.5S = 1.5(22000) = 33000\ psi \end{array}\right\} \left\{\begin{array}{l} True \\ True \end{array}\right\}$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lB} = 991.2 + 6973.5 = 7964.7\ psi \\ S_m^l + S_{bo}^{lB} = 991.2 + (-19966.9) = -18975.7\ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(23200) = 34800\ psi \\ 1.5S = 1.5(22000) = 33000\ psi \end{array}\right\} \left\{\begin{array}{l} True \\ True \end{array}\right\}$$

The allowable stress of the shell plate and stiffener is limited by the stiffener. Therefore, at the interface of the shell plate and stiffener, the allowable stress used in the acceptance criteria is that of the stiffener.

$$\left\{S_m^s + S_{bi}^{sC} = 1982.4 + 2199.4 = 4181.8\ psi\right\} \le \left\{1.5S = 1.5(22000) = 33000\ psi\right\} \qquad True$$

$$\left\{S_m^s + S_{bi}^{sB} = 1982.4 + 4390.2 = 6372.6\ psi\right\} \le \left\{1.5S = 1.5(22000) = 33000\ psi\right\} \qquad True$$

$$\left\{S_m^l + S_{bi}^{lA} = 991.2 + (-4372.9) = -3381.7\ psi\right\} \le \left\{1.5S = 1.5(22000) = 33000\ psi\right\} \qquad True$$

$$\left\{S_m^l + S_{bi}^{lB} = 991.2 + 4390.2 = 5381.4\ psi\right\} \le \left\{1.5S = 1.5(22000) = 33000\ psi\right\} \qquad True$$

The acceptance criteria for membrane and membrane plus bending stresses are satisfied at all locations; therefore, the design is complete.

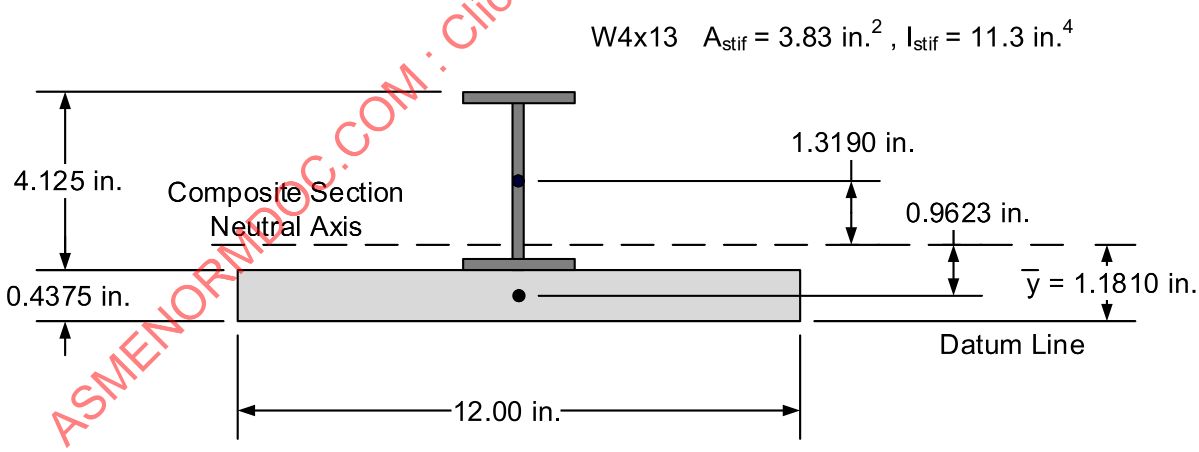

Figure E4.12.2 – Composite Section Details

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4.13 Layered Vessels

4.13.1 Example E4.13.1 – Layered Cylindrical Shell

Determine the required total thickness of the layered cylindrical shell for the following design conditions. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with Part 7, paragraph 7.4.11.

Vessel Data:

- Material = $SA-724,\ Grade\ B$
- Design Conditions = $5400\ psig\ @\ 300°F$
- Inside Diameter = $84.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $39600\ psi$
- Weld Joint Efficiency = 1.0
- Thickness of each layer = $0.3125\ in$

In accordance with Part 4, paragraph 4.13.4.1, determine the total thickness of the layered cylindrical shell using Part 4, paragraph 4.3.3.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{84}{2}\left(\exp\left[\frac{5400}{39600(1.0)}\right]-1\right) = 6.1361\ in$$

The required thickness for all layers is $6.1361\ in$.

Per paragraph 4.13.4.4, the minimum thickness of any layer shall not be less than $0.125\ in$.

$$\left\{t_{layer} = 0.3125\ in\right\} \geq \left\{t_{layer,\min} = 0.125\ in\right\} \qquad True$$

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4.13.2 Example E4.13.2 – Layered Hemispherical Head

Determine the required total thickness of the layered hemispherical head for the following design conditions. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with Part 7, paragraph 7.4.11.

Vessel Data:

- Material = $SA-724,\ Grade\ B$
- Design Conditions = $5400\ psig\ @\ 300°F$
- Inside Diameter = $84.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $39600\ psi$
- Weld Joint Efficiency = 1.0
- Thickness of each layer = $0.3125\ in$

In accordance with Part 4, paragraph 4.13.4.1, determine the total thickness of the layered hemispherical head using Part 4, paragraph 4.3.5.

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right]-1\right) = \frac{84.0}{2}\left(\exp\left[\frac{0.5(5400)}{39600(1.0)}\right]-1\right) = 2.9635\ in$$

The required thickness for all layers is $2.9635\ in$.

Per paragraph 4.13.4.4, the minimum thickness of any layer shall not be less than $0.125\ in$.

$$\{t_{layer} = 0.3125\ in\} \geq \{t_{layer,\min} = 0.125\ in\} \qquad True$$

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4.13.3 Example E4.13.3 – Maximum Permissible Gap in a Layered Cylindrical Shell

Determine if the anticipated maximum permissible gap between any layers, in accordance with paragraph 4.13.12.3 for the cylindrical shell in Example Problem E4.13.1 is adequate, given the specified design cycles. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with Part 7, paragraph 7.4.11.

Vessel Data:

- Material = $SA-724,\ Grade\ B$
- Design Conditions = $5400\ psig\ @\ 300°F$
- Inside Diameter = $84.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $39600\ psi$
- Weld Joint Efficiency = 1.0
- Thickness per Layer = $0.3125\ in$
- Number of Layers = 20
- Specified Design Cycles = 75000
- Anticipated Maximum Gap Height = $0.015\ in$
- Anticipated Maximum Gap Length = $0.25\ in$
- Minimum Ultimate Tensile Strength = $95\ ksi$
- Elastic Modulus at Temperature = $28.3E+06\ psi$

Commentary: In layered vessel design, the permissible gap height between layers is limited by the number of pressure cycles applied during operation. The opening and closure of the gap due to the applied pressure cycle creates a bending moment from which a bending stress and resulting total stress range and stress amplitude can be calculated. Using an appropriate fatigue curve with the calculated stress amplitude, a permissible number of pressure cycles can be determined. A comparison between the specified design cycles and the permissible cycles, based on the gap height, is made to determine acceptance.

The procedure to determine the maximum gap height is iterative as an initial value of the gap height is assumed, the resulting bending stress and stress amplitude are calculated, and the permissible number of cycles is determined using an appropriate fatigue curve. If the specified design cycles are less than the determined permissible cycles, the gap can be increased until the specified design cycles equal the permissible number of cycles. Conversely, if the specified design cycles are greater than the calculated permissible cycles, the gap can be reduced until the specified design cycles equal the permissible number of cycles.

In accordance with paragraph 4.13.12.3, the maximum number and size of gaps permitted in any cross section of a layered vessel shall be evaluated as follows. For this example, conservatively consider the gap between layers to be located at the outermost layer of the vessel.

The circumferential stress of the shell and the bending stress due to the gap can be calculated as follows.

$$S=\frac{R_o^2+R_1^2}{R_o^2-R_1^2}P=\frac{(48.25)^2+(42.0)^2}{(48.25)^2-(42.0)^2}(5400)=39175.0\ psi$$

where,

$$R_o = \begin{Bmatrix} Inside\ Radius + Number\ of\ Layers\left(Layer\ Thickness\right) \\ 42.0 + 20\left(0.3125\right) = 48.25\ in \end{Bmatrix}$$

$$R_1 = R_i = 42.0\ in$$

and,

$$S_b = \frac{1.812 E_y h}{R_g} = \frac{1.812\left(28.3E+06\right)\left(0.015\right)}{48.25} = 15941.8\ psi$$

where,

$$R_g = R_o = 48.25\ in$$

Since $\{S_b = 15941.8\, psi\} < \{0.71S = 0.71(39175.0) = 27814.3\, psi\}$, the total stress range, ΔS_n, is calculated as follows.

$$\Delta S_n = S + 0.3S_b + P = 39175.0 + 0.3\left(15941.8\right) + 5400 = 49357.5\ psi$$

The stress amplitude for fatigue analysis at the gap is calculated as follows.

$$S_{ag} = \frac{K_e \Delta S_n}{2} = \frac{1.0\left(49357.5\right)}{2} = 24678.8\ psi$$

Since $\{\Delta S_n = 49357.5\} < \{3S_m = 3(39600) = 118800\, psi\}$, the fatigue penalty factor, K_e, is calculated as follows.

$$K_e = 1.0$$

The fatigue analysis to determine the permissible number of cycles is in accordance with Annex 3F, using the smooth bar design fatigue models per paragraph 3F.1.2(a) for carbon steel not exceeding 700°F and paragraph 3F.1.3. Since $UTS = 95\, ksi$, interpolation of the permissible number of cycles is required using the equations found in paragraph 3F.1.2(a)(1) and paragraph 3F.1.2(a)(2).

Paragraph 3F.1.2(a) and substituting $S_a = S_{ag} = 24678.8\, psi \rightarrow 24.6788\, ksi$,

$$Y = \log\left[28.3E+03\left(\frac{S_a}{E_T}\right)\right] = \log\left[28.3E+03\left(\frac{24.6788}{28.3E+03}\right)\right] = 1.3923$$

Paragraph 3F.1.2(a)(1), with $\{10^Y = 10^{1.3923} = 24.6788\} > 20$, the number of permissible cycles is determined as follows.

$$X = \begin{Bmatrix} -4706.5245 + 1813.6228Y + \dfrac{6785.5644}{Y} - 368.12404Y^2 \\ -\dfrac{5133.7345}{Y^2} + 30.708204Y^3 + \dfrac{1596.1916}{Y^3} \end{Bmatrix}$$

$$X = \begin{Bmatrix} -4706.5245 + 1813.6228\left(1.3923\right) + \dfrac{6785.5644}{1.3923} - 368.12404\left(1.3923\right)^2 \\ -\dfrac{5133.7345}{\left(1.3923\right)^2} + 30.708204\left(1.3923\right)^3 + \dfrac{1596.1916}{\left(1.3923\right)^3} \end{Bmatrix} = 4.5953$$

Paragraph 3F.1.3, the number of design cycles, N can be computed based on the parameter X calculated for the applicable materials as follows.

$$N_{80} = 10^{X} = 10^{4.5953} = 39382\ cycles$$

Paragraph 3F.1.2(a)(2), with $\{10^{Y} = 10^{1.3923} = 24.6778\} < 43$, the number of permissible cycles is determined as follows.

$$X = \frac{-9.41749 + 14.7982Y - 5.94Y^2}{1 - 3.46282Y + 3.63495Y^2 - 1.21849Y^3}$$

$$X = \frac{-9.41749 + 14.7982(1.3923) - 5.94(1.3923)^2}{1 - 3.46282(1.3923) + 3.63495(1.3923)^2 - 1.21849(1.3923)^3}$$

$$X = 5.1667$$

Paragraph 3F.1.3, the number of design cycles, N can be computed based on the parameter X calculated for the applicable materials as follows.

$$N_{115} = 10^{X} = 10^{5.1667} = 146791\ cycles$$

Performing linear interpolation to determine the permissible number of cycles for $UTS = 95\ ksi$.

$$N_{95} = N_{80} + (UTS_{95} - UTS_{80})\frac{(N_{115} - N_{80})}{(UTS_{115} - UTS_{80})}$$

$$N_{95} = 39382 + (95 - 80)\frac{(146791 - 39382)}{(115 - 80)}$$

$$N_{95} = 85414\ cycles$$

Since $\{N_{95} = 85414\} \geq \{N_D = 75000\}$, the anticipated gap height at the outermost layer, $h = 0.015\ in.$, is acceptable.

The maximum permissible number of gaps and their corresponding arc lengths at any cross section of a layered vessel shall be calculated as follows.

$$F = 0.109\left(\frac{bh}{R_g^2}\right) = 0.109\left(\frac{(0.25)(0.015)}{48.25^2}\right) = 1.7558E-07$$

$$20\ layers \rightarrow F_{20} = 20(1.7558E-07) = 3.5116E-06$$

The total sum of the calculated F values shall not exceed the quantity,

$$F_T = \frac{1-\upsilon^2}{E_y}\left(\frac{2S_a}{K_e} - \frac{2PR_o^2}{R_o^2 - R_i^2}\right)$$

$$F_T = \frac{1-(0.3)^2}{28.3E+06}\left(\frac{2(39600)}{1.0} - \frac{2(5400)(48.25)^2}{(48.25)^2 - (42.0)^2}\right) = 0.001$$

Since $\{F_{20} = 3.5116E-06\} \leq \{F_T = 0.001\}$, the anticipated gap length at the outermost layer, $b = 0.25\ in.$, is acceptable.

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4.14 Evaluation of Vessels Outside of Tolerance

4.14.1 Example Problem E4.14.1 – Shell Tolerances

During construction examination of the vessel shell indicated angular weld misalignment at the long seam weld. The shell tolerances did not satisfy the fabrication tolerances given in paragraph 4.3.2. Determine if the design may be qualified using paragraph 4.14.1. The vessel is not in cyclic operation based on the screening criteria performed in accordance with paragraph 5.5.2.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $325\ psig @ 600°F$
- Inside Diameter = $30\ in$
- Wall Thickness = $0.5\ in$
- Joint Efficiency = $100\ \%$
- Corrosion Allowance = $0.063\ in$
- Allowable Stress = $19400\ psi @ 600°F$
- Yield Strength = $29100\ psi @ 600°F$
- Modulus of Elasticity = $26.5E+06\ psi @ 600°F$

Examination Data:

- Peaking distortion, δ = $0.33\ in$

Per paragraph 4.14.1, the user has agreed to permit the assessment procedures in API 579-1/ASME FFS-1 to be used to qualify the design. When API 579-1/ASME FFS-1 is used for the assessment, a Remaining Strength Factor of 0.95 shall be used in the calculations unless another value is agreed to by the user. However, the Remaining Strength Factor shall not be less than 0.90.

API 579-1/ASME FFS-1 – The Level 2 Assessment procedure for the evaluation of angular weld misalignment at a longitudinal weld joint is provided in Part 8. Paragraph 8.3.4.1 provides a technique to measure the angular misalignment (peaking) of the vessel and the resulting weld peaking has been documented as provided above, reference Figure 8.4(a) and Figure 8.8. Paragraph 8.4.3.2 provides a step-by-step procedure from this paragraph as shown below.

a) STEP 1 – Identify the component and weld misalignment type (see Table 8.10) and determine the following variables as applicable (see Figure 8.4). The weld misalignment is identified as angular misalignment on a longitudinal weld seam.

$$R = \frac{R_o + R_i}{2} = \frac{15.5 + 15.063}{2} = 15.2815\ in$$

$$\nu = 0.3$$

$$H_f = 3.0$$

$$RSF_a = 0.95$$

b) STEP 2 – Determine the wall thickness to be used in the assessment.

$$t_c = t_{nom} - LOSS - FCA = 0.5 - 0.0 - 0.063 = 0.437 \ in$$

c) STEP 3 – Determine the membrane stress from pressure σ_m, (see Annex 2C). For cylindrical shells, σ_m^c should be used for misalignment of longitudinal joints. Note that API 579-1/ASME FFS-1 permits the use of the equations from Part 4, paragraph 4.3 in lieu of those shown from VIII-1, as shown in paragraph 2C.3.3.1(a), Equation (2C.11). Rearranging equation (4.3.1) from VIII-2 to solve for stress results in the following.

$$\sigma_m^c = \frac{P}{E \cdot \ln\left[\frac{2t}{D}+1\right]} = \frac{325}{1.0 \cdot \ln\left[\frac{2(0.437)}{(30.126)}+1\right]} = 11364 \ psi$$

d) STEP 4 – Calculate the ratio of the induced bending stress to the applied membrane stress, R_b using the equations in Table 8.10 based on angular weld misalignment (local peaking).

$$S_p = \sqrt{\frac{12(1-\nu^2)PR^3}{E_y t_c^3}} = \sqrt{\frac{12(1-(0.3)^2)(325)(15.2815)^3}{(26.5E+06)(0.437)^3}} = 2.3931$$

And,

$$\frac{\delta}{R} = \frac{0.33}{15.2815} = 0.0216$$

From Figure 8.13, with

$$\begin{Bmatrix} S_p = 2.3931 \\ \frac{\delta}{R} = 0.0216 \end{Bmatrix} \Rightarrow C_f \approx 0.84$$

Therefore, for cylinders – Longitudinal Joints:

$$R_b^{clja} = 6\left(\frac{\delta}{t_c}\right)C_f = 6\left(\frac{0.33}{0.437}\right)(0.84) = 3.81$$

$$R_b = R_b^{cljc} + R_b^{clja} = 0.0 + 3.81$$

$$\sigma_{ms} = 0.0$$

e) STEP 5 – Determine the Remaining Strength Factor.

$$RSF = \min\left[\frac{H_f S_a}{\sigma_m(1+R_b)+\sigma_{ms}(1+R_{bs})}, 1.0\right]$$

$$RSF = \min\left[\frac{(3.0)(19400)}{(11364)(1+3.81)+(0.0)(1+1.0)}, 1.0\right] = \min[1.0647, 1.0] = 1.0$$

f) STEP 6 – Evaluate the results.

$$\{RSF = 1.0\} \geq \{RSF_a = 0.95\} \qquad\qquad True$$

The Level 2 Assessment Criterion is satisfied; therefore, the angular weld misalignment (peaking) is acceptable for the specified design conditions.

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4.14.2 Example Problem E4.14.2 – Shell Tolerances and Fatigue Evaluation

Determine if the vessel in the Example Problem E4.14.1 can operate for 2000 cycles. In accordance with paragraph 4.14.1, a fatigue analysis shall be performed using API 579-1/ASME FFS-1, as applicable. The fatigue analysis procedure is given in Part 8, paragraph 8.4.3.8. A fatigue analysis may be performed since the vessel satisfies the Level 2 criterion for the assessment of the weld misalignment as shown in Example Problem E4.14.1.

Operational Data

- Condition 1 = $0\ psig\ @\ 70°F$
- Condition 2 = $325\ psig @\ 600°F$
- Average Temperature of Cycle = $335°F$

Material Properties

- Ultimate Tensile Strength, S_{UTS} = $70000\ psi @\ 335°F$
- Yield Strength, S_y = $33215\,psi @\ 335°F$
- Allowable Stress, S_a = $22190\,psi @\ 335°F$
- Modulus of Elasticity, E_T = $28.16E+06\,psi @\ 335°F$
- Modulus of Elasticity, E_{ya} = $28.16E+06\,psi @\ 335°F$
- Modulus of Elasticity, E_A = $29.05E+06\,psi @ 100°F$

Additional Data

- Examination Group = $1b$
- Surface Examination = $None$
- Weld Prep = $As-Welded$
- Environmental Factor, f_e = 4.0

The step-by-step procedure from API 579-1/ASME FFS-1, Part 8, paragraph 8.3.4.2 is shown below.

a) STEP 1 – Determine the nature of the loading, the associated membrane stress, and the number of operating cycles.
   - The loading consists of pressure loading. From Example E4.14.1, $\sigma_m = 11364\,psi$.
   - The desired number of operating cycles, $N = 2000\ cycles$.

b) STEP 2 – Determine the ratio of the induced bending stress to the membrane stress resulting from weld misalignment. From Example E14.1.1,

$$R_b = R_b^{clja} = 3.81$$

c) STEP 3 – Using the loading history and membrane stress from STEP 1 and R_b from STEP 2, calculate the stress range for the fatigue analysis using Table 8.12 for a cylinder with a longitudinal weld joint with weld misalignment.

   Elastic Stress Analysis using welded joint fatigue curves:

$$\Delta\sigma_m = \sigma_m = 11364\ psi$$

$$\Delta\sigma_b = \sigma_m \left(R_b^{cljc} + R_b^{clja} + R_b^{or}\right) = (11364)(0.0+3.81+0.0) = 43297\ psi$$

Elastic Stress Analysis using smooth bat fatigue curves. The fatigue strength reduction factor will be applied when computing the alternating stress range; therefore, set $K_f = 1.0$ in the equation for ΔS_P.

$$\Delta S_P = \sigma_m \left(1 + R_b^{cljc} + R_b^{clja} + R_b^{or}\right)\left(K_f\right)$$

$$\Delta S_P = \left(11364\right)\left(1 + 0.0 + 3.81 + 0.0\right)\left(1.0\right) = 54661\ psi$$

d) STEP 4 – Compute the number of allowed cycles using the stress range determined in STEP 3 using Part 14, as applicable. Per paragraph 14.1.6, fatigue curves are presented in two forms: fatigue curves that are based on smooth bar test specimens and fatigue curves that are based on test specimens that include wed details with quality consistent with the fabrication and inspection requirements for pressure containing equipment. The fatigue curves to be used with this Part are contained in Annex 14B.

Paragraph 14.4.3 provides three methods for determining the permissible number of cycles:

1) Method A: Elastic Stress Analysis and Equivalent Stresses, paragraph 14.4.3.2,

2) Method B: Elastic-Plastic Stress Analysis and Equivalent Strain, paragraph 14.4.3.3, and

3) Method C: Elastic Stress Analysis and Structural Stress, paragraph 14.4.3.4.

Since an elastic-plastic stress analysis has not been conducted, the permitted number of cycles will be determined using Methods A and C. In both cases the stresses considered consist of those due to pressure loading only, i.e., supplementary loads and thermal gradients negligible.

Method A: Part 14, paragraph 14.4.3.2

1) STEP 1 through STEP 3 – For a fatigue assessment using an elastic stress analysis and equivalent stresses, the STEPS are similar to STEP 1 through STEP 3 in paragraph 8.4.3.8 with the exception that the elastic stress range is calculated from the stress tensors. Also, two options are provided to determine the equivalent stress range.

   i) OPTION 1 – Local thermal stress range is not separated from the total stress range.

   ii) OPTION 2 – Local thermal stress range is separated from the total stress range.

   For this example, OPTION 1 is selected and the stress range due to thermal loading is considered negligible and the mechanical loading consists of internal pressure only. Therefore, the stress range for internal pressure loading is given by STEP 3 from Part 8, paragraph 8.4.3.8.

2) STEP 4 – Perform the following for each cyclic stress range in the loading time history.

   i) STEP 4.1 – Obtain the stress tensors at the start and end of the cycle.

   Not required for this example.

   ii) STEP 4.2 – Calculate the equivalent primary plus secondary plus peak stress ranges for the cycle.

   As thermal loads are not applicable, $\Delta S_{LT} = 0.0$, the equivalent stress range for the pressure cycle is as shown in STEP 3 of Part 8, paragraph 8.4.3.8.

$$\Delta S_P = 54661\ psi$$

iii) STEP 4.3 – Calculate the effective alternating equivalent stress amplitude for the cycle using the results form STEP 4.2.

$$S_{alt} = \frac{K_f \cdot K_e \cdot \Delta S_P}{2} = \frac{(2.0)(1.0)(54661)}{2} = 54661 \ psi$$

The fatigue strength reduction factor, K_f, is determined from Table 14.5 based on type of weld and the quality level determined from Table 14.6. The quality level in Table 14.6 is based on the type of inspection performed on the weld. For the vessel material, the specification called for full volumetric and full visual examination, but neither MT nor PT were performed on the weld due to thickness limitations. Therefore, from Table 14.6 the quality level is 4. The weld being assessed is an as-welded full penetration weld. For an as-welded full penetration weld inspected to quality level 4, Table 14.5 stipulated a weld fatigue reduction factor of $K_f = 2.0$.

The factor K_e is a fatigue penalty factor that may be determined from Equations (14.25) to (14.27) depending on the value of the stress range ΔS_P compared to the permitted primary plus secondary stress range, S_{PS}. The allowable limit on the primary plus secondary stress range, S_{PS}, is determined from Equation (14.77), where the values of S_{UTS}, S_a, and S_y are the average values at the highest and lowest temperatures of the cycle.

$$S_{PS} = \min\left[S_{UTS}, \max\left[3.0S_a, 2S_y\right]\right]$$

$$S_{PS} = \min\left[70000, \max\left[(3)(22190), (2)(33215)\right]\right] = 66570 \ psi$$

Since $\{\Delta S_P = 54661 \ psi\} \le \{S_{PS} = 66570 \ psi\}$, K_e is determined per Equation (14.25) where $K_e = 1.0$.

iv) STEP 4.4 – If $K_e > 1.0$ and the material of construction for the component satisfies $YS/UTS \le 0.80$; then proceed to STEP 4.5.

Not required for this example.

v) STEP 4.5 – Determine the permitted number of cycles, N, for the alternating equivalent stress amplitude computed in STEP 4.3 using a fatigue curve based on the materials of construction in Annex 14B for the cycle.

Paragraph 14B.1.3 – The design fatigue curves in this paragraph are from VIII-2, Annex 3F.

Paragraph 14B1.4 – The fatigue analysis to determine the permissible number of cycles using the smooth bar design fatigue models is in accordance with paragraph 14B.1.4(a) for carbon steel not exceeding 700°F and paragraph 14B.1.5. Since $\sigma_{uts} \le 80 \ ksi$, the permissible number of cycles is required using the equations found in paragraph 14B.1.4(a)(1).

Paragraph 3-F.1.2(a) and substituting $S_a = S_{alt} = 54.661 \ ksi$,

$$Y = \log\left[28.3E+03\left(\frac{S_a}{E_T}\right)\right] = \log\left[28.3E+03\left(\frac{54.661}{28.16E+03}\right)\right] = 1.7398$$

Paragraph 14B.1.4(a)(1), with $\{10^Y = 10^{1.7398} = 54.9\} \ge 20$, the number of permissible cycles is determined as follows, using Equation (14B.2).

$$X=\begin{Bmatrix} -4706.5245+1813.6228Y+\dfrac{6785.5644}{Y}-368.12404Y^2 \\ -\dfrac{5133.7345}{Y^2}+30.708204Y^3+\dfrac{1596.1916}{Y^3} \end{Bmatrix}$$

$$X=\begin{Bmatrix} -4706.5245+1813.6228(1.7398)+\dfrac{6785.5644}{1.7398}-368.12404(1.7398)^2 \\ -\dfrac{5133.7345}{(1.7398)^2}+30.708204(1.7398)^3+\dfrac{1596.1916}{(1.7398)^3} \end{Bmatrix}=3.5201$$

Paragraph 14B.1.5, the number of design cycles, N, can be computed based on the parameter X calculated for the applicable material as follows.

$$N=10^X=10^{3.5201}=3312\ cycles$$

Method C: Part 14, paragraph 14.4.3.4

1) STEP 1 – Determine the load history for the component, considering all significant operating loads.

   See Operational Data:

2) STEP 2 – For the weld joint subject to fatigue evaluation determine the individual number of stress-strain cycles.

   The desired number of cycles, $N=2000\ cycles$.

3) STEP 3 – Determine the cyclic stress ranges at the location under consideration based on the elastic stress analysis in STEP 2.

4) STEP 4 – Perform the following for each cyclic stress rage in the loading time history.

   i) STEP 4.1 – Determine the elastic membrane and bending stresses normal to the assumed hypothetical crack lane at the start and end points for the cycle. Using this data, calculate the membrane and bending stress ranges, and the maximum, minimum, and mean stress for the cycle.

   From Example E14.1, the maximum membrane stress for the cycle occurs at a pressure of 325 psig, and the minimum membrane stress for the cycle occurs at zero pressure. Similarly, the maximum bending stress for the cycle occurs at a pressure of 325 psig, and the minimum bending stress for the cycle occurs at zero pressure. The values of the stress ranges are given in Equation (14.44) through (14.47).

$$\Delta\sigma_b^e=\left|{}^m\sigma_b^e-{}^n\sigma_b^e\right|=\left|(R_b)\left({}^m\sigma_m^e-{}^n\sigma_m^e\right)\right|=\left|(3.81)(11.364-0)\right|=43.297\ ksi$$

$$\sigma_{\max}=\max\left[\left({}^m\sigma_m^e+{}^mP_c+{}^m\sigma_b^e\right),\left({}^n\sigma_m^e+{}^nP_c+{}^n\sigma_b^e\right)\right]$$
$$\sigma_{\max}=\max\left[(11.364+0+43.297),(0+0+0)\right]=54.661\ ksi$$

$$\sigma_{\min}=\min\left[\left({}^m\sigma_m^e+{}^mP_c+{}^m\sigma_b^e\right),\left({}^n\sigma_m^e+{}^nP_c+{}^n\sigma_b^e\right)\right]$$
$$\sigma_{\min}=\min\left[(11.364+0+43.297),(0+0+0)\right]=0.0\ ksi$$

$$\sigma_{mean} = \frac{\sigma^e_{\max} + \sigma^e_{\min}}{2} = \frac{54.661+0}{2} = 27.331\ ksi$$

ii) STEP 4.2 – Calculate the elastic structural stress range, $\Delta\sigma^e$, for the cycle using Equation (14.48).

$$\Delta\sigma^e = \sigma_{\max} + \sigma_{\min} = 54.661 + 0.0 = 54.661\ ksi$$

iii) STEP 4.3 – Calculate the elastic structural strain, $\Delta\varepsilon^e$, from the elastically calculated structural stress range, $\Delta\sigma^e$, using Equation (14.49).

$$\Delta\varepsilon^e = \frac{\Delta\sigma^e}{E_{ya}} = \frac{54.661}{28.16E+03} = 1.9411E-03\ in/in$$

iv) STEP 4.4 – Calculate the pseudo-elastic structural strain range, $\Delta\varepsilon$, for the cycle by solving the nonlinear algebraic Equation (14.50).

$$\Delta\varepsilon = \frac{\Delta\sigma^e \cdot \Delta\varepsilon^e}{E_{ya} \cdot \Delta\varepsilon} + 2\left(\frac{\Delta\sigma^e \cdot \Delta\varepsilon^e}{2K_{css} \cdot \Delta\varepsilon}\right)^{\frac{1}{n_{css}}}$$

Note that Equation (14.50) is derived from Neuber's rule expressed by Equation (14.51) and a model for the material hysteresis loop stress-strain curve given by Equation (14.52). Equation (14.50) is derived by substituting Equation (14.51) in Equation (14.52).

Determine the stress range, $\Delta\sigma$, and strain range, $\Delta\varepsilon$, by correcting the elastically computed values of $\Delta\sigma^e$ and $\Delta\varepsilon^e$ by solving Equations (14.51) and (14.52) simultaneously, these equations are shown below.

$$\Delta\sigma \cdot \Delta\varepsilon = \Delta\sigma^e \cdot \Delta\varepsilon^e$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{E_{ya}} + 2\left(\frac{\Delta\sigma}{K_{css}}\right)^{\frac{1}{n_{css}}}$$

The parameters K_{css} and n_{css} are determined from Part 2, Annex 2E, Table 2E.9 for the average temperature during the cycle. The values for Carbon Steel (0.75 in. – weld metal) are:

$$T = 70°F \quad \rightarrow \quad n_{css} = 0.110 \quad K_{css} = 100.8\ ksi$$

$$T = 390°F \quad \rightarrow \quad n_{css} = 0.118 \quad K_{css} = 99.6\ ksi$$

The average temperature of the cycle is 335°F. Therefore, the values of n_{css} and K_{css} are determined as follows.

$$T = 335°F \quad \rightarrow \quad n_{css} = 0.11663 \quad K_{css} = 99.80625\ ksi$$

Substituting the above values into the equations results in the following simultaneous equations.

$$\Delta\sigma \cdot \Delta\varepsilon = \Delta\sigma^e \cdot \Delta\varepsilon^e = (54.661)(1.9411E-03) = 1.0601E-01$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{28.16E+03} + 2\left(\frac{\Delta\sigma}{2(99.80625)}\right)^{\frac{1}{0.11663}}$$

The solution of these equations is:

$$\Delta\sigma = 54.265\ ksi$$

$$\Delta\varepsilon = 1.9553E-03\ in/in$$

v) STEP 4.5 – Calculate the pseudo-elastic structural stress range $\Delta\sigma$ for the cycle using Equation (14.53).

$$\Delta\sigma = E_{ya}\cdot\Delta\varepsilon = (28.16E+03)(1.9553E-03) = 55.061\ ksi$$

vi) STEP 4.6 – Calculate the equivalent structural stress range parameter ΔS_{ess} for the cycle using Equation (14.54).

$$\Delta S_{ess} = \frac{\Delta\sigma}{t_{ess}^{\left(\frac{2-m_{ss}}{2\cdot m_{ss}}\right)}\cdot I^{\frac{1}{m_{ss}}}\cdot f_M} = \frac{55.061}{(0.625)^{\left(\frac{2-3.6}{2\cdot 3.6}\right)}\cdot(1.2789)\cdot(1)} = 38.784\ ksi$$

where,

$$m_{ss} = 3.6$$

$$t_{ess} = 0.625\ in \qquad \left(\text{for } \{t = 0.437\ in\} \le 0.625\ in\right)$$

$$I^{\frac{1}{m_{ss}}} = \frac{1.23 - 0.364R_b - 0.17R_b^2}{1.007 - 0.306R_b - 0.178R_b^2}$$

$$I^{\frac{1}{m_{ss}}} = \frac{1.23 - 0.364(0.7921) - 0.17(0.7921)^2}{1.007 - 0.306(0.7921) - 0.178(0.7921)^2} = 1.2789$$

$$R_b = \frac{\Delta\sigma_b^e}{\Delta\sigma^e} = \frac{43.297}{54.661} = 0.7921$$

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} = \frac{0.0}{54.611} = 0.0$$

$$f_M = 1.0 \qquad \left(since\ \{R = 0\} \le 0\right)$$

vii) STEP 4.7 – Determine the permissible number of cycles, N, based on the equivalent structural stress range parameter, ΔS_{ess}, for the cycle computed in STEP 4.6 using the fatigue curve based on the materials of construction in Annex 14B.

Paragraph 14B.4.2(a) – The constants C and h for use in Equation (14B.28) are provided in Table 14B.5. The Lower 99% prediction interval (-3σ) shall be used for design per VIII-2.

$$N = \frac{f_I}{f_E}\left(\frac{f_{MT}\cdot C}{\Delta S_{ess}}\right)^{\frac{1}{h}} = \frac{1.0}{4.0}\left(\frac{(0.9694)(818.3)}{38.784}\right)^{\frac{1}{0.3195}} = 3165\ cycles$$

where,

$$C = 818.3 \qquad h = 0.3195 \qquad \rightarrow for\ \ ferritic\ steels\ (-3\sigma)$$

$$f_I = 1.0 \qquad \left(\text{fatigue improvement techniques have not been used}\right)$$

$$f_E = 4.0 \qquad \begin{pmatrix} \text{environmental modification factor} \\ \text{the process fluid is considered mildly aggressive} \end{pmatrix}$$

$$f_{MT} = \frac{E_T}{E_A} = \frac{28.16E+03}{29.05E+03} = 0.9694$$

The component is acceptable for cyclic operation at the specified design conditions.

In summary, the fatigue life is satisfied by both Method 1 and Method 3.

Method 1:

$$\{N = 3312\} \geq \{Specified\ Design\ Cycles = 2000\} \qquad (True)$$

Method 3:

$$\{N = 3165\} \geq \{Specified\ Design\ Cycles = 2000\} \qquad (True)$$

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4.14.3 Example Problem E4.14.3 - Local Thin Area

For the vessel in Example Problem E4.14.1, radiographic examination of a Category A weld seam identified a grouping of linear indications which were removed during fabrication by blend grinding that has resulted in a region of local metal loss. The resulting region of local metal loss was made to conform to the requirements of paragraph 4.14.2.2. The user has agreed to apply the assessment procedures of Part 5 of API 579-1/ASME FFS-1 to determine whether the local thin area is acceptable without the need for weld repair.

Vessel Data:

- Material = $SA-516, Grade\,70$
- Design Conditions = $325\ psig@600°F$
- Inside Diameter = $30\ in$
- Wall Thickness = $0.5\ in$
- Joint Efficiency = $100\,\%$
- Corrosion Allowance = $0.063\ in$
- Allowable Stress = $19400\ psi@600°F$
- Supplemental Loads = Negligible

Examination Data:

Based on inspection data, the Critical Thickness Profile (CTP) in the longitudinal direction has a length, s of 2.0-inches and a uniform measured thickness of 0.437-inch. The critical thickness profile in the circumferential direction has a length c of 1.0-inch with the same uniform thickness. The region of local metal loss is located 45-inches away from the nearest structural discontinuity and is the only region of local metal loss under consideration.

As noted in paragraph 4.14.2.1, when API 579-1/ASME FFS-1 is used in the assessment, a Remaining Strength Factor of 0.98 shall be used in the calculations unless another value is agreed to by the user.

The Level 1 Assessment procedure for evaluation of a local thin area is provided in API 579-1/ASME FFS-1, Part 5, paragraph 5.4.2. The step-by-step procedure of paragraph 5.4.2.2 is shown below.

a) STEP 1 – Determine the CTP (Critical Thickness Profiles) (See Inspection Data above).

b) STEP 2 – Determine the wall thickness to be used in the assessment using Equation (5.3).

$$t_{nom} = 0.5\ in$$
$$LOSS = 0.0\ in$$
$$FCA = 0.063\ in$$
$$t_c = t_{nom} - LOSS - FCA = 0.5 - 0.0 - 0.063 = 0.437\ in$$

c) STEP 3 – Determine the minimum measured thickness, t_{mm}, and the dimension, s and c for the CTP.

$$t_{mm} = 0.437\ in$$
$$s = 2.0\ in$$
$$c = 1.0\ in$$

d) STEP 4 – Determine the remaining thickness ratio and the longitudinal flaw length parameter using Equations (5.5) and (5.6), respectively.

$$R_t = \frac{t_{mm} - FCA}{t_c} = \frac{0.437 - 0.063}{0.437} = 0.8558$$

$$\lambda = \frac{1.285s}{\sqrt{Dt_c}} = \frac{1.285(2.0)}{\sqrt{30.126(0.437)}} = 0.7083$$

where,

$$D = 30.0 + 2(LOSS + FCA) = 30.0 + 2(0.0 + 0.063) = 30.126\ in$$

e) STEP 5 – Check the limiting flaw size criteria for a Level 1 Assessment using Equations (5.7), (5.8), and (5.9).

$$\{R_t = 0.8558\} \geq 0.20 \qquad True$$

$$\{t_{mm} - FCA = 0.437 - 0.063 = 0.374\ in\} \geq 0.10\ in \qquad True$$

$$\{L_{msd} = 45\ in\} \geq \left\{1.8\sqrt{Dt_c} = 1.8\sqrt{30.126(0.437)} = 6.5311\ in\right\} \qquad True$$

f) STEP 6 – Check the criteria for a groove-like flaw. This step is not applicable because the region of localized metal loss is categorized as an LTA.

g) STEP 7 – Determine the $MAWP$ for the component using the thickness from STEP 2, see Annex 2C, paragraph 2C.2. Note that API 579-1/ASME FFS-1 permits the use of the equations from Part 4, paragraph 4.3 in lieu of those shown from VIII-1, as shown in paragraph 2C.3.3.1(a), Equation (2C.10). Rearranging equation (4.3.1) from VIII-2 to solve for the $MAWP$ results in the following.

$$MAWP^C = SE \cdot \ln\left[\frac{2t}{D} + 1\right] = (19400)(1.0) \cdot \ln\left[\frac{2(0.437)}{(30.126)} + 1\right] = 554.8\ psi$$

h) STEP 8 – Evaluate the longitudinal extent of the flaw.

$$RSF = \frac{R_t}{1 - \frac{1}{M_t}(1 - R_t)} = \frac{0.8558}{1 - \frac{1}{1.1074}(1 - 0.8558)} = 0.9839$$

where the parameter, M_t is determined from Table 5.2.

$$M_t = \begin{pmatrix} 1.0010 - 0.014195\lambda + 0.29090\lambda^2 - 0.096420\lambda^3 + 0.020890\lambda^4 - \\ 0.0030540\lambda^5 + 2.9570E-04\lambda^6 - 1.8462E-05\lambda^7 + 7.1553E-07\lambda^8 - \\ 1.5631E-08\lambda^9 + 1.4656E-10\lambda^{10} \end{pmatrix}$$

$$M_t = \begin{pmatrix} 1.0010 - 0.014195(0.7083) + 0.29090(0.7083)^2 - 0.096420(0.7083)^3 + \\ 0.020890(0.7083)^4 - 0.0030540(0.7083)^5 + 2.9570E-04(0.7083)^6 - \\ 1.8462E-05(0.7083)^7 + 7.1553E-07(0.7083)^8 - \\ 1.5631E-08(0.7083)^9 + 1.4656E-10(0.7083)^{10} \end{pmatrix}$$

$$M_t = 1.1074$$

Since $\{RSF = 0.9839\} \geq \{RSF_a = 0.98\}$, the longitudinal extent of the flaw is acceptable.

i) STEP 9 – Evaluate circumferential extent of the flaw.

1) STEP 9.1 – If Equation (5.13) I satisfied, the circumferential extent is acceptable, and no further evaluation is required.

$$\{c = 1.0\ in\} \le \left\{2s\left(\frac{E_c}{E_L}\right) = 2(2.0)\left(\frac{1.0}{1.0}\right) = 4.0\ in\right\} \qquad True$$

Since the above criteria is satisfied, the circumferential extent of the flaw is acceptable.

The Level 1 Assessment Criteria are satisfied. Therefore, the LTA is permissible and weld repair is not required.

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4.15 Supports and Attachments

4.15.1 Example E4.15.1 – Horizontal Vessel Supported by Two Saddles

Determine if the stresses in the horizontal vessel induced by the proposed saddle supports are with acceptable limits. The vessel is supported by two symmetric equally spaced saddles welded to the vessel, without reinforcing plates or stiffening rings. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined. See Figure E4.15.1.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $1800\ psig\ @175°F$
- Inside Cylinder Diameter = $60.0\ in$
- Cylinder Thickness = $3.0\ in$
- Corrosion Allowance = $0.125\ in$
- Formed Head Type = $2:1\ Elliptical$
- Head Thickness = $3.0\ in$
- Allowable Stress = $23500\ psi$
- Yield Stress at Design Temperature = $35250\ psi$
- Weld Joint Efficiency = 1.0
- Shell Tangent to Tangent Length = $292.0\ in$

Saddle Data:

- Material = $SA-516,\ Grade\ 70$
- Saddle Center Line to Head Tangent Line = $41.0\ in$
- Saddle Contact Angle = $123.0\ deg$
- Width of Saddles = $8.0\ in$
- Vessel Load per Saddle = $50459.0\ lbs$

Adjust the vessel inside diameter and thickness by the corrosion allowance.

$$ID = ID_{uc} + 2(Corrosion\ \ Allowance) = 60.0 + 2(0.125) = 60.25\ in$$

$$t = t - Corrosion\ \ Allowance = 3.0 - 0.125 = 2.875\ in$$

$$t_h = t_h - Corrosion\ \ Allowance = 3.0 - 0.125 = 2.875\ in$$

$$R_m = \frac{OD + ID}{4} = \frac{66.0 + 60.25}{4} = 31.5625\ in$$

$$R_{mh} = \frac{OD + ID}{4} = \frac{66.0 + 60.25}{4} = 31.5625\ in$$

$$h_m = \left(0.25\left(ID_{uc}\right) + Corrosion\ \ Allowance\right) + 0.5t_h$$

$$h_m = \left(0.25(60.0) + 0.125\right) + 0.5(2.875) = 16.5625\ in$$

Per Paragraph 4.15.3.

Paragraph 4.15.3.1, Application of Rules:

a) The stress calculation method is based on linear elastic mechanics and covers modes of failure by excessive deformation and elastic instability.

b) Saddle supports for horizontal vessels shall be configured to provide continuous support for at least one-third of the shell circumference, or $\theta = 120.0\ deg$.

Since $\{\theta = 123.0\ deg\} \geq \{\theta_{req} = 120.0\ deg\}$ the geometry is acceptable.

Paragraph 4.15.3.2, Moment and Shear Force:

The vessel is composed of a cylindrical shell with formed heads at each end that is supported by two equally spaced saddle supports. The moment at the saddle, M_1, the moment at the center of the vessel, M_2, and the shear force at the saddle, T, may be computed if the distance between the saddle centerline and head tangent line satisfies the following limit.

$$\{a = 41.0\ in\} \leq \{0.25L = 0.25(292.0) = 73.0\ in\} \qquad Satisfied$$

Bending Moment at the Saddle:

$$M_1 = -Qa\left(1 - \frac{1 - \frac{a}{L} + \frac{R_m^2 - h_m^2}{2aL}}{1 + \frac{4h_m}{3L}}\right)$$

$$M_1 = -(50459.0)(41.0)\left(1 - \frac{1 - \left(\frac{41.0}{292.0}\right) + \frac{(31.5625)^2 - (16.5625)^2}{2(41.0)(292.0)}}{1 + \frac{4(16.5625)}{3(292.0)}}\right)$$

$$M_1 = -357533.9\ in-lbs$$

Bending Moment at the Center of the Vessel:

$$M_2 = \frac{QL}{4}\left(\frac{1+\dfrac{2\left(R_m^2-h_m^2\right)}{L^2}}{1+\dfrac{4h_m}{3L}} - \frac{4a}{L}\right)$$

$$M_2 = \frac{50459.0(292.0)}{4}\left(\frac{1+\dfrac{2\left[(31.5625)^2-(16.5625)^2\right]}{(292.0)^2}}{1+\dfrac{4(16.5625)}{3(292.0)}} - \frac{4(41.0)}{292.0}\right)$$

$$M_2 == 1413685.4\ in-lbs$$

Shear Force at the Saddle:

$$T = \frac{Q(L-2a)}{L+\dfrac{4h_m}{3}} = \frac{50459.0\left[292.0-2(41.0)\right]}{292.0+\dfrac{4(16.5625)}{3}} = 33737.5\ lbs$$

Paragraph 4.15.3.3, Longitudinal Stress:

a) The longitudinal membrane plus bending stresses in the cylindrical shell between the supports are given by the following equations.

At the top of shell:

$$\sigma_1 = \frac{PR_m}{2t} - \frac{M_2}{\pi R_m^2 t} = \frac{1800(31.5625)}{2(2.875)} - \frac{1413685.4}{\pi(31.5625)^2(2.875)} = 9723.3\ psi$$

Note: A load combination that includes zero internal pressure and the vessel full of contents would provide the largest compressive stress at the top of the shell and should be checked as part of the design.

At the bottom of the shell:

$$\sigma_2 = \frac{PR_m}{2t} + \frac{M_2}{\pi R_m^2 t} = \frac{1800(31.5625)}{2(2.875)} + \frac{1413685.4}{\pi(31.5625)^2(2.875)} = 10037.6\ psi$$

b) The longitudinal stresses in the cylindrical shell at the support location are given by the following equations. The values of these stresses depend on the rigidity of the shell at the saddle support. The cylindrical shell may be considered as suitably stiffened if it incorporates stiffening rings at, or on both sides of the saddle support, or if the support is sufficiently close defined as $a \le 0.5R_m$ to the elliptical head.

Since $\{a = 41.0\ in\} > \{0.5R_m = 0.5(31.5625) = 15.7813\ in\}$, the criterion is not satisfied.

Therefore, for an unstiffened shell, calculate the maximum values of longitudinal membrane plus bending stresses at the saddle support as follows.

At points A and B in Figure 4.15.5:

$$\sigma_3^* = \frac{PR_m}{2t} - \frac{M_1}{K_1 \pi R_m^2 t} = \frac{1800(31.5625)}{2(2.875)} - \frac{-357533.9}{0.1114(\pi)(31.5625)^2(2.875)} = 10237.1\ psi$$

where the coefficient K_1 is found in Table 4.15.1,

$$K_1 = \frac{\Delta + \sin\Delta \cdot \cos\Delta - \dfrac{2\sin^2\Delta}{\Delta}}{\pi\left(\dfrac{\sin\Delta}{\Delta} - \cos\Delta\right)} = \frac{1.4181 + \sin[1.4181] \cdot \cos[1.4181] - \dfrac{2\sin^2[1.4181]}{1.4181}}{\pi\left(\dfrac{\sin[1.4181]}{1.4181} - \cos[1.4181]\right)}$$

$$K_1 = 0.1114$$

$$\Delta = \frac{\pi}{6} + \frac{5\theta}{12} = \frac{\pi}{6} + \frac{5\left[(123.0)\left(\dfrac{\pi}{180}\right)\right]}{12} = 1.4181\ rad$$

At the bottom of the shell:

$$\sigma_4^* = \frac{PR_m}{2t} + \frac{M_1}{K_1^* \pi R_m^2 t} = \frac{1800(31.5625)}{2(2.875)} + \frac{-357533.9}{0.2003(\pi)(31.5625)^2(2.875)} = 9682.1\ psi$$

where the coefficient K_1^* is found in Table 4.15.1,

$$K_1^* = \frac{\Delta + \sin\Delta \cdot \cos\Delta - \dfrac{2\sin^2\Delta}{\Delta}}{\pi\left(1 - \dfrac{\sin\Delta}{\Delta}\right)} = \frac{1.4181 + \sin[1.4181] \cdot \cos[1.4181] - \dfrac{2\sin^2[1.4181]}{1.4181}}{\pi\left(1 - \dfrac{\sin[1.4181]}{1.4181}\right)}$$

$$K_1^* = 0.2003$$

c) Acceptance Criteria:

$$\left\{|\sigma_1| = |9723.3|\ psi\right\} \le \left\{SE = 23500(1.0) = 23500\ psi\right\} \qquad True$$

$$\left\{|\sigma_2| = |10037.6|\ psi\right\} \le \left\{SE = 23500(1.0) = 23500\ psi\right\} \qquad True$$

$$\left\{|\sigma_3^*| = |10237.1|\ psi\right\} \le \left\{SE = 23500(1.0) = 23500\ psi\right\} \qquad True$$

$$\left\{|\sigma_4^*| = |9682.1|\ psi\right\} \le \left\{SE = 23500(1.0) = 23500\ psi\right\} \qquad True$$

Since all calculated stresses are positive (tensile), the compressive stress check per paragraph 4.15.3.3.c.2 is not required.

Paragraph 4.15.3.4, Shear Stresses:

The shear stress in the cylindrical shell without stiffening ring(s) that is not stiffened by a formed head, $\{a = 41.0\ in\} > \{0.5R_m = 0.5(31.5625) = 15.7813\ in\}$, is calculated as follows.

$$\tau_2 = \frac{K_2 T}{R_m t} = \frac{1.1229(33737.5)}{31.5625(2.875)} = 417.5 \ psi$$

where the coefficient K_2 is found in VIII-2, Table 4.15.1,

$$K_2 = \frac{\sin\alpha}{\pi - \alpha + \sin\alpha \cdot \cos\alpha} = \frac{\sin[1.9648]}{\pi - (1.9648) + \sin[1.9648] \cdot \cos[1.9648]} = 1.1229$$

$$\alpha = 0.95\left(\pi - \frac{\theta}{2}\right) = 0.95\left(\pi - \frac{123.0\left(\frac{\pi}{180}\right)}{2}\right) = 1.9648 \ rad$$

Acceptance Criteria:

$$|\tau_2| \le \min\left[0.8S, 0.533S_y\right]$$

$$|417.5| \ psi \le \left\{\min\left[0.8(23500), 0.533(35250)\right] = 18800 \ psi\right\} \quad True$$

Paragraph 4.15.3.5, Circumferential Stress:

a) Maximum circumferential bending moment – the distribution of the circumferential bending moment at the saddle support is dependent on the use of stiffeners at the saddle location. For a cylindrical shell without a stiffening ring, the maximum circumferential bending moment is shown in Figure 4.15.6 Sketch (a) and is calculated as follows.

$$M_\beta = K_7 Q R_m = (0.0504)(50459.0)(31.5625) = 80267.7 \ in-lbs$$

where the coefficient K_7 is found in Table 4.15.1,

when $a/R_m \ge 1.0$, $K_7 = K_6$,

$$\left\{\frac{a}{R_m} = \frac{41.0}{31.5625} = 1.2990\right\} \ge 1.0 \rightarrow K_7 = K_6 = 0.0504$$

$$K_6 = \left(\frac{\begin{array}{c}\dfrac{3\cos\beta}{4}\left(\dfrac{\sin\beta}{\beta}\right)^2 - \dfrac{5\sin\beta\cos^2\beta}{4\beta} + \dfrac{\cos^3\beta}{2} - \dfrac{\sin\beta}{4\beta} + \\ \dfrac{\cos\beta}{4} - \beta\sin\beta\left[\left(\dfrac{\sin\beta}{\beta}\right)^2 - \dfrac{1}{2} - \dfrac{\sin 2\beta}{4\beta}\right]\end{array}}{2\pi\left[\left(\dfrac{\sin\beta}{\beta}\right)^2 - \dfrac{1}{2} - \dfrac{\sin 2\beta}{4\beta}\right]} \right)$$

$$K_6 = \left(\frac{\begin{gathered}\frac{3\cos[2.0682]}{4}\left(\frac{\sin[2.0682]}{2.0682}\right)^2 - \frac{5\sin[2.0682]\cos^2[2.0682]}{4(2.0682)} + \\ \frac{\cos^3[2.0682]}{2} - \frac{\sin[2.0682]}{4(2.0682)} + \frac{\cos[2.0682]}{4} - \\ (2.0682)\sin[2.0682]\left[\left(\frac{\sin[2.0682]}{2.0682}\right)^2 - \frac{1}{2} - \left(\frac{\sin[2(2.0682)]}{4(2.0682)}\right)\right]\end{gathered}}{2\pi\left[\left(\frac{\sin[2.0682]}{2.0682}\right)^2 - \frac{1}{2} - \left(\frac{\sin[2(2.0682)]}{4(2.0682)}\right)\right]} \right) = 0.0504$$

$$\beta = \pi - \frac{\theta}{2} = \pi - \frac{123.0\left(\frac{\pi}{180}\right)}{2} = 2.0682\ rad$$

b) Width of cylindrical shell – the width of the cylindrical shell that contributes to the strength of the cylindrical shell at the saddle location shall be determined as follows.

$$\{x_1, x_2\} \le \left\{0.78\sqrt{R_m t} = 0.78\sqrt{31.5625(2.875)} = 7.4302\ in\right\}$$

If the width $(0.5b + x_1)$ extends beyond the limit of a, as shown in Figure 4.15.2, then the width x_1 shall be reduced such as not to exceed a.

$$\{(0.5b + x_1) = 0.5(8.0) + 7.4302 = 11.4302\ in\} \le \{a = 41.0\ in\} \qquad \textit{Satisfied}$$

c) Circumferential stresses in the cylindrical shell without stiffening ring(s).

The maximum compressive circumferential membrane stress in the cylindrical shell at the base of the saddle support shall be calculated as follows.

$$\sigma_6 = \frac{-K_5 Q k}{t(b + x_1 + x_2)} = \frac{-0.7492(50459.0)(0.1)}{2.875(8.0 + 7.4302 + 7.4302)} = -57.5\ psi$$

where the coefficient K_5 is found in Table 4.15.1,

$$K_5 = \frac{1 + \cos\alpha}{\pi - \alpha + \sin\alpha \cdot \cos\alpha} = \frac{1 + \cos[1.9648]}{\pi - (1.9648) + \sin[1.9648] \cdot \cos[1.9648]} = 0.7492$$

$$k = 0.1 \qquad \textit{when the vessel is welded to the saddle support}$$

The circumferential compressive membrane plus bending stress at Points G and H of Figure 4.15.6 Sketch (a) is determined as follows.

If $L \geq 8R_m$, then the circumferential compressive membrane plus bending stress shall be computed using Equation (4.15.24).

Since $\{L = 292.0\ in\} \geq \{8R_m = 8(31.5625) = 252.5\ in\}$, the criterion is satisfied.

$$\sigma_7 = \frac{-Q}{4t(b+x_1+x_2)} - \frac{3K_7Q}{2t^2}$$

$$\sigma_7 = \frac{-(50459.0)}{4(2.875)(8+7.4302+7.4302)} - \frac{3(0.0504)(50459.0)}{2(2.875)^2} = -653.4\ psi$$

The stresses at σ_6 and σ_7 may be reduced by adding a reinforcement or wear plate at the saddle location that is welded to the cylindrical shell.

A wear plate was not specified in this problem.

Acceptance Criteria:

$$\{|\sigma_6| = |57.5|\ psi\} \leq \{S = 23500\ psi\} \quad True$$

$$\{|\sigma_7| = |653.4|\ psi\} \leq \{1.25S = 1.25(23500) = 29375\ psi\} \quad True$$

Paragraph 4.15.3.6, Horizontal Splitting Force:

The horizontal force at the minimum section at the low point of the saddle is given by Equation (4.15.42). The saddle shall be designed to resist this force.

$$F_h = Q\left(\frac{1+\cos\beta - 0.5\sin^2\beta}{\pi - \beta + \sin\beta\cdot\cos\beta}\right)$$

$$F_h = (50459.0)\left(\frac{1+\cos[2.0682] - 0.5\sin^2[2.0682]}{\pi - (2.0682) + \sin[2.0682]\cdot\cos[2.0682]}\right) = 10545.1\ lbs$$

Commentary: The horizontal splitting force is equal to the sum of all the horizontal reactions at the saddle due to the weight loading of the vessel. The splitting force is used to calculate tension stress and bending stress in the web of the saddle. The following provides one possible method of calculating the tension and bending stress in the web and its acceptance criteria. However, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

The membrane stress is given by,

$$\left\{\sigma_t = \frac{F_h}{A_s}\right\} \leq \{0.6S_y\}$$

where A_2 is the cross-sectional area of the web at the low point of the saddle with units of in^2, and S_y is the yield stress of the saddle material with units of psi.

The bending stress is given by,

$$\left\{\sigma_b = \frac{F_h \cdot d \cdot c}{I}\right\} \le \left\{0.66 S_y\right\}$$

where d is the moment arm of the horizontal splitting force, measured from the center of gravity of the saddle arc to the bottom of the saddle baseplate with units of in, c is the distance from the centroid of the saddle composite section to the extreme fiber with units of in, I is the moment of inertia of the composite section of the saddle with units of in^4, and S_y is the yield stress of the saddle material with units of psi.

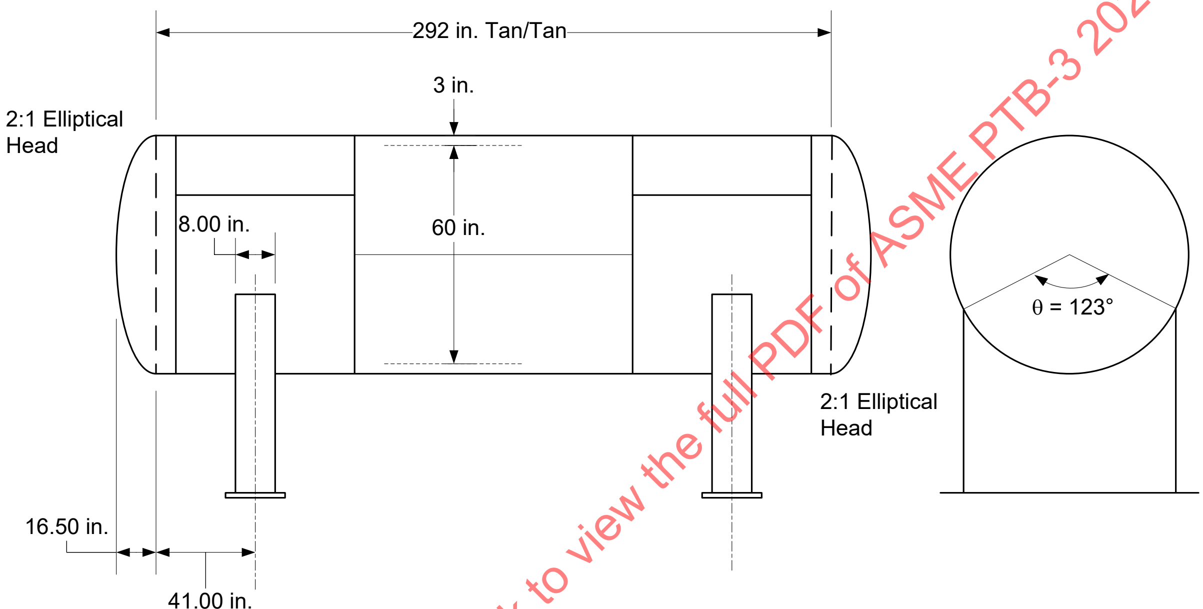

Figure E4.15.1 – Saddle Details

4.15.2 Example E4.15.2 – Vertical Vessel, Skirt Design

Determine if the proposed cylindrical vessel skirt is adequately designed considering the following loading conditions.

Skirt Data:

- Material = $SA-516,\ Grade\ 70$
- Design Temperature = $300°F$
- Skirt Inside Diameter = $150.0\ in$
- Thickness = $0.625\ in$
- Corrosion Allowance = $0.0\ in$
- Length of Skirt = $147.0\ in$
- Allowable Stress at Design Temperature = $22400\ psi$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength at Design Temperature = $33600\ psi$
- Design Loads = See Table E4.15.2.3

Adjust variable for corrosion and determine outside dimensions.

$$D = 150.0 + 2(Corrosion\ Allowance) = 150.0 + 2(0.0) = 150.0\ in$$

$$R = 0.5D = 0.5(150.0) = 75.0\ in$$

$$t = 0.625 - Corosion\ Allowance = 0.625 - 0.0 = 0.625\ in$$

$$D_o = 150.0 + 2(Uncorroded\ Thickness) = 150.0 + 2(0.625) = 151.25\ in$$

$$R_o = 0.5D_o = 0.5(151.25) = 75.625\ in$$

Per Paragraph 4.1.5.3.

This example uses paragraph 4.1.5.3 which provides specific requirements to account for both design loads and design load combinations used in the design of a vessel. These design loads and design load combinations (Table 4.1.1 and Table 4.1.2, respectively) are shown in this example problem in Table E4.15.2.1 and Table E4.15.2.2 for reference. The load factor, Ω_p, shown in Table 4.1.2 is used to simulate the maximum anticipated operating pressure acting simultaneously with the occasional loads specified. For this example, problem, $\Omega_p = 1.0$.

Per Paragraph 4.3.10

In accordance with paragraph 4.3.10.1, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. Determine applicability of the rules of paragraph 4.3.10.2 based on satisfaction of the following requirements.

- The section of interest is at least $2.5\sqrt{Rt}$ away from any major structural discontinuity.

$$2.5\sqrt{Rt} = 2.5\sqrt{(75.0)(0.625)} = 17.1163\ in \qquad True$$

- Shear force is not applicable.
- The shell R/t ratio is greater than 3.0, or:

$$\left\{R/t=\frac{75.0}{0.625}=120.0\right\}>3.0 \qquad\qquad True$$

In accordance with Paragraph 4.3.10.2, the following procedure shall be used to determine the acceptance criteria for stresses developed in cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. By inspection of the results shown in Table E4.15.2.3 and Table E4.15.2.4, Design Load Combination 5 is determined to the governing load combination. The pressure, net section axial force, and bending moment at the location of interest for Design Load Combination 5 are:

$$\Omega P+P_s=1.0P+P_s=0.0\ psi$$
$$F_5=-363500\ lbs$$
$$M_5=29110000\ in-lbs$$

a) STEP 1 – Calculate the membrane stress for the cylindrical shell. For the skirt, weld joint efficiency is set as $E=1.0$.

Note: θ is defined as the angle measured around the circumference from the direction of the applied bending moment to the point under consideration. For this example, problem $\theta=0.0\ deg$ to maximize the bending stress.

$$\sigma_{\theta m}=\frac{P}{E\left(D_o-D\right)}=\frac{0.0}{1.0\left(151.25-150.0\right)}=0.0\ psi$$

$$\sigma_{sm}=\frac{1}{E}\left(\frac{PD^2}{D_o^2-D^2}+\frac{4F}{\pi\left(D_o^2-D^2\right)}\pm\frac{32MD_o\cos\left[\theta\right]}{\pi\left(D_o^4-D^4\right)}\right)$$

$$\sigma_{sm}=\frac{1}{1.0}\left(\begin{array}{l}\left(\dfrac{0.0\left(150.0\right)^2}{\left(151.25\right)^2-\left(150.0\right)^2}\right)+\dfrac{4\left(-363500\right)}{\pi\left(\left(151.25\right)^2-\left(150.0\right)^2\right)}\pm\\ \dfrac{32\left(29110000\right)\left(151.25\right)\cos\left[0.0\right]}{\pi\left(\left(151.25\right)^4-\left(150.0\right)^4\right)}\end{array}\right)$$

$$\sigma_{sm}=\left\{\begin{array}{l}0.0+\left(-1229.0724\right)+2624.6357=1395.5633\ psi\\ 0.0+\left(-1229.0724\right)-2624.6357=-3853.7081\ psi\end{array}\right\}$$

$$\tau=\frac{16M_tD_o}{\pi\left(D_o^4-D^4\right)}=\frac{16\left(0.0\right)\left(151.25\right)}{\pi\left(\left(151.25\right)^4-\left(150.0\right)^4\right)}=0.0\ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\left(\tau\right)^2}\right)$$

$$\sigma_1 = \left\{ \begin{array}{l} 0.5\left(0 + (1395.5633) + \sqrt{\left(0 - (1395.5633)\right)^2 + 4(0)^2}\right) = 1395.5633\ psi \\ 0.5\left(0 + (-3853.7081) + \sqrt{\left(0 - (-3853.7081)\right)^2 + 4(0)^2}\right) = 0.0\ psi \end{array} \right\}$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2}\right)$$

$$\sigma_2 = \left\{ \begin{array}{l} 0.5\left(0 + (1395.5633) - \sqrt{\left(0 - (1395.5633)\right)^2 + 4(0)^2}\right) = 0.0\ psi \\ 0.5\left(0 + (-3853.7081) - \sqrt{\left(0 - (-3853.7081)\right)^2 + 4(0)^2}\right) = -3853.7081\ psi \end{array} \right\}$$

$$\sigma_3 = \sigma_r = 0.0\ psi \qquad For\ stress\ on\ the\ outside\ surface$$

c) STEP 3 – Check the allowable stress acceptance criteria.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]^{0.5}$$

$$\sigma_e = \left\{ \begin{array}{l} \dfrac{1}{\sqrt{2}}\left[\begin{array}{l} \left(0 - (1395.5633)\right)^2 + \left((1395.5633) - 0\right)^2 + \\ \left((0) - (0)\right)^2 \end{array} \right]^{0.5} = 1395.6\ psi \\ \dfrac{1}{\sqrt{2}}\left[\begin{array}{l} \left(0 - (-3853.7081)\right)^2 + \left((-3853.7081) - 0\right)^2 + \\ \left((0) - (0)\right)^2 \end{array} \right]^{0.5} = 3853.7\ psi \end{array} \right\}$$

$$\left\{ \begin{array}{l} \sigma_e = 1395.6 \\ \sigma_e = 3853.7 \end{array} \right\} \le \left\{S = 22400\ psi\right\} \qquad True$$

Since the maximum tensile principal stress is less than the acceptance criteria, the skirt section is adequately designed.

d) STEP 4 – For cylindrical and conical shells, if the meridional stress, σ_{sm} is compressive, then check the allowable compressive stress using paragraph 4.4.12.2 with $\lambda = 0.15$.

Since σ_{sm} is compressive, $\{\sigma_{sm} = -3853.7\ psi < 0\}$, a compressive stress check is required.

In accordance with paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

1) STEP 4.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$$F_{xe} = \frac{C_x E_y t}{D_o} = \frac{0.6482(28.3E+06)(0.625)}{151.25} = 75801.9008\ psi$$

where,

$$\frac{D_o}{t} = \frac{151.25}{0.625} = 242.0$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{147.0}{\sqrt{75.625(0.625)}} = 21.3818$$

Since $D_o/t < 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \frac{D_o}{t}\right)},\ 0.9\right] = \min\left[\frac{409(1.0)}{389 + \frac{151.25}{0.625}},\ 0.9\right] = 0.6482$$

Since $M_x \geq 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

2) STEP 4.2 – Calculate the predicted inelastic buckling stress, F_{ic}, per paragraph 4.4.3.

The equations for the allowable compressive stress consider both the predicted elastic buckling stress and predicted inelastic buckling stress. The predicted elastic buckling stress, F_{xe}, is determined based on the geometry of the component and the loading under consideration as provided in subsequent applicable paragraphs. The predicted inelastic buckling stress, F_{ic}, is determined using the following procedure.

i) STEP 4.2.1 – Calculate the predicted elastic buckling stress, F_{xe}.

$F_{xe} = 75801.9008\ psi$ (as determined in STEP 2 above)

ii) STEP 4.2.2 – Calculate the elastic buckling ratio factor, A_e.

$$A_e = \frac{F_{xe}}{E} = \frac{75801.9008}{28.3E+06} = 0.00267851$$

iii) STEP 4.2.3 – Solve for the predicted inelastic buckling stress, F_{ic}, through the determination of the material's tangent modulus, E_t, based on the stress-strain curve model at the design temperature per paragraph 3-D.5.1. The value of F_{ic} is solved for using an iterative procedure such that the following relationship is satisfied (see Table 4.4.2).

SEE EXAMPLE PROBLEM E4.4.1 FOR THE ITERATIVE PROCEDURE.

$$F_{ic} = 24624.7292\ psi$$

3) STEP 4.3 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{24624.7292}{33600}\right) = 1.8639$$

4) STEP 4.4 – Calculate the allowable axial compressive membrane stress as follows:

$$F_{xa} = \frac{F_{ic}}{FS} = \frac{24624.7292}{1.8639} = 13211.4004\ psi$$

5) STEP 4.5 – Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial

compressive membrane stress, F_{xa} per following criteria.

$$\{\sigma_{sm} = 3853.7\ psi\} \le \{F_{xa} = 13211.4\ psi\} \qquad True$$

The allowable compressive stress criterion is satisfied.

Table E4.15.2.1 – Design Loads

Table 4.1.1 – Design Loads	
Design Load Parameter	**Description**
P	Internal of External Specified Design Pressure (see paragraph 4.1.5.2.a)
P_s	Static head from liquid or bulk materials (e.g., catalyst)
D	Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following: • Weight of vessel including internals, supports (e.g., skirts, lugs, saddles, and legs), and appurtenances (e.g., platforms, ladders, etc.) • Weight of vessel contents under design and test conditions • Refractory linings, insulation • Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping • Transportation loads (the static forces obtained as equivalent to the dynamic loads experienced during normal operation of a transport vessel [see paragraph 1.2.1.2(b)]
L	• Appurtenance live loading • Effects of fluid flow, steady state or transient • Loads resulting from wave action
E	Earthquake loads [see paragraph 4.1.5.3(b)]
W	Wind loads [see paragraph 4.1.5.3(b)]
S_s	Snow loads
F	Loads due to deflagration

Table E4.15.2.2 – Design Load Combinations

Table 4.1.2 – Design Load Combinations	
Design Load Combination [Note (1) and (2)]	**General Primary Membrane Allowable Stress [Note (3)]**
$P+P_s+D$	S
$P+P_s+D+L$	S
$P+P_s+D+S_s$	S
$\Omega P+P_s+D+0.75L+0.75S_s$	S
$\Omega P+P_s+D+(0.6W\ or\ 0.7E)$	S
$\Omega P+P_S+D+0.75(0.6W\ or\ 0.7E)+0.75L+0.75S_s$	S
$0.6D+(0.6W\ or\ 0.7E)$ [Note (4)]	S
P_s+D+F	See Annex 4-D
Other load combinations as defined in the UDS	S

Notes:

1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.
2) See paragraph 4.1.5.3 for additional requirements.
3) S is the allowable stress for the load case combination [see paragraph 4.1.5.3(c)].
4) This load combination addresses an overturning condition for foundation design. It does not apply to design of anchorage (if any) to the foundation. Refer to ASCE/SEI 7, 2.4.1 Exception 2 for an additional reduction to W that may be applicable.

Table E4.15.2.3 – Design Loads (Net-Section Axial Force and Bending Moment) at the Base of The Skirt

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a); The skirt is not pressurized.	$P = 0.0$
P_s	Static head from liquid or bulk materials (e.g., catalyst); The skirt does not contain liquid head.	$P_s = 0.0$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest.	$D_F = -363500\ lbs$ $D_M = 0.0\ in - lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = -85700\ lbs$ $L_M = 90580\ in - lbs$
E	Earthquake loads	$E_F = 0.0\ lbs$ $E_M = 18550000\ in - lbs$
W	Wind Loads	$W_F = 0.0\ lbs$ $W_M = 48516667\ in - lbs$
S_s	Snow Loads	$S_{sF} = 0.0\ lbs$ $S_{sM} = 0.0\ in - lbs$
F	Loads due to Deflagration	$F_F = 0.0\ lbs$ $F_M = 0.0\ in - lbs$

Based on these loads, the skirt is required to be designed for the design load combinations shown in Table E4.15.2.4. Note that this table is given in terms of the design load combinations shown Table 4.1.2 (Table E4.15.2.2 of this example).

Table E4.15.2.4 – Design Load Combination at the Base of the Skirt

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P+P_s+D$	$P=P_s=0.0\ psi$ $F_1=-363500\ lbs$ $M_1=0.0\ in-lbs$	S
2	$P+P_s+D+L$	$P=P_s=0.0\ psi$ $F_2=-449200\ lbs$ $M_2=90580\ in-lbs$	S
3	$P+P_s+D+S_s$	$P=P_s=0.0\ psi$ $F_3=-363500\ lbs$ $M_3=0.0\ in-lbs$	S
4	$\Omega P+P_s+D+0.75L+0.75S_s$	$\Omega P=P_s=0.0\ psi$ $F_4=-427775\ lbs$ $M_4=67935\ in-lbs$	S
5	$\Omega P+P_s+D+(0.6W\ or\ 0.7E)$	$\Omega P=P_s=0.0\ psi$ $F_5=-363500\ lbs$ $M_5=29110000\ in-lbs$	S
6	$\Omega P+P_S+D+0.75(0.6W\ or\ 0.7E)+$ $0.75L+0.75S_s$	$\Omega P=P_s=0.0\ psi$ $F_6=-427775\ lbs$ $M_6=21900435\ in-lbs$	S
7	$0.6D+(0.6W\ or\ 0.7E)$	$F_7=-218100\ lbs$ $M_7=29110000\ in-lbs$	S
8	P_s+D+F	$P=P_s=0.0\ psi$ $F_8=-363500\ lbs$ $M_8=0.0\ in-lbs$	See Annex 4-D

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4.16 Flanged Joints

4.16.1 Example E4.16.1 – Integral Type

Determine if the stresses in the heat exchanger girth flange are with acceptable limits, considering the following design conditions. The flange is of an integral type and is attached to a cylindrical shell with a Category C, Type 1 butt weld and has been 100% radiographically examined. See Figure E4.16.1.

General Data:

- Cylinder Material = $SA-516,\ Grade\ 70$
- Design Conditions = $135\ psig\ @650°F$
- Allowable Stress at Design Temperature = $18800\ psi$
- Allowable Stress at Ambient Temperature = $25300\ psi$
- Corrosion Allowance = $0.125\ in$

Flange Data:

- Material = $SA-105$
- Allowable Stress at Design Temperature = $17800\ psi$
- Allowable Stress at Ambient Temperature = $24000\ psi$
- Modulus of Elasticity at Design Temperature = $26.0E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$

Bolt Data:

- Material = $SA-193,\ Grade\ B7$
- Allowable Stress at Design Temperature = $25000\ psi$
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Diameter = $0.75\ in$
- Number of Bolts = 44
- Root area = $0.302\ in^2$

Gasket Data:

- Material = Flat Metal Jacketed (Iron/Soft Steel)
- Gasket Factor = 3.75
- Seating Stress = $7600\ psi$
- Inside Diameter = $29.0\ in$
- Outside Diameter = $30.0\ in$

Evaluate the girth flange in accordance with paragraph 4.16.

Paragraph 4.16.6, Design Bolt Load. The procedure to determine the bolt loads for the operating and gasket seating conditions is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

$$P = 135\ psig\ at\ 650°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 4.16.1.

$$m = 3.75$$
$$y = 7600\ psi$$

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5(GOD - GID) = 0.5(30.0 - 29.0) = 0.500\ in$$

from Table 4.16.3, Facing Sketch Detail 2, Column II,

$$b_o = \frac{w + 3N}{8} = \frac{0.125 + 3(0.500)}{8} = 0.2031\ in$$

where,

$$w = raised\ nubbin\ width = 0.125\ in$$

for $b_o \leq 0.25\ in$,

$$b = b_o = 0.2031\ in$$

therefore, from Figure 4.16.8 the location of the gasket reaction is calculated as follows.

$$G = mean\ diameter\ of\ the\ gasket\ contact\ face$$
$$G = 0.5(30.0 + 29.0) = 29.5\ in$$

d) STEP 4 – Determine the design bolt load for the operating condition.

$$W_o = H + H_p = 0.785G^2P + (2b \cdot \pi GmP) \qquad for\ non-self-energized\ gaskets$$
$$W_o = 0.785(29.5)^2(135) + 2(0.2031)(\pi)(29.5)(3.75)(135) = 111282.7\ lbs$$

e) STEP 5 – Determine the design bolt load for the gasket seating condition.

$$W_g = \left(\frac{A_m + A_b}{2}\right)S_{bg} = \left(\frac{5.7221 + 13.2880}{2}\right)25000 = 237626.3\ lbs$$

where, the parameter A_b is the actual cross-sectional area of the bolts that is selected such that $A_b \geq A_m$.

$$A_b = (Number\ of\ bolts)(Root\ area\ of\ one\ bolt) = 44(0.302) = 13.2880\ in^2$$

$$A_m = \max\left[\left(\frac{W_o + F_A + \frac{4M_E}{G}}{S_{bo}}\right), \left(\frac{W_{gs}}{S_{bg}}\right)\right] = \max\left[\left(\frac{111282.7 + 0.0 + 0.0}{25000}\right), \left(\frac{143052.5}{25000}\right)\right]$$

$$A_m = \max[4.4513, 5.7221] = 5.7221\ in^2$$

and,

$$W_{gs} = \pi b G y \qquad for\ non-self-energized\ gaskets$$

$$W_{gs} = \pi(0.2031)(29.5)(7600) = 143052.5\ lbs$$

and $F_A = 0$ and $M_E = 0$ since there are no externally applied net-section forces and bending moments.

Paragraph 4.16.7, Flange Design Procedure. The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings. The procedure incorporates both a strength check and a rigidity check for flange rotation.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint and the external net-section axial force F_A and bending moment M_E.

$$P = 135\ psig\ at\ 650°F$$

$$F_A = 0$$

$$M_E = 0$$

b) STEP 2 – Determine the design bolt loads for operating condition W_o, and the gasket seating condition W_g, and the corresponding actual bolt load area A_b, from paragraph 4.16.6.

$$W_o = 111282.7\ lbs$$

$$W_g = 237626.3\ lbs$$

$$A_b = 13.2880\ in^2$$

c) STEP 3 – Determine an initial flange geometry (see Figure E4.16.1) in addition to the information required to determine the bolt load, the following geometric parameters are required.

1) Flange bore

$$B = \left[26.0 + 2(Corrosion\ Allowance)\right] = \left[26.0 + 2(0.125)\right] = 26.25\ in$$

2) Bolt circle diameter

$$C = 31.25\ in$$

3) Outside diameter of the flange

$$A = 32.875\ in$$

4) Flange thickness

$$t = 1.625 - 0.1875 = 1.4375\ in$$

5) Thickness of the hub at the large end

$$g_1 = \left[0.5(Hub\ OD\ at\ Back\ of\ Flange - Uncorroded\ Bore) - Corrosion\ Allowance\right]$$

$$g_1 = \left[0.5(27.625 - 26.0) - 0.125\right] = 0.6875\ in$$

6) Thickness of the hub at the small end

$$g_0 = (Hub\ Thickness\ at\ Cylinder\ Attachment - Corrosion\ Allowance)$$

$$g_0 = (0.4375 - 0.125) = 0.3125\ in$$

7) Hub length

$$h = 2.125\ in$$

d) STEP 4 – Determine the flange stress factors using the equations in Tables 4.16.4 and 4.16.5.

Table 4.16.4:

$$K = \frac{A}{B} = \frac{32.875}{26.25} = 1.2524$$

$$Y = \frac{1}{K-1}\left[0.66845 + 5.71690\left(\frac{K^2 \log_{10} K}{K^2 - 1}\right)\right]$$

$$Y = \frac{1}{1.2524-1}\left[0.66845 + 5.71690\frac{(1.2524)^2 \log_{10}[1.2524]}{(1.2524)^2 - 1}\right] = 8.7565$$

$$T = \frac{K^2(1 + 8.55246\log_{10} K) - 1}{(1.04720 + 1.9448K^2)(K-1)} = \frac{(1.2524)^2(1 + 8.55246\log_{10}[1.2524]) - 1}{(1.04720 + 1.9448(1.2524)^2)(1.2524-1)} = 1.8175$$

$$U = \frac{K^2(1 + 8.55246\log_{10} K) - 1}{1.36136(K^2 - 1)(K-1)} = \frac{(1.2524)^2(1 + 8.55246\log_{10}[1.2524]) - 1}{1.36136((1.2524)^2 - 1)(1.2524-1)} = 9.6225$$

$$Z = \frac{(K^2 + 1)}{(K^2 - 1)} = \frac{((1.2524)^2 + 1)}{((1.2524)^2 - 1)} = 4.5180$$

$$h_o = \sqrt{Bg_0} = \sqrt{(26.25)(0.3125)} = 2.8641\ in$$

$$X_g = \frac{g_1}{g_0} = \frac{0.6875}{0.3125} = 2.2000$$

$$X_h = \frac{h}{h_o} = \frac{2.125}{2.8641} = 0.7419$$

$$d = \frac{Ug_0^2 h_o}{V} = \frac{(9.6225)(0.3125)^2(2.8641)}{0.1578} = 17.0557\ in^3$$

$$e = \frac{F}{h_o} = \frac{0.7695}{2.8641} = 0.2687\ in^{-1}$$

$$L = \frac{te+1}{T} + \frac{t^3}{d} = \frac{1.4375(0.2687)+1}{1.8175} + \frac{(1.4375)^3}{17.0557} = 0.9369$$

Where from Table 4.16.5,

$$F=\begin{pmatrix}0.897697-0.297012\ln\left[X_g\right]+9.5257\left(10^{-3}\right)\ln\left[X_h\right]+\\0.123586\left(\ln\left[X_g\right]\right)^2+0.0358580\left(\ln\left[X_h\right]\right)^2-\\0.194422\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right)-0.0181259\left(\ln\left[X_g\right]\right)^3+\\0.0129360\left(\ln\left[X_h\right]\right)^3-0.0377693\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right)^2+\\0.0273791\left(\ln\left[X_g\right]\right)^2\left(\ln\left[X_h\right]\right)\end{pmatrix}$$

$$F=\begin{pmatrix}0.897697-0.297012\ln\left[2.20\right]+9.5257\left(10^{-3}\right)\ln\left[0.7419\right]+\\0.123586\left(\ln\left[2.20\right]\right)^2+0.0358580\left(\ln\left[0.7419\right]\right)^2-\\0.194422\left(\ln\left[2.20\right]\right)\left(\ln\left[0.7419\right]\right)-0.0181259\left(\ln\left[2.20\right]\right)^3+\\0.0129360\left(\ln\left[0.7419\right]\right)^3-0.0377693\left(\ln\left[2.20\right]\right)\left(\ln\left[0.7419\right]\right)^2+\\0.0273791\left(\ln\left[2.20\right]\right)^2\left(\ln\left[0.7419\right]\right)\end{pmatrix}$$

$$F=0.7695$$

$$For\ 0.5\le X_h\le 2.0,$$

$$V=\begin{pmatrix}0.0144868-\dfrac{0.135977}{X_g}-\dfrac{0.0461919}{X_h}+\dfrac{0.560718}{X_g^2}+\dfrac{0.0529829}{X_h^2}+\\\dfrac{0.244313}{X_gX_h}+\dfrac{0.113929}{X_g^3}-\dfrac{0.00928265}{X_h^3}-\dfrac{0.0266293}{X_gX_h^2}-\dfrac{0.217008}{X_g^2X_h}\end{pmatrix}$$

$$V=\begin{pmatrix}0.0144868-\dfrac{0.135977}{2.20}-\dfrac{0.0461919}{0.7419}+\dfrac{0.560718}{(2.20)^2}+\dfrac{0.0529829}{(0.7419)^2}+\\\dfrac{0.244313}{(2.20)(0.7419)}+\dfrac{0.113929}{(2.20)^3}-\dfrac{0.00929265}{(0.7419)^3}-\dfrac{0.0266293}{(2.20)(0.7419)^2}-\dfrac{0.217008}{(2.20)^2(0.7419)}\end{pmatrix}$$

$$V=0.1578$$

$$f = \max\left[1.0, \left(\frac{0.0927779 - 0.0336633X_g + 0.964176X_g^2 + 0.0566286X_h + 0.347074X_h^2 - 4.18699X_h^3}{1 - 5.96093\left(10^{-3}\right)X_g + 1.62904X_h + 3.49329X_h^2 + 1.39052X_h^3}\right)\right]$$

$$f = \max\left[1.0, \left(\frac{0.0927779 - 0.0336633(2.20) + 0.964176(2.20)^2 + 0.0566286(0.7419) + 0.347074(0.7419)^2 - 4.18699(0.7419)^3}{1 - 5.96093\left(10^{-3}\right)(2.20) + 1.62904(0.7419) + 3.49329(2.20)^2 + 1.39052(0.7419)^3}\right)\right]$$

$$f = 1.0$$

e) STEP 5 – Determine the flange forces.

$$H_D = 0.785B^2P = 0.785(26.25)^2(135) = 73023.4\ lbs$$

$$H = 0.785G^2P = 0.785(29.5)^2(135) = 92224.7\ lbs$$

$$H_T = H - H_D = 92224.7 - 73023.4 = 19201.3\ lbs$$

$$H_G = W_o - H = 111282.7 - 92224.7 = 19058.0\ lbs$$

f) STEP 6 – Determine the flange moment for the operating condition. When specified by the user or his designated agent, the maximum bolt spacing, B_{smax}, and the bolt spacing correction factor, B_{SC}, shall be applied in calculating the flange moment for internal pressure using the equations in Table 4.16.11. The flange moment M_o for the operating condition and flange moment M_g for the gasket seating condition without correction for bolt spacing $B_{SC} = 1$ is used for the calculation of the rigidity index in STEP 10. In these equations, h_D, h_T, and h_G are determined from Table 4.16.6.

$$M_o = abs\left[\left(\left(H_D h_D + H_T h_T + H_G h_G\right)B_{SC} + M_{oe}\right)F_s\right] \quad Internal\ Pressure$$

$$M_o = abs\left[\left(\left(73023.4(2.1563) + 19201.3(1.6875) + 19058.0(0.875)\right)\cdot 1.0 + 0.0\right)1.0\right]$$

$$M_o = 206538.3\ in - lbs$$

Where, per Table 4.16.6:

$$h_D = \frac{C - B - g_1}{2} = \frac{31.25 - 26.25 - 0.6875}{2} = 2.1563\ in$$

$$h_G = \frac{C - G}{2} = \frac{31.25 - 29.5}{2} = 0.875\ in$$

$$h_T = \frac{1}{2}\left[\frac{C - B}{2} + h_G\right] = \frac{1}{2}\left[\frac{31.25 - 26.25}{2} + 0.875\right] = 1.6875\ in$$

And, per Table 4.16.11, the maximum bolt spacing B_{smax} and the bolt spacing correction factor B_{SC} are calculated as follows. This calculation is only required when specified by the user or his designated agent.

$$B_{smax} = 2a + \frac{6t}{m+0.5} = 2(0.75) + \frac{6(1.4375)}{3.75+0.5} = 3.5294\ in$$

$$B_{sc} = \max\left[1,\ \sqrt{\frac{B_s}{2a+t}}\right] = \left[1,\ \left\{\sqrt{\frac{2.2312}{2(0.75)+1.4375}} = 0.8715\right\}\right] = 1$$

The actual bolt spacing is determined using the following equation.

$$B_s = \frac{\pi C}{No.\ of\ \ bolts} = \frac{\pi(31.25)}{44} = 2.2312\ in$$

And the procedure provides the designer the ability to add an externally applied net-section axial force and bending moment to the bolt load for the operating condition. These externally applied loads induce a bending moment, referenced as M_{oe}, which is calculated from Equation 4.16.16.

$F_A = 0$ and $M_E = 0$ since there are no externally applied net-section forces and bending. Therefore, the flange cross-section bending moment of inertia, I and polar moment of inertia, I_p, from Table 4.16.7 do need not be calculated. Therefore,

$$M_{oe} = 4M_E\left[\frac{I}{0.3846I_p + I}\right]\cdot\left[\frac{h_D}{(C-2h_D)}\right] + F_A h_D = 0\ in-lbs$$

And $F_S = 1.0$ for non-split rings, see paragraph 4.16.8.

g) STEP 7 – Determine the flange moment for gasket seating condition.

$$M_g = \frac{W_g(C-G)B_{SC}F_S}{2} \qquad Internal\ \ Pressure$$

$$M_g = \frac{237626.3(31.25-29.5)\cdot 1\cdot 1}{2} = 207923.0\ in-lbs$$

h) STEP 8 – Determine the flange stresses for the operating and gasket seating conditions using the equations in Table 4.16.8.

Note: As provided in paragraph 4.16.13 for the definition of B – If $B < 20g_1$, the designer may substitute the value of B_1 for B in the equation for S_H, where,

For integral flanges when $f \geq 1.0$,

$$B_1 = B + g_o$$

Since $\{B = 26.25\ in\} \geq \{20g_1 = 20(0.6875) = 13.75\ in\}$, the value of B shall be used to determine the value of S_H.

Operating Condition:

$$S_H = \frac{fM_o}{Lg_1^2 B} = \frac{(1.0)(206538.3)}{(0.9369)(0.6875)^2(26.25)} = 17767.8\ psi$$

$$S_R = \frac{(1.33te+1)M_o}{Lt^2B} = \frac{[(1.33)(1.4375)(0.2687)+1](2065338.3)}{(0.9369)(1.4375)^2(26.25)} = 6151.9\ psi$$

$$S_T = \frac{YM_o}{t^2B} - ZS_R = \frac{(8.7565)(206538.3)}{(1.4375)^2(26.25)} - 4.5180(6151.9) = 5547.3\ psi$$

Gasket Seating Condition:

$$S_H = \frac{fM_o}{Lg_1^2B} = \frac{(1.0)(207923.0)}{(0.9369)(0.6875)^2(26.25)} = 17886.9\ psi$$

$$S_R = \frac{(1.33te+1)M_o}{Lt^2B} = \frac{[(1.33)(1.4375)(0.2680)+1](207923.0)}{(0.9369)(1.4375)^2(26.25)} = 6193.1\ psi$$

$$S_T = \frac{YM_o}{t^2B} - ZS_R = \frac{(8.7565)(207923.0)}{(1.4375)^2(26.25)} - 4.5180(6193.1) = 5584.7\ psi$$

i) STEP 9 – Check the flange stress acceptance criteria. The criteria below shall be evaluated. If the stress criteria are satisfied, go to STEP 10. If the stress criteria are not satisfied, re-proportion the flange dimensions and go to STEP 4.

Allowable normal stress – The criteria to evaluate the normal stresses for the operating and gasket seating conditions are shown in Table 4.16.9, for integral-type flanges.

Operating Condition:

$$S_H \le \min[1.5S_f, 2.5S_n]$$

$$\{S_H = 17767.8\ psi\} \le \{\min[1.5(17800), 2.5(18800)] = 26700\ psi\} \qquad True$$

$$\{S_R = 6151.9\ psi\} \le \{S_f = 17800\ psi\} \qquad True$$

$$\{S_T = 5547.3\ psi\} \le \{S_f = 17800\ psi\} \qquad True$$

$$\left\{\frac{(S_H + S_R)}{2} = \frac{(17767.8 + 6151.9)}{2} = 11959.9\ psi\right\} \le \{S_{fo} = 17800\ psi\} \qquad True$$

$$\left\{\frac{(S_H + S_T)}{2} = \frac{(17767.8 + 5547.3)}{2} = 11657.6\ psi\right\} \le \{S_{fo} = 17800\ psi\} \qquad True$$

Gasket Seating Condition:

$$S_H \le \min[1.5S_f, 2.5S_n]$$

$$\{S_H = 17886.9\ psi\} \le \{\min[1.5(24000), 2.5(25300)] = 36000\ psi\} \qquad True$$

$$\{S_R = 6193.1\ psi\} \le \{S_f = 24000\ psi\} \qquad True$$

$$\{S_T = 5584.7\ psi\} \le \{S_f = 24000\ psi\} \qquad True$$

$$\left\{\frac{(S_H+S_R)}{2}=\frac{(17886.9+6193.1)}{2}=12040.0\ psi\right\}\le\left\{S_f=24000\ psi\right\}\qquad True$$

$$\left\{\frac{(S_H+S_T)}{2}=\frac{(17886.9+5584.7)}{2}=11735.8\ psi\right\}\le\left\{S_f=24000\ psi\right\}\qquad True$$

j) STEP 10 – Check the flange rigidity criterion in Table 4.16.10. If the flange rigidity criterion is satisfied, then the design is complete. If the flange rigidity criterion is not satisfied, then re-proportion the flange dimensions and go to STEP 3.

Operating Condition:

$$J=\frac{52.14VM_o}{LE_y g_0^2 K_I h_o}\le 1.0$$

$$\left\{J=\frac{52.14(0.1578)(206538.3)}{(0.9369)(26.0E+06)(0.3125)^2(0.3)(2.8641)}=0.8314\right\}\le 1.0\qquad True$$

where,

$$K_I=0.3\ for\ integral\ \ flanges$$

Gasket Seating Condition:

$$J=\frac{52.14VM_o}{LE_y g_o^2 K_I h_o}\le 1.0$$

$$\left\{J=\frac{52.14(0.1578)(207923.0)}{(0.9369)(29.4E+06)(0.3125)^2(0.3)(2.8641)}=0.7402\right\}\le 1.0\qquad True$$

where,

$$K_I=0.3\ for\ integral\ \ flanges$$

Since the acceptance criteria are satisfied, the design is complete.

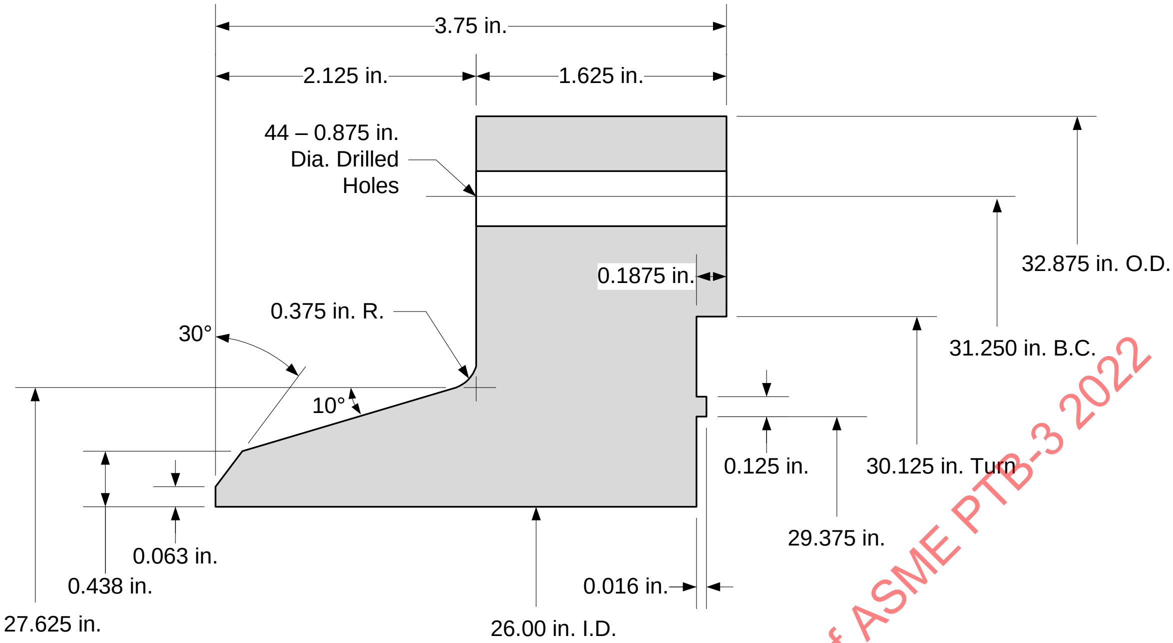


Figure E4.16.1 – Flanged Joints

4.16.2 Example E4.16.2 – Loose Type

Determine if the stresses in the ASME B16.5, Class 300, NPS 20 Slip–on Flange are with acceptable limits, considering the following design conditions. The flange is of a loose type with hub and is attached to a cylindrical shell with Category C fillet welds, see Figure 4.16.5 Sketch (a).

General Data:

- Cylinder Material = $SA-516,\ Grade\ 70$
- Design Conditions = $450\ psig @ 650°F$
- Allowable Stress at Design Temperature = $18800\ psi$
- Allowable Stress at Ambient Temperature = $25300\ psi$
- Corrosion Allowance = $0.0\ in$

Flange Data:

- Material = $SA-105$
- Allowable Stress at Design Temperature = $17800\ psi$
- Allowable Stress at Ambient Temperature = $24000\ psi$
- Modulus of Elasticity at Design Temperature = $26.0E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$

Bolt Data:

- Material = $SA-193,\ Grade\ B7$
- Allowable Stress at Design Temperature = $25000\ psi$
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Diameter = $1.25\ in$
- Number of Bolts = 24
- Root area = $0.929\ in^2$

Gasket Data:

- Material = Kammprofile
- Gasket Factor = 2.0
- Seating Stress = $2500\ psi$
- Inside Diameter = $20.875\ in$
- Outside Diameter = $22.875\ in$

Evaluate the flange in accordance with paragraph 4.16.

Paragraph 4.16.6, Design Bolt Load. The procedure to determine the bolt loads for the operating and gasket seating conditions is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint

$$P = 450\ psig\ at\ 650°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 4.16.1.

$$m = 2.0$$

$$y = 2500\ psi$$

Note: Table 4.16.1 provides a list of many commonly used gasket materials and contact facings with suggested design values of m and y that have generally proved satisfactory in actual service when using effective seating width b given in Table 4.16.3. The design values and other details given in this table are suggested only and are not mandatory.

For this example, gasket manufacturer's suggested m and y values were used.

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5(GOD - GID) = 0.5(22.875 - 20.875) = 1.0\ in$$

from Table 4.16.3, Facing Sketch Detail 1a, Column II,

$$b_o = \frac{N}{2} = \frac{1.0}{2} = 0.500\ in$$

for $b_o > 0.25\ in$,

$$b = C_b\sqrt{b_o} = (0.5)\sqrt{0.500} = 0.3536\ in$$

$$G = G_C - 2b = 22.875 - 2(0.3536) = 22.1678\ in$$

where,

$$C_b = 0.5,\ for\ US\ Customary\ Units$$

$$G_C = \min[Gasket\ OD,\ Flange\ Face\ OD] = \min[22.875,\ 23.0] = 22.875\ in$$

d) STEP 4 – Determine the design bolt load for the operating condition.

$$W_o = H + H_p = 0.785G^2P + (2b \cdot \pi GmP) \qquad for\ non-self-energized\ gaskets$$

$$W_o = 0.785(22.1678)^2(450) + 2(0.3536)(\pi)(22.1678)(2.0)(450) = 217916.9\ lbs$$

e) STEP 5 – Determine the design bolt load for the gasket seating condition.

$$W_g = \frac{(A_m + A_b)S_a}{2} = \frac{(8.7167 + 22.2960)25000}{2} = 387658.8\ lbs$$

where, the parameter A_b is the actual cross-sectional area of the bolts that is selected such that $A_b \geq A_m$.

$$A_b = (Number\ of\ bolts)(Root\ area\ of\ one\ bolt) = 24(0.929) = 22.2960\ in^2$$

$$A_m = \max\left[\left(\frac{W_o + F_A + \frac{4M_E}{G}}{S_{bo}}\right), \left(\frac{W_{gs}}{S_{bg}}\right)\right] = \max\left[\left(\frac{217916.9 + 0.0 + 0.0}{25000}\right), \left(\frac{61563.7}{25000}\right)\right]$$

$$A_m = \max\left[8.7167,\ 2.4625\right] = 8.7167\ in^2$$

and,

$$W_{gs} = \pi bGy \qquad \text{for non-self-energized gaskets}$$

$$W_{gs} = \pi(0.3536)(22.1678)(2500) = 61563.7\ lbs$$

and $F_A = 0$ and $M_E = 0$ since there are no externally applied net-section forces and bending moments.

Paragraph 4.16.7, Flange Design Procedure. The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings. The procedure incorporates both a strength check and a rigidity check for flange rotation.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

$$P = 450\ psig\ at\ 650°F$$

$$F_A = 0$$

$$M_E = 0$$

b) STEP 2 – Determine the design bolt loads for operating condition W_o, and the gasket seating condition W_g, and the corresponding actual bolt load area A_b, from paragraph 4.16.6.

$$W_o = 217916.9\ lbs$$

$$W_g = 387658.8\ lbs$$

$$A_b = 22.2960\ in^2$$

c) STEP 3 – Determine an initial flange geometry, in addition to the information required to determine the bolt load, the following geometric parameters are required. The flange is an ASME B16.5, Class 300, NPS 20 Slip–on Flange.

1) Flange bore

$$B = 20.20\ in$$

2) Bolt circle diameter

$$C = 27.0\ in$$

3) Outside diameter of the flange

$$A = 30.5\ in$$

4) Flange thickness

$$t = 2.44\ in$$

5) Thickness of the hub at the large end

$$g_1 = 1.460\ in$$

6) Thickness of the hub at the small end

$$g_0 = 1.460\ in$$

7) Hub length

$$h = 1.25\ in$$

d) STEP 4 – Determine the flange stress factors using the equations in Tables 4.16.4 and 4.16.5.

Table 4.16.4:

$$K = \frac{A}{B} = \frac{30.5}{20.20} = 1.5099$$

$$Y = \frac{1}{K-1}\left[0.66845 + 5.71690\left(\frac{K^2 \log_{10} K}{K^2 - 1}\right)\right]$$

$$Y = \frac{1}{1.5099-1}\left[0.66845 + 5.71690\frac{(1.5099)^2 \log_{10}[1.5099]}{(1.5099)^2 - 1}\right] = 4.8850$$

$$T = \frac{K^2(1+8.55246\log_{10} K)-1}{(1.04720+1.9448K^2)(K-1)} = \frac{(1.5099)^2(1+8.55246\log_{10}[1.5099])-1}{(1.04720+1.9448(1.5099)^2)(1.5099-1)} = 1.7064$$

$$U = \frac{K^2(1+8.55246\log_{10} K)-1}{1.36136(K^2-1)(K-1)} = \frac{(1.5099)^2(1+8.55246\log_{10}[1.5099])-1}{1.36136((1.5099)^2-1)(1.5099-1)} = 5.3681$$

$$Z = \frac{(K^2+1)}{(K^2-1)} = \frac{((1.5099)^2+1)}{((1.5099)^2-1)} = 2.5627$$

$$h_o = \sqrt{Bg_0} = \sqrt{(20.20)(1.46)} = 5.4307$$

$$X_g = \frac{g_1}{g_0} = \frac{1.460}{1.460} = 1.0$$

$$X_h = \frac{h}{h_o} = \frac{1.25}{5.4307} = 0.2302$$

$$d = \frac{Ug_0^2 h_o}{V_L} = \frac{(5.3681)(1.460)^2(5.4307)}{11.2955} = 5.5014\ in^3$$

$$e = \frac{F_L}{h_o} = \frac{3.2556}{5.4307} = 0.5995 in^{-1}$$

$$L = \frac{te+1}{T} + \frac{t^3}{d} = \frac{2.44(0.5995)+1}{1.7064} + \frac{(2.44)^3}{5.5014} = 4.0838$$

Where from Table 4.16.5,

$$For\ 0.1 \le X_h \le 0.25,$$

$$F_L = \left(\frac{\left\{ \begin{array}{l} 0.941074 + 0.176139\left(\ln\left[X_g\right]\right) - 0.188556\left(\ln\left[X_h\right]\right) + \\ 0.0689847\left(\ln\left[X_g\right]\right)^2 + 0.523798\left(\ln\left[X_h\right]\right)^2 - \\ 0.513894\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right) \end{array} \right\}}{\left\{ \begin{array}{l} 1 + 0.379392\left(\ln\left[X_g\right]\right) + 0.184520\left(\ln\left[X_h\right]\right) - \\ 0.00605208\left(\ln\left[X_g\right]\right)^2 - 0.00358934\left(\ln\left[X_h\right]\right)^2 + \\ 0.110179\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right) \end{array} \right\}} \right)$$

$$F_L = \left(\frac{\left\{ \begin{array}{l} 0.941074 + 0.176139\left(\ln\left[1.0\right]\right) - 0.188556\left(\ln\left[0.2302\right]\right) + \\ 0.0689847\left(\ln\left[1.0\right]\right)^2 + 0.523798\left(\ln\left[0.2302\right]\right)^2 - \\ 0.513894\left(\ln\left[1.0\right]\right)\left(\ln\left[0.2302\right]\right) \end{array} \right\}}{\left\{ \begin{array}{l} 1 + 0.379392\left(\ln\left[1.0\right]\right) + 0.184520\left(\ln\left[0.2302\right]\right) - \\ 0.00605208\left(\ln\left[1.0\right]\right)^2 - 0.00358934\left(\ln\left[0.2302\right]\right)^2 + \\ 0.110179\left(\ln\left[1.0\right]\right)\left(\ln\left[0.2302\right]\right) \end{array} \right\}} \right)$$

$$F_L = 3.2556$$

$$For\ 0.1 \le X_h \le 0.25,$$

$$\ln[V_L] = \left(\begin{array}{l} 6.57683 - 0.115516 X_g + 1.39499\sqrt{X_g}\left(\ln[X_g]\right) + \\ 0.307340\left(\ln[X_g]\right)^2 - 8.30849\sqrt{X_g} + 2.62307\left(\ln[X_g]\right) + \\ 0.239498 X_h\left(\ln[X_h]\right) - 2.96125\left(\ln[X_h]\right) + \dfrac{7.035052\left(10^{-4}\right)}{X_h} \end{array} \right)$$

$$\ln[V_L] = \left(\begin{array}{l} 6.57683 - 0.115516(1.0) + 1.39499\sqrt{1.0}\left(\ln 1.0\right) + \\ 0.307340\left(\ln[1.0]\right)^2 - 8.30849\sqrt{1.0} + 2.62307\left(\ln[1.0]\right) + \\ 0.239498(0.2302)\left(\ln[0.2302]\right) - 2.96125\left(\ln[0.2302]\right) + \dfrac{7.035052\left(10^{-4}\right)}{0.2302} \end{array} \right)$$

$$\ln[V_L] = 2.4244$$

$$V_L = \exp[2.4244] = 11.2955$$

$$f = 1.0$$

e) STEP 5 – Determine the flange forces.

$$H_D = 0.785 B^2 P = 0.785(20.20)^2(450) = 144140.1\ lbs$$

$$H = 0.785 G^2 P = 0.785(22.1678)^2(450) = 173591.1\ lbs$$

$$H_T = H - H_D = 173591.1 - 144140.1 = 29451.0\ lbs$$

$$H_G = W_o - H = 217916.9 - 173591.1 = 44325.8\ lbs$$

f) STEP 6 – Determine the flange moment for the operating condition. When specified by the user or his designated agent, the maximum bolt spacing, B_{smax}, and the bolt spacing correction factor, B_{SC}, shall be applied in calculating the flange moment for internal pressure using the equations in Table 4.16.11. The flange moment M_o for the operating condition and flange moment M_g for the gasket seating condition without correction for bolt spacing $B_{SC} = 1$ is used for the calculation of the rigidity index in STEP 10. In these equations, h_D, h_T, and h_G are determined from Table 4.16.6.

$$M_o = abs\left[\left(\left(H_D h_D + H_T h_T + H_G h_G\right) B_{SC} + M_{oe}\right) F_s\right] \quad Internal\ Pressure$$

$$M_o = abs\left[\left(\left(144140.1(3.40) + 29451.0(2.9081) + 44325.8(2.4161)\right)\cdot 1.0 + 0.0\right) 1.0\right]$$

$$M_o = 682818.4\ in-lbs$$

Where, per Table 4.16.6:

$$h_D = \frac{C-B}{2} = \frac{27.0-20.20}{2} = 3.40\ in$$

$$h_G = \frac{C-G}{2} = \frac{27.0-22.1678}{2} = 2.4161\ in$$

$$h_T = \frac{h_D + h_G}{2} = \frac{3.40+2.4161}{2} = 2.9081\ in$$

And, per Table 4.16.11, the maximum bolt spacing B_{smax} and the bolt spacing correction factor B_{SC} are calculated as follows. This calculation is only required when specified by the user or his designated agent.

$$B_{smax} = 2a + \frac{6t}{m+0.5} = 2(1.25) + \frac{6(2.44)}{2.0+0.5} = 8.3560\ in$$

$$B_{sc} = \max\left[1,\ \sqrt{\frac{B_s}{2a+t}}\right] = \left[1,\ \left\{\sqrt{\frac{3.5343}{2(1.25)+2.44}} = 0.8458\right\}\right] = 1$$

The actual bolt spacing is determined using the following equation.

$$B_s = \frac{\pi C}{No.\ of\ bolts} = \frac{\pi(27.0)}{24} = 3.5343\ in$$

And the procedure provides the designer the ability to add an externally applied net-section axial force and bending moment to the bolt load for the operating condition. These externally applied loads induce a bending moment, referenced as M_{oe}, which is calculated from Equation 4.16.16.

$F_A = 0$ and $M_E = 0$ since there are no externally applied net-section forces and bending. Therefore, the flange cross-section bending moment of inertia, I and polar moment of inertia, I_p, from Table 4.16.7 do need not be calculated. Therefore,

$$M_{oe} = 4M_E\left[\frac{I}{0.3846 I_p + I}\right]\cdot\left[\frac{h_D}{(C-2h_D)}\right] + F_A h_D$$

And $F_S = 1.0$ for non-split rings, see paragraph 4.16.8.

g) STEP 7 – Determine the flange moment for gasket seating condition.

$$M_g = \frac{W_g(C-G)B_{SC}F_S}{2} \qquad Internal\ Pressure$$

$$M_g = \frac{387658.8(27.0-22.1678)\cdot 1\cdot 1}{2} = 936622.4\ in-lbs$$

h) STEP 8 – Determine the flange stresses for the operating and gasket seating conditions using the equations in Table 4.16.8.

Note: As provided in paragraph 4.16.13 for the definition of B – If $B < 20g_1$, the designer may substitute the value of B_1 for B in the equation for S_H, where, For integral flanges when $f < 1.0$ and for loose type flanges,

$$B_1 = B + g_1$$

Although, $\{B = 20.20\ in\} < \{20g_1 = 20(1.46) = 29.2\ in\}$, the value of B will be used to determine the value of S_H.

Operating Condition:

$$S_H = \frac{fM_o}{Lg_1^2 B} = \frac{(1.0)(682818.4)}{(4.1032)(1.460)^2(20.20)} = 3883.1\ psi$$

$$S_R = \frac{(1.33te+1)M_o}{Lt^2 B} = \frac{\left[(1.33)(2.44)(0.5995)+1\right](682818.4)}{(4.0838)(2.44)^2(20.20)} = 4095.1\ psi$$

$$S_T = \frac{YM_o}{t^2 B} - ZS_R = \frac{(4.8850)(682818.4)}{(2.44)^2(20.20)} - 2.5627(4095.1) = 17241.1\ psi$$

Gasket Seating Condition:

$$S_H = \frac{fM_g}{Lg_1^2 B} = \frac{(1.0)(936622.4)}{(4.0838)(1.460)^2(20.20)} = 5326.5\ psi$$

$$S_R = \frac{(1.33te+1)M_g}{Lt^2 B} = \frac{\left[(1.33)(2.44)(0.5995)+1\right](936622.4)}{(4.0838)(2.44)^2(20.20)} = 5617.3\ psi$$

$$S_T = \frac{YM_g}{t^2 B} - ZS_R = \frac{(4.8850)(936622.4)}{(2.44)^2(20.20)} - 2.5627(5617.3) = 23649.6\ psi$$

i) STEP 9 – Check the flange stress acceptance criteria. The criteria below shall be evaluated. If the stress criteria are satisfied, go to STEP 10. If the stress criteria are not satisfied, re-proportion the flange dimensions and go to STEP 4.

Allowable normal stress – The criteria to evaluate the normal stresses for the operating and gasket seating conditions are shown in Table 4.16.9, for loose type flanges with a hub.

Operating Condition:

$$S_H \le 1.5S_f$$

$$\{S_H = 3883.1\ psi\} \le \{1.5(17800) = 26700\ psi\} \qquad True$$

$$\{S_R = 4095.1\ psi\} \le \{S_f = 17800\ psi\} \qquad True$$

$$\{S_T = 17241.1\ psi\} \le \{S_f = 17800\ psi\} \qquad True$$

$$\left\{\frac{(S_H + S_R)}{2} = \frac{(3883.1+4095.1)}{2} = 3989.1\ psi\right\} \le \{S_f = 17800\ psi\} \qquad True$$

$$\left\{\frac{(S_H+S_T)}{2}=\frac{(3883.1+17241.4)}{2}=10562.3\ psi\right\}\le\left\{S_f=17800\ psi\right\}\qquad True$$

Gasket Seating Condition:

$$S_H \le 1.5S_f$$

$$\left\{S_H=5326.5\ psi\right\}\le\left\{1.5(24000)=36000\ psi\right\}\qquad True$$

$$\left\{S_R=5617.3\ psi\right\}\le\left\{S_f=24000\ psi\right\}\qquad True$$

$$\left\{S_T=23649.6\ psi\right\}\le\left\{S_f=24000\ psi\right\}\qquad True$$

$$\left\{\frac{(S_H+S_R)}{2}=\frac{(5326.5+5617.3)}{2}=5471.9\ psi\right\}\le\left\{S_f=24000\ psi\right\}\qquad True$$

$$\left\{\frac{(S_H+S_T)}{2}=\frac{(5326.5+23649.6)}{2}=14488.1\ psi\right\}\le\left\{S_f=24000\ psi\right\}\qquad True$$

j) STEP 10 – Check the flange rigidity criterion in Table 4.16.13. If the flange rigidity criterion is satisfied, then the design is complete. If the flange rigidity criterion is not satisfied, then re-proportion the flange dimensions and go to STEP 3.

Operating Condition:

$$J=\frac{52.14V_L M_o}{LE_y g_0^2 K_L h_o}\le 1.0$$

$$\left\{J=\frac{52.14(11.2955)(682818.4)}{(4.0838)(26.0E+06)(1.460)^2(0.2)(5.4307)}=1.6359\right\}\le 1.0\quad Not\ Satisfied$$

where,

$$K_L=0.2\ \ for\ loose\ type\ \ flanges$$

Gasket Seating Condition:

$$J=\frac{52.14V_L M_o}{LE_y g_o^2 K_L h_o}\le 1.0$$

$$\left\{J=\frac{52.14(11.2955)(936622.4)}{(4.0838)(29.4E+06)(1.460)^2(0.2)(5.4307)}=1.9844\right\}\le 1.0\quad Not\ Satisfied$$

where,

$$K_L=0.2\ \ for\ loose\ type\ \ flanges$$

Since the flange rigidity criterion is not satisfied for either the operating condition or the gasket seating condition, the flange dimensions should be re-proportioned, and the design procedure shall be performed beginning with STEP 3.

NOTE: Although the proposed ASME B16.5 slip-on flange is shown not to satisfy the flange rigidity acceptance criteria of VIII-2 paragraph 4.16 Design Rules for Flanged Joints, Table 4.16.10, ASME B16.5–2020, Table 2–1.1C, Pressure–Temperature Ratings for Group 1.1 Materials, states an ASME Class 300 flange is permitted to operate at a pressure of 550 psi for a coincident temperature of 650°F.

4.17 Clamped Connections

4.17.1 Example E4.17.1 – Flange and Clamp Design Procedure

Using the data shown below, see Figure E4.17.1, determine if the clamp design satisfied paragraph 4.17, Design Rules for Clamped Connections.

General Data:

- Design Conditions = $3000\ psi@200°F$
- Corrosion Allowance = $0.0\ in$

Clamp:

- Material = $SA-216,\ Grade\ WCB$
- Inside Diameter = $43.75\ in$
- Thickness = $7.625\ in$
- Width = $28.0\ in$
- Gap = $14.0\ in$
- Lug height = $15.0\ in$
- Lug Width = $28.0\ in$
- Lip Length = $2.75\ in$
- Radial Distance from Connection Centerline to Bolts = $32.25\ in$
- Distance from W to the point where the clamp lug joins the clamp body = $3.7\ in$
- Allowable Stress @ Design Temperature = $22000\ psi$
- Allowable Stress @ Ambient Temperature = $24000\ psi$

Hub:

- Material = $SA-105$
- Inside Diameter = $18.0\ in$
- Pipe End Neck Thickness = $12.75\ in$
- Shoulder End Neck Thickness = $12.75\ in$
- Shoulder Thickness = $7.321\ in$
- Shoulder Height = $2.75\ in$
- Friction Angle = $5\ deg$
- Shoulder Transition Angle = $10\ deg$
- Allowable Stress @ Design Temperature = $22000\ psi$
- Allowable Stress @ ambient Temperature = $24000\ psi$

Bolt Data:

- Material = $SA-193,\ Grade\ B7$
- Allowable Stress @ Design Temperature = $23000\ psi$

- Allowable Stress @ Gasket Temperature = $23000\ psi$
- Diameter = $1.75\ in$
- Number of Bolts = 2
- Root area = $1.980\ in^2$

Gasket Data:

- Material = Self Energizing O-ring Type
- Gasket Reaction Location = $19.0\ in$
- Gasket Factor = 0
- Seating Stress = $0\ psi$

Evaluate the clamp in accordance with Paragraph 4.17.

Paragraph 4.17.4, Design Bolt Loads. The procedure to determine the bolt loads for the operating and gasket seating conditions are shown below.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint

$$P = 3000\ psig\ at\ 200°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 4.16.1.

$$m = 0.0 \qquad for\ self-energized\ gaskets$$
$$y = 0.0$$

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.0 \qquad for\ self-energized\ gaskets$$

From Table 4.16.3, Facing Sketch Detail (not required because gasket is self-energized).

$$b_o = \frac{N}{2} = \frac{0.0}{2} = 0.0\ in$$

For $b_o \le 0.25\ in.$,

$$b = b_o = 0.0\ in$$

Therefore, the location of the gasket reaction is calculated as follows.

$$G = mean\ diameter\ of\ the\ gasket\ contact\ face$$
$$G = 19.0\ in$$

d) STEP 4 – Determine the flange forces for the bolt load calculation.

$$H = 0.785G^2P = 0.785(19.0)^2(3000) = 850155.0\ lbs$$
$$H_p = 0.0 \qquad (for\ self-energized\ gaskets)$$
$$H_m = 0.0 \qquad (for\ self-energized\ gaskets)$$

e) STEP 5 – Determine the design bolt load for the operating condition.

$$W_o = \frac{2}{\pi}\left(H + H_p\right)\tan\left[\phi - \mu\right] = \frac{2}{\pi}\left(850155.0 + 0\right)\cdot\tan\left[10 - 5\right] = 47351.1\ lbs$$

f) STEP 6 – Determine the minimum required total bolt load for the gasket seating and assembly conditions.

$$W_{g1} = \frac{2}{\pi}H_m\tan\left[\phi + \mu\right] = \frac{2}{\pi}\left(0.0\right)\tan\left[10 + 5\right] = 0.0\ lbs$$

$$W_{g2} = \frac{2}{\pi}\left(H + H_p\right)\tan\left[\phi + \mu\right] = \frac{2}{\pi}\left(850155.0 + 0\right)\tan\left[10 + 5\right] = 145020.9\ lbs$$

g) STEP 7 – Determine the design bolt load for the gasket seating and assembly condition.

$$W_g = \left(A_m + A_b\right)S_{bg} = \left(3.1526 + 3.96\right)23000 = 163589.8\ lbs$$

The total cross-sectional area of bolts A_m required for the operating condition, gasket seating, and assembly condition is determined as follows.

$$A_m = \max\left[\frac{W_o}{2S_{bo}},\ \frac{W_{g1}}{2S_{bg}},\ \frac{W_{g2}}{2S_{bg}}\right]$$

$$A_m = \max\left[\left\{\frac{47531.1}{2\left(23000\right)} = 1.0294\right\},\ \left\{\frac{0.0}{2\left(23000\right)} = 0.0\right\},\left\{\frac{145020.9}{2\left(23000\right)} = 3.1526\right\}\right] = 3.1526\ in^2$$

The actual bolt area is calculated as follows (using two 1.75-inches diameter bolts).

$$A_b = \left(Number\ of\ bolts\right)\left(Root\ area\ of\ one\ bolt\right) = 2\left(1.980\right) = 3.96\ in^2$$

Verify that the actual bolt area is equal to or greater than the total required area.

$$\left\{A_b = 3.96\ in^2\right\} \geq \left\{A_m = 3.1526\ in^2\right\} \qquad True$$

Alternatively, if controlled bolting (e.g., bolt tensioning or torque control) techniques are used to assemble the clamp, assembly design bolt load may be calculated as follows. Note: This calculation is shown for informational purposes only and will not be used in the example problem.

$$W_g = 2A_m\cdot S_{bg} = 2\left(3.1526\right)23000 = 145019.6\ lbs$$

Paragraph 4.17.5 Flange and Clamp Design Procedure. The procedure to design a clamp connection is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flange joint.

See above data.

b) STEP 2 – Determine an initial flange and clamp geometry see Figures 4.17.1(c) and 4.17.2(a) and Figure E4.17.1 of this example.

c) STEP 3 – Determine the design bolt loads for operating condition, W_o, and the gasket seating and assembly condition, W_g, from paragraph 4.17.4.2.

$$W_o = 47351.1\ lbs$$
$$W_g = 163589.8\ lbs$$

d) STEP 4 – Determine the flange forces, H, H_p, and H_m from paragraph 4.17.4.2, Step 4.

$$H_D = 0.785B^2P = 0.785(18.0)^2(3000) = 763020.0\ lbs$$
$$H_G = \frac{1.571W_o}{\tan[\phi+\mu]} - (H + H_p) = \frac{1.571(47351.1)}{\tan[10+5]} - (850155.0 + 0) = -572533.0\ lbs$$
$$H_T = H - H_D = 850155.0 - 763020.0 = 87135.0\ lbs$$

e) STEP 5 – Determine the flange moment for the operating condition.

$$M_o = M_D + M_G + M_T + M_F + M_P + M_R$$
$$M_o = 5961093.8 + 0.0 + 1214444.1 + 0.0 + 25957.3 + (-254998.8) = 6946496.4\ in-lbs$$

where,

$$M_D = H_D\left[\frac{C-(B+g_1)}{2}\right] = 763020.0\left[\frac{46.375-(18.0+12.75)}{2}\right] = 5961093.8\ lbs$$

$$M_G = H_G h_G = -572533.0(0.0) = 0.0\ lbs$$

$$M_T = H_T\left[\frac{C}{2} - \frac{(B+G)}{4}\right] = 87135.0\left[\frac{46.375}{2} - \frac{(18.0+19.0)}{4}\right] = 1214444.1\ in-lbs$$

$$M_F = H_D\left(\frac{g_1-g_0}{2}\right) = 763020.0\left(\frac{12.75-12.75}{2}\right) = 0.0\ in-lbs$$

$$M_P = PBT\pi\left(\frac{T}{2} - \bar{h}\right) = 3000(18.0)(7.321)(\pi)\left(\frac{7.321}{2} - 3.6396\right) = 25957.3\ lbs$$

$$M_R = 1.571W_o\left(\bar{h} - T + \frac{(C-N)\tan[\phi]}{2}\right)$$

$$M_R = 1.571(47351.1)\left(3.6396 - 7.321 + \frac{(46.375-43.5)\tan[10]}{2}\right) = -254998.8\ lbs$$

and,

$$A = B + 2(g_1 + g_2) = 18.0 + 2(12.75 + 2.75) = 49.0\ in$$

$$N = B + 2g_1 = 18.0 + 2(12.75) = 43.5\ in$$

$$C = \frac{(A + C_i)}{2} = \frac{(49 + 43.75)}{2} = 46.375\ in$$

$$\bar{h} = \frac{T^2 g_1 + h_2^2 g_2}{2(Tg_1 + h_2 g_2)} = \frac{(7.321)^2 12.75 + (7.0785)^2 2.75}{2(7.321(12.75) + 7.0785(2.75))} = 3.6396\ in$$

$$h_2 = T - \frac{g_2 \tan[\phi]}{2} = 7.321 - \frac{2.75\tan[10]}{2} = 7.0786\ in$$

f) STEP 6 – Determine the flange moment for the gasket seating condition.

$$M_g = \frac{0.785 W_g (C-G)}{\tan[\phi+\mu]} = \frac{0.785(163589.8)(46.375-19.0)}{\tan[10+5]} = 13119810.2\ \ in-lbs$$

g) STEP 7 – Determine the hub factors.

$$F_H = 1 + \frac{1.818}{\sqrt{Bg_1}}\left[T - \bar{h} + \frac{3.305 I_h}{g_1^2(0.5B+\bar{g})}\right]$$

$$F_H = 1 + \frac{1.818}{\sqrt{18(12.75)}}\left[7.321 - 3.6396 + \frac{3.305(498.4148)}{(12.75)^2(0.5(18.0)+7.7123)}\right] = 1.5146$$

$$I_h = \frac{g_1 T^3}{3} + \frac{g_2 h_2^3}{3} - (g_2 h_2 + g_1 T)\bar{h}^2$$

$$I_h = \frac{12.75(7.321)^3}{3} + \frac{2.75(7.0785)^3}{3} - (2.75(7.0786) + 12.75(7.321))(3.6396)^2 = 498.4148\ in^4$$

$$\bar{g} = \frac{Tg_1^2 + h_2 g_2(2g_1+g_2)}{2(Tg_1+h_2g_2)} = \frac{7.321(12.75)^2 + 7.0786(2.75)(2(12.75)+2.75)}{2(7.321(12.75)+7.0786(2.75))} = 7.7123\ in$$

h) STEP 8 – Determine the reaction shear force at the hub neck for the operating condition.

$$Q_o = \frac{1.818 M_o}{F_H\sqrt{Bg_1}} = \frac{1.818(6946496.4)}{1.5146\sqrt{(18.0)(12.75)}} = 550389.8\ lbs$$

i) STEP 9 – Determine the reaction shear force at the hub neck for the gasket seating condition.

$$Q_g = \frac{1.818 M_g}{F_H\sqrt{Bg_1}} = \frac{1.818(13119810.2)}{1.5146\sqrt{(18.0)(12.75)}} = 1039518.3\ lbs$$

j) STEP 10 – Determine the clamp factors.

$$e_b = B_c - \frac{C_i}{2} - l_c - X = 32.25 - \frac{43.75}{2} - 2.75 - 2.7009 = 4.9241\ in$$

and,

$$I_c = \left(\frac{A_1}{3} + \frac{A_2}{4}\right)C_t^2 + \frac{A_3 l_c^2}{3} - A_c X^2$$

$$I_c = \left(\frac{97.2188}{3} + \frac{91.3389}{4}\right)(7.625)^2 + \frac{38.5(2.75)^2}{3} - 227.0577(2.7009)^2 = 1652.4435\ in^4$$

where,

$$X = \frac{\left(\frac{C_w}{2}-\frac{C_t}{3}\right)C_t^2 - 0.5\left(C_w - C_g\right)l_c^2}{A_c} = \frac{\left(\frac{28}{2}-\frac{7.625}{3}\right)\left(7.625\right)^2 - 0.5\left(28.0-14.0\right)\left(2.75\right)^2}{227.0577}$$

$$X = 2.7009\ in$$

and,

$$A_c = A_1 + A_2 + A_3 = 97.2188 + 91.3389 + 38.5 = 227.0577\ in^2$$

where,

$$A_1 = \left(C_w - 2C_t\right)C_t = \left(28.0 - 2\left(7.625\right)\right)7.625 = 97.2188\ in^2$$

$$A_2 = 1.571C_t^2 = 1.571\left(7.625\right)^2 = 91.3389\ in^2$$

$$A_3 = \left(C_w - C_g\right)l_c = \left(28.0-14.0\right)2.75 = 38.5\ \ in^2$$

k) STEP 11 – Determine the hub stress correction factor, f, based on g_1, g_0, h, and B using Table 4.16.4 and Table 4.16.5 and l_m using the following equation.

$$X_g = \frac{g_1}{g_0} = \frac{12.75}{12.75} = 1.0$$

$$X_h = \frac{h}{h_o} = \frac{h}{\sqrt{Bg_o}} = \frac{0.0}{\sqrt{18.0\left(12.75\right)}} = 0.0$$

$$f = \max\left[1.0, \left(\frac{\begin{array}{c}0.0927779 - 0.0336633X_g + 0.964176X_g^2 + \\ 0.0566286X_h + 0.347074X_h^2 - 4.18699X_h^3\end{array}}{\begin{array}{c}1 - 5.96093\left(10\right)^{-3}X_g + 1.62904X_h + \\ 3.49329X_h^2 + 1.39052X_h^3\end{array}}\right)\right]$$

$$f = \max\left[1.0, \left(\frac{\begin{array}{c}0.0927779 - 0.0336633\left(1\right) + 0.964176\left(1\right)^2 + \\ 0.0566286\left(1\right) + 0.347074\left(0\right) - 4.18699\left(0\right)\end{array}}{\begin{array}{c}1 - 5.96093\left(10\right)^{-3}1 + 1.62904\left(0\right) + \\ 3.49329\left(0\right) + 1.39052\left(0\right)\end{array}}\right)\right]$$

$$f = \max\left[1.0,\ 1.0294\right] = 1.0294$$

and,

$$l_m = l_c - 0.5(C - C_i) = 2.75 - 0.5\left(46.375 - 43.75\right) = 1.4375\ in$$

l) STEP 12 – Determine the flange and clamp stresses for the operating and gasket seating conditions using the equations in Table 4.17.1.

Operating Condition – Location: Flange:

Longitudinal Stress:

$$S_{1o} = f\left[\frac{PB^2}{4g_1(B+g_1)} + \frac{1.91M_o}{g_1^2(B+g_1)F_H}\right]$$

$$S_{1o} = 1.0294\left[\frac{3000(18.0)^2}{4(12.75)(18.0+12.75)} + \frac{1.91(6946496.4)}{(12.75)^2(18.0+12.75)1.5146}\right] = 2442.0\ psi$$

Lame Hoop Stress:

$$S_{2o} = P\left(\frac{N^2+B^2}{N^2-B^2}\right) = 3000\left(\frac{(43.5)^2+(18.0)^2}{(43.5)^2-(18.0)^2}\right) = 4239.6\ psi$$

Axial Shear Stress:

$$S_{3o} = \frac{0.75W_o}{T(B+2g_1)\tan[\phi-\mu]} = \frac{0.75(47351.1)}{7.321(18.0+2(12.75))\tan[10-5]} = 1274.6\ psi$$

Radial Shear Stress:

$$S_{4o} = \frac{0.477Q_o}{g_1(B+g_1)} = \frac{0.477(550389.8)}{12.75(18.0+12.75)} = 669.6\ psi$$

Operating Condition – Location: Clamp:

Longitudinal Stress:

$$S_{5o} = \frac{W_o}{2C\tan[\phi-\mu]}\left[\frac{1}{C_t} + \frac{3(C_t+2l_m)}{C_t^2}\right]$$

$$S_{5o} = \frac{47351.1}{2(46.375)\tan[10-5]}\left[\frac{1}{7.625} + \frac{3(7.625+2(1.4375))}{(7.625)^2}\right] = 3926.8\ psi$$

Tangential Stress:

$$S_{6o} = \frac{W_o}{2}\left[\frac{1}{A_c} + \frac{|e_b|\cdot(C_t-X)}{I_c}\right]$$

$$S_{6o} = \frac{47351.1}{2}\left[\frac{1}{227.0577} + \frac{|4.9241|\cdot(7.625-2.7009)}{1652.4435}\right] = 451.7\ psi$$

Lip Shear Stress:

$$S_{7o} = \frac{1.5W_o}{(C_w-C_g)C\tan[\phi-\mu]} = \frac{1.5(47351.1)}{(28.0-14.0)(46.375)\tan[10-5]} = 1250.4\ psi$$

Lug Bending Stress:

$$S_{8o} = \frac{3W_oL_a}{L_wL_h^2} = \frac{3(47351.1)(3.7)}{28.0(15.0)^2} = 83.4\ psi$$

Bearing Stress at clamp-to-hub contact:

$$S_{9o} = \frac{W_o}{(A-C_i)C\tan[\phi-\mu]} = \frac{47351.1}{(49.0-43.75)(46.375)\tan[10-5]} = 2223.0\ psi$$

Gasket Seating/Assembly Condition – Location: Flange:

Longitudinal Stress:

$$S_{1g} = f\left[\frac{1.91M_g}{g_1^2(B+g_1)F_H}\right] = 1.0294\left[\frac{1.91(13119810.2)}{(12.75)^2(18.0+12.75)(1.5146)}\right] = 3407.0\ psi$$

Lame Hoop Stress:

$$S_{2g} = 0.0$$

Axial Shear Stress:

$$S_{3g} = \frac{0.75W_g}{T(B+2g_1)\tan[\phi+\mu]} = \frac{0.75(163589.8)}{7.321(18.0+2(12.75))\tan[10+5]} = 1437.8\ psi$$

Radial Shear Stress:

$$S_{4g} = \frac{0.477Q_g}{g_1(B+g_1)} = \frac{0.477(1039518.3)}{12.75(18.0+12.75)} = 1264.7\ psi$$

Gasket Seating/Assembly Condition – Location: Clamp:

Longitudinal Stress:

$$S_{5g} = \frac{W_g}{2C\tan[\phi+\mu]}\left[\frac{1}{C_t} + \frac{3(C_t+2l_m)}{C_t^2}\right]$$

$$S_{5g} = \frac{163589.8}{2(46.375)\tan[10+5]}\left[\frac{1}{7.625} + \frac{3(7.625+2(1.4375))}{7.625^2}\right] = 4429.6\ psi$$

Tangential Stress:

$$S_{6g} = \frac{W_g}{2}\left[\frac{1}{A_c} + \frac{|e_b|\cdot(C_t-X)}{I_c}\right]$$

$$S_{6g} = \frac{163589.8}{2}\left[\frac{1}{227.0577} + \frac{|4.9241|\cdot(7.625-2.7009)}{1652.4435}\right] = 1560.4\ psi$$

Lip Shear Stress:

$$S_{7g} = \frac{1.5W_g}{\left(C_w - C_g\right) C \tan\left[\phi + \mu\right]} = \frac{1.5\left(163589.8\right)}{\left(28.0 - 14.0\right)\left(46.375\right) \tan\left[10 + 5\right]} = 1410.5\ psi$$

Lug Bending Stress:

$$S_{8g} = \frac{3W_g L_a}{L_w L_h^2} = \frac{3\left(163589.8\right)\left(3.7\right)}{28.0\left(15.0\right)^2} = 288.2\ psi$$

Bearing Stress at clamp-to-hub contact:

$$S_{9g} = \frac{W_g}{\left(A - C_i\right) C \tan\left[\phi + \mu\right]} = \frac{163589.8}{\left(49.0 - 43.75\right)\left(46.375\right) \tan\left[10 + 5\right]} = 2507.6\ psi$$

m) STEP 13 – Check the flange stress acceptance criteria for the operating and gasket seating conditions are shown in Table 24-8.

Operating Condition – Location: Flange:

$\left\{S_{1o} = 2442.0\ psi\right\} \le \left\{1.5S_{ho} = 33000\ psi\right\}$ *True*

$\left\{S_{2o} = 4239.6\ psi\right\} \le \left\{S_{ho} = 22000\ psi\right\}$ *True*

$\left\{S_{3o} = 1274.6\ psi\right\} < \left\{0.8S_{ho} = 17600\ psi\right\}$ *True*

$\left\{S_{4o} = 669.6\ psi\right\} \le \left\{0.8S_{ho} = 17600\ psi\right\}$ *True*

Operating Condition – Location: Clamp:

$\left\{S_{5o} = 3926.8\ psi\right\} \le \left\{1.5S_{co} = 33000\ psi\right\}$ *True*

$\left\{S_{6o} = 451.7\ psi\right\} \le \left\{1.5S_{co} = 33000\ psi\right\}$ *True*

$\left\{S_{7o} = 1250.4\ psi\right\} < \left\{0.8S_{co} = 17600\ psi\right\}$ *True*

$\left\{S_{8o} = 83.4\ psi\right\} \le \left\{S_{co} = 22000\ psi\right\}$ *True*

$\left\{S_{9o} = 2223.0\ psi\right\} \le \left\{1.6\min\left[S_{ho},\ S_{co}\right] = 35200\ psi\right\}$ *True*

Gasket Seating/Assembly Condition – Location: Flange:

$\left\{S_{1g} = 3407.1\ psi\right\} \le \left\{1.5S_{hg} = 33000\ psi\right\}$ *True*

$\left\{S_{2g} = 0.0\ psi\right\} \le \left\{S_{hg} = 22000\ psi\right\}$ *True*

$\left\{S_{3g} = 1437.8\ psi\right\} \le \left\{0.8S_{hg} = 17600\ psi\right\}$ *True*

$\left\{S_{4g} = 1264.7\ psi\right\} \le \left\{0.8S_{hg} = 17600\ psi\right\}$ *True*

Gasket Seating/Assembly Condition – Location: Clamp:

$$\left\{S_{5g} = 4429.6 \ psi\right\} \le \left\{1.5S_{cg} = 33000 \ psi\right\} \qquad True$$

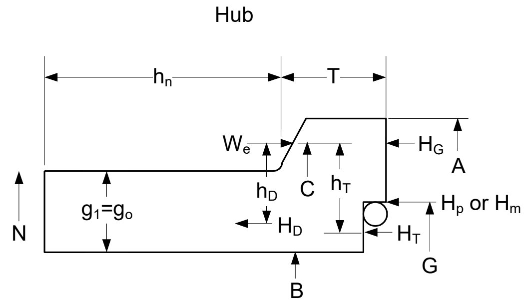
$$\left\{S_{6g} = 1560.4 \ psi\right\} \le \left\{1.5S_{cg} = 33000 \ psi\right\} \qquad True$$

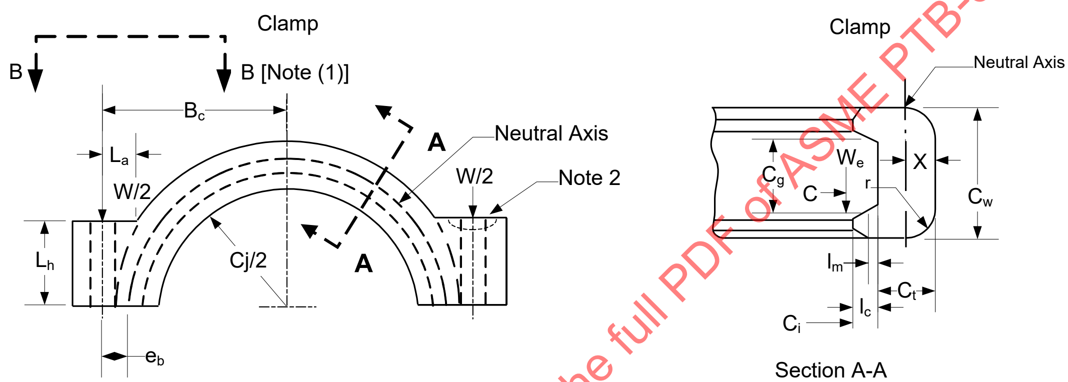
$$\left\{S_{7g} = 1410.5 \ psi\right\} \le \left\{0.8S_{cg} = 17600 \ psi\right\} \qquad True$$

$$\left\{S_{8g} = 288.2 \ psi\right\} < \left\{S_{cg} = 22000 \ psi\right\} \qquad True$$

$$\left\{S_{9g} = 2507.6 \ psi\right\} < \left\{1.6\min\left[S_{hg}, \ S_{cg}\right] = 35200 \ psi\right\} \qquad True$$

Since the acceptance criteria are satisfied, the design is complete.





Notes:
1) See Figure 4.17.2 for sction B-B
2) Clamp may have spherical depressions at bolt holes to facilitate the use of spherical nuts

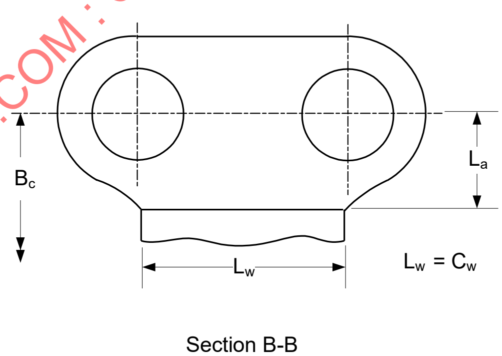


Figure E4.17.1 – Typical Hub and Clamp Configuration

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4.18 Tubesheets in Shell and Tube Heat Exchangers

4.18.1 Example E4.18.1 – U-Tube Tubesheet Integral with Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration a as shown in VIII-2, Figure 4.18.4, Configuration a.

- The shell side design condition is -10 to 60 psig at 500°F, and the tube side design condition is -15 to 140 psig at 500°F.
- The tube material is SA-249, Type 316 (S31600). The tubes are 0.75-inch outside diameter and 0.065-inch thick.
- The tubesheet material is SA-240, Type 316 (S31600). The tubesheet diameter is 12.939-inches. The tubesheet has 76 tube holes on a 1.0-inch square pattern with one centerline pass lane and no pass partition grooves. The largest center-to-center distance between adjacent tube rows is 2.25-inches, the length of the untubed lane is 11.626-inches, and the radius to the outermost tube hole center is 5.438-inches. There is no corrosion allowance on the tubesheet. The tubes are full-strength welded to the tubesheet with no credit taken for expansion.
- The shell material is SA-312, Type 316 (S31600) welded pipe. The shell inside diameter is 12.39-inches and the shell thickness is 0.18-inch.
- The channel material is SA-240, Type 316 (S31600). The channel inside diameter is 12.313-inches and the channel thickness is 0.313-inch.

<u>Data Summary:</u>

The data summary consists of those variables from the nomenclature (see VIII-2, paragraph 4.18.15) that are applicable to this configuration.

<u>Design Conditions:</u>

$P_{sd,\max} = 60\ psig$

$P_{sd,\min} = -10\ psig$

$P_{td,\max} = 140\ psig$

$P_{td,\min} = -15\ psig$

$T = 500^oF$

$T_c = 500^oF$

$T_s = 500^oF$

<u>Tubes:</u>

$d_t = 0.75\ in.$

$E_{tT} = 25.9E6\ psi$ from Table TM-1 of Section II, Part D at T

$S_{tT} = 18{,}000\ psi$ from Table 5A of Section II, Part D at T (for seamless tube, SA-213)

$t_t = 0.065\ in.$

The tubes are SA-249, Type 316 (welded). VIII-2, paragraph 4.18.15 requires the use of the allowable stress for the equivalent seamless product, which is SA-213, Type 316.

Tubesheet:

Tube Pattern: Square

$A = 12.939\ in.$

$A_L = 26.16\ in.^2$

$c_t = 0\ in.$

$E = 25.9E6\ psi$ from Table TM-1 of Section II, Part D at T

$h = 0.521\ in.$ (assumed)

$h_g = 0\ in.$

$L_{L1} = 11.626\ in.$

$p = 1.0\ in.$

$r_o = 5.438\ in.$

$S = 18{,}000\ psi$ from Table 5A of Section II, Part D at T

$S_y = 20{,}000\ psi$ from Table Y-1 of Section II, Part D at T

$U_{L1} = 2.25\ in.$

$\rho = 0$ for no tube expansion

Shell:

$D_s = 12.39\ in.$

$E_s = 25.9E6\ psi$ from Table TM-1 of Section II, Part D at T_s

$S_s = 18{,}000\ psi$ from Table 5A of Section II, Part D at T_s (for seamless pipe, SA-312)

$t_s = 0.180\ in.$

$\nu_s = 0.31$ from Table PRD of Section II, Part D for High Alloy Steels (300 Series)

The shell is SA-312, Type 316 welded pipe. VIII-2, paragraph 4.18.15 requires the use of the allowable stress for the equivalent seamless product, which is SA-312, Type 316 seamless pipe.

Channel:

$D_c = 12.313\ in.$

$E_c = 25.9E6\ psi$ from Table TM-1 of Section II, Part D at T_c

$S_c = 18{,}000\ psi$ from Table 5A of Section II, Part D at T_c

$t_c = 0.313\ in.$

$\nu_c = 0.31$ from Table PRD of Section II, Part D for High Alloy Steels (300 Series)

Calculation Procedure:

The calculation procedure for a U-tube heat exchanger tubesheet is given in VIII-2, paragraph 4.18.7.4. The calculation results are shown for the loading cases required to be analyzed (see paragraph 4.18.7.3).

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.6.4(a).

$D_o = 11.626\ in.$

$\mu = 0.2500$

$d^* = 0.7500\ in.$

$p^* = 1.152\ in.$

$\mu^* = 0.3489$

$h'_g = 0\ in.$

b) STEP 2 – Calculate ρ_s and ρ_c. For each loading case, list the tubesheet loads and the calculated value of M_{TS} for configuration a.

$\rho_s = 1.066$

$\rho_c = 1.059$

Summary Table for Tubesheet Loads and STEP 2				
Loading Case	P_s (psi)	P_t (psi)	W^* (lbf)	M_{TS} (in.-lb/in.)
1	-10	140	0	-160.1
2	60	-15	0	87.03
3	60	140	0	-77.14
4	-10	-15	0	4.030

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

$h/p = 0.5210$

$E^*/E = 0.4452$

$\nu^* = 0.2539$

$E^* = 11.53E6\ psi$

d) STEP 4 – For configuration a, calculate shell coefficients β_s, k_s, λ_s, δ_s and ω_s.

$\beta_s = 1.206\ in.^{-1}$

$k_s = 33.60E3\ lb$

$\lambda_s = 32.26E6\ psi$

$\delta_s = 6.956E\text{-}6\ in.^3/lb$

$\omega_s = 0.4894\ in.^2$

For configuration a, calculate channel coefficients β_c, k_c, λ_c, δ_c and ω_c.

$\beta_c = 0.9129\ in.^{-1}$

$k_c = 0.1337E6\ lb$

$\lambda_c = 111.0E6\ psi$

$\delta_c = 3.951E\text{-}6\ in.^3/lb$

$\omega_c = 0.7535\ in.^2$

e) STEP 5 – Calculate K and F for configuration a.

$K = 1.113$

$F = 9.446$

f) STEPS 6 thru 8 – For each loading case, calculate M^*, M_p, M_o, M and tubesheet bending stress σ.

Summary Table for STEPS 6 thru 8						
Loading Case	M^* (in.-lb/in.)	M_p (in.-lb/in.)	M_o (in.-lb/in.)	M (in.-lb/in.)	σ (psi)	$2S$ (psi)
1	-49.75	568.2	-462.6	568.2	35,990	36,000
2	46.36	-282.0	233.4	282.0	17,870	36,000
3	-1.014	305.5	-244.3	305.5	19,350	36,000
4	-2.378	-19.33	15.04	19.33	1,224	36,000

For Loading Cases 1-4 $|\sigma| \le 2S$. The bending stress criterion for the tubesheet is satisfied.

g) STEP 9 – Check the criterion below for the largest value of $|P_s - P_t|$ and calculate the shear stress, if required.

$$\{|P_s - P_t| = 150.0\ psi\} \le \left\{\frac{4\mu h}{D_o} \cdot \min\left[0.8S, 0.533S_y\right] = 477.7\ psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

h) STEP 10 – For each loading case, calculate the stresses in the shell and channel for configuration a, and check the acceptance criterion. The shell thickness shall be 0.18 in. for a minimum length of 2.688 in. adjacent to the tubesheet, and the channel thickness shall be 0.313 in. for a minimum length of 3.534 in. adjacent to the tubesheet.

Summary Table for STEP 10, Shell Results				
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)
1	-169.6	-17,600	17,770	27,000
2	1,018	12,210	13,230	27,000
3	1,018	-5,336	6,354	27,000
4	-169.6	-53.91	223.5	27,000

For Design Loading Cases 1 - 4 $|\sigma_s| \le 1.5S_s$. The stress criterion for the shell is satisfied.

Summary Table for STEP 10, Channel Results				
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)
1	1,343	25,290	26,640	27,000
2	-143.9	-11,690	11,830	27,000
3	1,343	14,630	15,970	27,000
4	-143.9	-1,023	1,167	27,000

For Design Loading Cases 1-4 $|\sigma_c| \leq 1.5S_c$. The stress criterion for the channel is satisfied.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

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4.18.2 Example E4.18.2 – U-Tube Tubesheet Gasketed with Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration d as shown in VIII-2, Figure 4.18.4, Configuration d.

- The shell side design condition is -15 to 10 psig at 300°F and the tube side design condition is 0 to 135 psig at 300°F.
- The tube material is SB-111, Admiralty (C44300). The tubes are 0.625 in. outside diameter and 0.065 in. thick.
- The tubesheet material is SA-285, Grade C (K02801). The tubesheet diameter is 20.0 in. The tubesheet has 386 tube holes on a 0.75 in. equilateral triangular pattern with one centerline pass lane and no pass partition grooves. The largest center-to-center distance between adjacent tube rows is 1.75 in., the length of the untubed lane is 16.813 in. and the radius to the outermost tube hole center is 8.094 in. There is a 0.125 in. corrosion allowance on the tube side. The tubes are expanded for the full thickness of the tubesheet.
- The channel flange gasket consists of a ring gasket with a centerline rib. The ring gasket outside diameter is 19.375 in., the inside diameter is 18.625 in., and the gasket factors are y = 10,000 psi and m = 3.0. The rib gasket width is 0.375 in., the length is 18.625 in., and the rib gasket factors are y = 9,000 psi and m = 3.75. The shell flange gasket outside diameter is 19.375 in., the inside diameter is 18.625 in., and the gasket factors are y = 10,000 psi and m = 3.0. The effective gasket width for both gaskets is per VIII-2 paragraph 4.16, Table 4.16.3, sketch (1a). There are (24) 0.75 in. diameter SA-193-B7 bolts on a 20.875 in. bolt circle.

Data Summary:

The data summary consists of those variables from the nomenclature (see VIII-2, paragraph 4.18.15) that are applicable to this configuration.

Design Conditions:

$P_{sd,\max} = 10\ psig$

$P_{sd,\min} = -15\ psig$

$P_{td,\max} = 135\ psig$

$P_{td,\min} = 0\ psig$

$T = 300^oF$

$T_{fe} = 300^oF$

Tubes:

$d_t = 0.625\ in.$

$E_{tT} = 15.3E6\ psi$ from Table TM-3 of Section II, Part D at T

$S_{tT} = 10{,}000\ psi$ from Table 5B of Section II, Part D at T

$t_t = 0.065\ in.$

Tubesheet:

Tube Pattern: Triangular

Assume an uncorroded tubesheet thickness of 1.405 inches.

$A = 20\ in.$

$A_L = 29.42\ in.^2$

$c_t = 0.125\ in.$

$D_E = 18.625\ in.$

$E = 28.3E6\ psi$ from Table TM-1 of Section II, Part D at T

$G_c = 19.00\ in.$ (G per VIII-2 paragraph 4.16)

$G_s = 19.00\ in.$ (G per VIII-2 paragraph 4.16)

$h = 1.405\ in. - 0.125\ in. = 1.28\ in.$ (assumed)

$h_g = 0\ in.$

$L_{L1} = 16.813\ in.$

$p = 0.75\ in.$

$r_o = 8.094\ in.$

$S = 17{,}700\ psi$ from Table 5A of Section II, Part D at T

$S_{fe} = 17{,}700\ psi$ from Table 5A of Section II, Part D at T_{fe}

$S_y = 26{,}500\ psi$ from Table Y-1 of Section II, Part D at T

$U_{L1} = 1.75\ in.$

$W_{m1c} = 50{,}854\ lb$ (W_o per VIII-2 paragraph 4.16)

$W_{m1s} = 3{,}505\ lb$ (W_o per VIII-2 paragraph 4.16)

$\rho = 1.0$ for full length tube expansion

Calculation Procedure:

The tubesheet is not extended as a flange but has an unflanged extension. The calculation procedure for this extension is given in VIII-2, paragraph 4.18.5.4(c). The minimum required thickness of this extension calculated at T_{fe} is:

$h_r = 0.05561\ in.$

The calculation procedure for a U-tube heat exchanger tubesheet is given in VIII-2, paragraph 4.18.7.4. The calculation results are shown for the loading cases required to be analyzed (see paragraph 4.18.7.3).

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.6.4(a).

$D_o = 16.813\ in.$

$\mu = 0.1667$

$d^* = 0.5802\ in.$

$p^* = 0.8053\ in.$

$\mu^* = 0.2794$

$h'_g = 0\ in.$

b) STEP 2 – Calculate ρ_s and ρ_c. For each loading case, list the tubesheet loads and the calculated value of M_{TS} for configuration d.

$\rho_s = 1.130$

$\rho_c = 1.130$

Summary Table for Tubesheet Loads and STEP 2				
Loading Case	P_s (psi)	P_t (psi)	W^* (lbf)	M_{TS} (in.-lb/in.)
1	-15	135	50,854	-785.0
2	10	0	3,505	52.33
3	10	135	50,854	-654.1
4	-15	0	0	-78.50

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

$h/p = 1.707$

$E^*/E = 0.2647$

$\nu^* = 0.3579$

$E^* = 7.491E6\ psi$

d) STEP 4 – For configuration d, skip STEP 4 and proceed to STEP 5.

e) STEP 5 – Calculate the diameter ratio K and the coefficient F for configuration d.

$K = 1.190$

$F = 0.4211$

f) STEPS 6 thru 8 – For each loading case, calculate M^*, M_p, M_o, M and tubesheet bending stress σ.

Summary Table for STEPS 6 thru 8						
Loading Case	M^* (in.-lb/in.)	M_p (in.-lb/in.)	M_o (in.-lb/in.)	M (in.-lb/in.)	σ (psi)	$2S$ (psi)
1	-785.0	-159.8	-2,384	2,384	31,250	35,400
2	52.33	10.65	159.0	159.0	2,083	35,400
3	-654.1	-133.1	-1,987	1,987	26,040	35,400
4	-78.50	-15.98	-238.4	238.4	3,125	35,400

For Loading Cases 1-4 $|\sigma| \le 2S$. The bending stress criterion for the tubesheet is satisfied.

g) STEP 9 – Check the criterion below for the largest value of $|P_s - P_t|$ and calculate the shear stress, if required.

$$\{|P_s - P_t| = 150.0\,psi\} \le \left\{\frac{4\mu h}{D_o} \cdot \min\left[0.8S, 0.533S_y\right] = 717.8\,psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

4.18.3 Example E4.18.3 – U-Tube Tubesheet Gasketed with Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration d as shown in VIII-2, Figure 4.18.4UHX-12.1 Configuration d.

- The shell side design condition is 0 to 375 psig at 500°F and the tube side design condition is 0 to 75 psig at 500°F.
- The tube material is SB-111, 90/10 Copper-Nickel (C70600). The tubes are 0.75 in. outside diameter and 0.049 in. thick.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet diameter is 48.88 in. The tubesheet has 1,534 tube holes on a 0.9375 in. equilateral triangular pattern with one centerline pass lane and a 0.1875 in. deep pass partition groove. The largest center-to-center distance between adjacent tube rows is 2.25 in., the length of the untubed lane 41.75 in., and the radius to the outermost tube hole center is 20.5 in. There is a 0.125 in. corrosion allowance on the tube side. The tubes are expanded for one-half of the tubesheet thickness.
- The channel flange gasket consists of a ring gasket with a centerline rib. The ring gasket outside diameter is 45.38 in., the inside diameter is 44.38 in., and the gasket factors are y = 10,000 psi and m = 3.0. The rib gasket width is 0.50 in., the length is 44.38 in., and the rib gasket factors are y = 9,000 psi and m = 3.75. The shell flange gasket outside diameter is 44.0 in., the inside diameter is 43.0 in., and the gasket factors are y = 10,000 psi and m = 3.0. The effective gasket width for both gaskets is per VIII-2 paragraph 4.16, Table 4.16.3, sketch (1a). There are (52) 1.0 in. diameter SA-193-B7 bolts on a 46.75 in. bolt circle.
- The tubesheet shall be designed for a differential design pressure of 300 psi.

Data Summary:

The data summary consists of those variables from the nomenclature (see VIII-2, paragraph 4.18.15) that are applicable to this configuration.

Design Conditions:

$$P_{sd,\max} = 375\ psig$$

$$P_{td,\max} = 75\ psig$$

$$T = 500^oF$$

$$T_{fe} = 500^oF$$

Tubes:

$$d_t = 0.75\ in.$$

$E_{tT} = 16.5E6\ psi$ from Table TM-3 of Section II, Part D at T

$S_{tT} = 8{,}000\ psi$ from Table 5B of Section II, Part D at T

$$t_t = 0.049\ in.$$

Tubesheet:

Tube Pattern: Triangular

Assume an uncorroded tubesheet thickness of 4.275 inches.

$A = 48.88\ in.$

$A_L = 93.94\ in.^2$

$c_t = 0.125\ in.$

$D_E = 44.38\ in.$

$E = 27.1E6\ psi$ from Table TM-1 of Section II, Part D at T

$G_c = 44.88\ in.$ (G per VIII-2 paragraph 4.16)

$G_s = 43.50\ in.$ (G per VIII-2 paragraph 4.16)

$h = 4.275\ in. - 0.125\ in. = 4.15\ in.$ (assumed)

$h_g = 0.1875\ in.$

$L_{L1} = 41.75\ in.$

$p = 0.9375\ in.$

$r_o = 20.5\ in.$

$S = 20{,}600\ psi$ from Table 5A of Section II, Part D at T

$S_{fe} = 20{,}600\ psi$ from Table 5A of Section II, Part D at T_{fe}

$S_y = 31{,}000\ psi$ from Table Y-1 of Section II, Part D at T

$U_{L1} = 2.25\ in.$

$W_{m1c} = 140{,}682\ lb$ (W_o per VIII-2 paragraph 4.16)

$W_{m1s} = 633{,}863\ lb$ (W_o per VIII-2 paragraph 4.16)

$\rho = 0.50$

Calculation Procedure:

The tubesheet is extended as a flange but has no bolt loads applied to the extension. The calculation procedure for this extension is given in VIII-2, paragraph 4.18.5.4(c). The minimum required thickness of the extension calculated at T_{fe} is:

$h_r = 0.2080\ in.$

The calculation procedure for a U-tube heat exchanger tubesheet is given in VIII-2, paragraph 4.18.7.4. The calculation results are shown for the loading cases required to be analyzed (see paragraph 4.18.7.3).

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.6.4(a).

$D_o = 41.75\ in.$

$\mu = 0.2000$

$d^* = 0.7381\ in.$

$p^* = 0.9714\ in.$

$\mu^* = 0.2402$

$h'_g = 0.06250\ in.$

b) STEP 2 – Calculate ρ_s and ρ_c. For Loading Case 3, list the tubesheet loads and the calculated value of M_{TS} for configuration d.

$\rho_s = 1.042$

$\rho_c = 1.075$

Summary Table for Tubesheet Loads and STEP 2				
Loading Case	P_s (psi)	P_t (psi)	W^* (lbf)	M_{TS} (in.-lb/in.)
1	N/A	N/A	N/A	N/A
2	N/A	N/A	N/A	N/A
3	375	75	633,863	2,251
4	N/A	N/A	N/A	N/A

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

$h/p = 4.427$

$E^*/E = 0.2043$

$\nu^* = 0.4069$

$E^* = 5.537E6\ psi$

d) STEP 4 – For configuration d, skip STEP 4 and proceed to STEP 5.

e) STEP 5 – Calculate the diameter ratio K and the coefficient F for configuration d.

$K = 1.171$

$F = 0.4577$

f) STEPS 6 thru 8 – For Loading Case 3, calculate M^*, M_p, M_o, M and tubesheet bending stress σ.

Summary Table for STEPS 6 thru 8						
Loading Case	M^* (in.-lb/in.)	M_p (in.-lb/in.)	M_o (in.-lb/in.)	M (in.-lb/in.)	σ (psi)	$2S$ (psi)
1	N/A	N/A	N/A	N/A	N/A	N/A
2	N/A	N/A	N/A	N/A	N/A	N/A
3	5,586	-1,299	26,540	26,540	39,670	41,200
4	N/A	N/A	N/A	N/A	N/A	N/A

For Loading Cases 1-4 $|\sigma| \le 2S$. The bending stress criterion for the tubesheet is satisfied.

g) STEP 9 – Check the criterion below for the largest value of $|P_s - P_t|$ and calculate the shear stress, if required.

$$\left\{|P_s - P_t| = 300.0\ psi\right\} \le \left\{\frac{4\mu h}{D_o} \cdot \min\left[0.8S, 0.533S_y\right] = 1310\ psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

4.18.4 Example E4.18.4 – U-Tube Tubesheet Gasketed with Shell and Integral with Channel, Extended as a Flange

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration e as shown in VIII-2, Figure 4.18.4, Configuration e.

- The shell side design condition is 0 to 650 psig at 400°F, and the tube side design condition is 0 to 650 psig at 400°F.
- The tube material is SA-179 (K10200). The tubes are 0.75 in. outside diameter and 0.085 in. thick.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet diameter is 37.25 in. The tubesheet has 496 tube holes on a 1.0 in. square pattern with one centerline pass lane and no pass partition grooves. The largest center-to-center distance between adjacent tube rows is 1.375 in., the length of the untubed lane is 26.25 in., and the radius to the outermost tube hole center is 12.75 in. There is a 0.125 in. corrosion allowance on the tube side. The tubes are expanded for the full thickness of the tubesheet.
- The shell flange gasket outside diameter is 32.875 in., the inside diameter is 31.875 in., and the gasket factors are y = 10,000 psi and m = 3.0. The effective gasket width is per VIII-2 paragraph 4.16, Table 4.16.3, sketch (1a). There are (36) 1.125 in. diameter SA-193-B7 bolts on a 35.0 in. bolt circle.
- The channel material is SA-516, Grade 70 (K02700). The channel inside diameter is 31 in. and the channel thickness is 0.625 in.

Data Summary:

The data summary consists of those variables from the nomenclature (see VIII-2, paragraph 4.18.15) that are applicable to this configuration.

Design Conditions:

$P_{sd,\max} = 650\ psig$

$P_{sd,\min} = 0\ psig$

$P_{td,\max} = 650\ psig$

$P_{td,\min} = 0\ psig$

$T = 400^oF$

$T_a = 70^oF$

$T_c = 400^oF$

$T_{fe} = 400^oF$

Tubes:

$d_t = 0.75\ in.$

$E_{tT} = 27.9E6\ psi$ from Table TM-1 of Section II, Part D at T

$S_{tT} = 13{,}400\ psi$ from Table 1A of Section II, Part D at T

$t_t = 0.085\ in.$

Tubesheet:

Tube Pattern: Square

Assume an uncorroded tubesheet thickness of 3.625 inches.

$A = 37.25\ in.$

$A_L = 36.09\ in.^2$

$C = 35.0\ in.$

$c_t = 0.125\ in.$

$E = 27.9E6\ psi$ from Table TM-1 of Section II, Part D at T

$G_s = 32.375\ in.$ (G per VIII-2 paragraph 4.16)

$h = 3.625\ in. - 0.125\ in. = 3.50\ in.$ (assumed)

$h_g = 0\ in.$

$L_{L1} = 26.25\ in.$

$p = 1.0\ in.$

$r_o = 12.75\ in.$

$S = 21{,}600\ psi$ from Table 5A of Section II, Part D at T

$S_{fe} = 21{,}600\ psi$ from Table 5A of Section II, Part D at T_{fe}

$S_y = 32{,}500\ psi$ from Table Y-1 of Section II, Part D at T

$U_{L1} = 1.375\ in.$

$W_{m1s} = 633{,}930\ lb$ (W_o per VIII-2 paragraph 4.16)

$W_s = 644{,}565\ lb$ (W_o per VIII-2 paragraph 4.16)

$\rho = 1.0$ for full length tube expansion

Channel:

$D_c = 31.0\ in.$

$E_c = 27.9E6\ psi$ from Table TM-1 of Section II, Part D at T_c

$S_c = 21{,}6000\ psi$ from Table 5A of Section II, Part D at T_c

$S_{y,c} = 32{,}500\ psi$ from Table Y-1 of Section II, Part D at T_c

$S_{PS,c} = 65{,}000\ psi$ $\max[3S, 2S_{y,c}]$ (per VIII-2 paragraph 4.1.6.3) at T_c

$t_c = 0.625\ in.$

$\nu_c = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Calculation Procedure:

The tubesheet is extended as a flange. The calculation procedure for a flanged extension is given in VIII-2, paragraph 4.18.5.4(a). The minimum required thickness of the flanged extension is the maximum of required thicknesses for the operating condition (at T_{fe}) and gasket seating condition (at T_a):

$$h_r = \max\,(1.563,\ 1.576) = 1.576\ in.$$

The calculation procedure for a U-tube heat exchanger tubesheet is given in VIII-2, paragraph 4.18.7.4. The calculation results are shown for the loading cases required to be analyzed (see paragraph 4.18.7.3).

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.7.6(a).

$$D_o = 26.25\ in.$$

$$\mu = 0.2500$$

$$d^* = 0.6361\ in.$$

$$p^* = 1.035\ in.$$

$$\mu^* = 0.3855$$

$$h'_g = 0\ in.$$

b) STEP 2 – Calculate ρ_s and ρ_c. For each loading case, list the tubesheet loads and the calculated value of M_{TS} for configuration e.

$$\rho_s = 1.233$$

$$\rho_c = 1.181$$

Summary Table for Tubesheet Loads and STEP 2				
Loading Case	P_s (psi)	P_t (psi)	W^* (lbf)	M_{TS} (in.-lb/in.)
1	0	650	0	-12,130
2	650	0	633,930	16,470
3	650	650	633,930	4,337

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

$$h/p = 3.500$$

$$E^*/E = 0.4413$$

$$\nu^* = 0.3179$$

$$E^* = 12.31E6\ psi$$

d) STEP 4 – For configuration e, shell coefficients $\beta_c = 0$, $k_s = 0$, $\lambda_s = 0$, $\delta_s = 0$ and $\omega_s = 0$.

For configuration e, calculate channel coefficients β_c, k_c, λ_c, δ_c and ω_c.

$$\beta_c = 0.4089\ in.^{-1}$$

$$k_c = 0.5101E6\ lb$$

$$\lambda_c = 7.646E6\ psi$$

$\delta_c = 11.71E\text{-}6\ in.^3/lb$

$\omega_c = 7.013\ in.^2$

e) STEP 5 – Calculate the diameter ratio K and the coefficient F for configuration e.

$K = 1.419$

$F = 0.9645$

f) STEPS 6 thru 8 – For each loading case, calculate M^*, M_p, M_o, M and tubesheet bending stress σ.

Summary Table for STEPS 6 thru 8						
Loading Case	M^* (in.-lb/in.)	M_p (in.-lb/in.)	M_o (in.-lb/in.)	M (in.-lb/in.)	σ (psi)	$2S$ (psi)
1	-7,572	3,018	-20,200	20,200	25,670	43,200
2	26,560	6,646	29,870	29,870	39,750	43,200
3	18,980	9,664	9,664	9,664	12,280	43,200

For Loading Cases 1-3 $|\sigma| \le 2S$. The bending stress criterion for the tubesheet is satisfied.

g) STEP 9 – Check the criterion below for the largest value of $|P_s - P_t|$ and calculate the shear stress, if required.

$$\{|P_s - P_t| = 650.0\ psi\} \le \left\{\frac{4\mu h}{D_o} \cdot \min\left[0.8S, 0.533S_y\right] = 2304\ psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

h) STEP 10 – For each loading case, calculate the stresses in the channel for configuration e, and check the acceptance criterion. The channel thickness shall be 0.625 in. for a minimum length of 7.923 in. adjacent to the tubesheet.

Summary Table for STEP 10, Channel Results					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	$S_{PS,c}$ (psi)
1	7,901	54,420	62,320	32,400	65,000
2	0	-56,470	56,470	32,400	65,000
3	7,901	-2,048	9,948	32,400	65,000

For Design Loading Cases 1 and 2 $|\sigma_c| > 1.5S_c$. For Design Loading Case 3 $|\sigma_c| \le 1.5S_c$. The stress criterion for the channel is not satisfied. For Design Loading Cases 1 and 2, since $|\sigma_c| \le S_{PS,c}$, Option 3 in STEP 11 is permitted.

i) STEP 11 – The design shall be reconsidered by using one or a combination of the following options.

- Option 1 – Increase the tubesheet thickness and return to STEP 1.
- Option 2 – Increase the integral channel thickness and return to STEP 1.
- Option 3 – Perform the simplified elastic-plastic calculation procedures as defined in VIII-2, paragraph 4.18.7.4, STEP 11 (c)(3) with a reduced effective modulus.

Since the total axial stress in the channel σ_c is between $1.5S_c$ and $S_{PS,c}$ for Design Condition Loading Cases 1 and 2, the procedure of VIII-2, paragraph 4.18.7.4, STEP 11 (c)(3) may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the channel occurs.

The results for the effect of plasticity for Design Condition Loading Cases 1 and 2 are shown below.

Summary Results for STEP 11, Elastic-Plastic Iteration Results per VIII-1, paragraph UHX-12.5.11, Option 3		
Design Condition Loading Case	1	2
$1.5S_c$, *psi*	32,400	32,400
σ_c, *psi*	62,320	56,470
E_c^*, *psi*	19.36*E*6	20.34*E*6
k_c, *lb*	0.3539*E6*	0.3718*E6*
λ_c	5.305*E*6	5.573*E*6
F	0.8348	0.8497
M_p, *in.-lb/in.*	2,242	7,928
M_o, *in.-lb/in.*	-20,980	31,150
M, *in.-lb/in.*	20,980	31,150
$\|\sigma\|$, *psi*	26,660	39,580

The final calculated tubesheet bending stresses of 26,660 psi (Loading Case 1) and 39,580 psi (Loading Case 2) are less than the allowable tubesheet bending stress of $2S = 43{,}000\ psi$. As such, this geometry meets the requirement of VIII-2, paragraph 4.18.7.4. The intermediate results for the elastic-plastic iteration are shown above.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

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4.18.5 Example E4.18.5 – Fixed Tubesheet Exchanger, Configuration b, Tubesheet Integral with Shell, Extended as a Flange and Gasketed on the Channel Side

A fixed tubesheet heat exchanger is to be designed with the tubesheet construction in accordance with configuration b as shown in VIII-2, Figure 4.18.5, Configuration b.

- For the Design Condition, the shell side design pressure is 0 to 150 psig at 700°F, and the tube side design pressure is 0 to 400 psig at 700°F.
- There is one operating condition. For Operating Condition 1, the operating pressures and operating metal temperatures are assumed to be the same as the Design Condition. The shell mean metal temperature is 550°F, and the tube mean metal temperature is 510°F.
- The tube material is SA-214 Welded (K01807). The tubes are 1 in. outside diameter and 0.083 in. thick. The unsupported tube span under consideration is between 2 tube supports, and the length of the unsupported tube span is 59 inches.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet diameter is 40.5 in. The tubesheet has 649 tube holes on a 1.25 in. equilateral triangular pattern. There is no pass partition lane, and the outermost tube radius from the tubesheet center is 16.625 in. The distance between the outer tubesheet faces is 168 in. The option for the effect of differential radial expansion is not required. There is no corrosion allowance on the tubesheet. The tubes are expanded to 95% of the tubesheet thickness.
- The shell material is SA-516, Grade 70 (K02700). The shell inside diameter is 34.75 in. and the shell thickness is 0.1875 in. There is no corrosion allowance on the shell. The shell contains an expansion joint that has an inside diameter of 38.5 in. and an axial rigidity of 11,388 lb/in. The efficiency of the shell circumferential welded joint (Category B) is 1.0.
- The channel flange gasket outside diameter is 37.3125 in., the inside diameter is 36.3125 in., and the gasket factors are y = 7,600 psi and m = 3.75. The effective gasket width is per VIII-2 paragraph 4.16, Table 4.16.3, sketch (1a). There are (68) 0.75 in. diameter SA-193-B7 bolts on a 38.875 in. bolt circle.

Data Summary:

The data summary consists of those variables from the nomenclature (see VIII-2, paragraph 4.18.15) that are applicable to this configuration.

Design Conditions:

$$P_{sd,\max} = 150\ psig$$

$$P_{sd,\min} = 0\ psig$$

$$P_{td,\max} = 400\ psig$$

$$P_{td,\min} = 0\ psig$$

$$T = 700^{o}F$$

$$T_a = 70^{o}F$$

$$T_{fe} = 700^{o}F$$

$$T_s = 700^{o}F$$

$$T_t = 700^{o}F$$

Operating Conditions:

$P_{so1,\max} = 150\ psig$

$P_{so1,\min} = 0\ psig$

$P_{to1,\max} = 400\ psig$

$P_{to1,\min} = 0\ psig$

$T_1 = 700^oF$

$T_{s1} = 700^oF$

$T_{t1} = 700^oF$

$T_{s,m1} = 550^oF$

$T_{t,m1} = 510^oF$

Tubes:

$d_t = 1.0\ in.$

$E_t = 25.5E6\ psi$ from Table TM-1 of Section II, Part D at T_t & T_{t1}

$E_{tT} = 25.5E6\ psi$ from Table TM-1 of Section II, Part D at T

$k = 1$ for an unsupported tube span between two tube supports

$\ell = 59\ in.$

$\ell_t = 59\ in.$

$S_t = 10{,}500\ psi$ from Table 1A of Section II, Part D at T_t & T_{t1} (see explanation below)

$S_{tT} = 10{,}500\ psi$ from Table 1A of Section II, Part D at T (see explanation below)

$S_{y,t} = 18{,}600\ psi$ from Table Y-1 of Section II, Part D at T_t & T_{t1}

$t_t = 0.083\ in.$

$\alpha_{t,m1} = 7.3E\text{-}6\ in./in./^oF$ from Table TE-1 of Section II, Part D at $T_{t,m1}$

$\nu_t = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Since the tubes are SA-214 (welded), VIII-2, paragraph 4.18.15 requires that the allowable stress for the welded tubes be divided by 0.85 if the equivalent seamless product is not available. In this case, SA-556, Grade A2 (K01807) could be used as the equivalent seamless product, but the alternative will be illustrated in this example. When the welded tube allowable stresses are divided by 0.85, the resulting allowable stresses are $S_t = 12{,}353\ psi$ and $S_{tT} = 12{,}353\ psi$.

Tubesheet:

Tube Pattern: Triangular

$A = 40.5$ *in.*

$A_L = 0$ $in.^2$ for no pass lanes

$C = 38.875$ *in.*

$c_t = 0$ *in.*

$E = 25.5E6$ *psi* from Table TM-1 of Section II, Part D at T & T_1

$G_c = 36.8125$ *in.* (G per VIII-2 paragraph 4.16)

$a_c = 18.41$ *in.*

$h = 3.0625$ *in.* (assumed)

$h_g = 0$ *in.*

$L_t = 168$ *in.*

$L = 161.875$ *in.*

$N_t = 649$

$p = 1.25$ *in.*

$r_o = 16.625$ *in.*

$S = 18{,}100$ *psi* from Table 5A of Section II, Part D at T & T_1

$S_a = 25{,}300$ *psi* from Table 5A of Section II, Part D at T_a

$S_{fe} = 18{,}100$ *psi* from Table 1A of Section II, Part D at T_{fe}

$S_y = 27{,}200$ *psi* from Table Y-1 of Section II, Part D at T & T_1

$S_{PS} = 54{,}400$ *psi* $\max[3S, 2S_y]$ (per VIII-2 paragraph 4.1.6.3) at T & T_1

$W_{m1c} = 512{,}473$ *lb* (W_o per VIII-2 paragraph 4.16)

$W_c = 512{,}937$ *lb* (W_o per VIII-2 paragraph 4.16)

$\rho = 0.9500$

Shell:

$D_J = 38.5$ *in.*

$D_s = 34.75$ *in.*

$a_s = 17.38$ *in.*

$E_s = 25.5E6$ *psi* from Table TM-1 of Section II, Part D at T_s & T_{s1}

$E_{s,w} = 1.0$

$K_J = 11{,}388$ *lb/in.*

$S_s = 18{,}100$ *psi* from Table 5A of Section II, Part D at T_s & T_{s1}

$S_{y,s} = 27{,}200$ *psi* from Table Y-1 of Section II, Part D at T_s & T_{s1}

$S_{PS,s}$ = 54,400 psi max[$3S_s$, $2S_{y,s}$] (per VIII-2 paragraph 4.1.6.3) at T_s & T_{s1}

$t_s = 0.1875\ in.$

$\alpha_{s,m1} = 7.30E\text{-}6\ in./in./^oF$ from Table TE-1 of Section II, Part D at $T_{s,m1}$

$\nu_s = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Calculation Procedure:

The tubesheet is extended as a flange. The calculation procedure for a flanged extension is given in VIII-2, paragraph 4.18.5.4(a). The minimum required thickness of the flanged extension is the maximum of required thicknesses for the operating condition (at T_{fe}) and gasket seating condition (at T_a):

$$h_r = \max(1.228, 1.168) = 1.228\ in.$$

The calculation procedure for a tubesheets of a fixed tubesheet heat exchanger is given in VIII-2, paragraph 4.18.8.4. The following results are for the design and operating loading cases required to be analyzed (see paragraph 4.18.8.3). This example illustrates the calculation of both the elastic and elastic-plastic solutions.

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.4.6(a). For the operating loading cases, $h'_g = 0$.

$D_o = 34.25\ in.$

$\mu = 0.2000$

$d^* = 0.8924\ in.$

$p^* = 1.250\ in.$

$\mu^* = 0.2861$

$h'_g = 0\ in.$

Calculate a_o, ρ_σ, ρ_c, x_s and x_t.

$a_o = 17.13\ in.$

$\rho_s = 1.015$

$\rho_c = 1.075$

$x_s = 0.4467$

$x_t = 0.6152$

b) STEP 2 – Calculate the shell axial stiffness K_s, tube axial stiffness K_t and stiffness factors $K_{s,t}$ and J.

$K_s = 3.242E6\ lb/in.$

$K_t = 37.67E3\ lb/in.$

$K_{s,t} = 0.1326$

$J = 3.500E\text{-}3$

For configuration b, calculate shell coefficients β_s, k_s, λ_s and δ_s.

$\beta_s = 0.7102\ in.^{-1}$

$k_s = 21.87E3\ lb$

$\lambda_s = 0.8794E6\ psi$

$\delta_s = 53.67E\text{-}6\ in.^3/lb$

For configuration b, channel coefficients $\beta_c = 0$, $k_c = 0$, $\lambda_c = 0$ and $\delta_c = 0$.

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

$h/p = 2.450$

$E^*/E = 0.2630$

$n^* = 0.3640$

$E^* = 6.706E6\ psi$

Calculate X_a and parameters Z_a, Z_d, Z_v, Z_w and Z_m from VIII-2, Table 4.18.3.

$X_a = 3.963$

$Z_a = 6.547$

$Z_d = 0.02461$

$Z_v = 0.06426$

$Z_w = 0.06426$

$Z_m = 0.3715$

d) STEP 4 – Calculate the diameter ratio K and the coefficient F.

$K = 1.182$

$F = 0.4888$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} and U.

$\Phi = 0.6667$

$Q_1 = -0.02264$

$Q_{z1} = 2.856$

$Q_{z2} = 6.888$

$U = 13.78$

e) STEP 5 – Calculate γ, $\omega_s, \omega_s^*, \omega_c, \omega_c^*$ and γ_b. Use the loads listed in the table below to calculate the results for an elastic solution in the corroded condition.

$\omega_s = 2.685\ in.^2$ $\quad\omega_s^* = -2.654\ in.^2$

$\omega_c = 0\ in^2$ $\quad\omega_c^* = 9.682\ in^2$

$\gamma_b = -0.06022$

Summary Table for Step 5 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	0	400	0	512,473
2	150	0	0	0
3	150	400	0	512,473
Summary Table for Step 5 – Operating Condition 1				
Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	0	400	-0.04727	512,937
2	150	0	-0.04727	512,937
3	150	400	-0.04727	512,937
4	0	0	-0.04727	512,937

f) STEP 6 – For each loading case, calculate P_s', P_t', P_γ, P_ω, P_W, P_{rim} and effective pressure P_e.

Summary Table for STEP 6 – Design Condition							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	0.8620E6	0	0	230.7	181.9	-399.4
2	-46,390	0	0	0	0	18.70	-21.50
3	-46,390	0.8620E6	0	0	230.7	200.6	-420.9

Summary Table for STEP 6 – Operating Condition 1							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	0.8620E6	-1,254	0	230.9	181.9	-400.0
2	-46,390	0	-1,254	0	230.9	18.70	-21.97
3	-46,390	0.8620E6	-1,254	0	230.9	200.6	-421.5
4	0	0	-1,254	0	230.9	0	-0.4744

g) STEP 7 – Elastic Iteration: Calculate Q_2, Q_3 and F_m, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (lbf)	Q_3	F_m	$h - h_g'$ (in)	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-7,041	0.09758	0.09751	3.0625	25,540	27,150
2	-319.0	0.07858	0.09006	3.0625	1,269	27,150
3	-7,360	0.09661	0.09713	3.0625	26,810	27,150

Summary Table for STEP 7 – Operating Condition 1						
Loading Case	Q_2 (lbf)	Q_3	F_m	h (in)	$\|\sigma\|$ (psi)	S_{PS} (psi)
1	-7,044	0.09747	0.09747	3.0625	25,570	54,400
2	-4,259	1.300	0.6705	3.0625	9,660	54,400
3	-7,363	0.09650	0.09709	3.0625	26,830	54,400
4	-3,940	56.63	28.42	3.0625	8,840	54,400

For Design Loading Cases 1-3 $|\sigma| \le 1.5S$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma| \le S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Check the criterion below for the largest value of P_e and calculate the shear stress, if required.

$$\{|P_e| = 421.5\ psi\} \le \left\{\frac{2\mu h}{a_o} \cdot \min\left[0.8S, 0.533S_y\right] = 1035\ psi\right\} \quad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

i) STEP 9 – For each loading case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$r_t = 0.3255\ in$

$F_t = 181.2$

$C_t = 164.5$

Summary Table for STEP 9 – Design Condition				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	-1.081	-4,024	3.808	7,570
2	-1.011	268.9	3.658	864.7
3	-1.077	-3,755	3.801	8,434

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	7,570	12,353	-4,024	1.346	5,693
2	864.7	12,353	---	---	---
3	8,434	12,353	-3,755	1.350	5,677

Summary Table for STEP 9 – Operating Condition 1				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	-1.081	-4,207	3.807	7,581
2	-5.520	-322.2	13.33	2,137
3	-1.077	-3,758	3.800	8,445
4	-213.2	-600.4	451.8	1,272

Summary Table for STEP 9 – Operating Condition 1 (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	$2S_t$ (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	7,581	24,706	-4,207	1.346	5,691
2	2,137	24,706	-322.2	1.250	6,129
3	8,445	24,706	-3,758	1.350	5,675
4	1,272	24,706	-600.4	1.250	6,129

For Design Loading Cases 1-3 $\sigma_{t,\max} \le S_t$, and for Operation Condition 1, Loading Cases 1-4 $\sigma_{t,\max} \le 2S_t$. The axial tension stress criterion for the tube is satisfied.

For all Loading Cases, $|\sigma_{t,\min}| \le S_{tb}$. The buckling criterion for the tube is satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Design Condition			
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{s,b}$ (psi)
1	26.08	18,100	---
2	-764.8	18,100	8,505
3	-738.7	18,100	8,505

Summary Table for STEP 10 – Operating Condition 1			
Loading Case	$\sigma_{s,m}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	0.05859	54,400	---
2	-786.1	54,400	8,505
3	-764.8	54,400	8,505
4	-21.24	54,400	8,505

For Design Loading Cases 1-3 $|\sigma_{s,m}| \le S_s E_{s,w}$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma_{s,m}| \le S_{PS,s}$. The axial membrane stress criterion for the shell is satisfied.

For all Loading Cases where the value of $\sigma_{s,m}$ is negative, $|\sigma_{s,m}| \le S_{s,b}$. The longitudinal compressive stress criterion for the shell is satisfied.

k) STEP 11 – For each loading case, calculate the stresses in the shell for configuration b, and check the acceptance criterion. The shell thickness shall be 0.1875 in. for a minimum length of 4.595 in. adjacent to the tubesheet.

Summary Table for STEP 11 – Design Condition					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)
1	26.08	-42,440	42,470	27,150	54,400
2	-764.8	19,210	19,980	27,150	54,400
3	-738.7	-23,230	23,970	27,150	54,400

Summary Table for STEP 11 – Operating Condition 1				
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$S_{PS,s}$ (psi)
1	0.05859	-42,480	42,480	54,400
2	-786.1	8,633	9,419	54,400
3	-764.8	-23,270	24,040	54,400
4	-21.24	-10,580	10,600	54,400

For Design Loading Case 1 $|\sigma_s| > 1.5S_s$. For Design Loading Cases 2 and 3 $|\sigma_s| \le 1.5S_s$ and for Operation Condition 1, Loading Cases 1-4 $|\sigma_s| \le S_{PS,s}$. The stress criterion for the shell is not satisfied. For Design Loading Case 1, since $|\sigma_s| \le S_{PS,s}$, Option 3 in STEP 12 is permitted.

l) STEP 12 – The design shall be reconsidered by using one or a combination of the following options.

- Option 1 – Increase the tubesheet thickness and return to STEP 1.
- Option 2 – Increase the integral shell and/or channel thickness and return to STEP 1.
- Option 3 – Perform the elastic-plastic calculation procedures as defined in VIII-2, paragraph 4.18.8.6(c).

Since the total axial stress in the shell $\sigma_{s,1}$ is between $1.5S_{s,1}$ and $S_{PS,s,1}$ for Design Condition Loading Case 1, the procedure of paragraph 4.18.8.6(c) may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

The results for the effect of plasticity for Design Condition Loading Case 1 are shown below.

Summary Results for STEP 12, Elastic-Plastic Iteration Results per VIII-2, paragraph 4.18.8.6(c).	
Design Condition Loading Case	1
S_s^*, psi	27,200
$fact_s$	0.7759
E_s^*, psi	19.78E6
k_s, lb	16,960
λ_s	0.6823E6
F	0.4701
ϕ	0.6412
Q_1	-0.02149
Q_{Z1}	2.865
Q_{Z2}	6.941
U	13.88
P_W, psi	232.5
P_{rim}, psi	183.3
P_e, psi	-399.4
Q_2, lb	-7,095
Q_3	0.09965
F_m	0.09832
$\lvert\sigma\rvert$, psi	25,750

The final calculated tubesheet bending stress of 25,750 psi (Loading Case 1) is less than the allowable tubesheet bending stress of 27,150 psi. As such, this geometry meets the requirement of VIII-2, paragraph 4.18.8.6. The intermediate results for the elastic-plastic iteration are shown above.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

4.18.6 Example E4.18.6 – Fixed Tubesheet Exchanger, Configuration b, Tubesheet Integral with Shell, Extended as a Flange and Gasketed on the Channel Side

A fixed tubesheet heat exchanger is to be designed with the tubesheet construction in accordance with configuration b as shown in VIII-2, Figure 4.18.5, Configuration b.

- For the Design Condition, the shell side design pressure is 0 to 335 psig at 675°F, and the tube side design pressure is 0 to 1040 psig at 650°F.
- There is one operating condition. For Operating Condition 1, the operating pressures and operating metal temperatures are assumed to be the same as the Design Condition. The shell mean metal temperature is 550°F, and the tube mean metal temperature is 490°F.
- The tube material is SA-214 (K01807). The tubes are 0.75 in. outside diameter and 0.083 in. thick. The unsupported tube span under consideration is between 2 tube supports, and the length of the unsupported tube span is 34 inches.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet diameter is 32.875 in. The tubesheet has 434 tube holes on a 0.9375 in. triangular pattern. There is no pass partition lane and the outermost tube radius from the tubesheet center is 10.406 in. The distance between the outer tubesheet faces is 144.375 in. The option for the effect of differential radial expansion is not required. There is a 0.125 in. corrosion allowance on both sides of the tubesheet. The tubes are expanded for a length of 4.374 in.
- The shell material is SA-516, Grade 70 (K02700). The shell outside diameter is 24 in. and the thickness is 0.5 in. There is a 0.125 in. corrosion allowance on the shell. There is also a shell band adjacent to each tubesheet. The shell bands are 1.25 in. thick and 9.75 in. long with a 0.125 in. corrosion allowance. The shell and shell band materials are the same. The shell contains an expansion joint that has an inside diameter of 29.46 in. and an axial rigidity of 14,759 lb/in. The efficiency of shell circumferential welded joint (Category B) is 0.85.
- The channel flange gasket outside diameter is 26.125 in., the inside diameter is 25.125 in., and the gasket factors are y = 26,000 psi and m = 6.5. The effective gasket width is per VIII-2 paragraph 4.16, Table 4.16.3, sketch (1a). There are (28) 1.375 in. diameter SA-193-B7 bolts on a 30.125 in. bolt circle.

Data Summary:

The data summary consists of those variables from the nomenclature (see VIII-1, paragraph UHX-5.1) that are applicable to this configuration.

Design Conditions:

$$P_{sd,\max} = 335\ psig$$

$$P_{sd,\min} = 0\ psig$$

$$P_{td,\max} = 1040\ psig$$

$$P_{td,\min} = 0\ psig$$

$$T = 675^oF$$

$$T_a = 70^oF$$

$$T_{fe} = 675^oF$$

$T_s = 675^oF$

$T_t = 675^oF$

Operating Conditions:

$P_{so1,\max} = 335\ psig$

$P_{so1,\min} = 0\ psig$

$P_{to1,\max} = 1040\ psig$

$P_{to1,\min} = 0\ psig$

$T_1 = 675^oF$

$T_{s1} = 675^oF$

$T_{t1} = 675^oF$

$T_{s,m1} = 550^oF$

$T_{t,m1} = 490^oF$

Tubes:

$d_t = 0.75\ in.$

$E_t = 25.75E6\ psi$ from Table TM-1 of Section II, Part D at T_t & T_{t1}

$E_{tT} = 25.75E6\ psi$ from Table TM-1 of Section II, Part D at T

$k = 1$ for an unsupported tube span between two tube supports

$\ell = 34\ in.$

$\ell_t = 34\ in.$

$S_t = 10{,}700\ psi$ from Table 1A of Section II, Part D at T_t & T_{t1} (see explanation below)

$S_{tT} = 10{,}700\ psi$ from Table 1A of Section II, Part D at T (see explanation below)

$S_{y,t} = 18{,}950\ psi$ from Table Y-1 of Section II, Part D at T_t & T_{t1}

$t_t = 0.083\ in.$

$\alpha_{t,m1} = 7.28E\text{-}6\ in./in./^oF$ from Table TE-1 of Section II, Part D at $T_{t,m1}$

$\nu_t = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Since the tubes are SA-214 (welded), VIII-2, paragraph 4.18.15 requires that the allowable stress for the welded tubes be divided by 0.85 if the equivalent seamless product is not available. In this case, SA-556, Grade A2 (K01807) could be used as the equivalent seamless product, but the alternative will be illustrated in this example. When the welded tube allowable stresses are divided by 0.85, the resulting allowable stresses are $S_t = 12{,}588\ psi$ and $S_{tT} = 12{,}588\ psi$.

Tubesheet:

Tube Pattern: Triangular

Assume an uncorroded tubesheet thickness of 4.75 inches.

$A = 32.875$ *in.*

$A_L = 0$ $in.^2$ for no pass lanes

$C = 30.125$ *in.*

$c_t = 0.125$ *in.*

$E = 25.575E6$ *psi* from Table TM-1 of Section II, Part D at T & T_1

$G_c = 25.625$ *in.* (G per VIII-2 paragraph 4.16)

$a_c = 12.81$ *in.*

$h = 4.75$ *in.* – 0.125 *in.* -0.125 *in.* = 4.500 *in.* (assumed)

$h_g = 0$ *in.*

$L_t = 144.125$ *in.*

$L = 144.125$ *in.* – 2(4.50 *in.*) = 135.125 *in.*

$\ell_{tx} = 4.374$ *in.*

$N_t = 434$

$p = 0.9375$ *in.*

$r_o = 10.406$ *in.*

$S = 18{,}450$ *psi* from Table 1A of Section II, Part D at T & T_1

$S_a = 20{,}000$ *psi* from Table 1A of Section II, Part D at T_a

$S_{fe} = 18{,}450$ *psi* from Table 1A of Section II, Part D at T_{fe}

$S_y = 27{,}700$ *psi* from Table Y-1 of Section II, Part D at T & T_1

$S_{PS} = 55{,}400$ *psi* max[$3S$, $2S_y$] (per VIII-2 paragraph 4.1.6.3) at T & T_1

$W_{m1c} = 808{,}456$ *lb* (W_{m1} per VIII-2 paragraph 4.16)

$W_c = 808{,}478$ *lb* (W per VIII-2 paragraph 4.16)

$\rho = 0.9720$

Shell Band (Adjacent to Tubesheet):

$D_s = 23.25$ *in.*

$a_s = 11.63$ *in.*

$E_{s,1} = 25.75E6$ *psi* from Table TM-1 of Section II, Part D at T_s & T_{s1}

$E_{s,w,1} = 0.85$

$\ell_1 = 9.75$ *in.* + 0.125 *in.* = 9.875 *in.*

$\ell'_1 = 9.75\ in. + 0.125\ in. = 9.875\ in.$

$S_{s,1} = 18{,}450\ psi$ from Table 1A of Section II, Part D at T_s & T_{s1}

$S_{y,s,1} = 27{,}700\ psi$ from Table Y-1 of Section II, Part D at T_s & T_{s1}

$S_{PS,s,1} = 55{,}400\ psi$ max$[3S_{s,1}, 2S_{y,s,1}]$ (per VIII-2 paragraph 4.1.6.3) at T_s & T_{s1}

$t_{s,1} = 1.125\ in.$

$\alpha_{s,m1,1} = 7.30E\text{-}6\ in./in./^oF$ from Table TE-1 of Section II, Part D at $T_{s,m1}$

$\nu_{s,1} = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Shell:

$D_J = 29.46\ in.$

$D_s = 23.25\ in.$

$E_s = 25.75E6\ psi$ from Table TM-1 of Section II, Part D at T_s & T_{s1}

$E_{s,w} = 0.85$

$K_J = 14{,}759\ lb/in.$

$S_s = 18{,}450\ psi$ from Table 1A of Section II, Part D at T_s & T_{s1}

$S_{y,s} = 27{,}700\ psi$ from Table Y-1 of Section II, Part D at T_s & T_{s1}

$S_{PS,s} = 55{,}400\ psi$ max$[3S_s, 2S_{y,s}]$ (per VIII-2 paragraph 4.1.6.3) at T_s & T_{s1}

$t_s = 0.375\ in.$

$\alpha_{s,m1} = 7.30E\text{-}6\ in./in./^oF$ from Table TE-1 of Section II, Part D at $T_{s,m1}$

$\nu_s = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Calculation Procedure:

The tubesheet is extended as a flange. The calculation procedure for a flanged extension is given in VIII-2, paragraph 4.18.5.4(a). The minimum required thickness of the flanged extension is the maximum of required thicknesses for the operating condition (at T_{fe}) and gasket seating condition (at T_a):

$h_r = \max(2.704, 2.597) = 2.704\ in.$

The calculation procedure for a tubesheets of a fixed tubesheet heat exchanger is given in VIII-2, paragraph 4.18.8.4. The following results are for the design and operating loading cases required to be analyzed (see paragraph 4.18.8.3). This example illustrates the calculation of both the elastic and elastic-plastic solutions for a shell that has a thickened shell band adjacent to the tubesheet.

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.6.4(a). For the operating loading cases, $h'_g = 0$.

$D_o = 21.562\ in.$

$\mu = 0.2000$

$d^* = 0.6392\ in.$

$p* = 0.9375\ in.$

$\mu^* = 0.3182$

$h'_g = 0\ in.$

Calculate a_o, ρ_s, ρ_c, x_s and x_t.

$a_o = 10.78\ in.$

$\rho_s = 1.078$

$\rho_c = 1.188$

$x_s = 0.4749$

$x_t = 0.6816$

b) STEP 2 – Calculate the shell axial stiffness K_s^*, tube axial stiffness K_t and stiffness factors $K_{s,t}$ and J.

K_s^* = 5.876E6 lb/in.

K_t = 33.14E3 $lb/in.$

$K_{s,t}$ = 0.4085

J = 2.505E-3

For configuration b, calculate shell coefficients β_s, k_s, λ_s and δ_s.

β_s = 0.3471 $in.^{-1}$

k_s = 2.331E6 lb

λ_s = 13.50E6 psi

δ_s = 3.965E-6 $in.^3/lb$

For configuration b, channel coefficients $\beta_c = 0$, $k_c = 0$, $\lambda_c = 0$ and $\delta_c = 0$.

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

$h/p = 4.800$

$E^*/E = 0.3051$

$\nu^* = 0.3423$

$E^* = 7.804E6\ psi$

Calculate X_a and parameters Z_a, Z_d, Z_v, Z_w and Z_m from VIII-2, Table 4.18.3.

$X_a = 1.995$

$Z_a = 0.8092$

$Z_d = 0.1745$

$Z_v = 0.1605$

$Z_w = 0.1605$

$Z_m = 0.6679$

d) STEP 4 – Calculate the diameter ratio K and the coefficient F.

K = 1.525

F = 2.047

Calculate Φ, Q_1, Q_{z1}, Q_{z2} and U.

Φ = 2.747

Q_1 = -0.1280

Q_{z1} = 1.221

Q_{z2} = 0.5952

U = 1.190

e) STEP 5 – Calculate γ, ω_s, ω_s^*, ω_c, ω_c^* and γ_b. Use the loads listed in the table below to calculate the results for an elastic solution in the corroded condition.

$\omega_s = 8.865\ in.^2$ $\omega_s^* = -8.495\ in.^2$

$\omega_c = 0\ in^2$ $\omega_c^* = 8.659\ in^2$

$\gamma_b = -0.2087$

Summary Table for Step 5 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	0	1040	0	808,456
2	335	0	0	0
3	335	1040	0	808,456

Summary Table for Step 5 – Operating Condition 1				
Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	0	1040	-0.06032	808,478
2	335	0	-0.06032	808,478
3	335	1040	-0.06032	808,478
4	0	0	-0.06032	808,478

f) STEP 6 – For each loading case, calculate P_s', P_t', P_γ, P_ω, P_W, P_{rim} and effective pressure P_e.

Summary Table for STEP 6 – Design Condition							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	1.017*E*6	0	0	275.0	92.23	-1,039
2	-0.1674*E*6	0	0	0	0	29.14	-171.0
3	-0.1674*E*6	1.017*E*6	0	0	275.0	121.4	-1,210

Summary Table for STEP 6 – Operating Condition 1							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	1.017*E*6	-2,376	0	275.0	92.23	-1,042
2	-0.1674*E*6	0	-2,376	0	275.0	29.14	-173.2
3	-0.1674*E*6	1.017*E*6	-2,376	0	275.0	121.4	-1,213
4	0	0	-2,376	0	275.0	0	-2.148

g) STEP 7 – Elastic Iteration: Calculate Q_2, Q_3 and F_m, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (lbf)	Q_3	F_m	$h - h_g'$ (in)	$\|\sigma\|$ (psi)	$1.5S$ (psi)
1	-12,650	0.08150	0.1986	4.500	22,330	27,675
2	-1,004	-0.02696	0.1574	4.500	2,913	27,675
3	-13,650	0.06617	0.1927	4.500	25,240	27,675

Summary Table for STEP 7 – Operating Condition 1						
Loading Case	Q_2 (lbf)	Q_3	F_m	h (in)	$\|\sigma\|$ (psi)	S_{PS} (psi)
1	-12,650	0.08101	0.1984	4.500	22,360	55,400
2	-10,480	0.9131	0.5333	4.500	9,995	55,400
3	-13,650	0.06578	0.1926	4.500	25,280	55,400
4	-9,473	75.77	37.94	4.500	8,817	55,400

For Design Loading Cases 1-3 $|\sigma| \le 1.5S$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma| \le S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Check the criterion below for the largest value of P_e and calculate the shear stress, if required.

$$\{|P_e| = 1213\ psi\} \le \left\{\frac{2\mu h}{a_o} \cdot \min\left[0.8S, 0.533S_y\right] = 2464\ psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

i) STEP 9 – For each loading case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$r_t = 0.2376\ in$

$F_t = 143.1$

$C_t = 163.8$

Summary Table for STEP 9 – Design Condition				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	0.4598	-1,118	1.487	4,047
2	0.5904	1,258	1.349	1,886
3	0.4782	140.2	1.468	5,933

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	4,047	12,588	-1,118	2.000	5,336
2	1,886	12,588	---	---	---
3	5,933	12,588	---	---	---

Summary Table for STEP 9 – Operating Condition 1				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	0.4604	-1,110	1.487	4,061
2	-0.5417	315.9	2.545	2,902
3	0.4787	148.5	1.467	5,947
4	-90.69	-942.2	97.82	1,016

Summary Table for STEP 9 – Operating Condition 1 (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	$2S_t$ (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	4,061	25,176	-1,110	2.000	5,336
2	2,902	25,176	---	---	---
3	5,947	25,176	---	---	---
4	1,016	25,176	-942.2	1.250	8,538

For Design Loading Cases 1-3 $\sigma_{t,\max} \le S_t$, and for Operation Condition 1, Loading Cases 1-4 $\sigma_{t,\max} \le 2S_t$. The axial tension stress criterion for the tube is satisfied.

For all Loading Cases $|\sigma_{t,\min}| \le S_{tb}$. The buckling criterion for the tube is satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Shell Band – Design Condition			
Loading Case	$\sigma_{s,m,1}$ (psi)	$S_{s,1}E_{s,w}$ (psi)	$S_{s,b,1}$ (psi)
1	3.369	15,683	---
2	-493.9	15,683	12,580
3	-490.5	15,683	12,580

Summary Table for STEP 10 – Shell Band – Operating Condition 1			
Loading Case	$\sigma_{s,m,1}$ (psi)	$S_{PS,s,1}$ (psi)	$S_{s,b,1}$ (psi)
1	-6.926	55,400	12,580
2	-503.0	55,400	12,580
3	-500.8	55,400	12,580
4	-9.103	55,400	12,580

For Design Loading Cases 1-3 $|\sigma_{s,m,1}| \leq S_{s,1}E_{s,w}$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma_{s,m,1}| \leq S_{PS,s,1}$. The axial membrane stress criterion for the shell band is satisfied.

For all Loading Cases where the value of $\sigma_{s,m,1}$ is negative, $|\sigma_{s,m,1}| \leq S_{s,b,1}$. The longitudinal compressive stress criterion for the shell band is satisfied.

Summary Table for STEP 10 – Main Shell – Design Condition			
Loading Case	$\sigma_{s,m}$ (psi)	$S_sE_{s,w}$ (psi)	$S_{s,b}$ (psi)
1	10.43	15,683	---
2	-1,529	15,683	10,800
3	-1,518	15,683	10,800

Summary Table for STEP 10 – Main Shell – Operating Condition 1			
Loading Case	$\sigma_{s,m}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	-21.44	55,400	10,800
2	-1,557	55,400	10,800
3	-1,550	55,400	10,800
4	-28.18	55,400	10,800

For Design Loading Cases 1-3 $|\sigma_{s,m}| \leq S_sE_{s,w}$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma_{s,m}| \leq S_{PS,s}$. The axial membrane stress criterion for the main shell is satisfied.

For all Loading Cases where the value of $\sigma_{s,m}$ is negative, $|\sigma_{s,m}| \leq S_{s,b}$. The longitudinal compressive stress criterion for the main shell is satisfied.

k) STEP 11 – For each loading case, calculate the stresses in the shell band for configuration b, and check the acceptance criterion. The shell band thickness shall be 1.125 in. for a minimum length of 9.206 in adjacent to the tubesheet.

Summary Table for STEP 11, Shell Band Results – Design Condition					
Loading Case	$\sigma_{s,m,1}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_{s,1}$ (psi)	$S_{PS,s,1}$ (psi)
1	3.369	-41,040	41,040	27,675	55,400
2	-493.9	617.7	1,112	27,675	55,400
3	-490.5	-40,420	40,910	27,675	55,400

Summary Table for STEP 11, Shell Band Results – Operating Condition 1				
Loading Case	$\sigma_{s,m,1}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$S_{PS,s,1}$ (psi)
1	-6.926	-41,070	41,080	55,400
2	-503.0	-19,410	19,920	55,400
3	-500.8	-40,460	40,960	55,400
4	-9.103	-20,030	20,040	55,400

For Design Loading Cases 1 and 3 $|\sigma_s| > 1.5S_{s,1}$. For Design Loading Case 2 $|\sigma_s| \le 1.5S_{s,1}$ and for Operation Condition 1, Loading Cases 1-4 $|\sigma_s| \le S_{PS,s,1}$. The stress criterion for the shell band is not satisfied. For Design Loading Cases 1 and 3, since $|\sigma_s| \le S_{PS,s,1}$, Option 3 in STEP 12 is permitted.

l) STEP 12 – The design shall be reconsidered by using one or a combination of the following options.

- Option 1 – Increase the tubesheet thickness and return to STEP 1.
- Option 2 – Increase the integral shell and/or channel thickness and return to STEP 1.
- Option 3 – Perform the elastic-plastic calculation procedures as defined in VIII-2, paragraph 4.18.8.6(c).

Since the total axial stress in the shell band $\sigma_{s,1}$ is between $1.5S_{s,1}$ and $S_{PS,s,1}$ for Design Condition Loading Cases 1 and 3, the procedure of VIII-2, paragraph 4.18.8.6(c) may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

The results for the effect of plasticity for Design Condition Loading Cases 1 and 3 are shown below.

Summary Results for STEP 12, Elastic-Plastic Iteration Results per VIII-2, paragraph 4.18.8.6(c)		
Design Condition Loading Case	1	3
S_S^*, psi	27,700	27,700
$fact_s$	0.8074	0.8163
E_S^*, psi	20.79*E*6	21.02*E*6
k_s, *lb*	1.882*E6*	1.903*E*6
λ_s	10.90*E*6	11.02*E*6
F	1.828	1.838
ϕ	2.453	2.467
Q_1	-0.1196	-0.1200
Q_{Z1}	1.231	1.231
Q_{Z2}	0.6395	0.6373
U	1.279	1.275
P_W, *psi*	295.5	294.5
P_{rim}, *psi*	99.10	130.0
P_e, *psi*	-1,039	-1,210
Q_2, *lb*	-13,590	-14,620
Q_3	0.1055	0.08787
F_m	0.2077	0.2010
$\|\sigma\|$, *psi*	23,350	26,320

The final calculated tubesheet bending stresses of 23,350 psi (Loading Case 1) and 26,320 psi (Loading Case 3) are less than the allowable tubesheet bending stress of 27,675 psi. As such, this geometry meets the requirement of VIII-2, paragraph 4.18.8.6. The intermediate results for the elastic-plastic iteration are shown above.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

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4.18.7 Example E4.18.7 – Fixed Tubesheet Exchanger, Configuration a

A fixed tubesheet heat exchanger with the tubesheet construction in accordance with configuration a as shown in VIII-2, Figure 4.18.5, Configuration a.

- For the Design Condition, the shell side design pressure is 0 to 325 psig at 400°F, and the tube side design pressure is 0 to 200 psig at 300°F. The tube design temperature is 300°F.
- There is one operating condition. For Operating Condition 1, the operating pressures and operating metal temperatures are assumed to be the same as those for the Design Condition. The shell mean metal temperature is 151°F, and the tube mean metal temperature is 113°F.
- The tube material is SA-249, Type 304L (S30403). The tubes are 1 in. outside diameter and 0.049 in. thick. The unsupported tube span under consideration is between 2 tube supports, and the length of the unsupported tube span is 48 inches.
- The tubesheet material is SA-240, Type 304L (S30403). The tubesheet diameter is 43.125 in. The tubesheet has 955 tube holes on a 1.25 in. equilateral triangular pattern. There is no pass partition lane and the outermost tube radius from the tubesheet center is 20.125 in. The distance between the outer tubesheet faces is 240 in. The option for the effect of differential radial expansion is not required. There is no corrosion allowance on the tubesheet. The tubes are expanded from the tube side face of the tubesheet to 0.125 in. from the shell side face of the tubesheet.
- The shell material is SA-240, Type 304L (S30403). The shell inside diameter is 42 in. and the shell thickness is 0.5625 in. There is no corrosion allowance on the shell and no expansion joint in the shell. The efficiency of shell circumferential welded joint (Category B) is 0.85.
- The channel material is SA-516, Grade 70 (K02700). The inside diameter of the channel is 42.125 in. and the channel thickness is 0.375 in. There is no corrosion allowance on the channel.

For this example, first assume a value of 1.375 in. for the tubesheet thickness and perform the calculation procedure described below starting at STEP 1. The data shown below will be the same except as follows:

$$h = 1.375\ in.$$

$$\ell_{tx} = 1.25\ in.$$

$$\rho = 0.9091$$

$$L = 237.25\ in.$$

In STEP 7, the calculated bending stresses for the tubesheet are less than the allowable stresses for all the Design Loading Cases and for all the Operating Condition 1 Loading Cases. The maximum Design Loading Case bending stress is 23,480 psi, which is less than the allowable of 23,700 psi and the maximum Operating Case bending stress 40,360 psi, which is less than the allowable of 47,400 psi. The bending stress criterion for the tubesheet is satisfied.

In STEP 8, check the criterion below for the largest value of P_e and calculate the shear stress, if required.

$$\left\{|P_e| = 262\ psi\right\} \le \left\{\frac{2\mu h}{a_o} \cdot \min\left[0.8S, 0.533S_y\right] = 249\ psi\right\} \qquad False$$

The above criterion is not satisfied.

Calculate the shear stress τ for Operating Condition 1, Loading Case 1.

$A_p = 1406.25\ in^2$

$C_p = 135\ in$

$\tau = \text{-}9906\ psi$

$$\{|\tau| = 9906\ psi\} \leq \left\{\left(\frac{1}{4\mu}\right)\left(\frac{1}{h}\left\{\frac{4A_p}{C_p}\right\}\right)P_e = 9432\ psi\right\} \qquad False$$

The tubesheet is overstressed for Operating Condition 1, Loading Case 1. Increase the tubesheet thickness to 1.500 in and return to STEP 1 of the calculation procedure in paragraph 4.18.8.4.

Data Summary:

The data summary consists of those variables from the nomenclature (see VIII-2, paragraph 4.18.15) that are applicable to this configuration.

Design Conditions:

$P_{sd,max}$ = 325 *psig*

$P_{sd,min}$ = 0 psig

$P_{td,max}$ = 200 *psig*

$P_{td,min}$ = 0 psig

T = 400°*F*

T_a = 70°F

T_c = 300°*F*

T_s = 400°*F*

T_t = 300°*F*

Operating Conditions:

$P_{so1,max}$ = 325 *psig*

$P_{so1,min}$ = 0 psig

$P_{to1,max}$ = 200 *psig*

$P_{to1,min}$ = 0 psig

T_1 = 400°*F*

T_{c1} = 300°*F*

T_{s1} = 400°*F*

T_{t1} = 300°*F*

$T_{s,m1}$ = 151°*F*

$T_{t,m1}$ = 113°*F*

Tubes:

d_t = 1.0 *in.*

E_t = 27.0*E*6 *psi* from Table TM-1 of Section II, Part D at T_t & T_{t1}

E_{tT} = 26.4*E*6 *psi* from Table TM-1 of Section II, Part D at T

k = 1 for an unsupported tube span between two tube supports

ℓ = 48 *in.*

ℓ_t = 48 in.

S_t = 16,700 *psi* from Table 5A of Section II, Part D at T_t & T_{t1} (see explanation below)

S_{tT} = 15,800 *psi* from Table 5A of Section II, Part D at T (see explanation below)

$S_{y,t}$ = 19,200 *psi* from Table Y-1 of Section II, Part D at T_t & T_{t1}

t_t = 0.049 *in.*

$\alpha_{t,m1}$ = 8.652*E*-6 *in./in./°F* from Table TE-1 of Section II, Part D at $T_{t,m1}$

ν_t = 0.31 from Table PRD of Section II, Part D for High Alloy Steels (300 Series)

Since the tubes are SA-249, Type 304L (welded), VIII-2, paragraph 4.18.15 requires that the allowable stress for the welded tubes be divided by 0.85 if the equivalent seamless product is not available. In this case, SA-213, Type 304L could be used as the equivalent seamless product, but the alternative will be illustrated in this example. When the welded tube allowable stresses are divided by 0.85, the resulting allowable stresses are $S_t = 16{,}706\ psi$ and $S_{tT} = 15{,}765\ psi$.

Tubesheet:

Tube Pattern: Triangular

A = 43.125 *in.*

A_L = 0 $in.^2$ for no pass lanes

c_t = 0 in.

E = 26.4*E*6 *psi* from Table TM-1 of Section II, Part D at T & T_1

h = 1.500 *in.* (assumed)

h_g = 0 in.

L_t = 240 in.

L = 237 *in.*

ℓ_{tx} = 1.375 *in.*

N_t = 955

p = 1.25 *in.*

r_o = 20.125 in.

S = 15,800 *psi* from Table 5A of Section II, Part D at T & T_1

S_y = 17,500 *psi* from Table Y-1 of Section II, Part D at T & T_1

S_{PS} = 47,400 psi max[3S, 2Sy] (per VIII-2 paragraph 4.1.6.3) at T & T_1

ρ = 0.9167

Shell:

D_s = 42 in.

a_s = 21.00 *in.*

E_s = 26.4*E*6 *psi* from Table TM-1 of Section II, Part D at T_s & T_{s1}

$E_{s,w}$ = 0.85

S_s = 15,800 *psi* from Table 5A of Section II, Part D at T_s & T_{s1}

$S_{y,s}$ = 17,500 *psi* from Table Y-1 of Section II, Part D at T_s & T_{s1}

$S_{PS,s}$ = 47,400 *psi* max[$3S_s$, $2S_{y,s}$] (per VIII-2 paragraph 4.1.6.3) at T_s & T_{s1}

t_s = 0.5625 *in.*

$\alpha_{s,m1}$ = 8.802*E*-6 *in./in./°F* from Table TE-1 of Section II, Part D at $T_{s,m1}$

ν_s = 0.31 from Table PRD of Section II, Part D for High Alloy Steels (300 Series)

Channel:

D_c = 42.125 *in.*

a_c = 21.06 *in.*

E_c = 28.3*E*6 *psi* from Table TM-1 of Section II, Part D at T_c & T_{c1}

S_c = 22,400 *psi* from Table 5A of Section II, Part D at T_c & T_{c1}

$S_{y,c}$ = 33,600 *psi* from Table Y-1 of Section II, Part D at T_c & T_{c1}

$S_{PS,c}$ = 67,200 *psi* max[$3S_c$, $2S_{y,c}$] (per VIII-2 paragraph 4.1.6.3) at T_c & T_{c1}

t_c = 0.375 *in.*

ν_c = 0.30 from Table PRD of Section II, Part D for Carbon Steels

Calculation Procedure:

The calculation procedure for the tubesheets of a fixed tubesheet heat exchanger is given in VIII-2, paragraph 4.18.8.4. The following results are for the design and operating loading cases required to be analyzed (see VIII-2, paragraph 4.18.8.3). This example illustrates the calculation of both the elastic and elastic-plastic solutions.

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.6.4(a). For the operating loading cases, $h'_g = 0$.

D_o = 41.25 *in.*

μ = 0.2000

d^* = 0.9104 *in.*

p^* = 1.250 *in.*

μ^* = 0.2717

h'_g = 0 in.

Calculate a_o, ρ_s, ρ_c, x_s and x_t.

a_o = 20.63 *in.*

ρ_s = 1.018

ρ_c = 1.021

x_s = 0.4388

x_t = 0.5434

b) STEP 2 – Calculate the shell axial stiffness K_s, tube axial stiffness K_t and stiffness factors $K_{s,t}$ and J.

$K_s = 8.378E6\ lb/in.$

$K_t = 16.68E3\ lb/in.$

$K_{s,t} = 0.5260$

$J = 1.0$

For configuration a, calculate shell coefficients β_s, k_s, λ_s and δ_s.

$\beta_s = 0.3709\ in.^{-1}$

$k_s = 0.3213E6\ lb$

$\lambda_s = 41.05E6\ psi$

$\delta_s = 25.09E\text{-}6\ in.^3/lb$

For configuration a, calculate channel coefficients β_c, k_c, λ_c and δ_c.

$\beta_c = 0.4554\ in.^{-1}$

$k_c = 0.1245E6\ lb$

$\lambda_c = 17.86E6\ psi$

$\delta_c = 35.53E\text{-}6\ in.^3/lb$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

h/p = 1.200

E^*/E = 0.2723

ν^* = 0.3439

E^* = 7.188*E*6 *psi*

Calculate X_a and parameters Z_a, Z_d, Z_v, Z_w and Z_m from VIII-2, Table 4.18.3.

X_a = 6.586

Z_a = 170.6

Z_d = 5.246*E*-3

Z_v = 0.02339

Z_w = 0.02339

Z_m = 0.2203

d) STEP 4 – Calculate the diameter ratio K and the coefficient F.

$K = 1.045$

$F = 5.484$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} and U.

$\Phi = 7.371$

$Q_1 = -0.05879$

$Q_{z1} = 3.641$

$Q_{z2} = 9.822$

$U = 19.64$

e) STEP 5 – Calculate γ, $\omega_s, \omega_s^*, \omega_c, \omega_c^*$ and γ_b. Use the loads listed in the table below to calculate the results for an elastic solution in the corroded condition.

$\omega_s = 4.739\ in.^2$ $\quad\quad \omega_s^* = -4.668\ in.^2$

$\omega_c = 3.461\ in^2$ $\quad\quad \omega_c^* = -2.720\ in^2$

$\gamma_b = 0.0$

Summary Table for Step 5 – Design Condition

Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	0	200	0	0
2	325	0	0	0
3	325	200	0	0

Summary Table for Step 5 – Operating Condition 1

Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	0	200	-0.08080	0
2	325	0	-0.08080	0
3	325	200	-0.08080	0
4	0	0	-0.08080	0

f) STEP 6 – For each loading case, calculate P_s', P_t', P_γ, P_ω, P_W, P_{rim} and effective pressure P_e.

Summary Table for STEP 6 – Design Condition

Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	545.5	0	0	0	-25.12	-99.74
2	630.1	0	0	0	0	70.06	122.4
3	630.1	545.5	0	0	0	44.94	22.65

Summary Table for STEP 6 – Operating Condition 1							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	545.5	-963.0	0	0	-25.12	-268.1
2	630.1	0	-963.0	0	0	70.06	-45.93
3	630.1	545.5	-963.0	0	0	44.94	-145.7
4	0	0	-963.0	0	0	0	-168.3

g) STEP 7 – Elastic Iteration: Calculate Q_2, Q_3 and F_m, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (lbf)	Q_3	F_m	$h - h_g'$ (in)	$\|\sigma\|$ (psi)	$1.5S$ (psi)
1	207.3	-0.06856	0.03428	1.500	14,280	23,700
2	-578.3	-0.08100	0.04050	1.500	20,700	23,700
3	-371.0	-0.1358	0.06790	1.500	6,420	23,700

Summary Table for STEP 7 – Operating Condition 1						
Loading Case	Q_2 (lbf)	Q_3	F_m	h (in)	$\|\sigma\|$ (psi)	S_{PS} (psi)
1	207.3	-0.06242	0.03121	1.500	34,930	47,400
2	-578.3	0.4075E-3	0.03703	1.500	7,101	47,400
3	-371.0	-0.04681	0.02341	1.500	14,240	47,400
4	0	-0.05879	0.02939	1.500	20,660	47,400

For Design Loading Cases 1-3 $|\sigma| \leq 1.5S$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma| \leq S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Check the criterion below for the largest value of P_e and calculate the shear stress, if required.

$$\{|P_e| = 268.1\ psi\} \leq \left\{\frac{2\mu h}{a_o} \cdot \min\left[0.8S, 0.533S_y\right] = 271\ psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

i) STEP 9 – For each loading case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$r_t = 0.3367\ in$

$F_t = 142.6$

$C_t = 166.6$

Summary Table for STEP 9 – Design Condition				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	-0.2819	-1,308	3.426	2,228
2	-0.2569	1,664	3.152	-2,325
3	-0.2187	371.6	2.117	-134.1

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	2,228	16,706	-1,308	1.537	7,148
2	2,325	16,706	-2,325	1.674	6,563
3	371.6	16,706	-134.1	2.000	5,493

Summary Table for STEP 9 – Operating Condition 1				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	-0.2967	-1,799	3.561	8,087
2	-0.5149	1,137	4.944	3,534
3	-0.3401	-149.3	3.905	5,762
4	-0.3059	-492.3	3.641	5,859

Summary Table for STEP 9 – Operating Condition 1 (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	$2S_t$ (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	8,087	33,412	-1,799	1.469	7,476
2	3,534	33,412	---	---	---
3	5,762	33,412	-149.3	1.298	8,465
4	5,859	33,412	-492.3	1.429	7,685

For Design Loading Cases 1-3 $\sigma_{t,\max} \le S_t$, and for Operation Condition 1, Loading Cases 1-4 $\sigma_{t,\max} \le 2S_t$. The axial tension stress criterion for the tube satisfied.

For all Loading Cases, $|\sigma_{t,\min}| \le S_{tb}$. The buckling criterion for the tube satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Main Shell – Design Condition			
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{s,b}$ (psi)
1	1,781	13,430	---
2	2,387	13,430	---
3	4,168	13,430	---

Summary Table for STEP 10 – Main Shell – Operating Condition 1			
Loading Case	$\sigma_{s,m}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	-1,210	47,400	6,730
2	-604.3	47,400	6,730
3	1,177	47,400	---
4	-2,991	47,400	6,730

For Design Loading Cases 1-3 $|\sigma_{s,m}| \leq S_s E_{s,w}$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma_{s,m}| \leq S_{PS,s}$. The axial membrane stress criterion for the shell satisfied.

For all Loading Cases where the value of $\sigma_{s,m}$ is negative, $|\sigma_{s,m}| \leq S_{s,b}$. The longitudinal compressive stress criterion for the shell satisfied.

k) STEP 11 – For each loading case, calculate the stresses in the shell and channel for configuration a, and check the acceptance criterion. The shell thickness shall be 0.5625 in. for a minimum length of 8.749 in. adjacent to the tubesheet, and the channel thickness shall be 0.375 in. for a minimum length of 7.154 in. adjacent to the tubesheet.

Summary Table for STEP 11, Shell Results – Design Condition					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)
1	1,781	-12,320	14,100	23,700	47,400
2	2,387	28,550	30,940	23,700	47,400
3	4,168	16,230	20,400	23,700	47,400

Summary Table for STEP 11, Shell Results – Operating Condition 1				
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$S_{PS,s}$ (psi)
1	-1,210	-38,510	39,720	47,400
2	-604.3	2,360	2,964	47,400
3	1,177	-9,961	11,140	47,400
4	-2,991	-26,190	29,180	47,400

For Design Loading Case 2 $|\sigma_s| > 1.5S_s$. For Design Loading Cases 1 and 3 $|\sigma_s| \le 1.5S_s$ and for Operation Condition 1, Loading Cases 1-4 $|\sigma_s| \le S_{PS,s}$. The stress criterion for the shell is not satisfied. For Design Loading Case 2, since $|\sigma_s| \le S_{PS,s}$, Option 3 in Step 12 is permitted.

Summary Table for STEP 11, Channel Results – Design Condition					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	$S_{PS,c}$ (psi)
1	5,567	28,450	34,020	33,600	67,200
2	0	-9,257	9,257	33,600	67,200
3	5,567	19,200	24,760	33,600	67,200

Summary Table for STEP 11, Channel Results – Operating Condition 1				
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$S_{PS,c}$ (psi)
1	5,567	52,410	57,980	67,200
2	0	14,700	14,700	67,200
3	5,567	43,150	48,720	67,200
4	0	23,960	23,960	67,200

For Design Loading Case 1 $|\sigma_c| > 1.5S_c$. For Design Loading Cases 2 and 3 $|\sigma_c| \le 1.5S_c$ and for Operation Condition 1, Loading Cases 1-4 $|\sigma_c| \le S_{PS,c}$. The stress criterion for the channel is not satisfied. For Design Loading Case 1, since $|\sigma_c| \le S_{PS,c}$, Option 3 in Step 12 is permitted.

l) STEP 12 – The design shall be reconsidered by using one or a combination of the following options.

- Option 1 – Increase the tubesheet thickness and return to STEP 1.
- Option 2 – Increase the integral shell and/or channel thickness and return to STEP 1.
- Option 3 – Perform the elastic-plastic calculation procedures as defined in VIII-2, paragraph 4.18.8.6(c).

Since the total axial stress in the shell σ_s is between $1.5S_s$ and $S_{PS,s}$ for Design Condition Loading Case 2, the procedure of VIII-2, paragraph 4.18.8.6(c) may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

Since the total axial stress in the channel σ_c is between $1.5S_c$ and $S_{PS,c}$ for Design Condition Loading Case 1, the procedure of VIII-2, paragraph 4.18.8.6(c) may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the channel occurs. The results are not presented for Design Condition Loading Case 1, because the calculated values of $fact_s$ and $fact_c$ equal 1.0 for this case and further plasticity calculations are not required.

The results for the effect of plasticity for Design Condition Loading Case 2 are shown below.

Summary Results for STEP 12, Elastic-Plastic Iteration Results per VIII-2, paragraph 4.18.8.6(c)	
Design Condition Loading Case	2
S_s^*, psi	17,500
S_c^*, psi	33,600
$fact_s$	0.7474
$fact_c$	1.000
E_s^*, psi	19.73E6
E_c^*, psi	28.30E6
k_s, lb	0.2402$E6$
λ_s	30.68E6
k_c, lb	0.1245$E6$
λ_c	17.86E6
F	4.538
ϕ	6.098
Q_1	-0.05312
Q_{Z1}	3.766
Q_{Z2}	11.00
U	21.99
P_W, psi	0
P_{rim}, psi	78.44
P_e, psi	120.8
Q_2, lb	-647.5
Q_3	-0.07832
F_m	0.03916
$\|\sigma\|$, psi	19,750

The final calculated tubesheet bending stress is 19,750 psi (Design Loading Case 1) is less than the allowable tubesheet bending stress of 23,700 psi. As such, this geometry meets the requirement of VIII-2, paragraph 4.18.8.6. The intermediate results for the elastic-plastic iteration are shown above.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

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4.18.8 Example E4.18.8 – Floating Tubesheet Heat Exchanger with an Immersed Floating Head

A floating tubesheet exchanger with an immersed floating head is to be designed as shown in VIII-2, Figure 4.18.10, sketch (a). The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in VIII-2, Figure 4.18.11, sketch (d) and not extended as a flange. The floating tubesheet is not extended as a flange in accordance with configuration C as shown in VIII-2, Figure 4.18.12, sketch (c).

- For the Design Condition, the shell side design pressure is 0 to 250 psig at 550°F, and the tube side design pressure is 0 to 150 psig at 550°F.
- For these configurations, the operating conditions are not required to be considered.
- The tube material is SA-179 (K01200). The tubes are 0.75 in. outside diameter and 0.083 in. thick. The largest equivalent unsupported buckling length of the tube is 15.375 in.
- The tubesheet material is SA-105 (K03504). The stationary tubesheet diameter is 29.875 in. and the floating tubesheet diameter is 26.875 in. The tubesheet has 466 tube holes on a 1.0 in. triangular pattern with one centerline pass lane. There is a 0.197 in. deep pass partition groove in the stationary tubesheet only. The largest center-to-center distance between adjacent tube rows is 2.5 in., the length of the untubed lane is 25.75 in., and the radius to the outermost tube hole center is 12.5 in. The distance between the outer tubesheet faces is 256 in. There is no corrosion allowance on the tubesheet. The tubes are expanded to 80% of the tubesheet.
- The channel flange gasket consists of a ring gasket with a centerline rib. The ring gasket outside diameter is 29.875 in., the inside diameter is 28.875 in., and the gasket factors are y = 4,000 psi and m = 3.0. The rib gasket width is 0.50 in., the length is 28.875 in., and the rib gasket factors are y = 4,000 psi and m = 3.0. The shell flange gasket outside diameter is 29.875 in., the inside diameter is 28.875 in., and the gasket factors are y = 4,000 psi and m = 3.0. The effective gasket width for both gaskets is per VIII-1 Appendix 2, Table 2-5.2, sketch (1a). There are (32) 0.75 in. diameter SA-193-B7 bolts on a 31.417 in. bolt circle.
- The floating head flange gasket outside diameter is 26.875 in., the inside diameter is 26.125 in., and the gasket factors are y = 4,000 psi and m = 3.0. The effective gasket width is per VIII-2 paragraph 4.16, Table 4.16.3, sketch (1a). There are (28) 0.625 in. diameter SA-193-B7 bolts on a 27.625 in. bolt circle.

For this example, first assume a value of 1.75 in. for the tubesheet thickness and perform the calculation procedure described below starting at STEP 1. The data shown below will be the same except as follows:

$$h = 1.75\ in.$$

$$L = 252.5\ in.$$

In STEP 7, the calculated bending stress of 29,830 psi for the stationary tubesheet exceeds the allowable stress of 28,500 psi for Design Loading Case 2.

The tubesheet is overstressed for Design Loading Case 2. Increase the tubesheet thickness to 1.8125 in and return to Step 1 of the calculation procedure in paragraph 4.18.9.4.

<u>Data Summary:</u>

The data summary consists of those variables from the nomenclature (see VIII-2, paragraph 4.18.15 that are applicable to these configurations.

Design Conditions:

$P_{sd,\max} = 250\ psig$

$P_{sd,\min} = 0\ psig$

$P_{td,\max} = 150\ psig$

$P_{td,\min} = 0\ psig$

$T = 550^{o}F$

$T_{fe} = 550^{o}F$

$T_{t} = 550^{o}F$

Tubes:

$d_t = 0.75\ in.$

$E_t = 26.9E6\ psi$ from Table TM-1 of Section II, Part D at T_t

$E_{tT} = 26.9E6\ psi$ from Table TM-1 of Section II, Part D at T

$\ell_t = 15.375\ in.$

$S_t = 13{,}350\ psi$ from Table 1A of Section II, Part D at T_t

$S_{tT} = 13{,}350\ psi$ from Table 1A of Section II, Part D at T

$S_{y,t} = 20{,}550\ psi$ from Table Y-1 of Section II, Part D at T_t

$t_t = 0.083\ in.$

Stationary and Floating Tubesheets (Common Data):

Tube Pattern: Triangular

$A_L = 26.38\ in.^2$

$c_t = 0\ in.$

$E = 26.75E6\ psi$ from Table TM-1 of Section II, Part D at T

$h = 1.8125\ in.$ (assumed)

$L_{L1} = 25.75\ in.$

$L_t = 256\ in.$

$L = 252.375\ in.$

$N_t = 466$

$p = 1.0\ in.$

$r_o = 12.5\ in.$

$S = 19{,}000\ psi$ from Table 5A of Section II, Part D at T

$S_{fe} = 19{,}000\ psi$ from Table 5A of Section II, Part D at T_{fe}

$S_y = 28{,}450\ psi$ from Table Y-1 of Section II, Part D at T

$U_{L1} = 2.5\ in.$

$\rho = 0.80$

Stationary Tubesheet:

$A = 29.875\ in.$

$C = 31.417\ in.$

$D_E = 29.375\ in.$

$G_c = 29.375\ in.$ (G per VIII-2 paragraph 4.16)

$a_c = 14.69\ in.$

$G_s = 29.375\ in.$ (G per VIII-2 paragraph 4.16)

$a_s = 14.69\ in.$

$h_g = 0.197\ in.$

$W_{m1c} = 128{,}856\ lb$ (W_o per VIII-2 paragraph 4.16)

$W_{m1s} = 203{,}931\ lb$ (W_o per VIII-2 paragraph 4.16)

Floating Tubesheet:

$A = 26.875\ in.$

$C = 27.625\ in.$

$D_E = 26.125\ in.$

$G_1 = 26.5\ in.$

$G_c = 26.5\ in.$ (G per VIII-2 paragraph 4.16)

$a_c = 13.25\ in.$

$a_s = 13.25\ in.$

$h_g = 0\ in.$

$W_{m1c} = 96{,}732\ lb$ (W_o per VIII-2 paragraph 4.16)

Calculation Procedure – Stationary Tubesheet:

The stationary tubesheet is not extended as a flange but has an unflanged extension. The calculation procedure for this extension is given in VIII-2, paragraph 4.18.5.4(c). The minimum required thickness of this extension calculated at T_{fe} is:

$h_r = 0.1208\ in.$

The calculation procedure for the stationary tubesheet of a floating tubesheet heat exchanger is given in VIII-2, paragraph 4.18.9.4. The following results are for the design loading cases required to be analyzed (see VIII-2, paragraph 4.18.9.3).

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.6.4(a).

$D_o = 25.75\ in.$

$\mu = 0.2500$

$d^* = 0.6562\ in.$

$p^* = 1.068\ in.$

$\mu^* = 0.3857$

$h'_g = 0.197\ in.$

Calculate a_o, ρ_s, ρ_c, x_s and x_t.

$a_o = 12.88\ in.$

$\rho_s = 1.141$

$\rho_c = 1.141$

$x_s = 0.6047$

$x_t = 0.7603$

b) STEP 2 – For configuration d, shell coefficients $\beta_s = 0$, $k_s = 0$, $\lambda_s = 0$ and $\delta_s = 0$.

For configuration d, channel coefficients $\beta_c = 0$, $k_c = 0$, $\lambda_c = 0$ and $\delta_c = 0$.

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

$h/p = 1.813$

$E^*/E = 0.4020$

$\nu^* = 0.3098$

$E^* = 10.75E6\ psi$

Calculate X_a and parameters Z_a, Z_d, Z_v, Z_w and Z_m from VIII-2, Table 4.18.3.

$X_a = 3.525$

$Z_a = 3.811$

$Z_d = 0.03511$

$Z_v = 0.08232$

$Z_w = 0.08232$

$Z_m = 0.4325$

d) STEP 4 – Calculate the diameter ratio K and the coefficient F.

$K = 1.160$

$F = 0.2551$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} and U.

$\Phi = 0.3341$

$Q_1 = 0.09897$

e) STEP 5 – Calculate γ, $\omega_s, \omega_s^*, \omega_c, \omega_c^*$ and γ_b.

$\omega_s = 0\ in.^2$ $\qquad \omega_s^* = 1.758\ in.^2$

$\omega_c = 0\ in^2$ $\qquad \omega_c^* = 1.758\ in^2$

$\gamma_b = 0.0$

f) STEP 6 – Use the loads listed in the table below to calculate effective pressure P_e and the results for an elastic solution in the corroded condition.

Summary Table for Step 6 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	P_e (psi)	W^* (lbf)
1	0	150	-150.0	128,856
2	250	0	250.0	203,931
3	250	150	100.0	203,931

g) STEP 7 – Elastic Iteration: Calculate Q_2, Q_3 and F_m, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (lbf)	Q_3	F_m	$h-h'_g$ (in)	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-230.4	0.1175	0.1127	1.6155	16,710	28,500
2	384.1	0.1175	0.1127	1.6155	27,840	28,500
3	-153.6	0.1175	0.1127	1.6155	11,140	28,500

For Design Loading Cases 1-3 $\lvert\sigma\rvert \le 1.5S$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Check the criterion below for the largest value of P_e and calculate the shear stress, if required.

$$\{|P_e| = 250\ psi\} \le \left\{\frac{2\mu h}{a_o}\cdot \min\left[0.8S, 0.533S_y\right] = 1067\ psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

i) STEP 9 – For each loading case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$r_t = 0.2376\ in$

$F_t = 64.70$

$C_t = 160.7$

Summary Table for STEP 9 – Design Condition				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	-1.070	-1,764	3.458	2,600
2	-1.070	2,690	3.458	-4,584
3	-1.070	926.1	3.458	-1,984

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	2,600	13,350	-1,764	1.521	10,790
2	4,584	13,350	-4,584	1.521	10,790
3	1,984	13,350	-1,984	1.521	10,790

For Design Loading Cases 1-3 $\sigma_{t,\max} \le S_t$. The axial tension stress criterion for the tube is satisfied.

For Design Loading Cases 1-3 $|\sigma_{t,\min}| \le S_{tb}$. The buckling criterion for the tube is satisfied.

Calculation Procedure – Floating Tubesheet:

The floating tubesheet is not extended as a flange but has an unflanged extension. The calculation procedure for this extension is given in VIII-2, paragraph 4.18.5.4(c). The minimum required thickness of this extension calculated at T_{fe} is:

$$h_r = 0.1074\ in.$$

The calculation procedure for the floating tubesheet of a floating tubesheet heat exchanger is given in VIII-2, paragraph 4.18.9.4. The following results are for the design loading cases required to be analyzed (see VIII-2, paragraph 4.18.9.3).

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.6.4(a).

$$D_o = 25.75\ in.$$

$$\mu = 0.2500$$

$$d^* = 0.6562\ in.$$

$$p^* = 1.068\ in.$$

$$\mu^* = 0.3857$$

$$h'_g = 0\ in.$$

Calculate a_o, ρ_s, ρ_c, x_s and x_t.

$$a_o = 12.88\ in.$$

$$\rho_s = 1.029$$

$$\rho_c = 1.029$$

$$x_s = 0.6047$$

$$x_t = 0.7603$$

b) STEP 2 – For configuration C, shell coefficients $\beta_s = 0$, $k = 0_s$, $\lambda_s = 0$ and $\delta_s = 0$.

For configuration C, channel coefficients $\beta_c = 0$, $k_c = 0$, $\lambda_c = 0$ and $\delta_c = 0$.

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

$$h/p = 1.813$$

$$E^*/E = 0.4020$$

$\nu^* = 0.3098$

$E^* = 10.75E6\ psi$

Calculate X_a and parameters Z_a, Z_d, Z_v, Z_w and Z_m from VIII-2, Table 4.18.3.

$X_a = 3.525$

$Z_a = 3.811$

$Z_d = 0.03511$

$Z_v = 0.08232$

$Z_w = 0.08232$

$Z_m = 0.4325$

d) STEP 4 – Calculate the diameter ratio K and the coefficient F.

$K = 1.044$

$F = 0.07341$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} and U.

$F = 0.09615$

$Q_1 = 0.02036$

e) STEP 5 – Calculate g, $\omega_s, \omega_s^*, \omega_c, \omega_c^*$ and gb.

$\omega_s = 0\ in.^2$ $\omega_s^* = 0.07134\ in.^2$

$\omega_c = 0\ in^2$ $\omega_c^* = 0.07134\ in^2$

$\gamma_b = 0.0$

f) STEP 6 – Use the loads listed in the table below to calculate effective pressure Pe and the results for an elastic solution in the corroded condition.

Summary Table for Step 6 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	P_e (psi)	W^* (lbf)
1	0	150	-150.0	96,732
2	250	0	250.0	0
3	250	150	100.0	96,732

g) STEP 7 – Elastic Iteration: Calculate Q2, Q3 and Fm, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (lbf)	Q_3	F_m	$h - h'_g$ (in)	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-10.27	0.02119	0.07702	1.8125	9,068	28,500
2	17.12	0.02119	0.07702	1.8125	15,110	28,500
3	6.849	0.02119	0.07702	1.8125	6,045	28,500

For Design Loading Cases 1-3 $|\sigma| \le 1.5S$. The bending stress criterion for the tubesheet is satisfied.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

4.18.9 Example E4.18.9 – Floating Tubesheet Exchanger with an Externally Sealed Floating Head

A floating tubesheet exchanger with an externally sealed (packed) floating head is to be designed as shown in VIII-2, Figure 4.18.10, sketch (b). The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in VIII-2, Figure 4.18.11, sketch (d) and not extended as a flange. The floating tubesheet is integral with the floating head in accordance with configuration A as shown in VIII-2, Figure 4.18.12, sketch (a). The floating head is an assembly consisting of a channel cylinder and a formed head, both of which are the same material.

- For the Design Condition, the shell side design pressure is 0 to 150 psig at 250°F, and the tube side design pressure is 0 to 30 psig at 250°F.
- There is one operating condition that shall be considered for the effect of the radial thermal expansion between the floating head and floating tubesheet. For Operating Condition 1, the operating pressures and operating metal temperatures are assumed to be the same as the design values. The floating head metal temperature at the floating tubesheet is 235°F and the floating tubesheet metal temperature at the rim is 200°F.
- The tube material is SB-338, Grade 2 Seamless (R50400). The tubes are 1.00 in. outside diameter, and 0.049 in. thick. The largest equivalent unsupported buckling length of the tube is 16 in.
- The tubesheet material is SB-265, Grade 2 (R50400). The stationary tubesheet diameter is 51 in. and the floating tubesheet diameter is 47.625 in. The tubesheet has 1189 tube holes on a 1.25 in. triangular pattern with no pass partition lanes. The radius to the outermost tube hole center is 22.605 in. The distance between the outer tubesheet faces is 144 in. There is no corrosion allowance on the tubesheet. The tubes are expanded to 95.8% of the tubesheet.
- The channel flange gasket outside diameter is 50.375 in., the inside diameter is 48 in., and the gasket factors are y = 1,600 psi and m = 2.0. The shell flange gasket outside diameter is 50.5 in., the inside diameter is 48 in., and the gasket factors are y = 1,600 psi and m = 2.0. The effective gasket width for both gaskets is per VIII-2 paragraph 4.16, Table 4.16.3, sketch (1a). There are (40) 0.875 in. diameter SA-193-B7 bolts on a 52 in. bolt circle.
- The floating head material is SB-265, Grade 2 (R50400). The floating head outside diameter is 47.625 in. and the thickness in 0.3125 in.

Data Summary:

The data summary consists of those variables from the nomenclature (see VIII-2, paragraph 4.18.15) that are applicable to this configuration.

Design Conditions:

$$P_{sd,\max} = 150\ psig$$

$$P_{sd,\min} = 0\ psig$$

$$P_{td,\max} = 30\ psig$$

$$P_{td,\min} = 0\ psig$$

$$T = 250^oF$$

$$T_a = 70^oF$$

$T_c = 250^oF$

$T_{fe} = 250^oF$

$T_t = 250^oF$

Operating Conditions (Floating Tubesheet Only):

$P_{so1,\max} = 150\ psig$

$P_{so1,\min} = 0\ psig$

$P_{to1,\max} = 30\ psig$

$P_{to1,\min} = 0\ psig$

$T_1 = 250^oF$

$T_{c1} = 250^oF$

$T_1' = 200^oF$

$T_{c1}' = 235^oF$

Tubes:

$d_t = 1.00\ in.$

$E_t = 14.8E6\ psi$ from Table TM-5 of Section II, Part D at T_t

$E_{tT} = 14.8E6\ psi$ from Table TM-5 of Section II, Part D at T & T_1

$\ell_t = 16\ in.$

$S_t = 11{,}300\ psi$ from Table 1B of Section II, Part D at T_t

$S_{tT} = 11{,}300\ psi$ from Table 1B of Section II, Part D at T & T_1

$S_{y,t} = 28{,}600\ psi$ from Table Y-1 of Section II, Part D at T_t

$t_t = 0.049\ in.$

Stationary and Floating Tubesheets:

Tube Pattern: Triangular

$A_L = 0\ in.^2$ for no pass lanes

$c_t = 0\ in.$

$E = 14.8E6\ psi$ from Table TM-5 of Section II, Part D at T & T_1

$h = 1.375\ in.$ (assumed)

$h_g = 0\ in.$

$L_t = 144\ in.$

$L = 141.25\ in.$

$N_t = 1189$

$p = 1.25\ in.$

$r_o = 22.605\ in.$

$S = 11{,}300\ psi$ from Table 1B of Section II, Part D at T & T_1

$S_y = 28{,}600\ psi$ from Table Y-1 of Section II, Part D at T & T_1

$\rho = 0.958$

Stationary Tubesheet:

$A = 51\ in.$

$C = 52\ in.$

$D_E = 48\ in.$

$G_c = 49.6044\ in.$ (G per VIII-2 paragraph 4.16)

$a_c = 24.8\ in.$

$G_s = 49.7094\ in.$ (G per VIII-2 paragraph 4.16)

$a_s = 24.85\ in.$

$W_{m1c} = 65{,}148\ lb$ (W_o per VIII-2 paragraph 4.16)

$W_{m1s} = 327{,}983\ lb$ (W_o per VIII-2 paragraph 4.16)

Floating Tubesheet:

$A = 47.625\ in.$

$S_{PS} = 33{,}900\ psi$ limited to $3S$, (per VIII-2 paragraph 4.1.6.3) at T & T_1

$\alpha'_1 = 4.7E\text{-}6\ in./in./^oF$ from Table TE-5 of Section II, Part D at T'_1

Floating Head Channel Cylinder:

$D_c = 47\ in.$

$a_c = 23.5\ in.$

$a_s = 23.5\ in.$

$E_c = 14.8E6\ psi$ from Table TM-5 of Section II, Part D at T_c & T_{c1}

$S_c = 11{,}300\ psi$ from Table 1B of Section II, Part D at T_c & T_{c1}

$S_{y,c} = 28{,}600\ psi$ from Table Y-1 of Section II, Part D at T_c & T_{c1}

$S_{PS,c} = 33{,}900\ psi$ limited to $3S_c$ (per VIII-2 paragraph 4.1.6.3) at T_c & T_{c1}

$t_c = 0.3125\ in.$

$\alpha'_{c1} = 4.77E\text{-}6\ in./in./^oF$ from Table TE-5 of Section II, Part D at T'_{c1}

$\nu_c = 0.32$ from Table PRD of Section II, Part D for Titanium UNS R50400

Calculation Procedure – Stationary Tubesheet:

The stationary tubesheet is not extended as a flange but has an unflanged extension. The calculation procedure for this extension is given in VIII-2, paragraph 4.18.5.4(c). The minimum required thickness of this extension calculated at T_{fe} is:

$h_r = 0.1991\ in.$

The calculation procedure for the stationary tubesheet of a floating tubesheet heat exchanger is given in VIII-2, paragraph 4.18.9.4. The following results are for the design loading cases required to be analyzed (see VIII-2, paragraph 4.18.9.3.

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.6.4(a).

$D_o = 46.21\ in.$

$\mu = 0.2000$

$d^* = 0.9061\ in.$

$p^* = 1.250\ in.$

$\mu^* = 0.2751$

$h'_g = 0\ in.$

Calculate a_o, ρ_s, ρ_c, x_s and x_t.

$a_o = 23.10\ in.$

$\rho_s = 1.076$

$\rho_c = 1.073$

$x_s = 0.4432$

$x_t = 0.5470$

b) STEP 2 – For configuration d, shell coefficients $\beta_c = 0$, $k = 0_s$, $\lambda_s = 0$ and $\delta_s = 0$.

For configuration d, channel coefficients $\beta_c = 0$, $k_c = 0$, $\lambda_c = 0$ and $\delta_c = 0$.

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

$h/p = 1.100$

$E^*/E = 0.2803$

$\nu^* = 0.3374$

$E^* = 4.149E6\ psi$

Calculate X_a and parameters Z_a, Z_d, Z_v, Z_w and Z_m from VIII-2, Table 4.18.3.

$X_a = 8.842$

$Z_a = 3162$

$Z_d = 2.143E\text{-}3$

$Z_v = 0.01300$

$Z_w = 0.01300$

$Z_m = 0.1634$

d) STEP 4 – Calculate the diameter ratio K and the coefficient F.

$K = 1.104$

$F = 0.2331$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} and U.

$\Phi = 0.3118$

$Q_1 = 0.06821$

e) STEP 5 – Calculate γ, $\omega_s, \omega_s^*, \omega_c, \omega_c^*$ and γ_b.

$\omega_s = 0\ in.^2$ $\qquad$ $\omega_s^* = 1.589\ in.^2$

$\omega_c = 0\ in^2$ $\qquad$ $\omega_c^* = 0.8867\ in^2$

$\gamma_b = -2.272E\text{-}3$

f) STEP 6 – Use the loads listed in the table below to calculate effective pressure P_e and the results for an elastic solution in the corroded condition.

Summary Table for Step 6 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	P_e (psi)	W^* (lbf)
1	0	30	-30.00	65,148
2	150	0	-23.58	327,983
3	150	30	-53.58	327,983

g) STEP 7 – Elastic Iteration: Calculate Q_2, Q_3 and F_m, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (lbf)	Q_3	F_m	$h - h'_g$ (in)	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-47.73	0.07416	0.05572	1.375	10,290	16,950
2	113.9	0.05010	0.04573	1.375	6,640	16,950
3	88.58	0.06201	0.05050	1.375	16,660	16,950

For Design Loading Cases 1-3 $\lvert\sigma\rvert \le 1.5S$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Check the criterion below for the largest value of P_e and calculate the shear stress, if required.

$$\{|P_e| = 53.58\ psi\} \le \left\{\frac{2\mu h}{a_o} \cdot \min[0.8S, 0.533S_y] = 215\ psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

i) STEP 9 – For each loading case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$r_t = 0.3367\ in$

$F_t = 47.52$

$C_t = 101.1$

Summary Table for STEP 9 – Design Condition				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	-1.190	-502.2	9.494	2,586
2	-0.9846	416.8	8.539	2,580
3	-1.086	-78.31	9.011	5,134

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	2,586	11,300	-502.2	1.250	11,300
2	2580	11,300	---	---	---
3	5134	11,300	-78.31	1.250	11,300

For Design Loading Cases 1-3 $\sigma_{t,\max} \le S_t$. The axial tension stress criterion for the tube is satisfied.

For Design Loading Cases 1-3 $|\sigma_{t,\min}| \le S_{tb}$. The buckling criterion for the tube is satisfied.

Calculation Procedure – Floating Tubesheet:

The calculation procedure for the floating tubesheet of a floating tubesheet heat exchanger is given in VIII-2, paragraph 4.18.9.4. The following results are for the design and operating loading cases required to be analyzed (see VIII-2, paragraph 4.18.9.3).

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.6.4(a).

$D_o = 46.21\ in.$

$\mu = 0.2000$

$d^* = 0.9061\ in.$

$p^* = 1.250\ in.$

$\mu^* = 0.2751$

$h'_g = 0\ in.$

Calculate a_o, ρ_s, ρ_c, x_s and x_t.

$a_o = 23.10\ in.$

$\rho_s = 1.017$

$\rho_c = 1.017$

$x_s = 0.4432$

$x_t = 0.5470$

b) STEP 2 – For configuration A, shell coefficients $\beta_s = 0$, $k = 0_s$, $\lambda_s = 0$ and $\delta_s = 0$.

For configuration A, calculate channel coefficients β_c, k_c, λ_c and δ_c.

$\beta_c = 0.4711\ in.^{-1}$

$k_c = 39.51E3\ lb$

$\lambda_c = 7.962E6\ psi$

$\delta_c = .1003E\text{-}3\ in.^3/lb$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

$h/p = 1.100$

$E^*/E = 0.2803$

$\nu^* = 0.3374$

$E^* = 4.149E6\ psi$

Calculate X_a and parameters Z_a, Z_d, Z_v, Z_w and Z_m from VIII-2, Table 4.18.3.

$X_a = 8.842$

$Z_a = 3162$

$Z_d = 2.143E\text{-}3$

$Z_v = 0.01300$

$Z_w = 0.01300$

$Z_m = 0.1634$

d) STEP 4 – Calculate the diameter ratio K and the coefficient F.

$K = 1.031$

$F = 1.343$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} and U.

$\Phi = 1.796$

$Q_1 = -4.829E\text{-}3$

e) STEP 5 – Calculate γ, $\omega_s, \omega_s^*, \omega_c, \omega_c^*$, γ_b, P_s^* and P_c^*.

$\omega_s = 0\ in.^2$ $\omega_s^* = 0.07868\ in.^2$

$\omega_c = 3.130\ in^2$ $\omega_c^* = -3.051\ in^2$

$\gamma_b = 0.0$

Configuration A

According to paragraph 4.18.9.6(c),

$$T_r = \frac{T' + T_c'}{2} = 217.5°\text{F} \quad \text{and} \quad T_c^* = \frac{T_c' + T_r}{2} = 226.25°\text{F}$$

For this example, the conservative option permitted by paragraph 4.18.9.6(c) is used to calculate P_s^* and P_c^*.

$$T_r = T' = 200°\text{F} \quad \text{and} \quad T_c^* = T_c' = 235°\text{F}$$

f) STEP 6 – Use the loads listed in the table below to calculate effective pressure P_e and the results for an elastic solution in the corroded condition.

Summary Table for Step 6 – Design Condition						
Loading Case	P_s (psi)	P_t (psi)	P_e (psi)	W^* (lbf)	P_s^* (psi)	P_c^* (psi)
1	0	30	-30.00	0	0	0
2	150	0	-5.173	0	0	0
3	150	30	-35.17	0	0	0

Summary Table for Step 6 – Operating Condition 1						
Loading Case	P_s (psi)	P_t (psi)	P_e (psi)	W^* (lbf)	P_s^* (psi)	P_c^* (psi)
1	0	30	-30.00	0	0	34.65
2	150	0	-5.173	0	0	34.65
3	150	30	-35.17	0	0	34.65
4	0	0	0	0	0	34.65

g) STEP 7 – Elastic Iteration: Calculate Q_2, Q_3 and F_m, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (lbf)	Q_3	F_m	$h - h_g'$ (in)	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	70.75	-0.01366	0.02277	1.375	4,207	16,950
2	9.124	-0.01144	0.02348	1.375	747.9	16,950
3	79.88	-0.01334	0.02287	1.375	4,955	16,950

Summary Table for STEP 7 – Operating Condition 1						
Loading Case	Q_2 (lbf)	Q_3	F_m	h (in)	$\lvert\sigma\rvert$ (psi)	S_{PS} (psi)
1	154.6	-0.02413	0.01980	1.375	3,659	33,900
2	92.95	-0.07215	0.03607	1.375	1,149	33,900
3	163.7	-0.02227	0.02032	1.375	4,402	33,900
4	83.82	0	0	1.375	3,515	33,900

For Design Loading Cases 1-3 $|\sigma| \le 1.5S$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma| \le S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEPS 8 and 9 – For configuration A, skips STEPS 8 and 9 and proceed to STEP 10.

i) STEP 10 – For each loading case, calculate the stresses in the channel for configuration A, and check the acceptance criterion. The channel thickness shall be 0.3125 in. for a minimum length of 6.898 in. adjacent to the tubesheet.

Summary Table for STEP 10, Channel Results – Design Condition					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	$S_{PS,c}$ (psi)
1	1,121	9,752	10,870	16,950	33,900
2	0	1,125	1,125	16,950	33,900
3	1,121	10,880	12,000	16,950	33,900

Summary Table for STEP 10, Channel Results – Operating Condition 1				
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$S_{PS,c}$ (psi)
1	1,121	13,480	14,600	33,900
2	0	4,854	4,854	33,900
3	1,121	14,610	15,730	33,900
4	0	3,729	3,729	33,900

For Design Loading Cases 1-3 $|\sigma_c| \le 1.5S_c$ and for Operation Condition 1, Loading Cases 1-4 $|\sigma_c| \le S_{PS,c}$. The stress criterion for the channel is satisfied.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

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4.18.10 Example E4.18.10 – Floating Tubesheet Exchanger with an Internally Sealed Floating Tubesheet

A floating tubesheet exchanger with an internally sealed floating head is to be designed as shown in VIII-2, Figure 4.18.10, sketch (c). The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in VIII-2, Figure 4.18.11, sketch (d) and not extended as a flange. The floating tubesheet is packed and sealed on its edge in accordance with configuration D as shown in VIII-2, Figure 4.18.12, sketch (d).

- For the Design Condition, the shell side design pressure is 0 to 150 psig at 400°F, and the tube side design pressure is 0 to 175 psig at 400°F.
- For these configurations, the operating conditions are not required to be considered.
- The tube material is SA-213, Type 316L (S31603). The tubes are 0.75 in. outside diameter, and 0.065 in. The largest equivalent unsupported buckling length of the tube is 20.75 in.
- The tubesheet material is SA-240, Type 316L (S31603). The stationary tubesheet diameter is 39.875 in. and the floating tubesheet diameter is 36.875 in. The tubesheet has 1066 tube holes on a 0.9375 in. triangular pattern with no pass partition lanes. The radius to the outermost tube hole center is 15.563 in. The distance between the outer tubesheet faces is 155.875 in. There is no corrosion allowance on the tubesheet. The tubes are expanded to 88% of the tubesheet.
- The channel flange gasket outside diameter is 39.941 in., the inside diameter is 38.941 in., and the gasket factors are y = 4,000 psi and m = 3.0. The shell flange gasket outside diameter is 39.941 in., the inside diameter is 38.941 in., and the gasket factors are y = 4,000 psi and m = 3.0. The effective gasket width for both gaskets is per VIII-2 paragraph 4.16, Table 4.16.3, sketch (1a). There are (40) 0.75 in. diameter SA-193-B7 bolts on a 41.625 in. bolt circle.

Data Summary:

The data summary consists of those variables from the nomenclature (see VIII-2, paragraph 4.18.15) that are applicable to these configurations.

Design Conditions:

$$P_{sd,\max} = 150\ psig$$

$$P_{sd,\min} = 0\ psig$$

$$P_{td,\max} = 175\ psig$$

$$P_{td,\min} = 0\ psig$$

$$T = 400^oF$$

$$T_{fe} = 400^oF$$

$$T_t = 400^oF$$

Tubes:

$$d_t = 0.75\ in.$$

$$E_t = 26.4E6\ psi \text{ from Table TM-1 of Section II, Part D at } T_t$$

$$E_{tT} = 26.4E6\ psi \text{ from Table TM-1 of Section II, Part D at } T$$

$\ell_t = 20.75\ in.$

$S_t = 15{,}700\ psi$ from Table 5A of Section II, Part D at T_t

$S_{tT} = 15{,}700\ psi$ from Table 5A of Section II, Part D at T

$S_{y,t} = 17{,}500\ psi$ from Table Y-1 of Section II, Part D at T_t

$t_t = 0.065\ in.$

Stationary and Floating Tubesheets (Common Data):

Tube Pattern: Triangular

$A_L = 0\ in.^2$ for no pass lanes

$c_t = 0\ in.$

$E = 26.4E6\ psi$ from Table TM-1 of Section II, Part D at T

$h = 1.188\ in.$ (assumed)

$h_g = 0\ in.$

$L_t = 155.875\ in.$

$L = 153.499\ in.$

$N_t = 1066$

$p = 0.9375\ in.$

$r_o = 15.563\ in.$

$S = 15{,}700\ psi$ from Table 5A of Section II, Part D at T

$S_y = 17{,}500\ psi$ from Table Y-1 of Section II, Part D at T

$\rho = 0.88$

Stationary Tubesheet:

$A = 39.875\ in.$

$C = 41.625\ in.$

$D_E = 38.941\ in.$

$G_c = 39.441\ in.$ (G per VIII-2 paragraph 4.16)

$a_c = 19.72\ in.$

$G_s = 39.441\ in.$ (G per VIII-2 paragraph 4.16)

$a_s = 19.72\ in.$

$W_{m1c} = 246{,}209\ lb$ (W_o per VIII-2 paragraph 4.16)

$W_{m1s} = 211{,}036\ lb$ (W_o per VIII-2 paragraph 4.16)

Floating Tubesheet:

$A = 36.875\ in.$

$a_c = 18.44\ in.$

$a_s = 18.44\ in.$

Calculation Procedure – Stationary Tubesheet:

The stationary tubesheet is not extended as a flange but has an unflanged extension. The calculation procedure for this extension is given in VIII-2, paragraph 4.18.5.4(c). The minimum required thickness of this extension calculated at T_{fe} is:

$h_r = 0.1356\ in.$

The calculation procedure for the stationary tubesheet of a floating tubesheet heat exchanger is given in VIII-2, paragraph 4.18.9.4. The following results are for the design loading cases required to be analyzed (VIII-2, paragraph 4.18.9.3).

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.6.4(a).

$D_o = 31.876\ in.$

$\mu = 0.2000$

$d^* = 0.6356\ in.$

$p^* = 0.9375\ in.$

$\mu^* = 0.3220$

$h'_g = 0\ in.$

Calculate a_o, ρ_s, ρ_c, x_s and x_t.

$a_o = 15.94\ in.$

$\rho_s = 1.237$

$\rho_c = 1.237$

$x_s = 0.4099$

$x_t = 0.5967$

b) STEP 2 – For configuration d, shell coefficients $\beta_s = 0$, $k_s = 0$, $\lambda_s = 0$ and $\delta_s = 0$.

For configuration d, channel coefficients $\beta_c = 0$, $k_c = 0$, $\lambda_c = 0$ and $\delta_c = 0$.

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

$h/p = 1.267$

$E^*/E = 0.3376$

$\nu^* = 0.3161$

$E^* = 8.913E6\ psi$

Calculate X_a and parameters Z_a, Z_d, Z_v, Z_w and Z_m from VIII-2, Table 4.18.3.

$X_a = 7.399$

$Z_a = 482.2$

$Z_d = 3.691E\text{-}3$

$Z_v = 0.01864$

$Z_w = 0.01864$

$Z_m = 0.1967$

d) STEP 4 – Calculate the diameter ratio K and the coefficient F.

$K = 1.251$

$F = 0.4535$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} and U.

$\Phi = 0.5969$

$Q_1 = 0.2024$

e) STEP 5 – Calculate γ, $\omega_s, \omega_s^*, \omega_c, \omega_c^*$ and γ_b.

$\omega_s = 0 \ in.^2$ $\qquad \omega_s^* = 8.003 \ in.^2$

$\omega_c = 0 \ in^2$ $\qquad \omega_c^* = 8.003 \ in^2$

$\gamma_b = 0$

f) STEP 6 – Use the loads listed in the table below to calculate effective pressure P_e and the results for an elastic solution in the corroded condition.

Summary Table for Step 6 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	P_e (psi)	W^* (lbf)
1	0	175	92.92	246,209
2	150	0	-79.65	211,036
3	150	175	13.27	246,209

g) STEP 7 – Elastic Iteration: Calculate Q_2, Q_3 and F_m, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (lbf)	Q_3	F_m	$h - h'_g$ (in)	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-1,253	0.09623	0.07020	1.188	21,880	23,550
2	1,074	0.09623	0.07020	1.188	18,750	23,550
3	-179.0	0.09623	0.07020	1.188	3,125	23,550

For Design Loading Cases 1-3 $\lvert\sigma\rvert \le 1.5S$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Check the criterion below for the largest value of P_e and calculate the shear stress, if required.

$$\{\lvert P_e\rvert = 92.92 \ psi\} \le \left\{\frac{2\mu h}{a_o} \cdot \min\left[0.8S, 0.533S_y\right] = 278 \ psi\right\} \qquad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

i) STEP 9 – For each loading case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$r_t = 0.2433\ in$

$F_t = 85.30$

$C_t = 172.6$

Summary Table for STEP 9 – Design Condition				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	-1.129	2.485	8.220	-4,647
2	-1.129	-152.1	8.220	3,833
3	-1.129	-149.6	8.220	-813.8

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	4,647	15,700	-4,647	1.250	10,540
2	3,833	15,700	-152.1	1.250	10,540
3	813.8	15,700	-813.8	1.250	10,540

For Design Loading Cases 1-3 $\sigma_{t,\max} \le S_t$. The axial tension stress criterion for the tube is satisfied.

For Design Loading Cases 1-3 $|\sigma_{t,\min}| \le S_{tb}$. The buckling criterion for the tube is satisfied.

Calculation Procedure – Floating Tubesheet:

The calculation procedure for a floating tubesheet of a floating tubesheet heat exchanger is given in VIII-2, paragraph 4.18.9.4. The following results are for the 3 Design Loading Cases required to be analyzed (see VIII-2, paragraph 4.18.9.3.

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.6.4(a).

$D_o = 31.876\ in.$

$\mu = 0.2000$

$d^* = 0.6356\ in.$

$p^* = 0.9375\ in.$

$\mu^* = 0.3220$

$h'_g = 0\ in.$

Calculate a_o, ρ_s, ρ_c, x_s and x_t.

$a_o = 15.94\ in.$

$\rho_s = 1.157$

$\rho_c = 1.157$

$x_s = 0.4099$

$x_t = 0.5967$

b) STEP 2 – For configuration D, shell coefficients $\beta_s = 0$, $k_s = 0$, $\lambda_s = 0$ and $\delta_s = 0$.

For configuration D, channel coefficients $\beta_c = 0$, $k_c = 0$, $\lambda_c = 0$ and $\delta_c = 0$.

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

$h/p = 1.267$

$E^*/E = 0.3376$

$\nu^* = 0.3161$

$E^* = 8.913E6\ psi$

Calculate X_a and parameters Z_a, Z_d, Z_v, Z_w and Z_m from VIII-2, Table 4.18.3.

$X_a = 7.399$

$Z_a = 482.2$

$Z_d = 3.691E\text{-}3$

$Z_v = 0.01864$

$Z_w = 0.01864$

$Z_m = 0.1967$

d) STEP 4 – Calculate the diameter ratio K and the coefficient F.

$K = 1.157$

$F = 0.2951$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} and U.

$\Phi = 0.3884$

$Q_1 = 0.1390$

e) STEP 5 – Calculate γ, ω_s, ω_s^*, ω_c, ω_c^* and γ_b.

$\omega_s = 0\ in.^2$ $\quad \omega_s^* = 3.369\ in.^2$

$\omega_c = 0\ in^2$ $\quad \omega_c^* = 3.369\ in^2$

$\gamma_b = 0$

f) STEP 6 – Use the loads listed in the table below to calculate effective pressure P_e and the results for an elastic solution in the corroded condition.

Summary Table for Step 6 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	P_e (psi)	W^* (lbf)
1	0	175	59.19	0
2	150	0	-50.74	0
3	150	175	8.456	0

g) STEP 7 – Elastic Iteration: Calculate Q_2, Q_3 and F_m, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (lbf)	Q_2	F_m	$h - h'_g$ (in)	$\|\sigma\|$ (psi)	$1.5S$ (psi)
1	-547.7	0.06612	0.05745	1.188	11,400	23,550
2	469.4	0.06612	0.05745	1.188	9,775	23,550
3	-78.24	0.06612	0.05745	1.188	1,629	23,550

For Design Loading Cases 1-3 $|\sigma| \leq 1.5S$. The bending stress criterion for the tubesheet is satisfied.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

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4.18.11 Example E4.18.11 – Fixed Tubesheet Kettle Exchanger, Configuration a, Tubesheet Integral with Shell and Channel

A fixed tubesheet kettle heat exchanger is to be designed with the tubesheet construction in accordance with configuration a as shown in VIII-2, Figure 4.18.5, Configuration a.

- For the Design Condition, the shell side design pressure is FV to 150 psig at 400°F, and the tube side design pressure is FV to 180 psig at 400°F.
- There is one operating condition. For Operating Condition 1, the shell side operating pressure is 0 to 150 at 400°F and the tube side operating pressure is 0 to 180 psig at 400°F. The shell mean metal temperature is 195 °F, and the tube mean metal temperature is 220°F.
- The tube material is SA-179 (K01200). The tubes are 0.75 in. outside diameter, and 0.083 in. thick. The unsupported tube span under consideration is between 2 tube supports, and the length of the unsupported tube span is 41.1429 in.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet diameter is 31 in. There are 641 tube holes on a 1 in. square pattern. There is no pass partition lane and the outermost tube radius from the tubesheet center is 14.25 in. The distance between the outer tubesheet faces is 287.875 in. The option for the effect of differential radial expansion is not required. There is a 0.125 in. corrosion allowance on both sides. The tubes are expanded to 100% of the tubesheet thickness.
- The shell material adjacent to the tubesheet (small cylinder) is SA-516, Grade 70 (K02700). The inside diameter is 30 in., the thickness is 0.50 in., and the length is 7.0625 in. There is a 0.125 in. corrosion allowance. There is no expansion joint. The efficiency of circumferential welded joint (Category B) is 0.85.
- The kettle shell material (large cylinder) is SA-516, Grade 70 (K02700). The inside diameter is 42 in., the thickness is 0.50 in., and the length is 228.125 in. There is a 0.125 in. corrosion allowance. The efficiency of circumferential welded joint (Category B) is 0.85.
- The eccentric cone material is SA-516, Grade 70 (K02700). The inside diameter at the large end is 42 in., the inside diameter at the small end is 30 in., the thickness is 0.50 in, the half-apex angle is 30º, and the length is 20.8125 in. There is a 0.125 in. corrosion allowance. The efficiency of circumferential welded joint (Category B) is 0.85.
- The channel material is SA-516, Grade 70 (K02700). The inside diameter of the channel is 30 in. and the channel is 0.50 in. thick. There is a 0.125 in. corrosion allowance on the channel.

Data Summary:

The data summary consists of those variables from the nomenclature (see VIII-2, paragraph 4.18.15) that are applicable to this configuration.

Design Conditions:

$$P_{sd,\max} = 150\ psig$$

$$P_{sd,\min} = \text{-}15\ psig$$

$$P_{td,\max} = 180\ psig$$

$$P_{td,\min} = \text{-}15\ psig$$

$T = 400^oF$

$T_a = 70^oF$

$T_c = 400^oF$

$T_s = 400^oF$

$T_t = 400^oF$

Operating Conditions:

$P_{so1,\max} = 150\ psig$

$P_{so1,\min} = 0\ psig$

$P_{to1,\max} = 180\ psig$

$P_{to1,\min} = 0\ psig$

$T_1 = 400^oF$

$T_{c1} = 400^oF$

$T_{s1} = 400^oF$

$T_{t1} = 400^oF$

$T_{s,m1} = 195^oF$

$T_{t,m1} = 220^oF$

Tubes:

$d_t = 0.75\ in.$

$E_t = 27.9E6\ psi$ from Table TM-1 of Section II, Part D at T_t & T_{t1}

$E_{tT} = 27.9E6\ psi$ from Table TM-1 of Section II, Part D at T

$k = 1$ for an unsupported tube span between two tube supports

$\ell = 41.1429\ in.$

$\ell_t = 41.1429\ in.$

$S_t = 13{,}400\ psi$ from Table 1A of Section II, Part D at T_t & T_{t1} (see explanation below)

$S_{tT} = 13{,}400\ psi$ from Table 1A of Section II, Part D at T (see explanation below)

$S_{y,t} = 22{,}200\ psi$ from Table Y-1 of Section II, Part D at T_t & T_{t1}

$t_t = 0.083\ in.$

$\alpha_{t,m1} = 6.74E\text{-}6\ in./in./^oF$ from Table TE-1 of Section II, Part D at $T_{t,m1}$

$\nu_t = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Tubesheet:

Tube Pattern: Square

Assume an uncorroded tubesheet thickness of 2 inches.

$A = 31.0\ in.$

$A_L = 0\ in.^2$ for no pass lanes

$c_t = 0.125\ in.$

$E = 27.9E6\ psi$ from Table TM-1 of Section II, Part D at T & T_1

$h = 2\ in. - 0.125\ in.\ -0.125\ in. = 1.75\ in.$ (assumed)

$h_g = 0\ in.$

$L_t = 287.625\ in.$

$L = 287.625\ in. - 2(1.75\ in.) = 284.125\ in.$

$N_t = 641$

$p = 1.0\ in.$

$r_o = 14.25\ in.$

$S = 21{,}600\ psi$ from Table 5A of Section II, Part D at T & T_1

$S_y = 32{,}500\ psi$ from Table Y-1 of Section II, Part D at T & T_1

$S_{PS} = 65{,}000\ psi$ max[$3S$, $2S_y$] (per VIII-2 paragraph 4.1.6.3) at T & T_1

$\rho = 1.0$

Shell (Small Cylinder):

$D_s = 30.25\ in.$

$a_s = 15.13\ in.$

$E_s = 27.9E6\ psi$ from Table TM-1 of Section II, Part D at T_s & T_{s1}

$E_{s,w} = 0.85$

$L_s = 7.0625\ in. + 0.125\ in. = 7.1875\ in.$

$S_s = 21{,}600\ psi$ from Table 5A of Section II, Part D at T_s & T_{s1}

$S_{y,s} = 32{,}500\ psi$ from Table Y-1 of Section II, Part D at T_s & T_{s1}

$S_{PS,s} = 65{,}000\ psi$ max[$3S_s$, $2S_{y,s}$] (per VIII-2 paragraph 4.1.6.3) at T_s & T_{s1}

$t_s = 0.375\ in.$

$\alpha_{s,m1} = 6.69E\text{-}6\ in./in./^oF$ from Table TE-1 of Section II, Part D at $T_{s,m1}$

$\nu_s = 0.30$ from Table PRD of Section II, Part D for Carbon Steels

Kettle Shell (Large Cylinder):

$D_{s,L} = 42.25\ in.$

$E_{s,L} = 27.9E6\ psi$ from Table TM-1 of Section II, Part D at T_s & T_{s1}

$E_{s,L,w} = 0.85$

$L_{s,L} = 228.125\ in.$

$S_{s,L} = 21{,}600\ psi$ from Table 5A of Section II, Part D at T_s & T_{s1}

$S_{y,s,L}$= 32,500 *psi* from Table Y-1 of Section II, Part D at T_s & T_{s1}

$S_{PS,s,L}$ = 65,000 *psi* max[$3S_{s,L}$, $2S_{y,s,L}$] (per VIII-2 paragraph 4.1.6.3) at T_s & T_{s1}

$t_{s,L}$ = 0.375 *in.*

$\alpha_{s,m1,L}$ = 6.69*E*-6 *in./in./°F* from Table TE-1 of Section II, Part D at $T_{s,m1}$

$\nu_{s,L}$ = 0.30 from Table PRD of Section II, Part D for Carbon Steels

Eccentric Cone:

$D_{ecc,S}$ = 30.25 *in.*

$D_{ecc,L}$ = 42.25 *in*

E_{ecc} = 27.9*E*6 *psi* from Table TM-1 of Section II, Part D at T_s & T_{s1}

$E_{ecc,w}$ = 0.85

L_{ecc} = 20.8125 *in.*

S_{ecc} = 21,600 *psi* from Table 5A of Section II, Part D at T_s & T_{s1}

$S_{y,ecc}$ = 32,500 *psi* from Table Y-1 of Section II, Part D at T_s & T_{s1}

$S_{PS,ecc}$ = 65,000 *psi* max[$3S_{ecc}$, $2S_{y,ecc}$] (per VIII-2 paragraph 4.1.6.3) at T_s & T_{s1}

t_{ecc} = 0.375 *in.*

α_{ecc} = 30°

$\alpha_{ecc,m1}$ = 6.69*E*-6 *in./in./°F* from Table TE-1 of Section II, Part D at $T_{s,m1}$

ν_{ecc} = 0.30 from Table PRD of Section II, Part D for Carbon Steels

Channel:

D_c = 30.25 *in.*

a_c = 15.13 *in.*

E_c = 27.9*E*6 *psi* from Table TM-1 of Section II, Part D at T_c & T_{c1}

S_c = 21,600 *psi* from Table 1A of Section II, Part D at T_c & T_{c1}

$S_{y,c}$ = 32,500 *psi* from Table Y-1 of Section II, Part D at T_c & T_{c1}

$S_{PS,c}$ = 65,000 *psi* max[$3S_c$, $2S_{y,c}$] (per VIII-2 paragraph 4.1.6.3) at T_c & T_{c1}

t_c = 0.375 *in.*

ν_c = 0.30 from Table PRD of Section II, Part D for Carbon Steels

Calculation Procedure:

The calculation procedure for the tubesheets of a kettle type fixed tubesheet heat exchanger is given in VIII-2, paragraph 4.18.8.4 with supplemental data provided in paragraph 4.18.8.9. The following results are for the design and operating loading cases required to be analyzed (see VIII-2, paragraph 4.18.8.3). This example illustrates the calculation of the elastic solution.

a) STEP 1 – Calculate D_o, μ, μ^* and h'_g from VIII-2, paragraph 4.18.6.4(a).

$D_o = 29.25\ in.$

$\mu = 0.2500$

$d^* = 0.6388\ in.$

$p^* = 1.000\ in.$

$\mu^* = 0.3612$

$h'_g = 0\ in.$

Calculate a_o, ρ_s, ρ_c, x_s and x_t.

$a_o = 14.63\ in.$

$\rho_s = 1.034$

$\rho_c = 1.034$

$x_s = 0.5786$

$x_t = 0.7445$

b) STEP 2 – Calculate the shell axial stiffnesses K_{ecc}, $K_{s,L}$, K_s, K_s^*, tube axial stiffness K_t and stiffness factors $K_{s,t}$ and J.

$K_{ecc} = 38.69E6\ lb/in.$

$K_{s,L} = 6.142E6\ lb/in.$

$K_s = 140.1E6\ lb/in.$

$K_s^* = 4.371E6\ lb/in.$

$K_t = 17.08E3\ lb/in.$

$K_{s,t} = 0.3993$

$J = 1.0$

For configuration a, calculate shell coefficients β_s, k_s, λ_s and δ_s.

$\beta_s = 0.5364\ in.^{-1}$

$k_s = 0.1445E6\ lb$

$\lambda_s = 11.65E6\ psi$

$\delta_s = 18.59E\text{-}6\ in.^3/lb$

For configuration a, calculate channel coefficients β_c, k_c, λ_c and δ_c.

$\beta_c = 0.5364\ in.^{-1}$

$k_c = 0.1445E6\ lb$

$\lambda_c = 11.65E6\ psi$

$\delta_c = 18.59E\text{-}6\ in.^3/lb$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-2, paragraph 4.18.6.4(b) and calculate E^*.

$h/p = 1.750$

$E^*/E = 0.4204$

$\nu^* = 0.3155$

$E^* = 11.73E6\ psi$

Calculate X_a and parameters Z_a, Z_d, Z_v, Z_w and Z_m from VIII-2, Table 4.18.3.

$X_a = 4.001$

$Z_a = 6.731$

$Z_d = 0.02411$

$Z_v = 0.06420$

$Z_w = 0.06420$

$Z_m = 0.3743$

d) STEP 4 – Calculate the diameter ratio K and the coefficient F.

$K = 1.060$

$F = 1.454$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} and U.

$\Phi = 1.913$

$Q_1 = -0.05165$

$Q_{z1} = 2.664$

$Q_{z2} = 5.749$

$U = 11.50$

e) STEP 5 – Calculate γ, ω_s, ω_s^*, ω_c, ω_c^* and γ_b. Use the loads listed in the table below to calculate the results for an elastic solution in the corroded condition.

$\omega_s = 2.889\ in.^2$ $\omega_s^* = -2.762\ in.^2$

$\omega_c = 2.889\ in^2$ $\omega_c^* = -2.762\ in^2$

$\gamma_b = 0$

Summary Table for Step 5 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	-15	180	0	0
2	150	-15	0	0
3	150	180	0	0
4	-15	-15	0	0

Summary Table for Step 5 – Operating Condition 1				
Loading Case	P_s (psi)	P_t (psi)	γ (in)	W^* (lbf)
1	0	180	0.04965	0
2	150	0	0.04965	0
3	150	180	0.04965	0
4	0	0	0.04965	0

f) STEP 6 – Calculate v_s^* and for each loading case, calculate P_s', P_t', P_γ, P_ω, P_W, P_{rim} and effective pressure P_e.

$$A_s = 926.4\ in.^2 \qquad A_{s,L} = 1801\ in.^2$$

$$\Delta_{ecc} = 12.00\ in. \qquad v_s^* = 0.4756$$

Summary Table for STEP 6 – Design Condition							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	-48.08	612.4	0	0	0	-28.96	-128.5
2	480.8	-51.04	0	0	0	24.50	103.7
3	480.8	612.4	0	0	0	-4.455	-25.37
4	-48.08	-51.04	0	0	0	0.2051E-6	0.5512

Summary Table for STEP 6 – Operating Condition 1							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	612.4	808.9	0	0	-26.73	31.63
2	480.8	0	808.9	0	0	22.27	244.5
3	480.8	612.4	808.9	0	0	-4.455	125.4
4	0	0	808.9	0	0	0	150.8

g) STEP 7 – Elastic Iteration: Calculate Q_2, Q_3 and F_m, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (lbf)	Q_3	F_m	$h - h_g'$ (in)	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	313.9	-0.07449	0.03953	1.750	5,893	32,400
2	-265.6	-0.07560	0.03927	1.750	4,723	32,400
3	48.29	-0.06945	0.04072	1.750	1,198	32,400
4	-2.223E-6	-0.05165	0.04521	1.750	28.91	32,400

Summary Table for STEP 7 – Operating Condition 1						
Loading Case	Q_2 (lbf)	Q_3	F_m	h (in)	$\|\sigma\|$ (psi)	S_{PS} (psi)
1	289.8	0.03401	0.07243	1.750	2,658	65,000
2	-241.5	-0.06088	0.04282	1.750	12,150	65,000
3	48.29	-0.04805	0.04618	1.750	6,718	65,000
4	0	-0.05165	0.04521	1.750	7,906	65,000

For Design Loading Cases 1-4 $|\sigma| \le 1.5S$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma| \le S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Check the criterion below for the largest value of P_e and calculate the shear stress, if required.

$$\{|P_e| = 244.5\ psi\} \le \left\{\frac{2\mu h}{a_o} \cdot \min\left[0.8S, 0.533S_y\right] = 1033\ psi\right\} \quad True$$

Since the above criterion is satisfied, the shear stress is not required to be calculated.

i) STEP 9 – For each loading case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$r_t = 0.2376\ in$

$F_t = 173.1$

$C_t = 157.5$

Summary Table for STEP 9 – Design Condition				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	-0.4590	-1,216	2.477	1,058
2	-0.4548	874.6	2.467	-951.6
3	-0.4778	-357.7	2.518	100.4
4	-0.5442	16.81	2.664	6.147

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	1,216	13,400	-1,216	2.000	4,593
2	951.6	13,400	-951.6	2.000	4,593
3	357.7	13,400	-357.7	1.991	4,618
4	16.81	13,400	---	---	---

Summary Table for STEP 9 – Operating Condition 1				
Loading Case	$F_{t,\min}$	$\sigma_{t,1}$ (psi)	$F_{t,\max}$	$\sigma_{t,2}$ (psi)
1	-0.8637	-643.0	3.369	-1,450
2	-0.5097	1,274	2.589	-3,292
3	-0.5576	136.8	2.694	-2,321
4	-0.5442	494.5	2.664	-2,421

Summary Table for STEP 9 – Operating Condition 1 (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	$2S_t$ (psi)	$\sigma_{t,\min}$ (psi)	F_s	S_{tb} (psi)
1	1,450	26,800	-1,450	1.565	5,868
2	3,292	26,800	-3,292	1.956	4,697
3	2,321	26,800	-2,321	1.903	4,828
4	2,421	26,800	-2,421	1.918	4,790

For Design Loading Cases 1-4 $\sigma_{t,\max} \le S_t$, and for Operation Condition 1, Loading Cases 1-4 $\sigma_{t,\max} \le 2S_t$. The axial tension stress criterion for the tube is satisfied.

For all Loading Cases $|\sigma_{t,\min}| \le S_{tb}$. The buckling criterion for the tube is satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Small Cylinder:

Summary Table for STEP 10 – Design Condition			
Loading Case	$\sigma_{s,m}$ (psi)	$S_sE_{s,w}$ (psi)	$S_{s,b}$ (psi)
1	939.8	18,360	---
2	1,846	18,360	---
3	3,074	18,360	---
4	-288.5	18,360	14,320

Summary Table for STEP 10 – Operating Condition 1			
Loading Case	$\sigma_{s,m}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	3,941	65,000	---
2	4,748	65,000	---
3	5,882	65,000	---
4	2,808	65,000	---

For Design Loading Cases 1-4 $|\sigma_{s,m}| \le S_s E_{s,w}$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma_{s,m}| \le S_{PS,s}$. The axial membrane stress criterion for the small cylinder is satisfied.

For all Loading Cases where the value of $\sigma_{s,m}$ is negative, $|\sigma_{s,m}| \le S_{s,b}$. The longitudinal compressive stress criterion for the small cylinder is satisfied.

Eccentric Cone:

Summary Table for STEP 10 – Design Condition				
Loading Case	$\sigma_{ecc,S,m}$ (psi)	$\sigma_{ecc,L,m}$ (psi)	$S_{ecc}E_{ecc,w}$ (psi)	$S_{ecc,b}$ (psi)
1	1,085	544.0	18,360	---
2	2,132	3,888	18,360	---
3	3,550	4,907	18,360	---
4	-333.2	-475.0	18,360	13460

Summary Table for STEP 10 – Operating Condition 1				
Loading Case	$\sigma_{ecc,S,m}$ (psi)	$\sigma_{ecc,L,m}$ (psi)	$S_{PS,ecc}$ (psi)	$S_{ecc,b}$ (psi)
1	4,551	3,270	65,000	---
2	5,483	6,296	65,000	---
3	6,792	7,237	65,000	---
4	3,242	2,329	65,000	---

For Design Loading Cases 1-4 $|\sigma_{ecc,S,m}|$ and $|\sigma_{ecc,L,m}| \le S_{ecc}E_{ecc,w}$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma_{ecc,S,m}|$ and $|\sigma_{ecc,L,m}| \le S_{PS,ecc}$. The axial membrane stress criterion for the eccentric cone is satisfied.

For all Loading Cases where the values of $\sigma_{ecc,S,m}$ or $\sigma_{ecc,L,m}$ are negative, $|\sigma_{ecc,S,m}|$ and $|\sigma_{ecc,L,m}| \le S_{ecc,b}$. The longitudinal compressive stress criterion for the eccentric cone is satisfied.

Large Cylinder:

Summary Table for STEP 10 – Design Condition			
Loading Case	$\sigma_{s,L,m}$ (psi)	$S_{s,L}E_{s,L,w}$ (psi)	$S_{s,L,b}$ (psi)
1	471.1	18,360	---
2	3,367	18,360	---
3	4,250	18,360	---
4	-411.4	18,360	13,460

Summary Table for STEP 10 – Operating Condition 1			
Loading Case	$\sigma_{s,L,m}$ (psi)	$S_{PS,s,L}$ (psi)	$S_{s,L,b}$ (psi)
1	2,832	65,000	---
2	5,452	65,000	---
3	6,267	65,000	---
4	2,017	65,000	---

For Design Loading Cases 1-4 $|\sigma_{s,L,m}| \leq S_{s,L}E_{s,L,w}$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma_{s,L,m}| \leq S_{PS,s,L}$. The axial membrane stress criterion for the large cylinder is satisfied.

For all Loading Cases where the value of $\sigma_{s,L,m}$ is negative, $|\sigma_{s,L,m}| \leq S_{s,L,b}$. The longitudinal compressive stress criterion for the large cylinder is satisfied.

k) STEP 11 – For each loading case, calculate the stresses in the shell and channel for configuration a, and check the acceptance criterion. The shell thickness shall be 0.375 in. for a minimum length of 6.137 in. adjacent to the tubesheet, and the channel thickness shall be 0.375 in. for a minimum length of 6.137 in. adjacent to the tubesheet.

Summary Table for STEP 11, Shell Results – Design Condition					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)
1	939.8	-12,290	13,230	32,400	65,000
2	1,846	18,290	20,140	32,400	65,000
3	3,074	6,861	9,936	32,400	65,000
4	-288.5	-862.0	1,151	32,400	65,000

Summary Table for STEP 11, Shell Results – Operating Condition 1				
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$S_{PS,s}$ (psi)
1	3,941	5,929	9,870	65,000
2	4,748	33,900	38,640	65,000
3	5,882	23,340	29,230	65,000
4	2,808	16,480	19,290	65,000

For Design Loading Cases 1-4 $|\sigma_s| \leq 1.5S_s$, and for Operation Condition 1, Loading Cases 1-4 $|\sigma_s| \leq S_{PS,s}$. The stress criterion for the shell is satisfied.

Summary Table for STEP 11, Channel Results – Design Condition					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	$S_{PS,c}$ (psi)
1	3,586	22,440	26,030	32,400	65,000
2	-298.8	-9,993	10,290	32,400	65,000
3	3,586	13,430	17,010	32,400	65,000
4	-298.8	-982.6	1,281	32,400	65,000

Summary Table for STEP 11, Channel Results – Operating Condition 1				
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$S_{PS,c}$ (psi)
1	3,586	5,139	8,724	65,000
2	0	-24,670	24,670	65,000
3	3,586	-3,053	6,639	65,000
4	0	-16,480	16,480	65,000

For Design Loading Cases 1-4 $|\sigma_c| \le 1.5S_c$ and for Operation Condition 1, Loading Cases 1-4 $|\sigma_c| \le S_{PS,c}$. The stress criterion for the channel is satisfied.

The calculation procedure is complete, and the unit geometry is acceptable for the given design conditions and assumed tubesheet thickness.

4.19 Bellows Expansion Joints

4.19.1 Example E4.19.1 – U-Shaped Un-reinforced Bellows Expansion Joint and Fatigue Evaluation

Check the acceptability of a U-shaped unreinforced bellows expansion joint for the given design conditions in accordance with paragraph 4.19.

Design Conditions:

- Pressure (Internal) at Temperature = $50\ psig\ @650°F$
- Axial Movements in Compression and Extension = 2 *Independent* Operating Conditions

Operating Condition 1

- Axial Movement (Compression) = $4.5\ in$
- Angular Deflection = *None*
- Lateral Deflection = *None*
- Specified Number of Cycles = 1000

Operating Condition 2

- Axial Movement (Extension) = $0.75\ in$
- Angular Deflection = *None*
- Lateral Deflection = *None*
- Specified Number of Cycles = 5000

Bellows:

- Material = $SA-240,\ Type\ 321$
- Allowable Stress = $17900\ psi$
- Yield Strength = $19800\ psi$
- Modulus of Elasticity at Design Temperature = $25.05E+06\ psi$
- Modulus of Elasticity at Room Temperature = $28.3E+06\ psi$
- Inside Diameter of Convolution = $48.0\ in$
- Outside Diameter of Convolution = $52.0\ in$
- Convolution Height = $2.0\ in$
- Number of Convolutions = 12
- Number of Plies = 1
- Nominal Ply Thickness = $0.048\ in$
- Convolution Pitch = $1.0\ in$
- Mean Radius of Convolution = $0.25\ in$
- Crest Convolution Inside Radius = $0.226\ in$
- Root Convolution Inside Radius = $0.226\ in$
- End Tangent Length = $1.25\ in$
- Installed without Cold Spring = *Yes*
- Circumferential welds = *No*

The bellows was formed with a mandrel from a cylinder with an inside diameter of $48.0\ in$ and preformed 100% to the outside of the cylinder. The bellows is in as-formed condition. It is attached externally to the shell.

Collar:

- Collar = $None$

Cylindrical shell on which the bellows is attached:

- Inside Diameter of Shell = $47.25\ in$
- Thickness of Shell = $0.375\ in$
- Minimum Length of Shell on each Side of the Bellows = $10.5\ in$

Evaluate per paragraph 4.19.

a) STEP 1 – Check applicability of design rules per paragraph 4.19.2.

1) Bellows length must satisfy $Nq \le 3D_b$:

$$\{Nq = 12(1.0) = 12\ in\} \le \{3D_b = 3(48.0) = 144\ in\} \qquad True$$

2) Bellows thickness must satisfy $nt \le 0.2\ in$:

$$\{nt = 1(0.048) = 0.048\ in\} \le \{0.2\ in\} \qquad True$$

3) Number of plies must satisfy $n \le 5$:

$$\{n = 1\} \le \{5\} \qquad True$$

4) Displacement shall be essentially axial.

No angular or lateral deflection is specified, so the condition is satisfied.

5) The rules are valid for design temperatures up to the temperatures shown in Table 4.19.1.

The material specification $SA-240,\ Type\ 321$ is an austenitic stainless steel and falls under the material classification of Table 3-A.3.

$$\{T = 650°F\} \le \{Table\ 3A.3 \rightarrow 800°F\} \qquad True$$

6) The fatigue equations of paragraph 4.19.5.7 are valid for austenitic stainless steels.

$$SA-240,\ Type\ 321 \rightarrow UNS\ S32100$$

7) The length of the cylindrical shell on each side of the bellows shall not be less than $1.8\sqrt{D_s t_s}$.

$$\{10.5\ in\} \ge \left\{1.8\sqrt{D_s t_s} = 1.8\sqrt{(47.25)(0.375)} = 7.577\ in\right\} \qquad True$$

b) STEP 2 – Check the applicability of paragraph 4.19.5.2.

1) A variation of 10% between the crest convolution radius, r_{ic}, and the root convolution radius, r_{ir}, is permitted.

$$\{0.9r_{ir} = 0.9(0.226) = 0.203\ in\} \le \{r_{ic} = 0.226\ in\} \le \{1.1r_{ir} = 1.1(0.226) = 0.249\ in\} \quad True$$

2) Torus radius shall satisfy $r_i \geq 3t$.

$$\left\{ r_i = \frac{r_{ic} + r_{ir}}{2} = \frac{0.226 + 0.226}{2} = 0.226\ in \right\} \geq \left\{ 3t = 3(0.048) = 0.144\ in \right\} \qquad True$$

3) Sidewall offset angle shall meet $-15\ deg \leq \alpha \leq 15\ deg$.

$$\alpha = \text{atan}\left[\frac{\left(\frac{q}{2} - 2r_m \right)}{(w - 2r_m)} \right] = \text{atan}\left[\frac{\left(\frac{1.0}{2} - 2(0.25) \right)}{(2.0 - 2(0.25))} \right] = 0.0\ deg$$

$$-15\ deg \leq \{ \alpha = 0.0 \} \leq 15\ deg \qquad True$$

4) Convolution height shall meet $w \leq D_b/3$.

$$\{ w = 2.0\ in \} \leq \left\{ \frac{D_b}{3} = \frac{48.0}{3} = 16.0\ in \right\} \qquad True$$

c) STEP 3 – Check stresses in bellows at design conditions per paragraph 4.19.5.3. Since the bellows are subject to internal pressure, calculations and acceptability criteria are per Table 4.19.2. The following values are calculated.

$$D_m = D_b + w + nt = 48.0 + 2.0 + 1(0.048) = 50.048\ in$$

$$t_p = t\sqrt{\frac{D_b}{D_m}} = 0.048\sqrt{\frac{48.0}{50.048}} = 0.0470\ in$$

$$k = \min\left[\left(\frac{L_t}{1.5\sqrt{D_b t}} \right), 1.0 \right] = \min\left[\left(\frac{1.25}{1.5\sqrt{48.0(0.048)}} \right), 1.0 \right] = 0.5490$$

$$A = \left[2\pi r_m + 2\sqrt{\left\{ \frac{q}{2} - 2r_m \right\}^2 + \{ w - 2r_m \}^2} \right] n t_p$$

$$A = \left[2\pi(0.25) + 2\sqrt{\left\{ \frac{1.0}{2} - 2(0.25) \right\}^2 + \{ 2.0 - 2(0.25) \}^2} \right] (1.0)(0.0470) = 0.2148\ in^2$$

Table 4.19.3 is used to determine C_p. The following values are calculated.

$$C_1 = \frac{2r_m}{w} = \frac{2(0.25)}{(2.0)} = 0.250 \quad \text{with} \quad 0.0 \leq C_1 \leq 1.0$$

$$C_2 = \frac{1.82 r_m}{\sqrt{D_m t_p}} = \frac{1.82(0.25)}{\sqrt{(50.048)(0.0470)}} = 0.2966 \quad \text{with} \quad 0.2 \leq C_2 \leq 4.0$$

The coefficients, $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are interpolated.

$$\alpha_0 = 1.000 \qquad \alpha_3 = 0.711$$
$$\alpha_1 = -0.587 \qquad \alpha_4 = 0.662$$
$$\alpha_2 = -0.589 \qquad \alpha_5 = -0.646$$

$$C_p = \alpha_0 + \alpha_1 C_1 + \alpha_2 C_1^2 + \alpha_3 C_1^3 + \alpha_4 C_1^4 + \alpha_5 C_1^5$$

$$C_p = \left(\begin{array}{l} 1.000 + (-0.587)(0.25) + (-0.589)(0.25)^2 + \\ (0.711)(0.25)^3 + (0.662)(0.25)^4 + (-0.646)(0.25)^5 \end{array} \right) = 0.8295$$

Calculate Stresses (Externally Attached Bellows):

Circumferential Membrane stress in bellows tangent due to pressure (S_1).

$$S_1 = \frac{P(D_b + nt)^2 L_t E_b k}{2\left[nt(D_b + nt) L_t E_b + t_c D_c L_c E_c k \right]}$$

$$S_1 = \frac{50(48.0 + 1(0.048))^2 (1.25)(25.05E+06)(0.5490)}{2\left[1(0.048)(48.0 + 1(0.048))(1.25)(25.05E+06) + 0 \right]} = 13738.7 \ psi$$

Circumferential Membrane stress in bellows end convolutions due to pressure ($S_{2,E}$).

$$S_{2,E} = \frac{1}{2} \frac{P\left[qD_m + L_t (D_b + nt) \right] E_b}{(A + nt_p L_t) E_b + t_c L_c E_c}$$

$$= \frac{1}{2} \frac{50\left[1.0(50.048) + 1.25(48.0 + 1(0.048)) \right](25.05E+06)}{(0.2148 + 1(0.0470)(1.25))(25.05E+06) + 0} = 10062.9 \ psi$$

Circumferential Membrane stress in bellows intermediate convolutions due to pressure ($S_{2,I}$).

$$S_{2,I} = \frac{PqD_m}{2A} = \frac{50(1.0)(50.048)}{2(0.2148)} = 5824.9 \ psi$$

Meridional Membrane stress in bellows due to pressure (S_3).

$$S_3 = \frac{Pw}{2nt_p} = \frac{50(2.0)}{2(1)(0.0470)} = 1063.8 \ psi$$

Meridional Bending stress in bellows due to pressure (S_4).

$$S_4 = \frac{PC_p}{2n} \left(\frac{w}{t_p} \right)^2 = \frac{50(0.8295)}{2(1)} \left(\frac{2.0}{0.0470} \right)^2 = 37550.9 \ psi$$

Acceptance Criteria:

$$\{S_1 = 13738.7\ psi\} \le \{S = 17900\ psi\} \qquad True$$

$$\{S_{2,E} = 10062.9\ psi\} \le \{S = 17900\ psi\} \qquad True$$

$$\{S_{2,I} = 5824.9\ psi\} \le \{S = 17900\ psi\} \qquad True$$

$$\{S_3 + S_4 = 1063.8 + 37550.9 = 38614.7\ psi\} \le \{K_m S = 3.0(17900) = 53700\ psi\} \quad True$$

where,

$$K_m = 1.5Y_{sm} = 1.5(2.0) = 3.0 \qquad for\ As-Formed\ Bellows$$

and, since material $SA-240,\ Type\ 321$ is an austenitic stainless steel, the yield strength multiplier Y_{sm} is calculated as follows.

$$Y_{sm} = 1 + 9.94(K_f \varepsilon_f) - 7.59(K_f \varepsilon_f)^2 - 2.4(K_f \varepsilon_f)^3 + 2.21(K_f \varepsilon_f)^4$$

$$Y_{sm} = 1 + 9.94(1.0(0.1203)) - 7.59(1.0(0.1203))^2 - 2.4(1.0(0.1203))^3 + 2.21(1.0(0.1203))^4$$

$$Y_{sm} = 2.0517$$

However, if $\{Y_{sm} = 2.0517\} > 2.0 \rightarrow Y_{sm} = 2.0$.

and, the forming method factor, K_f for expanding mandrel or roll forming,

$$K_f = 1.0$$

and the forming strain ε_f for bellows formed 100% to the outside of the initial cylinder is:

$$\varepsilon_f = \sqrt{\left[\ln\left(1 + \frac{2w}{D_b}\right)\right]^2 + \left[\ln\left(1 + \frac{nt_p}{2r_m}\right)\right]^2}$$

$$\varepsilon_f = \sqrt{\left[\ln\left(1 + \frac{2(2.0)}{(48.0)}\right)\right]^2 + \left[\ln\left(1 + \frac{(1)(0.0470)}{2(0.25)}\right)\right]^2} = 0.1203\ in/in$$

Therefore, the bellows meet internal pressure acceptance criteria at design conditions.

d) STEP 4 – Check column instability due to internal pressure per paragraph 4.19.5.4.

$$P_{sc} = \frac{0.34\pi K_b}{Nq} = \frac{0.34\pi(1649.0)}{12(1.0)} = 146.8\ psi$$

where, the axial stiffness, K_b, is calculated using Equation 4.19.25.

$$K_b = \frac{\pi E_b D_m}{2(1-\nu_b^2)C_f}\left(\frac{n}{N}\right)\left(\frac{t_p}{w}\right)^3 = \frac{\pi(25.05E+06)(50.048)}{2(1-0.3^2)(1.4193)}\left(\frac{1}{12}\right)\left(\frac{0.0470}{2.0}\right)^3$$

$$K_b = 1649.0\ lb/in$$

and Table 4.19.4 is used to determine C_f. The following values are calculated.

$$C_2 = 0.2966$$

The coefficients, $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are interpolated.

$$\alpha_0 = 1.006 \qquad \alpha_3 = 5.719$$
$$\alpha_1 = 2.106 \qquad \alpha_4 = -5.501$$
$$\alpha_2 = -2.930 \qquad \alpha_5 = 2.067$$

$$C_f = \alpha_0 + \alpha_1 C_1 + \alpha_2 C_1^2 + \alpha_3 C_1^3 + \alpha_4 C_1^4 + \alpha_5 C_1^5$$

$$C_f = \begin{pmatrix} 1.006 + (2.106)(0.25) + (-2.930)(0.25)^2 + \\ (5.719)(0.25)^3 + (-5.501)(0.25)^4 + (2.067)(0.25)^5 \end{pmatrix} = 1.4193$$

Acceptance Criterion:

$$\{P = 50\ psi\} \le \{P_{sc} = 146.8\ psi\} \qquad True$$

Therefore, the bellows meet columns instability criterion at design conditions.

e) STEP 5 – Check in-plane instability due to internal pressure per paragraph 4.19.5.5.

$$P_{si} = \frac{AS_y^*(\pi - 2)}{D_m q\left[1 + 2\delta^2 + \left(1 - 2\delta^2 + 4\delta^4\right)^{0.5}\right]^{0.5}}$$

$$P_{si} = \left(\frac{0.2148(45540)(\pi - 2)}{50.048(1.0)\left[1 + 2(2.1489)^2 + \left(1 - 2(2.1489)^2 + 4(2.1489)^4\right)^{0.5}\right]^{0.5}}\right) = 51.2\ psi$$

where,

$$\delta = \frac{S_4}{3S_{2,I}} = \frac{37550.9}{3(5824.9)} = 2.1489$$

and,

$$S_y^* = 2.3S_y = 2.3(19800) = 45540\ psi \qquad for\ As-Formed\ \ Bellows$$

Acceptance Criterion:

$$\{P = 50\ psi\} \le \{P_{si} = 51.2\ psi\} \qquad True$$

Therefore, the bellows meet in-plane instability criterion at design conditions.

f) STEP 6 – Perform a fatigue evaluation per paragraph 4.19.5.7.

There are 2 independent operating conditions. Therefore, 2 elementary evaluations are first performed, and the cumulative damage is then calculated for the overall specified cycles.

1) STEP 6.1 – Damage for Operating Condition 1.

Calculate the equivalent axial displacement range:

The axial displacement range, Δq, is calculated using the procedure shown in paragraph 4.19.8.

$$\{x = -4.5 \ in\} \rightarrow \quad See \ design \ data$$

$$\Delta q_x = \frac{x}{N} = \frac{-4.5}{12} = -0.375 \ in$$

The corresponding axial force, F_x, applied to the ends of the bellows is calculated as follows.

$$F_x = K_b x = 1648.7(-4.5) = -7419.2 \ lbs$$

Since there is no lateral deflection or angular rotation.

$$\Delta q_y = \Delta q_\theta = 0.0 \ in$$

Total equivalent axial displacement per convolution:

$$\Delta q_e = \Delta q_x + \Delta q_y + \Delta q_\theta = -0.375 \ in$$

$$\Delta q_c = \Delta q_x - \Delta q_y - \Delta q_\theta = -0.375 \ in$$

Note that, since there is no lateral deflection or angular rotation, there is no difference between the "extended" side and the "compressed" side of the bellows. Both sides are in the same state (extended or compressed). The so-called "extended" side is actually compressed.

Since the bellows was installed without cold spring:

Initial Position	Operating Position
$\Delta q_{e,0} = 0.0 \ in$	$\Delta q_{e,1} = \Delta q_{x,1} = -0.375 \ in$
$\Delta q_{c,0} = 0.0 \ in$	$\Delta q_{c,1} = \Delta q_{x,1} = -0.375 \ in$

Total equivalent axial displacement range per convolution:

$$\Delta q = \max\left[\left|\Delta q_{e,1}\right|, \left|\Delta q_{c,1}\right|\right] = \max\left[\left|-0.375\right|, \left|-0.375\right|\right] = 0.375 \ in$$

Calculate stresses due to equivalent axial displacement range of each convolution:

Meridional membrane stress (S_5).

$$S_5 = \frac{E_b t_p^2 \Delta q}{2w^3 C_f} = \frac{(25.05E+06)(0.0470)^2(0.375)}{2(2.0)^3(1.4193)} = 913.8 \ psi$$

Meridional bending stress (S_6).

$$S_6 = \frac{5E_b t_p \Delta q}{3w^2 C_d} = \frac{5(25.05E+06)(0.0470)(0.375)}{3(2.0)^2(1.3474)} = 136530.3 \ psi$$

where,

$$C_f = 1.4193 \text{ (see STEP 4)}$$

and Table 4.19.5 is used to determine C_d. The following values are calculated.

$$C_2 = 0.2966$$

The coefficients, $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are interpolated.

$$\alpha_0 = 1.000 \qquad \alpha_3 = -3.441$$
$$\alpha_1 = 1.228 \qquad \alpha_4 = 3.453$$
$$\alpha_2 = 1.309 \qquad \alpha_5 = -1.190$$

$$C_d = \alpha_0 + \alpha_1 C_1 + \alpha_2 C_1^2 + \alpha_3 C_1^3 + \alpha_4 C_1^4 + \alpha_5 C_1^5$$

$$C_d = \begin{pmatrix} 1.000 + (1.228)(0.25) + (1.309)(0.25)^2 + \\ (-3.441)(0.25)^3 + (3.453)(0.25)^4 + (-1.190)(0.25)^5 \end{pmatrix} = 1.3474$$

Total stress range due to cyclic displacement (S_t).

$$S_t = 0.7[S_3 + S_4] + [S_5 + S_6]$$
$$S_t = 0.7(1063.8 + 37550.9) + (913.8 + 136530.3) = 164474.4 \ psi$$

<u>Calculate allowable number of cycles</u>, N_{alw}, using the equations from Table 4.19.6.

$$\left\{ K_g \left(\frac{E_o}{E_b} \right) S_t = 1.0 \left(\frac{28.3E+06}{25.05E+06} \right) (164474.4) = 185813.4 \ psi \right\} \geq 65000 \ psi$$

therefore,

$$N_{alw} = \left(\frac{5.2E+06}{K_g \left(\frac{E_0}{E_b} \right) S_t - 38300} \right)^2 = \left(\frac{5.2E+06}{1.0 \left(\frac{28.3E+06}{25.05E+06} \right) (164474.4) - 38300} \right)^2 = 1242 \, cycles$$

<u>Calculate the usage factor</u> per paragraph 4.19.3.2.a.

$$U_1 = N_{spe} / N_{alw} = 1000 / 1242 = 0.805$$

2) STEP 6.2 – Damage for Operating Condition 2.

Calculate the equivalent axial displacement range:

The axial displacement range, Δq, is calculated using the procedure shown in paragraph 4.19.8.

$$\{x = 0.75 \ in\} \rightarrow \quad See \ design \ data$$

$$\Delta q_x = \frac{x}{N} = \frac{0.75}{12} = 0.0625 \ in$$

The corresponding axial force, F_x, applied to the ends of the bellows is calculated as follows.

$$F_x = K_b x = 1648.7(0.75) = 1236.5 \ lbs$$

Since there is no lateral deflection or angular rotation.

$$\Delta q_y = \Delta q_\theta = 0.0 \ in$$

Total equivalent axial displacement per convolution:

$$\Delta q_e = \Delta q_c = \Delta q_x = +0.0625 \ in$$

Note that, since there is no lateral deflection or angular rotation, there is no difference between the "extended" side and the "compressed" side of the bellows. Both sides are in the same state (extended or compressed). The so-called "compressed" side is actually extended.

Since the bellows was installed without cold spring:

Initial Position	*Operating Position*
$\Delta q_{e,0} = 0.0\ in$	$\Delta q_{e,1} = \Delta q_{x,1} = +0.0625\ in$
$\Delta q_{c,0} = 0.0\ in$	$\Delta q_{c,1} = \Delta q_{x,1} = +0.0625\ in$

Total equivalent axial displacement range per convolution:

$$\Delta q = \max\left[\left|\Delta q_{e,1}\right|, \left|\Delta q_{c,1}\right|\right] = \max\left[\left|+0.0625\right|, \left|+0.0625\right|\right] = 0.0625\ in$$

Calculate stresses due to equivalent axial displacement range of each convolution:

Meridional membrane stress (S_5).

$$S_5 = \frac{E_b t_p^2 \Delta q}{2w^3 C_f} = \frac{(25.05E+06)(0.0470)^2(0.0625)}{2(2.0)^3(1.4193)} = 152.3\ psi$$

Meridional bending stress (S_6).

$$S_6 = \frac{5E_b t_p \Delta q}{3w^2 C_d} = \frac{5(25.05E+06)(0.0470)(0.0625)}{3(2.0)^2(1.3474)} = 22755.1\ psi$$

where $C_f = 1.4193$ and $C_d = 1.3474$ (see STEP 6.1),

Total stress range due to cyclic displacement (S_t).

$$S_t = 0.7\left[S_3 + S_4\right] + \left[S_5 + S_6\right]$$

$$S_t = 0.7(1063.8 + 37550.9) + (152.3 + 22755.1) = 49937.6\ psi$$

Calculate allowable number of cycles, N_{alw}, using the equations from Table 4.19.6.

$$\left\{K_g\left(\frac{E_o}{E_b}\right)S_t = 1.0\left(\frac{28.3E+06}{25.05E+06}\right)(49937.6) = 56416.6\ psi\right\} < 65000\ psi$$

therefore,

$$N_{alw} = \left(\frac{6.7E+06}{K_g\left(\frac{E_0}{E_b}\right)S_t - 30600}\right)^2 = \left(\frac{6.7E+06}{1.0\left(\frac{28.3E+06}{25.05E+06}\right)(49937.6) - 30600}\right)^2 = 67352\ cycles$$

Calculate the usage factor per paragraph 4.19.3.2.a.

$$U_2 = N_{spe} / N_{alw} = 5000 / 67352 = 0.074$$

3) STEP 6.3 – Cumulative Damage.

$$\left\{U = U_1 + U_2 = 0.805 + 0.074 = 0.879\right\} \le 1.0 \qquad True$$

The bellows meets fatigue design criterion at design conditions.

Therefore, the bellows meets all of the design requirements of paragraph 4.19 at design conditions.

4.19.2 Example E4.19.2 – Toroidal Bellows Expansion Joint and Fatigue Evaluation

Check the acceptability of a toroidal bellows expansion joint for the given design conditions in accordance with paragraph 4.19.

Design Conditions:

- Pressure (Internal) at Temperature = $400\ psig\ @650°F$
- Axial Movements in Compression and Extension = *2 Concurrent Operating Conditions*

Operating Condition 1

- Axial Displacement (Compression) = $0.25\ in$
- Angular Deflection = $None$
- Lateral Deflection = $None$
- Specified Number of Cycles = 1000

Operating Condition 2

- Axial Displacement (Extension) = $0.745\ in$
- Angular Deflection = $None$
- Lateral Deflection = $None$
- Specified Number of Cycles = 1000

Bellows:

- Material = $SA-240,\ Type\ 321$
- Allowable Stress = $17900\ psi$
- Modulus of Elasticity at Design Temperature = $25.05E+06\ psi$
- Modulus of Elasticity at @ Room Temperature = $28.3E+06\ psi$
- Inside Diameter of Bellows = $36.0\ in$
- Mean Diameter of Bellows = $40.0\ in$
- Number of Convolutions = 2
- Convolution Pitch = $4.000\ in$
- Mean Radius of Convolutions = $1.5\ in$
- Number of Plies = 1
- Ply Thickness = $0.078\ in$
- Installed without Cold Spring = Yes
- Circumferential welds = No

The bellows is attached to the shell externally on both sides.

Reinforcing and Tangent Collars:

- Material = $SA-240,\ Type\ 321$
- Allowable Stress = $17900\ psi$
- Modulus of Elasticity at Design Temperature = $25.05E+06\ psi$

Tangent Collars:

- Tangent Collar Joint Efficiency = 1.0
- Tangent Collar Thickness = $0.75\ in$
- Cross Sectional Metal Area of one Tangent Collar = $1.034\ in^2$
- Length from Attachment Weld to the Center of the First Convolution = $2.0\ in$

Reinforcing Collars:

- Reinforcing Collar Joint Efficiency = 1.0
- Reinforcing Collar Thickness = $0.75\ in$
- Overall Length of one Reinforcing Collar = $3.094\ in$
- Cross Sectional Metal Area of one Reinforcing Collar based on Overall Length = $2.068\ in^2$

Cylindrical shell on which the bellows is attached:

- Inside Diameter of Shell = $35.0\ in$
- Thickness of Shell = $0.50\ in$
- Minimum Length of Shell on each Side of the Bellows = $10.5\ in$

Evaluate per paragraph 4-19.

a) STEP 1 – Check applicability of design rules per paragraph 4.19.2.

1) Bellows length must satisfy $(N-1)q+2L_d \le 3D_b$.

$$\left\{(N-1)q+2L_d=(2-1)(4)+2(2.0)=8\ in\right\} \le \left\{3D_b=3(36)=108\ in\right\} \qquad True$$

2) Bellows thickness must satisfy $nt \le 0.2\ in$.

$$\left\{nt=1(0.078)=0.078\ in\right\} \le \left\{0.2\ in\right\} \qquad True$$

3) Number of plies must satisfy $n \le 5$.

$$\left\{n=1\right\} \le \left\{5\right\} \qquad True$$

4) Displacement shall be essentially axial.

No angular or lateral deflection is specified, so the condition is satisfied.

5) The rules are valid for design temperatures up to the temperatures shown in Table 4.19.1.

The material specification $SA-240,\ Type\ 321$ is an austenitic stainless steel and falls under the material classification of Table 3-A.3.

$$\left\{T=650°F\right\} \le \left\{Table\ 3A.3 \rightarrow 800°F\right\} \qquad True$$

6) The fatigue equations of paragraph 4.19.7.7 are valid for austenitic stainless steels.

$$SA-240,\ Type\ 321 \rightarrow UNS\ \ S32100$$

7) The length of the cylindrical shell on each side of the bellows shall not be less than $1.8\sqrt{D_s t_s}$.

$$\left\{10.5\ in\right\} \geq \left\{1.8\sqrt{D_s\, t_s} = 1.8\sqrt{(35.0)(0.50)} = 7.530\ in\right\} \qquad \textit{True}$$

b) STEP 2 – Check the applicability of paragraph 4.19.7.2.

1) The type of attachment to the shell shall be the same on both sides.

The bellows is attached to the shell externally on both sides, so the condition is satisfied.

2) Distance L_g shall be less than $0.75r$ in the maximum extended position.

The distance across the inside opening in the neutral position is calculated,

$$L_{g0} = q - \left(L_{rt} + 2nt\right) = 4.0 - \left(3.094 + 2\left(1\right)\left(0.078\right)\right) = 0.750\, in$$

The only movement is in the axial direction. The maximum opening corresponds to the maximum extension.

$$L_g = L_{g0} + \Delta q_{x,e} = L_{g0} + \frac{x_e}{N} = 0.750 + \left(\frac{0.745}{2}\right) = 1.1225\, in$$

therefore,

$$\left\{L_g = 1.1225\ in\right\} < \left\{0.75r = 0.75\left(1.5\right) = 1.1250\ in\right\} \qquad \textit{True}$$

3) The third condition applies to internally attached bellows.

Not applicable.

c) STEP 3 – Check stresses in bellows at design conditions per paragraph 4.19.7.3. Since the bellows are subject to internal pressure, calculations and acceptability criteria are per Table 4.19.9.

$$D_c = D_b + 2nt + t_c = 36.0 + 2\left(1\right)\left(0.078\right) + 0.75 = 36.906\ in$$

$$D_r = D_b + 2nt + t_r = 36.0 + 2\left(1\right)\left(0.078\right) + 0.75 = 36.906\ in$$

$$t_p = t\sqrt{\frac{D_b}{D_m}} = 0.078\sqrt{\frac{36.0}{40.0}} = 0.0740\ in$$

Calculate stresses (Externally Attached Bellows):

Circumferential Membrane stress in end tangent due to internal pressure (S_1).

$$S_1 = \frac{P\left(D_b + nt\right)^2 L_d E_b}{2 D_c E_c A_{tc}}$$

$$S_1 = \frac{\left(400\right)\left(36.0 + \left(1\right)\left(0.078\right)\right)^2 \left(2.0\right)\left(25.05E+06\right)}{2\left(36.906\right)\left(25.05E+06\right)\left(1.034\right)} = 13643.5\, psi$$

Circumferential Membrane stress in tangent collar due to internal pressure (S_1').

$$S_1' = \frac{P D_c L_d}{2 A_{tc}} = \frac{\left(400\right)\left(36.906\right)\left(2.0\right)}{2\left(1.034\right)} = 14277.0\ psi$$

Circumferential Membrane stress in bellows due to internal pressure (S_2).

$$S_2 = \frac{Pr}{2nt_p} = \frac{(400)(1.5)}{2(1)(0.074)} = 4054.1\ psi$$

Meridional membrane stress in bellows due to internal pressure (S_3).

$$S_3 = \frac{Pr}{nt_p}\left(\frac{D_m - r}{D_m - 2r}\right) = \frac{(400)(1.5)}{(1)(0.074)}\left(\frac{40.0-1.5}{40.0-2(1.5)}\right) = 8436.8\ psi$$

Circumferential Membrane stress in reinforcing collar due to internal pressure (S_2').

Since $\{L_{rt} = 3.094\ in\} \le \left\{2/3\sqrt{D_r t_r} = 2/3\sqrt{(36.906)(0.75)} = 3.5074\ in\right\}$.

$$S_2' = \frac{D_r\,(L_{rt} + L_g + 2nt)}{2\,A_{rt}}\ P$$

$$S_2' = \frac{36.906\left(3.094+1.1225+2(1)(0.078)\right)}{2(2.068)}(400) = 15606.5\ psi$$

Acceptance Criteria:

$\{S_1 = 13643.5\ psi\} \le \{S = 17900\ psi\}$	*True*
$\{S_1' = 14277.0\ psi\} \le \{C_{wc}S_c = (1.0)(17900) = 17900\ psi\}$	*True*
$\{S_2 = 4054.1\ psi\} \le \{S = 17900\ psi\}$	*True*
$\{S_3 = 8436.8\ psi\} \le \{S = 17900\ psi\}$	*True*
$\{S_2' = 15606.5\ psi\} \le \{C_{wr}S_r = (1.0)(17900) = 17900\ psi\}$	*True*

Therefore, the bellows meets internal pressure stress acceptance criteria at design conditions.

d) STEP 4 – Check column instability due to internal pressure per paragraph 4.19.7.4.

$$P_{sc} = \frac{0.15\pi K_b}{Nr} = \frac{0.15\pi(12751.7)}{2(1.5)} = 2003.0\ psi$$

where, the axial stiffness, K_b is calculated using Equation 4.19.36.

$$K_b = \frac{E_b D_m B_3}{12\ (1-\nu_b{}^2)}\left(\frac{n}{N}\right)\left(\frac{t_p}{r}\right)^3 = \frac{(25.05E+06)(40.0)(2.3149)}{12\left(1-(0.3)^2\right)}\left(\frac{1}{2}\right)\left(\frac{0.074}{1.5}\right)^3$$

$$K_b = 12751.7\ lb/in$$

and Table 4.19.10 is used to determine B_3. The following values are calculated.

$$C_3 = \frac{6.61r^2}{D_m t_p} = \frac{6.61(1.5)^2}{40.0(0.074)} = 5.0245$$

$$B_3 = \frac{0.99916 - 0.091665C_3 + 0.040635C_3^2 - 0.0038483C_3^3 + 0.00013392C_3^4}{1 - 0.1527C_3 + 0.013446C_3^2 - 0.00062724C_3^3 + 1.4374(10)^{-5}C_3^4}$$

$$B_3 = \left(\frac{\begin{array}{c} 0.99916 - 0.091665(5.0245) + 0.040635(5.0245)^2 - \\ 0.0038483(5.0245)^3 + 0.00013392(5.0245)^4 \end{array}}{\begin{array}{c} 1 - 0.1527(5.0245) + 0.013446\left((5.0245)^2\right) - \\ 0.00062724(5.0245)^3 + 1.4374(10)^{-5}(5.0245)^4 \end{array}} \right) = 2.3149$$

Acceptance Criterion:

$$\{P = 400\ psi\} \le \{P_{sc} = 2003.0\ psi\} \qquad True$$

Therefore, the bellows meets column instability criterion at design conditions.

e) STEP 5 – Perform a fatigue evaluation per paragraph 4.19.7.7.

There are 2 concurrent operating conditions. Both have the same specified number of cycles. Therefore, the fatigue evaluation is performed considering cycles between the 2 operating conditions with a specified number of cycles equal to their common specified value.

Calculate the equivalent axial displacement range

The axial displacement range, Δq, is calculated using the procedure shown in paragraph 4.19.8.

$$\left\{ \begin{array}{l} x_{c,1} = -0.25\ in \\ x_{e,2} = 0.745\ in \end{array} \right\} \rightarrow See\ design\ data$$

$$\Delta q_{x,1} = \frac{x_{c,1}}{N} = \frac{-0.25}{2} = -0.125\ in$$

$$\Delta q_{x,2} = \frac{x_{e,2}}{N} = \frac{0.745}{2} = 0.3725\ in$$

The corresponding axial force, F_x, applied to the ends of the bellows is calculated as follows.

$$F_{x,1} = K_b x_{c,1} = 12751.7(-0.125) = -1594.0\ lbs$$
$$F_{x,2} = K_b x_{e,2} = 12751.7(0.3725) = +4750.0\ lbs$$

Since there are no lateral deflections or angular rotations.

$$\Delta q_y = \Delta q_\theta = 0.0\ in$$

Total equivalent axial displacement per convolution:

$$\Delta q_{e,1} = \Delta q_{c,1} = \Delta q_{x,1} = -0.125\ in$$

$$\Delta q_{e,2} = \Delta q_{c,2} = \Delta q_{x,2} = 0.3725\ in$$

Note that, since there are no lateral deflection or angular rotation, there is no difference between the "extended" side and the "compressed" side of the bellows. Both sides are in the same state (extended or compressed).

The bellows is operating between 2 operating conditions:

$$\begin{array}{ll} Operating\ \ Position\ 1 & Operating\ \ Position\ 2 \\ \Delta q_{e,1} = \Delta q_{x,1} = -0.125\ in & \Delta q_{e,2} = \Delta q_{x,2} = +0.3725\ in \\ \Delta q_{c,1} = \Delta q_{x,1} = -0.125\ in & \Delta q_{c,2} = \Delta q_{x,2} = +0.3725\ in \end{array}$$

Total equivalent axial displacement range per convolution, (axial displacement only):

$$\Delta q = \left|\Delta q_{x,2} - \Delta q_{x,1}\right| = \left|(0.3725) - (-0.125)\right| = 0.4975\ in$$

Calculate stresses due to equivalent axial displacement range of each convolution:

Meridional membrane stress (S_5).

$$S_5 = \frac{E_b t_p^2 B_1 \Delta q}{34.3 r^3} = \frac{(25.05E+06)(0.074)^2(3.6431)(0.4975)}{34.3(1.5)^3} = 2147.7\ psi$$

Meridional bending stress (S_5).

$$S_6 = \frac{E_b t_p B_2 \Delta q}{5.72 r^2} = \frac{(25.05E+06)(0.074)(0.9971)(0.4975)}{5.72(1.5)^2} = 71448.5\ psi$$

where, Table 4.19.10 is used to determine $B_1,\ B_2$. The following values are calculated.

$$C_3 = 5.0245$$

$$B_1 = \frac{1.00404 + 0.028725C_3 + 0.18961C_3^2 - 0.00058626C_3^3}{1 + 0.14069C_3 - 0.0052319C_3^2 + 0.00029867C_3^3 - 6.2088(10)^{-6}C_3^4}$$

$$B_1 = \frac{\begin{pmatrix} 1.00404 + 0.028725(5.0245) + \\ 0.18961(5.0245)^2 - 0.00058626(5.0245)^3 \end{pmatrix}}{\begin{pmatrix} 1 + 0.14069(5.0245) - 0.0052319(5.0245)^2 + \\ 0.00029867(5.0245)^3 - 6.2088(10)^{-6}(5.0245)^4 \end{pmatrix}} = 3.6431$$

Since $\{C_3 = 5.0245\} > 5$,

$$B_2 = \frac{0.049198 - 0.77774C_3 - 0.13013C_3^2 + 0.080371C_3^3}{1 - 2.81257C_3 + 0.63815C_3^2 + 0.0006405C_3^3}$$

$$B_2 = \frac{\begin{pmatrix} 0.049198 - 0.77774(5.0245) - \\ 0.13013(5.0245)^2 + 0.080371(5.0245)^3 \end{pmatrix}}{\begin{pmatrix} 1 - 2.81257(5.0245) + \\ 0.63815(5.0245)^2 + 0.0006405(5.0245)^3 \end{pmatrix}} = 0.9971$$

Total stress range due to cyclic displacement (S_t):

$$S_t = 3S_3 + S_5 + S_6 = 3(8436.8) + 2147.7 + 71448.5 = 98906.6\ psi$$

Calculate the allowable number of cycles, N_{alw}, using the equations from Table 4.19.11.

$$\left\{K_g\left(\frac{E_o}{E_b}\right)S_t = 1.0\left(\frac{28.3E+06}{25.05E+06}\right)(98906.6) = 111738.8\ psi\right\} \geq \{65000\ psi\}$$

therefore,

$$N_{alw} = \left(\frac{5.2E+06}{\left(K_g\left(\frac{E_o}{E_b}\right)S_t - 38300\right)}\right)^2 = \left(\frac{5.2E+06}{1.0\left(\frac{28.3E+06}{25.05E+06}\right)(98906.6) - 38300}\right)^2 = 5013\ cycles$$

$$\{N_{alw} = 5013\ cycles\} \geq \{N_{spe} = 1000\ cycles\}$$

The bellows meets fatigue design criterion at design conditions.

Therefore, the bellows meets all the design requirements of paragraph 4.19 at design conditions.

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4.20 Design Rules for Flexible Shell Element Expansion Joints

4.20.1 Example E4.20.1 – Review of Requirements for Flexible Shell Element Expansion Joints

An engineer is tasked with developing a design specification for a heat exchanger to be equipped with a flexible shell element expansion joint that is to be constructed in accordance with paragraph 4.20. As part of developing the design specification, the following items need to be considered.

a) The User's Design Specification and Manufacturer's Design Report shall be certified by a Certifying Engineer in accordance with Annex 2A and Annex 2B, respectively. See paragraphs 2.2.1.1 and 2.2.1.2 and paragraphs 2.3.3.1 and 2.3.3.2, respectively. In addition, the competency requirements and qualification requirements as outlined in Annex 2J shall be verified.

b) Scope

These rules apply to single-layer flexible shell element expansion joints shown in Figure 4.20.1 and are limited to applications involving axial displacement only. The rules in paragraph 4.20 cover the common types of flexible shell element expansion joints but are not intended to limit the configurations or details to those illustrated or otherwise described herein. Designs that differ from those covered in paragraph 4.20 (e.g., multilayer, asymmetric geometries or loadings having a thick liner or other attachments) shall be in accordance with paragraph 4.1.1.2.

c) Conditions of Applicability

1) For carbon and low-alloy steels, the minimum thickness, exclusive of corrosion allowance, shall be 3 mm (0.125 in.) for all pressure-retaining components.

2) For high-alloy and nonferrous steels, the minimum thickness shall conform to the requirements of paragraph 4.1.2.

3) The knuckle radius, r_a or r_b, of any formed element shall not be less than three times the element thickness, t, as shown in Figure 4.20.1.

4) Extended straight flanges between the inner torus and the shell and between both outer tori are permissible. An outer shell element between the outer tori is permissible. Extended straight flanges between the inner torus and the shell, between the outer tori and an outer shell element, and between both outer tori that do not have an intermediate outer shell element with lengths in excess of $0.5\sqrt{R\,t_f}$ shall satisfy all the requirements of 4.3.3.

5) Nozzles or other attachments located in the outer straight flange or outer shell element shall satisfy the axial spacing requirements of Figure 4.20.2.

d) Design Considerations

1) Expansion joints shall be designed to provide flexibility for thermal expansion and to function as pressure-containing elements.

2) The vessel Manufacturer shall specify the design conditions and requirements for the detailed design and manufacture of the expansion joint.

3) Thinning of any flexible element as a result of forming operations shall be considered in the design and specification of material thickness.

4) In all vessels with integral expansion joints, the hydrostatic end force caused by the pressure or the joint spring force, or both, shall be resisted by adequate restraining elements (e.g., exchanger tubes,